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Two-echelon production routing problem with simultaneous pickup and delivery

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Abstract : This paper introduces a novel variant of the Production Routing Problem (PRP) in a Two-Echelon supply chain involving multiple production plants, distribution centers (DCs), and retailers. Each plant produces a unique item, different from what the other plants produce. We consider reverse logistics through recyclable packaging collection from retailers to the plants through the DCs. The objective is to minimize the total cost, which includes production, inventory, and transportation costs, over a multi-period and finite horizon. The problem incorporates two-echelon distribution systems, one between the plants and DCs, and one between the DCs and the retailers, in which we consider a heterogeneous fleet of vehicles and model a Vehicle Routing Problem with Simultaneous Pickup and Delivery (VRPSPD). Additionally, inventory management is considered at all facilities for pickups and deliveries. In this study, we develop a mixed integer linear programming (MILP) model, which is then solved using a commercial solver for small problem instances. We also analyze the complexity of the problem and the impact of different parameters on the structure of the solution.

Keywords: Production routing problem, supply chain integration, vehicle routing problem, simultaneous pickup and delivery, reverse logistics, heuristics

Résumé : Cet article présente une nouvelle variante du problème de production et de distribution intégré dans une chaîne d'approvisionnement à deux niveaux impliquant plusieurs usines de production, centres de distribution (CDs) et détaillants. Chaque usine fabrique un article unique, différent de ceux produits par les autres usines. Nous prenons en compte la logistique inverse à travers la collecte d'emballages recyclables auprès des détaillants jusqu'aux usines via les centres de distribution. L'objectif est de minimiser le coût total, qui comprend les coûts de production, d'inventaire et de transport, sur un horizon multi-périodes et fini. Le problème intègre des systèmes de distribution à deux niveaux, l'un entre les usines et les CDs, et l'autre entre les CDs et les détaillants, dans lesquels nous considérons une flotte hétérogène de véhicules et modélisons un problème de tournées de véhicules avec ramassage et livraison simultanés. De plus, la gestion des stocks est prise en compte dans toutes les installations, tant pour les ramassages que pour les livraisons. Dans cette étude, nous développons un modèle de programmation linéaire en nombres entiers mixtes, qui est ensuite résolu à l'aide d'un solveur commercial pour de petites instances du problème. Nous analysons également la complexité du problème ainsi que l'impact de différents paramètres sur la structure de la solution.

Mots clés : Problème de production et de distribution intégré, intégration de la chaîne d'approvisionnement, problème de tournées de véhicules, ramassage et livraison simultanés, logistique inverse, heuristiques

1 Introduction

Supply chain integration (SCI) refers to the alignment of all processes and activities in the supply chain by connecting suppliers, manufacturers, distributors, and retailers (Adulyasak et al., 2015). SCI aims to improve service levels, reduce costs, optimize resource utilization, and increase market responsiveness. The integration of production and distribution activities, through routes, also known as the Production Routing Problem (PRP), is one of the most well-known problems in SCI. It involves the simultaneous optimization of production, inventory, distribution, and routing decisions over a given planning horizon. The primary objective of the PRP is to minimize the total operational costs, which include production costs, inventory holding costs, and transportation costs (Adulyasak et al., 2015; Hrabec et al., 2022).

Chandra and Fisher (1994) are the first to examine the advantages of integrating production and routing decisions, showing that solving the PRP leads to cost savings of 3 to 20% compared to solving the individual problems separately (Habibi et al., 2017; Golsefidi and Jokar, 2020). Many solution methods have been developed to solve large size instances of PRP (Habibi et al., 2024). Among these methods: Lagrangian heuristics (Fumero and Vercellis, 1999), Memetic Algorithm (Boudia and Prins, 2009), Branch & Price (Bard and Nananukul, 2010), adaptive large neighborhood search (Adulyasak et al., 2014b), branch & cut (Adulyasak et al., 2014a), two-phase iterative heuristics (Absi et al., 2015), multi-phase heuristic (Solyalı and Süral, 2017), variable neighborhood search (Qiu et al., 2018b), a combination of a two-phase iterative method, a repairing strategy and a fix-and-optimize procedure (Li et al., 2019), multi-start matheuristic (Avci and Yildiz, 2019; Vadseth et al., 2023), benders decomposition for PRP with oredr-up-to-level policy (Zhang et al., 2021), parallelized branch & cut algorithm (Schenekemberg et al., 2021), and two-phase infeasible space matheuristic (Manousakis et al., 2022).

Beyond its traditional scope, PRP can be expanded to incorporate reverse logistics (Bouanane and Benadada, 2022). In this extended framework, the optimization not only addresses the forward flow of goods from production plants to retailers, but also considers the backward flow of defective products, used items, or recyclable packaging from retailers to the plants. These returned items may undergo refurbishment, remanufacturing, or recycling processes, thereby enhancing sustainability and reducing waste (Bouanane and Benadada, 2022).

In that spirit, Habibi et al. (2017) introduced the Collection-Disassembly Problem (CDP), in which vehicles collect products from retailers and transport them to a disassembly center. The goal of CDP is to minimize the total cost of collection, transportation, and disassembly. The authors developed a Two-Phase Iterative Heuristic to solve this problem. This work was later extended by Habibi et al. (2019) to incorporate demand uncertainty. Liu et al. (2021) proposed a bi-objective CDP that aims to minimize total costs while maximizing service level, defined as the average probability of meeting each demand under demand uncertainty. Their model addresses cases where historical demand data may be unreliable, using partially known distributional information. The service level objective is modeled via a chance constraint, and a deterministic equivalent mixed-integer program (MIP) is developed.

Recent literature highlights the rising interest in reverse logistics due to economic changes and environmental awareness. As a result, the VRP with simultaneous pickup and delivery has gained increasing attention for its critical role in optimizing reverse logistics operations (Golsefidi and Jokar, 2020). The pickup and delivery operations in VRP naturally extend to the PRP, where managing these two operations simultaneously introduces additional logistical challenges.

Qiu et al. (2018a) addressed the single-product PRP, incorporating reverse logistics and remanufacturing, with simultaneous pickup and delivery. They proposed a MIP model that includes multiple manufacturing and remanufacturing centers where manufacturing centers produce new products and remanufacturing centers restore used products. To solve the problem, the authors developed a branch-and-cut algorithm. Golsefidi and Jokar (2020) introduced a MILP model for the PRP with simultaneous pickup and delivery, incorporating reverse product flow and reproduction operations at the plant. The authors developed a robust MILP formulation under multiple uncertainty conditions and proposed two metaheuristics, Simulated Annealing and Genetic Algorithm, to solve the problem. An integrated production-routing model for a three-echelon supply chain containing a two-echelon transportation system is presented by Beheshtinia et al. (2021). The model accounts for multi-site manufacturing, simultaneous pickup and delivery, and uncertain demand, costs, and production capacity. The performance of the proposed model is evaluated using real data from an Iranian pharmaceutical production center. Bouanane et al. (2020) proposed a model for the multiple plant PRP in a reverse logistics setting, incorporating simultaneous pickup and delivery. Bouanane and Benadada (2022) extended this work by integrating pollution in the model presented in Bouanane et al. (2020). The authors investigated the reduction of carbon emissions under the cap & trade carbon policy.

Recently, Borumand et al. (2024) addressed the PRP within a Closed-Loop Supply Chain (CLSC) for beverage glass bottles, considering uncertainties in both the demand for filled bottles and the quantity of empty bottles returned. The model integrates simultaneous delivery and pickup routing. The authors developed a MILP model and adopted a multi-stage adjustable robust optimization (ARO) formulation to address uncertainties. To solve the ARO problem, they developed an exact oracle-based algorithm and introduced a heuristic search method to improve computational efficiency. Habibi et al. (2024) proposed a novel PRP model in a CLSC, incorporating remanufacturing and disassembly decisions for end-of-life returned products. The authors developed novel hybrid heuristics based on two-phase iterative and relax-and-fix heuristics to tackle this problem. The developed methods outperform branch-and-cut algorithm for large size instances with a small vehicle capacity.

Table 1 classifies the existing literature based on production, routing, inventory, and simultaneous pickup and delivery aspects. Most studies focus on single-echelon models with uncertain data, while some of them consider remanufacturing operations, which is beyond the scope of this study. However, the work most closely related to the present research is the problem discussed by Beheshtinia et al. (2021), which addresses a robust PRP involving multiple plants, DCs, and retailers. They studied a robust PRP that incorporates a heterogeneous vehicle fleet and a two-echelon transportation network, with pickup operations limited to DCs and inventory management applied only to products. In contrast, the current study expands the existing literature by considering multiple plants, each producing a distinct product type, along with multiple DCs and retailers. In addition, it incorporates inventory management for both products and packaging materials in all facilities, while also allowing pickup operations at both DCs and retailers. Similarly to Beheshtinia et al. (2021), we consider a heterogeneous fleet and a two-echelon transportation structure.

Table 1: A summary of the PRPSPD literature.

Authors	Period	Plant	Product	Echelon	Fleet	DCs pickup	Retailer pickup	Inventory capacity	Inventory type	Data
Qiu et al. (2018a)	M	M	S	S	Het[c]		✓	P,R	Pk,Dl	Det
Golsefidi and Jokar (2020)	M	S	S	S	Hom		✓	P,R	Pk,Dl	Un
Bouanane et al. (2020)	M	M	S	S	Hom[b]		✓	P,R	Pk,Dl	Det
Beheshtinia et al. (2021)	M	M	M	M	Het[c]	✓		D,R	Dl	Un
Borumand et al. (2024)	M	M	S	S	Het		✓	P,R	Pk,Dl	Un
Habibi et al. (2024)	M	M	M	S	Hom[b]		✓	P,R	Pk,Dl	Det
This work	**M**	**M[a]**	**M**	**M**	**Het[c]**	✓	✓	P,D,R	**Pk,Dl**	**Det**

Note. M: multiple, S: single, Hom: homogeneous, Het: heterogeneous, P: plants, D: DCs, R: retailers, Pk: pickup. Dl: delivery, Un: uncertain, Det: deterministic.
[a] Each plant produces a unique product. [b] One fleet at each plant. [c] One fleet at each echelon.

The contribution of this paper is threefold: (a) Contribution to the SCI area by proposing a MILP model for the Two-Echelon Production Routing Problem with Simultaneous Pickup and Delivery (2EPRPSPD). In this problem, we address a combination of five decisions: production, inventory management, distribution, routing, and reverse logistics. The novelty of our approach lies in two key aspects: (1) Each plant is assumed to produce a distinct type of product, which is consistent with an efficient strategy where each plant is equipped with specialized machines adapted to a specific product type. (2) The pickup process occurs in two distinct stages, first from retailers, then from distribution centers (DCs). (b) Contribution to the literature on reverse logistics by considering the simultaneous execution of packaging pickup and product delivery operations. We distinguish between two types of packaging, foldable and unfoldable, based on a packaging type factor, which makes the problem more realistic. (c) Analysis of the impact of critical parameters, specifically the production capacity factor and the packaging type factor, on the structure of the solution.

The rest of the paper is organized as follows. Section 2 describes the 2EPRPSPD and gives a MILP formulation of the problem. This is followed by some numerical experiments and results analysis in Section 3. Conclusion and research perspectives are presented in Section 4.

2 Problem statement

In this section, we describe and formulate the 2EPRPSPD. This problem is NP-hard, as its complexity trivially derives from the PRP, which is itself known to be NP-hard. It arises in large grocery and supermarket supply chains where products like vegetables, meat, dairy, bread, and drinks are distributed to retail stores. Simultaneously, unsold or expired products, returnable containers and pallets, or recyclable packaging are collected for return to production plants.

2.1 Description of the 2EPRPSPD

The considered problem is a two-echelon supply chain containing a set of plants P, a set of distribution centers D, a set of retailers R, a set of periods T, and two-echelon transportation systems. We define by N_1 the set of nodes in the first-echelon (plants + DCs), $N_1 = P \cup D$, and by N_2 the set of nodes in the second-echelon (DCs + retailers), $N_2 = D \cup R$. Each production plant p has a capacity pc_{pt} for the amount that can be produced in period t with an associated production setup cost sc_{pt} and unit variable production cost vc_{pt}. The setup cost refers to the fixed cost incurred each time a production process is initiated or reconfigured for a new product batch. This includes the material installation and preparation costs. We consider that each plant p produces a unique type of product, also indexed by p. At each period t, items can be transported from plants to DCs using the first echelon vehicle fleet, denoted K_1. Each DC can be visited at most once by each plant and is responsible for serving a set of retailers. Additionally, each retailer r can be served by at most one DC per period and has a demand d_{rpt} for product p in period t that must be satisfied on time. Items can be transported from DCs to retailers using the second echelon vehicle fleet, denoted K_2.

In this problem, we also consider reverse logistics, where the packaging of products delivered to retailers in period $t-1$ becomes available for pickup in period t in the quantity p_{rpt}, i.e., $p_{rpt} = c' * d_{rp,t-1}$ with c' being a packaging type factor indicating how much the packaging can be compressed or folded during return logistics. These packages and those available in pickup inventories are transported to DCs, sorted by product type, and then become available for pickup in period $t+1$ for first echelon vehicles. As we must satisfy the demand on time, we put the focus on deliveries. Therefore, we consider that the packages picked up at a certain facility are only available in the next period to go upstream in the supply chain. This way, a vehicle does not need to wait for the arrival of picked-up items before continuing its delivery route. It should be noted that, in both echelons, we consider a heterogeneous fleet of vehicles and model a Vehicle Routing Problem with Simultaneous Pickup and Delivery (VRPSPD). Each vehicle k_1 (k_2) in the first echelon (second echelon) has an associated capacity Q^{k_1} (Q^{k_2}) and a fixed cost for using it f^{k_1} (f^{k_2}). Vehicle fixed cost refers to the set of expenses associated with owning a vehicle. These costs typically include insurance, licensing fees, and scheduled maintenance. The transportation cost from node i to node j in the first and second echelons is denoted by c_{ij}. Figure 1 illustrates an example of the considered problem for a given period t, highlighting the forward flow of products from production plants to DCs, and subsequently to retailers, as well as the reverse flow of packaging from retailers back to DCs and then back to the plants. In this example, there are three production plants, corresponding to three product types, along with two DCs and four retailers. In the first echelon, three routes (blue, orange, and green) connect the plants to the DCs, each operated by a different vehicle. Notably, the first DC is not serviced by the third plant (see the green route), which can be explained by the fact that this DC or its associated retailers already hold sufficient inventory of the third product. In the second echelon, two routes (black and light blue) of different sizes connect the DCs to the retailers, also operated by two distinct vehicles. Each vehicle in the network simultaneously delivers the required quantities of products and collects the available packaging materials for return to its origin facility, while respecting its capacity at each facility along its route.

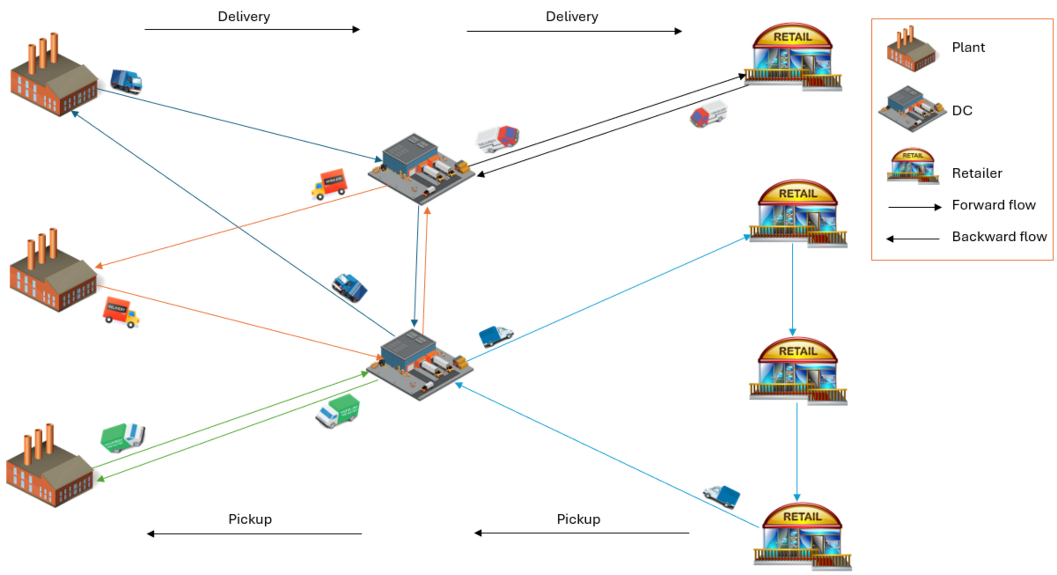

Figure 1: An example of a 2EPRPSPD.

Moreover, inventory management for pickups and deliveries is also considered at each facility type. We use $\mathscr{P}$ and $\mathscr{D}$ to differentiate between parameters and decision variables associated with pickup and delivery, respectively. We assume that in period $t = 0$, the initial inventories, for both products and packaging at all facilities, are equal to zero. For deliveries, each inventory has an associated unit holding cost for product p ($h^{\mathscr{D}}_{pt}$, $h^{\mathscr{D}}_{dpt}$, and $h^{\mathscr{D}}_{rpt}$) and holding capacity ($L^{\mathscr{D}}_{p}$, $L^{\mathscr{D}}_{d}$, and $L^{\mathscr{D}}_{r}$). For pickups, we consider unlimited inventories at plants and inventories with an associated unit holding cost for packaging of product p ($h^{\mathscr{P}}_{dpt}$ and $h^{\mathscr{P}}_{rpt}$) and holding capacity ($L^{\mathscr{P}}_{d}$ and $L^{\mathscr{P}}_{r}$) at DCs and retailers.

The aim of this problem is to find a feasible production and distribution plan that meets all the demands of the retailers, while minimizing the overall operational costs.

In this problem, several decisions need to be made. First, the amount produced in each plant p in period t, denoted by x_{pt}. Second, the amount of each product p sent to (picked up at) distribution center d using a vehicle of fleet K_1 and sent to (picked up at) retailer r using a vehicle of fleet K_2, denoted by $q^{k_1}_{pdt}$ ($b^{k_1}_{dpt}$) and $q^{k_2}_{drpt}$ ($b^{k_2}_{rdpt}$), respectively. Third, the inventory level of product p at plant p, distribution center d, and retailer r at the end of period t, denoted by $I^{\mathscr{D}}_{pt}$, $I^{\mathscr{D}}_{dpt}$, and $I^{\mathscr{D}}_{rpt}$, respectively, as well as the inventory level of packaging of product p at distribution center d and retailer r at the end of period t, denoted by $I^{\mathscr{P}}_{dpt}$ and $I^{\mathscr{P}}_{rpt}$, respectively. Fourth, the load of vehicles after leaving DCs and retailers in period t, denoted by $vl^{k_1}_{dt}$ and $vl^{k_2}_{rt}$, respectively. Fifth, a binary variable z_{pt} indicating if the plant p produces in period t. Sixth, binary variables $y^{k_1}_{dpt}$ indicating if a distribution center d is served by a plant p using vehicle k_1 and binary variables y_{rdt} indicating if a retailer r is served by a distribution center d, in period t. Seventh, binary variables $\alpha^{k_1}_{ijt}$ ($\beta^{k_2}_{ijt}$) indicating if the arc (i, j) is used by vehicle k_1 (k_2) in period t. Finally, variables indicating the position of a distribution center d (retailer r) in the route of vehicle k_1 (k_2) in period t, denoted by $u^{k_1}_{dt}$ ($u^{k_2}_{rt}$), respectively. Table 2 summarizes the different notations (sets, parameters, and decision variables) used in this problem.

Table 2: Symbols description of the PRPPD formulations.

Symbol	Description
	Sets
P	Set of plants, $p \in P$.
D	Set of distribution centers, $d \in D$.
R	Set of retailers, $r \in R$.
T	Set of periods, $t \in T$.
K_1	Set of vehicles of the first echelon, $k \in K_1$.
K_2	Set of vehicles of the second echelon, $k \in K_2$.
N_1	Set of nodes (plants + DCs), $N_1 = P \cup D$.
N_2	Set of nodes (DCs + retailers), $N_2 = D \cup R$.
	Parameters
sc_{pt}	Production setup cost at plant p in period t.
vc_{pt}	Production unit processing cost at plant p in period t.
pc_{pt}	Production capacity at plant p in period t.
d_{rpt}	Demand of retailer r of product p in period t.
p_{rpt}	Amount of packaging of product p available for pickup at retailer r in period t.
$h^{\mathscr{D}}_{pt}$	Inventory unit holding cost of product at plant p in period t.
$h^{\mathscr{D}}_{dpt}$	Inventory unit holding cost of product p at DC d in period t.
$h^{\mathscr{D}}_{rpt}$	Inventory unit holding cost of product p at retailer r in period t.
$h^{\mathscr{P}}_{dpt}$	Inventory unit holding cost of packaging of product p at DC d in period t.
$h^{\mathscr{P}}_{rpt}$	Inventory unit holding cost of packaging of product p at retailer r in period t.
$L^{\mathscr{D}}_{p}$	Inventory holding capacity of products at plant p.
$L^{\mathscr{D}}_{d}$	Inventory holding capacity of products at DC d.
$L^{\mathscr{D}}_{r}$	Inventory holding capacity of products at retailer r.
$L^{\mathscr{P}}_{d}$	Inventory holding capacity of packaging at DC d.
$L^{\mathscr{P}}_{r}$	Inventory holding capacity of packaging at retailer r.
Q^{k_1}	Capacity of vehicle k_1, $k_1 \in K_1$.
Q^{k_2}	Capacity of vehicle k_2, $k_2 \in K_2$.
f^{k_1}	Fixed cost of using vehicle k_1, $k_1 \in K_1$.
f^{k_2}	Fixed cost of using vehicle k_2, $k_2 \in K_2$.
c_{ij}	Transportation cost from node i to node j in the first and second echelons.
	Decision variables
x_{pt}	Production amount at plant p on period t.
z_{pt}	1 if there is production at plant p on period t, 0 otherwise.
$\alpha^{k_1}_{ijt}$	1 if vehicle k_1 goes from node i to node j in period t, $i, j \in N_1$.
$\beta^{k_2}_{ijt}$	1 if vehicle k_2 goes from node i to node j in period t, $i, j \in N_2$.
$y^{k_1}_{dpt}$	1 if DC d is served by plant p using vehicle k_1 in period t, 0 otherwise.
y_{rdt}	1 if retailer r is served by DC d in period t, 0 otherwise.
$q^{k_1}_{pdt}$	Quantity of products delivered from plant p to DC d by vehicle k_1 in period t.
$q^{k_2}_{drpt}$	Quantity of product p delivered from DC d to retailer r by vehicle k_2 in period t.
$b^{k_1}_{dpt}$	Quantity of product p picked up from DC d and sent to plant p in period t.
$b^{k_2}_{rdpt}$	Quantity of product p picked up from retailer r and sent to DC d in period t.
$vl^{k_1}_{dt}$	Load of vehicle k_1 after leaving DC d in period t.
$vl^{k_2}_{rt}$	Load of vehicle k_2 after leaving retailer r in period t.
$I^{\mathscr{D}}_{pt}$	Inventory level at plant p at the end of period t.
$I^{\mathscr{D}}_{dpt}$	Inventory level of product p at DC d at the end of period t.
$I^{\mathscr{D}}_{rpt}$	Inventory level of product p at retailer r at the end of period t.
$I^{\mathscr{P}}_{dpt}$	Inventory level of packaging of product p at DC d at the end of period t.
$I^{\mathscr{P}}_{rpt}$	Inventory level of packaging of product p at retailer r at the end of period t.
u^{k1}_{dt}	a variable indicating the position of DC d in a route.
v^{k2}_{rt}	a variable indicating the position of retailer r in a route.

2.2 Formulation of the 2EPRPSPD

In this section, we present the objective function and the different constraints that formulate the considered problem.

2.2.1 Objective function

The objective function aims at minimizing the overall cost, which is composed of production, transportation, and inventory holding costs.

$$\min Z = C_{prod} + C_{tran} + C_{inv}$$

- Equation (1) represents the production cost, including production setup cost and production variable cost.

$$C_{prod} = \sum_{p\in P}\sum_{t\in T} sc_{pt}.z_{pt} + \sum_{p\in P}\sum_{t\in T} vc_{pt}.x_{pt} \tag{1}$$

- Equation (2) represents the transportation cost, including vehicle transportation costs and vehicle fixed costs in both echelons.

$$\begin{aligned} C_{tran} = & \sum_{k_1\in K_1}\sum_{t\in T}\sum_{i\in N_1}\sum_{j\in N_1, j\neq i} c_{ij}.\alpha_{ijt}^{k_1} + \sum_{k_2\in K_2}\sum_{t\in T}\sum_{i\in N_2}\sum_{j\in N_2, j\neq i} c_{ij}.\beta_{ijt}^{k_2} \\ & + \sum_{k_1\in K_1}\sum_{t\in T}\sum_{p\in P}\sum_{d\in D} f^{k_1}.\alpha_{pdt}^{k_1} + \sum_{k_2\in K_2}\sum_{t\in T}\sum_{d\in D}\sum_{r\in R} f^{k_2}.\beta_{drt}^{k_2} \end{aligned} \tag{2}$$

- Equation (3) represents the total holding cost of deliveries at plants, DCs, and retailers, and the total holding cost of pickups at DCs and retailers.

$$\begin{aligned} C_{inv} = & \sum_{p\in P}\sum_{t\in T} h_{pt}^{\mathscr{D}}.I_{pt}^{\mathscr{D}} + \sum_{d\in D}\sum_{p\in P}\sum_{t\in T} h_{dpt}^{\mathscr{D}}.I_{dpt}^{\mathscr{D}} + \sum_{r\in R}\sum_{p\in P}\sum_{t\in T} h_{rpt}^{\mathscr{D}}.I_{rpt}^{\mathscr{D}} \\ & + \sum_{d\in D}\sum_{p\in P}\sum_{t\in T} h_{dpt}^{\mathscr{P}}.I_{dpt}^{\mathscr{P}} + \sum_{r\in R}\sum_{p\in P}\sum_{t\in T} h_{rpt}^{\mathscr{P}}.I_{rpt}^{\mathscr{P}} \end{aligned} \tag{3}$$

2.2.2 Constraints:

Constraints (4)–(6) enforce inventory balance for deliveries at each plant, DC, and retailer in period t, respectively, considering that each plant produces a unique product. The inventory balance for pickups at each DC and retailer in period t is imposed by Constraints (7)–(9). Additionally, Constraints (8) ensure that vehicles in the first echelon collect available packaging from the pickup inventory, as they cannot wait for the completion of second echelon operations. For $t = 0$, the inventory level decision variables do not appear in constraints (4)–(9) because they represent initial inventories, which are assumed to be zero as previously discussed. Constraints (10)–(12) impose the inventory holding capacity for original products at each plant, DC, and retailer, respectively. Similarly, the inventory holding capacity for packaging at each DC and retailer is enforced by Constraints (13) and (14), respectively.

$$I_{p,t-1}^{\mathscr{D}} + x_{pt} = \sum_{d\in D}\sum_{k_1\in K_1} q_{pdt}^{k1} + I_{pt}^{\mathscr{D}} \qquad \forall\, p\in P,\ t\in T \tag{4}$$

$$I_{dp,t-1}^{\mathscr{D}} + \sum_{k_1\in K_1} q_{pdt}^{k1} = \sum_{r\in R}\sum_{k_2\in K_2} q_{drpt}^{k2} + I_{dpt}^{\mathscr{D}} \qquad \forall\, d\in D,\ p\in P,\ t\in T \tag{5}$$

$$I_{rp,t-1}^{\mathscr{D}} + \sum_{d\in D}\sum_{k_2\in K_2} q_{drpt}^{k2} = d_{rpt} + I_{rpt}^{\mathscr{D}} \qquad \forall\, r\in R,\ p\in P,\ t\in T \tag{6}$$

$$I_{dp,t-1}^{\mathscr{P}} + \sum_{r\in R}\sum_{k_2\in K_2} b_{rdpt}^{k2} = \sum_{k_1\in K_1} b_{dpt}^{k1} + I_{dpt}^{\mathscr{P}} \qquad \forall\, d\in D,\ p\in P,\ t\in T \tag{7}$$

$$\sum_{k_1 \in K_1} b^{k1}_{dpt} \leq I^{\mathscr{P}}_{dp,t-1} \qquad \forall\, d \in D,\ p \in P,\ t \in T \tag{8}$$

$$I^{\mathscr{P}}_{rp,t-1} + p_{rpt} = \sum_{d \in D} \sum_{k_2 \in K_2} b^{k2}_{rdpt} + I^{\mathscr{P}}_{rpt} \qquad \forall\, r \in R,\ p \in P,\ t \in T \tag{9}$$

$$I^{\mathscr{D}}_{pt} \leq L^{\mathscr{D}}_{p} \qquad \forall\, p \in P,\ t \in T \tag{10}$$

$$\sum_{p \in P} I^{\mathscr{D}}_{dpt} \leq L^{\mathscr{D}}_{d} \qquad \forall\, d \in D,\ t \in T \tag{11}$$

$$\sum_{p \in P} I^{\mathscr{D}}_{rpt} \leq L^{\mathscr{D}}_{r} \qquad \forall\, r \in R,\ t \in T \tag{12}$$

$$\sum_{p \in P} I^{\mathscr{P}}_{dpt} \leq L^{\mathscr{P}}_{d} \qquad \forall\, d \in D,\ t \in T \tag{13}$$

$$\sum_{p \in P} I^{\mathscr{P}}_{rpt} \leq L^{\mathscr{P}}_{r} \qquad \forall\, r \in R,\ t \in T \tag{14}$$

Constraints (15) limit the production amount at each plant p to the minimum between its production capacity and the sum of demands in the remaining periods.

$$x_{pt} \leq \min\{pc_{pt}, \sum_{r \in R} \sum_{t' \in T,\ t' \geq t} d_{rpt'}\}.z_{pt} \qquad \forall\, p \in P,\ t \in T \tag{15}$$

Constraints (16) and (17) are related to flow conservation in the first and second echelon, respectively. Constraints (18) and (19) indicate that each vehicle is used at most once in the first and second echelon, respectively. Constraints (20) and (21) force a vehicle to leave the plant p if it visits a distribution center d, and to leave the distribution center d if it visits a retailer r. The assignment of DCs to plants and the assignment of retailers to DCs is defined by Constraints (22) and (23), respectively. Constraints (22) indicate that if a vehicle travels from production plant p to distribution center i (i.e., if $\sum_{i \in N_1} \alpha^{k_1}_{pit} = 1$) then to another distribution center d (i.e., if $\sum_{i \in N_1} \alpha^{k_1}_{idt} = 1$), the latter must be assigned to the plant p, thus $y^{k_1}_{dpt} = 1$. The same principle holds for Constraints (23). Constraints (24)–(26) indicate that each DC is served at most once by each plant in period t. Constraints (27) and (28) indicate that each retailer is served by at most one DC in period t. The subtour elimination constraints (MTZ constraints (Miller et al., 1960)) associated with the first and second echelons are represented by Constraints (29) and (30), respectively.

$$\sum_{j \in N_1,\ j \neq i} \alpha^{k_1}_{ijt} = \sum_{j \in N_1,\ j \neq i} \alpha^{k_1}_{jit} \qquad \forall\, i \in N_1,\ k_1 \in K_1,\ t \in T \tag{16}$$

$$\sum_{j \in N_2,\ j \neq i} \beta^{k_2}_{ijt} = \sum_{j \in N_2,\ j \neq i} \beta^{k_2}_{jit} \qquad \forall\, i \in N_2,\ k_2 \in K_2,\ t \in T \tag{17}$$

$$\sum_{p \in P} \sum_{d \in D} \alpha^{k_1}_{pdt} \leq 1 \qquad \forall\, k_1 \in K_1,\ t \in T \tag{18}$$

$$\sum_{d \in D} \sum_{r \in R} \beta^{k_2}_{drt} \leq 1 \qquad \forall\, k_2 \in K_2,\ t \in T \tag{19}$$

$$\sum_{d' \in D} \sum_{k_1 \in K_1} \alpha^{k_1}_{pd't} \geq \sum_{k_1 \in K_1} y^{k_1}_{dpt} \qquad \forall\, p \in P,\ d \in D,\ t \in T \tag{20}$$

$$\sum_{r' \in R} \sum_{k_2 \in K_2} \beta^{k_2}_{dr't} \geq y_{rdt} \qquad \forall\, d \in D,\ r \in R,\ t \in T \tag{21}$$

$$\sum_{i \in N_1} \alpha^{k_1}_{pit} + \sum_{i \in N_1} \alpha^{k_1}_{idt} - y^{k_1}_{dpt} \leq 1 \qquad \forall\, d \in D,\ p \in P,\ t \in T,\ k_1 \in K_1 \tag{22}$$

$$\sum_{i \in N_2} \beta^{k_2}_{dit} + \sum_{i \in N_2} \beta^{k_2}_{irt} - y_{rdt} \leq 1 \qquad \forall\, d \in D,\ r \in R,\ t \in T,\ k_2 \in K_2 \tag{23}$$

$$\sum_{p\in P} y^{k_1}_{dpt} \le 1 \qquad \forall\, d \in D,\ k_1 \in K_1,\ t \in T \tag{24}$$

$$\sum_{i\in N_1} \alpha^{k_1}_{idt} = \sum_{p\in P} y^{k_1}_{dpt} \qquad \forall\, d \in D,\ k_1 \in K_1,\ t \in T \tag{25}$$

$$\sum_{k_1\in K_1} y^{k_1}_{dpt} \le 1 \qquad \forall\, p \in P,\ d \in D,\ t \in T \tag{26}$$

$$\sum_{d\in D} y_{rdt} \le 1 \qquad \forall\, r \in R,\ t \in T \tag{27}$$

$$\sum_{i\in N_2}\sum_{k_2\in K_2} \beta^{k_2}_{irt} = \sum_{d\in D} y_{rdt} \qquad \forall\, r \in R,\ t \in T \tag{28}$$

$$u^{k1}_{dt} - u^{k1}_{d't} + |D|.\alpha^{k_1}_{dd't} \le |D| - 1 \qquad \forall\, d, d' \in D,\ d \neq d',\ k_1 \in K_1,\ t \in T \tag{29}$$

$$u^{k2}_{rt} - u^{k2}_{r't} + |R|.\beta^{k_2}_{rr't} \le |R| - 1 \qquad \forall\, r, r' \in R,\ r \neq r',\ k_2 \in K_2,\ t \in T \tag{30}$$

Constraints (31) and (32) ensure that the vehicle's capacity is respected in the first and second echelons, respectively. Constraints (33)–(36) limit the delivery quantities from plants to DCs and from DCs to retailers. Similarly, Constraints (37)–(40) restrict the pickup quantities from DCs to plants and from retailers to DCs. Additionally, constraints (33)–(40) link distribution and routing variables and force the use of the same vehicle in a route.

$$\sum_{p\in P}\sum_{d\in D} q^{k_1}_{pdt} \le Q^{k_1} \qquad \forall\, k_1 \in K_1,\ t \in T \tag{31}$$

$$\sum_{d\in D}\sum_{r\in R}\sum_{p\in P} q^{k_2}_{drpt} \le Q^{k_2} \qquad \forall\, k_2 \in K_2,\ t \in T \tag{32}$$

$$q^{k_1}_{pdt} \le Q^{k_1}.y^{k_1}_{dpt} \qquad \forall\, p \in P,\ d \in D,\ k_1 \in K_1,\ t \in T \tag{33}$$

$$q^{k_1}_{pdt} \le Q^{k_1}.(\alpha^{k_1}_{pdt} + \sum_{d'\in D} \alpha^{k_1}_{d'dt}) \qquad \forall\, p \in P,\ d \in D,\ k_1 \in K_1,\ t \in T \tag{34}$$

$$\sum_{p\in P} q^{k_2}_{drpt} \le \min\{Q^{k_2}, \sum_{p\in P}\sum_{t'\in T,\ t'\ge t} d_{rpt'}\}.y_{rdt} \qquad \forall\, d \in D,\ r \in R,\ k_2 \in K_2,\ t \in T \tag{35}$$

$$\sum_{p\in P} q^{k_2}_{drpt} \le \min\{Q^{k_2}, \sum_{p\in P}\sum_{t'\in T,\ t'\ge t} d_{rpt'}\}.(\beta^{k_2}_{drt} + \sum_{r'\in R} \beta^{k_2}_{r'rt}) \qquad \forall\, d \in D,\ r \in R,\ k_2 \in K_2,\ t \in T \tag{36}$$

$$b^{k_1}_{dpt} \le Q^{k_1}.y^{k_1}_{dpt} \qquad \forall\, p \in P,\ d \in D,\ k_1 \in K_1,\ t \in T \tag{37}$$

$$b^{k_1}_{dpt} \le Q^{k_1}.(\alpha^{k_1}_{pdt} + \sum_{d'\in D} \alpha^{k_1}_{d'dt}) \qquad \forall\, p \in P,\ d \in D,\ k_1 \in K_1,\ t \in T \tag{38}$$

$$\sum_{p\in P} b^{k_2}_{rdpt} \le Q^{k_2}.y_{rdt} \qquad \forall\, d \in D,\ r \in R,\ k_2 \in K_2,\ t \in T \tag{39}$$

$$\sum_{p\in P} b^{k_2}_{rdpt} \le Q^{k_2}.(\beta^{k_2}_{drt} + \sum_{r'\in R} \beta^{k_2}_{r'rt}) \qquad \forall\, d \in D,\ r \in R,\ k_2 \in K_2,\ t \in T \tag{40}$$

Constraints (41)–(43) define the vehicle load at each DC in period t. Similarly, Constraints (44)(46) specify the vehicle load at each retailer in period t.

$$vl^{k_1}_{dt} \le Q^{k_1} \qquad \forall\, d \in D,\ k_1 \in K_1,\ t \in T \tag{41}$$

$$vl^{k_1}_{dt} \ge \sum_{d'\in D} q^{k_1}_{pd't} - q^{k_1}_{pdt} + b^{k_1}_{dpt} - Q^{k_1}(1 - \alpha^{k_1}_{pdt}) \qquad \forall\, p \in P,\ d \in D,\ t \in T,\ k_1 \in K_1 \tag{42}$$

$$vl^{k_1}_{d't} \ge vl^{k_1}_{dt} - q^{k_1}_{pd't} + b^{k_1}_{d'pt} - Q^{k_1}(1 - \alpha^{k_1}_{dd't}) \qquad \forall\, p \in P,\ d \in D,\ d' \in D,\ t \in T,\ k_1 \in K_1 \tag{43}$$

$$vl^{k_2}_{rt} \le Q^{k_2} \qquad \forall\, r \in R,\ k_2 \in K_2,\ t \in T \tag{44}$$

$$vl^{k_2}_{rt} \ge \sum_{r'\in R}\sum_{p\in P} q^{k_2}_{dr'pt} - \sum_{p\in P} q^{k_2}_{drpt} + \sum_{p\in P} b^{k_2}_{rdpt} - Q^{k_2}(1 - \beta^{k_2}_{drt}) \qquad \forall\, d \in D,\ r \in R,\ t \in T,\ k_2 \in K_2 \tag{45}$$

$$vl^{k_2}_{r't} \ge vl^{k_2}_{rt} - \sum_{p\in P} q^{k_2}_{dr'pt} + \sum_{p\in P} b^{k_2}_{r'dpt} - Q^{k_2}(1 - \beta^{k_2}_{rr't}) \qquad \forall\, r \in R,\ r' \in R,\ d \in D,\ t \in T,\ k_2 \in K_2 \tag{46}$$

Constraints (47)–(59) define the domain definition of decision variables.

$$x_{pt} \geq 0 \qquad \forall\, p \in P,\ t \in T \tag{47}$$

$$q^{k_1}_{pdt},\ b^{k_1}_{dpt} \geq 0 \qquad \forall\, p \in P,\ d \in D,\ t \in T,\ k_1 \in K_1 \tag{48}$$

$$q^{k_2}_{drpt},\ b^{k_2}_{rdpt} \geq 0 \qquad \forall\, p \in P,\ d \in D,\ r \in R,\ t \in T,\ k_2 \in K_2 \tag{49}$$

$$vl^{k_1}_{dt},\ u^{k1}_{dt} \geq 0 \qquad \forall\, d \in D,\ t \in T,\ k_1 \in K_1 \tag{50}$$

$$vl^{k_2}_{rt},\ u^{k2}_{rt} \geq 0 \qquad \forall\, r \in R,\ t \in T,\ k_2 \in K_2 \tag{51}$$

$$I^{\mathscr{D}}_{pt} \geq 0 \qquad \forall\, p \in P,\ t \in T \tag{52}$$

$$I^{\mathscr{D}}_{dpt},\ I^{\mathscr{P}}_{dpt}, \geq 0 \qquad \forall\, p \in P,\ d \in D,\ t \in T \tag{53}$$

$$I^{\mathscr{D}}_{rpt},\ I^{\mathscr{P}}_{rpt}, \geq 0 \qquad \forall\, p \in P,\ r \in R,\ t \in T \tag{54}$$

$$z_{pt} \in \{0,1\} \qquad \forall\, p \in P,\ t \in T \tag{55}$$

$$\alpha^{k_1}_{ijt} \in \{0,1\} \qquad \forall\, t \in T,\ k_1 \in K_1,\ i,j \in N_1 \tag{56}$$

$$\beta^{k_2}_{ijt} \in \{0,1\} \qquad \forall\, t \in T,\ k_2 \in K_2,\ i,j \in N_2 \tag{57}$$

$$y^{k_1}_{dpt} \in \{0,1\} \qquad \forall\, p \in P,\ d \in D,\ t \in T,\ k_1 \in K_1 \tag{58}$$

$$y_{rdt} \in \{0,1\} \qquad \forall\, d \in D,\ r \in R,\ t \in T \tag{59}$$

3 Computational experiments

In this section, we present the results of the numerical experiments. The model presented in Section 2 has been coded in Python 3.12 and solved using the Gurobi solver (version 12.0), with a time limit set to three hours for each instance. The experiments have been performed on a single core equipped with a 2.65Ghz processor and 100 GB of RAM.

3.1 Instances data

For these experiments, we generate random instances inspired by benchmarks introduced in Gruson et al. (2019). The number of plants, DCs, retailers, vehicles, and periods in each instance are given in Table 3. For each class, we generate 10 instances randomly.

Table 3: Sets size.

Instance class	Class 1	Class 2	Class 3	Class 4
P	1	2	2	4
D	2	3	3	3
R	5	10	15	10
K_1	2	3	3	5
K_2	2	3	3	4
T	7	7	7	7

Table 4 shows the values of the parameters associated with production. The production setup costs, the production variable costs, and retailers demands are generated following a uniform distribution. For pickups, we test two types of returned packaging, foldable and unfoldable, that can be differentiated by a packaging type factor $c' \in \{0.1, 1\}$. The packaging of products delivered to retailers in period $t-1$ are available for pickup at period t, i.e. $p_{rpt} = c' * d_{rp,t-1}$, where $c' = 1$ for unfoldable packages like pallets and containers and 0.1 for foldable packages like cartons.

The plant production capacity for each period t is calculated using the following formulas presented in Gruson et al. (2019):

$$pc_{pt} = \frac{c}{|T|} * \sum_{r \in R} \sum_{t \in T} d_{rpt}$$

with c being the production capacity factor, $c \in \{1.25, 1.5, 1.75, 2.00\}$.

Table 4: Production and demand parameters.

Parameter	Setup cost (sc_{pt})	Variable cost (vc_{pt})	Demand (d_{rpt})	Pickup (p_{rpt})
Value	U[3000,5000]	U[30,50]	U[10,50]	$c' * d_{rp,t-1}$

For each class, we evaluate the 10 instances under every possible combination of the production capacity factor (c) and packaging type factor (c') values. This results in 80 instances per class, leading to a total of 320 instances evaluated in this study.

The parameters associated with inventories are given in Table 5. The holding costs for both pickups and deliveries as well as the holding capacity for deliveries at each plant, DC, and retailer, are generated following a uniform distribution. In contrast, the holding capacity for pickups depends on the type of packaging, either foldable or unfoldable.

Table 5: Inventory parameters.

Inventory	Cost	Plants	DCs	Retailers
Delivery	Holding cost ($h^{\mathscr{D}}$)	U[0.5,1]	U[0.5,1]	U[0.5,1]
	Holding cap ($L^{\mathscr{D}}$)	U[1300,1500]	U[600,1000]	U[150,250]
Pickup	Holding cost ($h^{\mathscr{P}}$)	/	U[0.25,0.5]	U[0.25,0.5]
	Holding cap ($L^{\mathscr{P}}$)	/	$c' * L^{\mathscr{D}}_d$	$c' * L^{\mathscr{D}}_r$

Table 6 presents the parameters related to the vehicle sets. Each set includes four types of vehicles, with each type characterized by its capacity and fixed cost. The vehicle fleets required for each instance class, at both the first and second echelons, are generated randomly in the first and second vehicle sets, respectively.

Table 6: Vehicles parameters.

Fleet	K1				K2			
Capacity (Q)	1700	1800	1900	2000	1300	1400	1500	1600
Fixed cost (f)	17	18	19	20	13	14	15	16

Finally, the positions of all facilities, including plants, CDs, and retailers, are randomly generated within the network space. The transportation cost between any pair of facilities is then computed as the Euclidean distance between their respective locations.

3.2 Results analysis

In this section, we present the results of the experiments conducted across the four instance classes for all possible combinations of parameters c and c' (results of 320 instances). These combinations allow us to systematically analyze the impact of varying production capacities and packaging types on the overall system performance.

The results of these experiments are summarized in Table 7. The first column represents the instance class. The second and third columns provide the values of factors c' and c. The factor c' represents the extent to which the package can be compressed, 1 for unfoldable packages and 0.1 for foldable packages. The factor c is associated with the production capacity, linking it to the average total demand (see Appendix A). The next three columns display the objective value, the lower bound, and the optimality gap. The seventh column reports the number of explored nodes in the Branch-and-Cut ($B\&C$) algorithm. The CPU time is given in the eighth column. The ninth column indicates the number of instances solved to proven optimality. Finally, the last column presents the objective values of the linear relaxation.

Table 7: Experimental results.

Class	c'	c	Obj	LB	Gap%	#Nodes	CPU time	#*Opt*	LR Obj
1	0.1	1.25	67125.11	67121.96	0.0	242170.3	141.32	10	63829.36
		1.5	62934.01	62932.69	0.0	26373.0	15.78	10	60597.80
		1.75	58834.09	58832.81	0.0	32129.5	24.53	10	56800.86
		2	55752.65	55750.69	0.0	17548.8	12.50	10	52722.94
	1	1.25	66404.00	66400.81	0.0	762260.0	471.14	10	63523.31
		1.5	62348.88	62346.31	0.0	396001.0	263.33	10	59410.54
		1.75	58271.81	58271.18	0.0	52792.5	27.32	10	56321.23
		2	57043.43	57042.92	0.0	27000.7	16.05	10	53987.49
2	0.1	1.25	211997.30	211834.35	0.08	307513.1	8375.20	3	205318.08
		1.5	207123.65	206948.21	0.08	440320.6	9872.31	1	201335.79
		1.75	198725.91	198552.25	0.09	271580.3	9050.60	2	193548.87
		2	192687.86	192598.30	0.05	328239.0	6094.29	6	185972.54
	1	1.25	211099.55	210976.06	0.06	219639.0	7722.67	4	204855.99
		1.5	204357.00	204143.24	0.10	276096.2	9881.56	2	198211.75
		1.75	198201.36	198043.32	0.08	252029.1	8524.71	5	192469.12
		2	197685.88	197545.89	0.07	255070.3	8919.24	2	190793.40
3	0.1	1.25	317460.60	317262.20	0.06	252799.0	10311.59	2	311294.72
		1.5	277157.30	276902.70	0.09	350259.3	9862.84	1	270874.48
		1.75	295725.75	295388.53	0.12	205630.2	10800.24	0	289699.94
		2	294223.64	293863.01	0.12	744320.7	10800.23	0	287055.91
	1	1.25	298422.16	297692.59	0.25	311380.4	10800.27	0	291781.35
		1.5	300227.54	299771.49	0.15	307955.5	10341.85	1	293348.06
		1.75	293601.70	293143.22	0.16	293409.1	10800.24	0	287343.79
		2	295926.80	295266.05	0.22	264164.2	10800.53	0	288584.22
4	0.1	1.25	422591.93	421730.02	0.20	78526.8	10800.18	0	410404.38
		1.5	409073.26	408021.65	0.25	126802.5	10800.16	0	396080.61
		1.75	394539.11	393437.39	0.28	86547.6	10800.17	0	382424.19
		2	382116.25	381127.80	0.26	62624.0	10800.11	0	368966.49
	1	1.25	441767.68	438308.31	0.77	142736.0	10800.18	0	428706.89
		1.5	407129.39	405125.10	0.49	167486.7	10800.21	0	393854.06
		1.75	397154.68	395438.64	0.43	148439.1	10800.15	0	382369.50
		2	380196.42	378258.65	0.51	121151.6	10800.16	0	364952.98

The analysis of Table 7 reveals that the objective value generally decreases as c increases within each class and for a fixed c', indicating that higher values of c lead to lower costs. The optimality gap (Gap) is small, mostly between 0.0% and 0.51%, demonstrating good convergence despite the difficulty of the problem. However, both the average number of explored nodes in the $B\&C$ algorithm and the CPU time increase considerably as the value of c' increases for all cases in classes 1 and 4, and for some cases in classes 2 and 3.

Interestingly, the number of explored nodes tends to decrease as the problem complexity increases, (i.e., when the number of facilities increases), particularly in Class 4. CPU time varies significantly across classes, with Classes 3 and 4 often reaching the maximum limit of 3 hours. This indicates that the solver struggles either to find optimal solutions or to prove optimality within the time limit, as most instances in these classes have a non-zero optimality gap at termination. In contrast, Class 1 has significantly lower CPU times, indicating that its instances are the simplest and reach optimality easily. The objective value of the relaxed model (LR Obj) follows a similar trend to the MILP objective, decreasing as c increases. Overall, larger values of c enhance optimization efficiency, whereas larger values of c' make the problem more challenging, with Class 1 being the easiest to solve and Classes 3 and 4 presenting significant computational challenges.

Figure 2 and Figure 3 illustrate, respectively, the average objective function values and the average CPU time values across different values of the capacity factor c for the four classes under the packaging

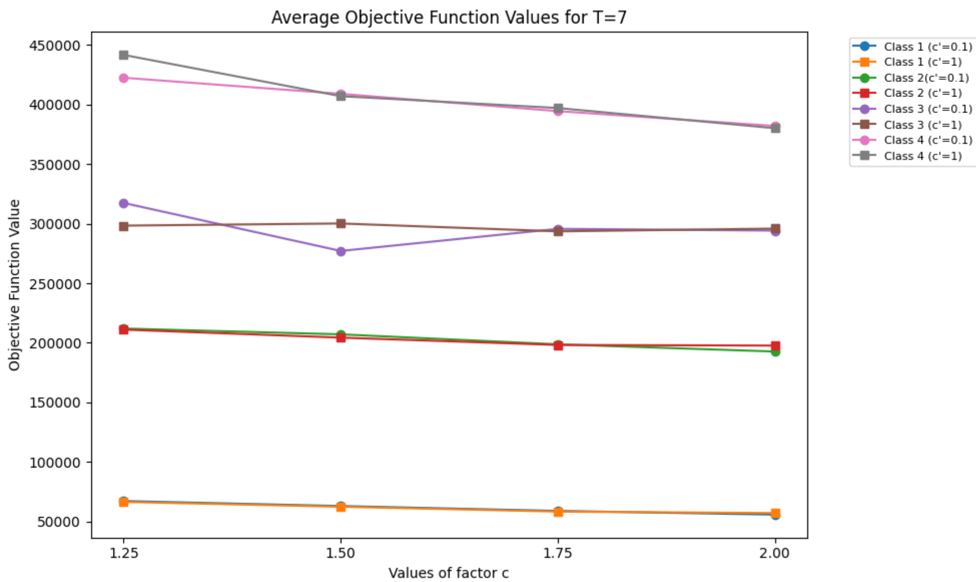

Figure 2: Average objective function values.

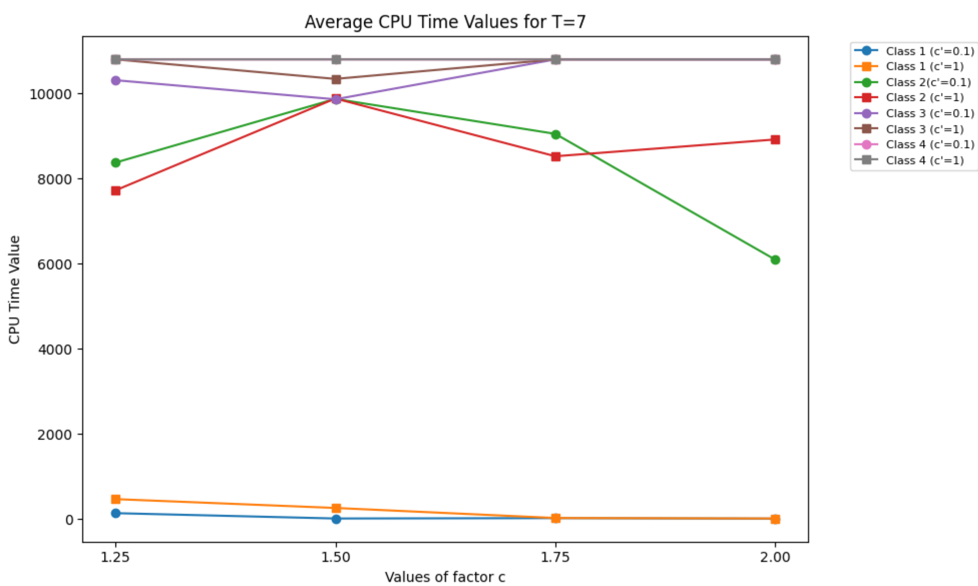

Figure 3: Average CPU time values.

type factor c', with a fixed planning horizon $T = 7$. Figure 2 confirms that, across all classes, increasing c leads to a consistent decrease in objective values, indicating improved cost efficiency with greater production capacity. Instances with $c' = 1$ systematically yield lower objective values than those with $c' = 0.1$, emphasizing the impact of the factor c' on the objective function values.

Figure 3 shows that CPU time generally decreases with increasing c for simpler problem classes (Class 1), reflecting improved computational efficiency. In contrast, for more complex classes (Classes 3 and 4), CPU time remains consistently high across all values of c, showing that these instances are computationally demanding. Additionally, instances with $c' = 1$ generally result in higher CPU times compared to $c' = 0.1$, particularly in Classes 1, 3, and 4.

4 Conclusion

In this paper, we introduced a formulation for the two-echelon PRP with simultaneous pickup and delivery. The results of numerical experiments demonstrate that the proposed model performs well for small instances, as most of them are solved to optimality. However, the problem becomes increasingly challenging as the number of facilities increases.

The proposed formulation offers operational and managerial benefits like optimized resource utilization, cost reduction, optimized routes by performing pickups and deliveries simultaneously, and improved coordination of production, inventory, and distribution operations.

Moving forward, we plan to evaluate the model on larger instances and extended planning horizons of 15 and 30 periods. Additionally, we aim to enhance solution quality by developing solution methods such as Top-down and Bottom-up matheuristics.

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