Les Cahiers du GERAD ISSN: 0711–2440

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G–2025–39

June 2025

La publication de ces rapports de recherche est rendue possible grâce au soutien de HEC Montréal, Polytechnique Montréal, Université McGill, Université du Québec à Montréal, ainsi que du Fonds de recherche du Québec – Nature et technologies.

Dépôt légal – Bibliothèque et Archives nationales du Québec, 2025
– Bibliothèque et Archives Canada, 2025

The publication of these research reports is made possible thanks to the support of HEC Montréal, Polytechnique Montréal, McGill University, Université du Québec à Montréal, as well as the Fonds de recherche du Québec – Nature et technologies.

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Cat-Suite: A collection of optimization problems with categorical and quantitative variables for benchmarking

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Abstract : Benchmarking new optimization methods on test problems is essential for assessing their performance and tuning their parameters. Yet, few problems are available in the literature for mixed-variable optimization. This manuscript introduces Cat-Suite, a collection of optimization problems involving categorical, integer, and continuous variables. Currently, Cat-Suite includes 32 analytical problems, half of which are constrained and half unconstrained. Each of these problems is either a modification of an existing one from the literature or derived from classical continuous test functions, such as Rosenbrock. Cat-Suite is an ongoing project, and additional problems will be added in the future. The goal is to facilitate benchmarking and testing of novel mixed-variable optimization methods with a diverse problem collection. The problems are implemented in Python and available at https://github.com/bbopt/Cat-Suite.

Keywords : Optimization, benchmarking, mixed-variable, categorical variables, constraints

Acknowledgements: This research is funded by a Natural Sciences and Engineering Research Council of Canada (NSERC) PhD Excellence Scholarship (PGS D), a Fonds de Recherche du Québec (FRQNT) PhD Excellence Scholarship and an Institut de l'Énergie Trottier (IET) PhD Excellence Scholarship, as well as by the NSERC discovery grants RGPIN–2020–04448 (Audet), RGPIN–2024–05093 (Diouane) and RGPIN–2024–05086 (Le Digabel).

1 Introduction

In mixed-variable optimization, categorical variables are notoriously hard to tackle due to their qualitative nature and lack of inherent structure. Such variables take values, called categories, in sets that lack well-defined metrics and may be unordered [5, 11]. Many practical applications involve these variables. Optimal engineering designs are often found through the optimization of physics-based models with variables that include material choices, such as steel or alloys. Designated methods tackling such problems form an active research field, notably in mixed-variable derivative-free optimization, where functions are frequently *expensive-to-evaluate* computer simulations without expressions [1, 3, 7, 8, 15, 16, 17, 18]. However, the current literature offers few test problems that are mixed-variable, making it difficult to thoroughly test the performance of novel solution methods. This manuscript introduces Cat-Suite, a collection of mixed-variable optimization problems.

The purpose of Cat-Suite is to facilitate *benchmarking* and *testing* of mixed-variable optimization methods. The project draws inspiration from CUTEst, a well-known collection of problems for continuous optimization [9]. Cat-Suite is an ongoing project. The collection will be expanded with additional and more diverse problems. Contributions and participation are welcome.

Currently, Cat-Suite contains 32 mixed-variable analytical problems, half of which are unconstrained and the other half constrained. Each problem contains at least one categorical and one continuous variables. Most of these problems also contains integer variables. The categorical variables influence the behavior of the functions involve via cases, tables or penalties taking continuous and integer variables. Approximately half of the problems are nonsmooth, *i.e.*, there is a least one function that is not differentiable for fixed categorical and integer variables. Nonsmoothness typically adds considerable challenge, but it is highly relevant in some optimization context, such as derivative-free or blackbox optimization where functions are often contaminated by noise or discontinuities [2, 4]. The current test problems proposed are either modifications of existing problems from the literature or adaptations of classic continuous optimization problems, such as the Rosenbrock and Rastrigin problems [12].

1.1 Problem statement

The problems in Cat-Suite are mixed-variable with inequality constraints, expressed as follows:

$$\min_{\boldsymbol{x}\in\Omega} f(\boldsymbol{x}) \tag{1}$$

where $f : \mathcal{X} \to \mathbb{R}$ is the objective function, $\mathcal{X}$ is the domain of the objective and constraint functions, $\Omega \coloneqq \{\boldsymbol{x} \in \mathcal{X} \ : \ g_j(\boldsymbol{x}) \leq 0, \ j \in \{1, 2, \ldots, m\}\}$ is the feasible set, $g_j : \mathcal{X} \to \overline{\mathbb{R}}$ is the j-th constraint function of the problem, with $j \in J$, and $m \in \mathbb{N}$ is the number of constraints.

The variables of a given type $t \in \{\text{cat}, \text{int}, \text{cont}\}$ are contained in a column vector expressed as

$$\boldsymbol{x}^t \coloneqq \left(x_1^t, x_2^t, \ldots, x_{n^t}^t\right) \in \mathcal{X}^t \coloneqq \mathcal{X}_1^t \times \mathcal{X}_2^t \times \ldots \times \mathcal{X}_{n^t}^t, \tag{2}$$

where $n^t \in \mathbb{N}$ is the number of variables of type t, $\mathcal{X}^t$ is the set of type t, $x_i^t \in \mathcal{X}_i^t$ denotes the i-th variable of type t, and $\mathcal{X}_i^t$ contains the bounds of x_i^t. The indices of the variables of type t are contained in the set $I^t \coloneqq \{1, 2, \ldots, n^t\}$. Following this, a point is formalized as a partition of these vectors by types and the domain is constructed with Cartesian products of the sets by types, such that

$$\boldsymbol{x} \coloneqq \left(\boldsymbol{x}^{\text{cat}}, \boldsymbol{x}^{\text{int}}, \boldsymbol{x}^{\text{cont}}\right) \in \mathcal{X} \coloneqq \mathcal{X}^{\text{cat}} \times \mathcal{X}^{\text{int}} \times \mathcal{X}^{\text{cont}}. \tag{3}$$

In this manuscript, the problems considered are not *hierarchical*. There are no *meta variables* that control the inclusion, bounds or admissible values of other variables, as described in [5, 10]. The number of variables is fixed, and the bounds of each variable is predefined.

The rest of this document is organized as follows. Section 2 summarizes the main characteristics of the problems, such as the number of variables or constraints, using tables. This section also discusses the techniques used to construct the problems. Afterwards, Sections 3 and 4 details the mathematical formulations of the unconstrained and constrained problems respectively.

2 Summary and characteristics of the test problems

The characteristics of the unconstrained and constrained test problems are presented in Table 1 and Table 2, respectively. The $\star$ symbol denotes a user-specified positive integer. The implementations are publicly available at `https://github.com/bbopt/Cat-Suite`. Additional information is also provided, such as the best feasible value known for problems.

Table 1: Unconstrained test problems.

Name	Sec.	n^{cat}	ℓ	n^{int}	n^{cont}	Smooth	Ref.	Original problem	Modifications
Cat-1	3.1	2	9	2	$\star$	No	[12]	Ackley	Fct-to-cases & penalty
Cat-2	3.2	2	9	2	3	No	[12]	Beale	Cont-to-cat
Cat-3	3.3	2	4	2	2	Yes	[15]	Augmented-Branin	Added integers & removed constraints
Cat-4	3.4	2	4	2	4	No	[12]	Bukin-6	Cont-to-cat
Cat-5	3.5	1	6	1	3	Yes	[13]	EVD-52	Added integers & minimax-to-cat
Cat-6	3.6	2	9	0	2	Yes	[15]	Goldstein	Removed constraints
Cat-7	3.7	3	16	3	2	No	[12]	Goldstein-Price	Penalty
Cat-8	3.8	1	4	1	5	Yes	[13]	HS78	Added integer, added category & minimax-to-cat
Cat-9	3.9	2	9	2	$\star$	No	[12]	Rastrigin	Cont-to-cat & fct-to-cat
Cat-10	3.10	2	6	2	$\star$	No	[12]	Rosenbrock	Fct-to-cat & penalty
Cat-11	3.11	1	4	1	4	Yes	[13]	Rosen-Suzuki	Added integer & minimax-to-cat
Cat-12	3.12	1	5	$\star$	$\star$	No	[12]	Styblinski-Tang	Cont-to-cat
Cat-13	3.13	1	10	0	4	Yes	[14]	Toy	Added variables & modified functions
Cat-14	3.14	1	10	0	8	No	[14]	Toy	Added variables & modified functions
Cat-15	3.15	1	5	3	4	Yes	[13]	Wong-1	Cont-to-int & minimax-to-cat
Cat-16	3.16	2	9	2	$\star$	No	[12]	Zakharov	Created

Table 2: Constrained test problems.

Name	Sec.	n^{cat}	ℓ	n^{int}	n^{cont}	m	Smooth	Ref.	Original problem	Modifications
Cat-cstrs-1	4.1	2	9	2	3	3	No	[12]	Beale	Cat-2 with added constraints
Cat-cstrs-2	4.2	2	4	2	2	1	Yes	[15]	Augmented-Branin	Added integers
Cat-cstrs-3	4.3	2	9	2	4	2	No	[12]	Bukin-6	Cat-4 with added constraints
Cat-cstrs-4	4.4	1	4	4	4	3	Yes	[13]	Dembo-5	Con-to-int & minimax-to-cat
Cat-cstrs-5	4.5	1	6	1	3	1	No	[13]	EVD-52	Added integers & added constraints
Cat-cstrs-6	4.6	2	9	2	3	4	Yes	[6]	G-09	Con-to-int, modified constraints & cat-tables
Cat-cstrs-7	4.7	2	9	0	2	1	Yes	[15]	Goldstein	None
Cat-cstrs-8	4.8	2	25	2	2	2	Yes	[12]	Himmelblau	Added constraints
Cat-cstrs-9	4.9	2	4	3	5	4	Yes	[13]	HS-114	Con-to-int & minimax-to-cat
Cat-cstrs-10	4.10	1	3	2	4	6	Yes	[13]	Pentagon	Con-to-int & minimax-to-cat
Cat-cstrs-11	4.11	1	8	2	2	3	Yes	[6]	Pressure-Vessel	Con-to-int, added integers & con-to-cat
Cat-cstrs-12	4.12	2	25	1	2	2	Yes	[6]	Reinforced-Concrete	Added continuous & simplified cat tables
Cat-cstrs-13	4.13	2	6	2	$\star$	1	No	[12]	Rosenbrock	Cat-10 with added constraints
Cat-cstrs-14	4.14	1	5	$\star$	$\star$	2	No	[12]	Styblinski–Tang	Cat-12 with added constraints
Cat-cstrs-15	4.15	1	10	0	4	2	Yes	[14]	Toy	Added variables & added constraints
Cat-cstrs-16	4.16	1	6	4	6	3	Yes	[13]	Wong-2	Con-to-int, minimax-to-cat & added cstrs

The problem statements, with the objective and constraints functions, are presented in their corresponding subsection. A problem is nonsmooth if there is a least one function that is not differentiable with respect to its continuous variables. For each problem, a reference and a summary of the modifications are provided.

Three techniques are employed to create mixed-variable problems from classic problems. The first technique replaces continuous variables with functions that depend on both categorical and continuous variables. For example, in Cat-12, detailed in Subsection 3.12, the function s in the summation replaces a continuous variable x_i^{cont} from the original Styblinski-Tang objective. This functions s now maps x_i^{cont}

differently depending on the category taken by the categorical variable $x^{\text{cat}} \in \{\text{A}, \text{B}, \text{C}, \text{D}, \text{E}\}$. This is referred to as "con-to-cat" in the tables. The second technique, similar to the first one, transforms a function into another one with cases, each assigned to a category. This is named "fct-to-cases". For example, in Cat-1, formulated in 3.1, the three function cases of the second term are variations of the original Ackley objective function. The last technique, called "penalty", simply adds a penalty function with cases assigned to categories. The first term in the objective function of the Cat-1 problem correspond to such penalty.

The problems modified from [13] are originally minimax problems on many functions. These problems are transformed into mixed-variable problems by assigning each function to a categorical component $\boldsymbol{x}^{\text{cat}} \in \mathcal{X}^{\text{cat}}$. This technique is taken from [14] and this is referred to as "minimax-to-cat" in the tables. Some problems are adapted from [14], with minor modifications such as introducing integer variables. Similarly, problems taken from [15] undergo small modifications. The Cat-23 constrained problem in Table 2 is the only unmodified problem taken directly from [15]. The minimax problems in [13] are strictly continuous. Some continuous variables are transformed into integer variables and this is referred to as "con-to-int" in the tables. The mixed-variable constrained problems from [6] have been modified with the different techniques discussed above.

3 Description of the unconstrained problems

The following subsections details the unconstrained problems.

3.1 Cat-1: modified Ackley [12]

$$
\begin{aligned}
&f\left(x_1^{\text{cat}}, x_2^{\text{cat}}, x_1^{\text{int}}, x_2^{\text{int}}, \boldsymbol{x}^{\text{cont}}\right) = s\left(x_2^{\text{cat}}, \boldsymbol{x}^{\text{cont}}\right)\\
&+\begin{cases}
-20\exp\left(-\frac{2}{10}\sqrt{\frac{1}{n^{\text{cont}}}\sum_{i=1}^{n^{\text{cont}}}\left(x_i^{\text{cont}} + x_1^{\text{int}}\right)^2}\right) - \exp\left(\frac{1}{n^{\text{cont}}}\sum_{i=1}^{n^{\text{cont}}}\cos\left(2\pi x_i^{\text{cont}} \times x_2^{\text{int}}\right)\right) & \text{if } x_1^{\text{cat}} = \text{A}\\
-10\exp\left(-\frac{1}{10}\sqrt{\frac{1}{n^{\text{cont}}}\sum_{i=1}^{n^{\text{cont}}}\left|x_i^{\text{cont}} + x_1^{\text{int}}\right|}\right) + 5\sum_{i=1}^{n^{\text{cont}}}\cos\left(2\pi x_i^{\text{cont}} - x_2^{\text{int}}\right) & \text{if } x_1^{\text{cat}} = \text{B}\\
-15\exp\left(-\sqrt{\frac{1}{n^{\text{cont}}}\sum_{i=1}^{n^{\text{cont}}}\left|x_i^{\text{cont}} \times x_1^{\text{int}}\right|}\right) - 5\exp\left(-\frac{1}{n^{\text{cont}}}\sum_{i=1}^{n^{\text{cont}}}\cos\left(2\pi x_i^{\text{cont}} + x_2^{\text{int}}\right)\right) & \text{if } x_1^{\text{cat}} = \text{C}
\end{cases}
\end{aligned}
$$

where

$$
s\left(x_2^{\text{cat}}, \boldsymbol{x}^{\text{cont}}\right) = \begin{cases}
\left|\sum_{i=1}^{n^{\text{cont}}} x_i^{\text{cont}}\right| \times 15 & \text{if } x_2^{\text{cat}} = \text{A}\\
\left|\sum_{i=1}^{n^{\text{cont}}} (x_i^{\text{cont}} - 1)\right| \times 15 & \text{if } x_2^{\text{cat}} = \text{B}\\
\left|\sum_{i=1}^{n^{\text{cont}}} x_i^{\text{cont}}\right| \times 10 & \text{if } x_2^{\text{cat}} = \text{C}
\end{cases}
$$

- $n^{\text{cat}} = 2$ and $x_1^{\text{cat}}, x_2^{\text{cat}} \in \{\text{A}, \text{B}, \text{C}\}$;
- $n^{\text{int}} = 2$ and $x_1^{\text{int}} \in \{1, 2, \ldots, 10\}$, $x_2^{\text{int}} \in \{-1, 0, 1\}$;
- $n^{\text{cont}} \in \mathbb{N}$ and $x_i^{\text{cont}} \in [-5, 5]$ for $i \in I^{\text{cont}}$.

3.2 Cat-2: modified Beale [12]

$$
\begin{aligned}
f\left(x_1^{\text{cat}}, x_2^{\text{cat}}, x_1^{\text{int}}, x_2^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}\right) &= \left(1.5 - s_1\left(x_1^{\text{cat}}, x_1^{\text{cont}}\right) + x_1^{\text{int}}\left(1 - s_2(x_2^{\text{cat}}, x_2^{\text{cont}})\right)\right)^2\\
&+ \left(2.25 - s_1\left(x_1^{\text{cat}}, x_1^{\text{cont}}\right) + x_2^{\text{int}}\left(1 - s_2(x_2^{\text{cat}}, x_2^{\text{cont}})\right)^2\right)^2\\
&+ \left(2.625 - s_1\left(x_1^{\text{cat}}, x_1^{\text{cont}}\right) + x_3^{\text{cont}}\left(1 - s_2(x_2^{\text{cat}}, x_2^{\text{cont}})\right)^3\right)^2
\end{aligned}
$$

where

$$s_1(x_1^{\text{cat}}, x_1^{\text{cont}}) = \begin{cases} \lfloor x_1^{\text{cont}} \rfloor & \text{if } x_1^{\text{cat}} = \text{A} \\ x_1^{\text{cont}} & \text{if } x_1^{\text{cat}} = \text{B} \\ \exp\left(\frac{x_1^{\text{cont}}}{2}\right) & \text{if } x_1^{\text{cat}} = \text{C} \end{cases} \quad \text{and} \quad s_2(x_2^{\text{cat}}, x_2^{\text{cont}}) = \begin{cases} \sqrt{|x_2^{\text{cont}}| + 1} & \text{if } x_2^{\text{cat}} = \text{A} \\ \left|x_2^{\text{cont}}\right| & \text{if } x_2^{\text{cat}} = \text{B} \\ \left(x_2^{\text{cont}}\right)^2 - 2 & \text{if } x_2^{\text{cat}} = \text{C} \end{cases}$$

- $n^{\text{cat}} = 2$ and $x_1^{\text{cat}}, x_2^{\text{cat}} \in \{\text{A}, \text{B}, \text{C}\}$;
- $n^{\text{int}} = 2$ and $x_1^{\text{int}}, x_2^{\text{int}} \in \{-2, -1, 0, 1, 2\}$;
- $n^{\text{cont}} = 3$ and $x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}} \in [-4.5, 4.5]$.

3.3 Cat-3: modified Branin [15]

$$f\left(x_1^{\text{cat}}, x_2^{\text{cat}}, x_1^{\text{int}}, x_2^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}\right) = \begin{cases} s\left(x_1^{\text{int}}, x_2^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}\right) & \text{if } x_1^{\text{cat}} = \text{A and } x_2^{\text{cat}} = \text{A} \\ 0.4s\left(x_1^{\text{int}}, x_2^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}\right) & \text{if } x_1^{\text{cat}} = \text{A and } x_2^{\text{cat}} = \text{B} \\ -0.75s\left(x_1^{\text{int}}, x_2^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}\right) & \text{if } x_1^{\text{cat}} = \text{B and } x_2^{\text{cat}} = \text{A} \\ -0.5s\left(x_1^{\text{int}}, x_2^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}\right) & \text{if } x_1^{\text{cat}} = \text{B and } x_2^{\text{cat}} = \text{B} \end{cases}$$

where

$$s\left(x_1^{\text{int}}, x_2^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}\right) =$$
$$\frac{1}{a}\left[\left(\left(15x_2^{\text{cont}} - \frac{x_1^{\text{int}}}{4\pi^2}\left(15x_1^{\text{cont}} - x_2^{\text{int}}\right)^2 + \frac{x_1^{\text{int}}}{\pi}\left(15x_1^{\text{cont}} - x_2^{\text{int}}\right)\right)^2 + \left(x_1^{\text{int}} - \frac{1}{8\pi}\right)\cos\left(15x_1^{\text{cont}} - 5\right) + x_1^{\text{int}}\right) - b\right]$$

with $a = 51.9496$, $b = 54.8104$,

- $n^{\text{cat}} = 2$ and $x_1^{\text{cat}}, x_2^{\text{cat}} \in \{\text{A}, \text{B}\}$;
- $n^{\text{int}} = 2$ and $x_1^{\text{int}}, x_2^{\text{int}} \in \{1, 2, 3, 4, 5\}$;
- $n^{\text{cont}} = 2$ and $x_1^{\text{cont}}, x_2^{\text{cont}} \in [0, 1]$.

3.4 Cat-4: modified Bukin-6 [12]

$$\begin{aligned} f\left(x_1^{\text{cat}}, x_2^{\text{cat}}, x_1^{\text{int}}, x_2^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}, x_4^{\text{cont}}\right) = 100\sqrt{\left|s_1\left(x_1^{\text{cat}}, x_2^{\text{cont}}, x_3^{\text{cont}}\right) - 0.01 s_2\left(x_2^{\text{cat}}, x_1^{\text{int}}, x_1^{\text{cont}}, x_4^{\text{cont}}\right)\right|} \\ + 0.01\left|s_2\left(x_2^{\text{cat}}, x_1^{\text{int}}, x_1^{\text{cont}}, x_4^{\text{cont}}\right) + x_2^{\text{int}}\right| \end{aligned}$$

where

$$s_1(x_1^{\text{cat}}, x_2^{\text{cont}}, x_3^{\text{cont}}) = \begin{cases} \sqrt{|x_2^{\text{cont}} + x_3^{\text{cont}}| + 2} & \text{if } x_1^{\text{cat}} = \text{A} \\ \left|x_2^{\text{cont}} + x_3^{\text{cont}}\right| & \text{if } x_1^{\text{cat}} = \text{B} \\ \frac{\left(x_2^{\text{cont}} + x_3^{\text{cont}}\right)^2}{1.25} + 1 & \text{if } x_1^{\text{cat}} = \text{C} \end{cases}$$

and

$$s_2(x_2^{\text{cat}}, x_1^{\text{int}}, x_1^{\text{cont}}, x_4^{\text{cont}}) = \begin{cases} \sqrt{|x_1^{\text{int}} + x_1^{\text{cont}} + x_4^{\text{cont}}| + 1.5} & \text{if } x_2^{\text{cat}} = \text{A} \\ \left|x_1^{\text{int}} + x_1^{\text{cont}} + x_4^{\text{cont}}\right| & \text{if } x_2^{\text{cat}} = \text{B} \\ \frac{x_1^{\text{int}} + \left(x_1^{\text{cont}} + x_4^{\text{cont}}\right)^2}{1.25} + 1 & \text{if } x_2^{\text{cat}} = \text{C} \end{cases}$$

- $n^{\text{cat}} = 2$ and $x_1^{\text{cat}}, x_2^{\text{cat}} \in \{\text{A}, \text{B}, \text{C}\}$;
- $n^{\text{int}} = 2$ and $x_1^{\text{int}}, x_2^{\text{int}} \in \{-5, -4, \ldots, 4, 5\}$;
- $n^{\text{cont}} = 4$ and $\left(x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}, x_4^{\text{cont}}\right) \in [-15, 5] \times [-3, 3] \times [-15, 5] \times [-3, 3]$.

3.5 Cat-5: modified EVD-52 [13]

$$f(x^{\text{cat}}, x^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}) = \begin{cases} \left(x_1^{\text{cont}}\right)^2 + \left(x_2^{\text{cont}}\right)^2 + \left(x_3^{\text{cont}}\right)^2 - 1 - \frac{x^{\text{int}}}{50} & \text{if } x^{\text{cat}} = \text{A} \\ \left(x_1^{\text{cont}}\right)^2 + \left(x_2^{\text{cont}}\right)^2 + \left(x_3^{\text{cont}} - 2\right)^2 - \frac{x^{\text{int}}}{50} & \text{if } x^{\text{cat}} = \text{B} \\ x_1^{\text{cont}} + x_2^{\text{cont}} + x_3^{\text{cont}} - 1 - \frac{x^{\text{int}}}{50} & \text{if } x^{\text{cat}} = \text{C} \\ x_1^{\text{cont}} + x_2^{\text{cont}} - x_3^{\text{cont}} + 1 + \frac{x^{\text{int}}}{50} & \text{if } x^{\text{cat}} = \text{D} \\ 2\left(x_1^{\text{cont}}\right)^3 + 6\left(x_2^{\text{cont}}\right)^2 + 2\left(5x_3^{\text{cont}} - x_1^{\text{cont}} + 1\right)^2 + \frac{x^{\text{int}}}{50} & \text{if } x^{\text{cat}} = \text{E} \\ \left(x_1^{\text{cont}}\right)^2 - 9x_3^{\text{cont}} + \frac{x^{\text{int}}}{50} & \text{if } x^{\text{cat}} = \text{F} \end{cases}$$

- $n^{\text{cat}} = 1$ and $x^{\text{cat}} \in \{\text{A}, \text{B}, \text{C}, \text{D}, \text{E}, \text{F}\}$;
- $n^{\text{int}} = 1$ and $x^{\text{int}} \in \{-25, -24, \ldots, , 24, 25\}$;
- $n^{\text{cont}} = 3$ and $x_i^{\text{cont}} \in [-25, 25]$ for $i \in I^{\text{cont}}$.

3.6 Cat-6: modified Goldstein [15]

$$\begin{aligned} f\left(x_1^{\text{cat}}, x_2^{\text{cat}}, x_1^{\text{cont}}, x_2^{\text{cont}}\right) = \\ & 53.3108 + 0.184901 x_1^{\text{cont}} \\ & - 5.02914\left(x_1^{\text{cont}}\right)^3 \times 10^{-6} + 7.72522\left(x_1^{\text{cont}}\right)^4 \times 10^{-8} \\ & - 0.0870775 x_2^{\text{cont}} - 0.106959 s_1(x_1^{\text{cat}}) \\ & + 7.98772\left(s_1(x_1^{\text{cat}})\right)^3 \times 10^{-6} + 0.00242482 s_2(x_2^{\text{cat}}) \\ & + 1.32851\left(s_2\left(x_2^{\text{cat}}\right)\right)^3 \times 10^{-6} - 0.00146393 x_1^{\text{cont}} x_2^{\text{cont}} \\ & - 0.00301588 x_1^{\text{cont}} s_1(x_1^{\text{cat}}) - 0.00272291 x_1^{\text{cont}} s_2(x_2^{\text{cat}}) \\ & + 0.0017004 x_2^{\text{cont}} s_1(x_1^{\text{cat}}) + 0.0038428 x_2^{\text{cont}} s_2(x_2^{\text{cat}}) \\ & - 0.000198969 s_1(x_1^{\text{cat}}) s_2(x_2^{\text{cat}}) \\ & + 1.86025 x_1^{\text{cont}} x_2^{\text{cont}} s_1(x_1^{\text{cat}}) \times 10^{-5} \\ & - 1.88719 x_1^{\text{cont}} x_2^{\text{cont}} s_2(x_2^{\text{cat}}) \times 10^{-6} \\ & + 2.50923 x_1^{\text{cont}} s_1(x_1^{\text{cat}}) s_2(x_2^{\text{cat}}) \times 10^{-5} \\ & - 5.62199 x_2^{\text{cont}} s_1(x_1^{\text{cat}}) s_2(x_2^{\text{cat}}) \times 10^{-5} \end{aligned}$$

where

$$s_1\left(x_1^{\text{cat}}\right) = \begin{cases} 20 & \text{if } x_1^{\text{cat}} = \text{A}, \\ 50 & \text{if } x_1^{\text{cat}} = \text{B}, \\ 80 & \text{if } x_1^{\text{cat}} = \text{C}, \end{cases} \qquad \text{and} \qquad s_2\left(x_2^{\text{cat}}\right) = \begin{cases} 20 & \text{if } x_2^{\text{cat}} = \text{A}, \\ 50 & \text{if } x_2^{\text{cat}} = \text{B}, \\ 80 & \text{if } x_2^{\text{cat}} = \text{C}. \end{cases}$$

- $n^{\text{cat}} = 2$ and $x_1^{\text{cat}}, x_2^{\text{cat}} \in \{\text{A}, \text{B}, \text{C}\}$;
- $n^{\text{int}} = 0$;
- $n^{\text{cont}} = 2$ and $x_1^{\text{cont}}, x_2^{\text{cont}} \in [0, 100]$.

3.7 Cat-7: modified Goldstein-Price [12]

$$\begin{aligned} f\left(x_1^{\text{cat}}, x_2^{\text{cat}}, x_3^{\text{cat}}, x_1^{\text{int}}, x_2^{\text{int}}, x_3^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}\right) &= s_1\left(x_1^{\text{cont}}, x_2^{\text{cont}}\right) + \\ & s_2\left(x_1^{\text{cat}}, x_2^{\text{cat}}, x_1^{\text{int}}, x_2^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}\right) + s_3\left(x_3^{\text{cat}}, x_3^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}\right) \end{aligned}$$

where

$$
\begin{aligned}
&s_1\left(x_1^{\text{cont}}, x_2^{\text{cont}}\right)\\
&\quad=\left(1+\left(x_1^{\text{cont}}+x_2^{\text{cont}}+1\right)^2\left(19-14x_1^{\text{cont}}+3\left(x_1^{\text{cont}}\right)^2-14x_2^{\text{cont}}+6x_1^{\text{cont}}x_2^{\text{cont}}+3\left(x_2^{\text{cont}}\right)\right)\right)\\
&\qquad\left(30+\left(2x_1^{\text{cont}}-3x_2^{\text{cont}}\right)^2\left(18-32x_1^{\text{cont}}+12\left(x_1^{\text{cont}}\right)^2+48x_2^{\text{cont}}-36x_1^{\text{cont}}x_2^{\text{cont}}+27\left(x_2^{\text{cont}}\right)^2\right)\right)
\end{aligned}
$$

and

$$
s_2\left(x_1^{\text{cat}}, x_2^{\text{cat}}, x_1^{\text{int}}, x_2^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}\right)=\begin{cases}
2+\frac{\left(x_1^{\text{cont}}+x_1^{\text{int}}\right)^2+\left(x_2^{\text{cont}}+x_2^{\text{int}}\right)^2}{2} & \text{if } x_1^{\text{cat}}=\text{A and } x_2^{\text{cat}}=\text{A}\\
1.5+\frac{\left(x_1^{\text{cont}}+x_1^{\text{int}}\right)^2+\left|x_2^{\text{cont}}+x_2^{\text{int}}\right|}{4} & \text{if } x_1^{\text{cat}}=\text{A and } x_2^{\text{cat}}=\text{B}\\
1.5+\frac{\left|x_1^{\text{cont}}+x_1^{\text{int}}\right|+\left(x_2^{\text{cont}}+x_2^{\text{int}}\right)^2}{4} & \text{if } x_1^{\text{cat}}=\text{B and } x_2^{\text{cat}}=\text{A}\\
1+\frac{\left|x_1^{\text{cont}}+x_1^{\text{int}}\right|+\left|x_2^{\text{cont}}+x_2^{\text{int}}\right|}{1} & \text{if } x_1^{\text{cat}}=\text{B and } x_2^{\text{cat}}=\text{B}
\end{cases}
$$

and

$$
s_3\left(x_3^{\text{cat}}, x_3^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}\right)=\begin{cases}
\frac{\left|x_3^{\text{int}}+x_2^{\text{cont}}\right|+2}{2} & \text{if } x_3^{\text{cat}}=\text{A}\\
\frac{\left|x_3^{\text{int}}-x_2^{\text{cont}}\right|+2}{2} & \text{if } x_3^{\text{cat}}=\text{B}\\
\frac{\left|-x_3^{\text{int}}+x_2^{\text{cont}}\right|+2}{2} & \text{if } x_3^{\text{cat}}=\text{C}\\
\frac{\left|-x_3^{\text{int}}-x_2^{\text{cont}}\right|+2}{2} & \text{if } x_3^{\text{cat}}=\text{D}
\end{cases}
$$

- $n^{\text{cat}}=3$ and $x_1^{\text{cat}}, x_2^{\text{cat}} \in \{\text{A}, \text{B}\}$ and $x_3^{\text{cat}} \in \{\text{A}, \text{B}, \text{C}, \text{D}\}$;
- $n^{\text{int}}=3$ and $x_i^{\text{int}} \in \{-2,-1,0,1,2\}$ for $i \in I^{\text{int}}$;
- $n^{\text{cont}}=2$ and $x_j^{\text{cont}} \in [-2,2]$ for $j \in I^{\text{cont}}$.

3.8 Cat-8: modified HS-78 [13]

$$
f(x^{\text{cat}}, x^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}, x_4^{\text{cont}}, x_5^{\text{cont}})=\prod_{i=1}^{5} x_i^{\text{cont}}+x^{\text{int}} s\left(x^{\text{cat}}, x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}, x_4^{\text{cont}}, x_5^{\text{cont}}\right)
$$

where

$$
\begin{aligned}
&s\left(x^{\text{cat}}, x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}, x_4^{\text{cont}}, x_5^{\text{cont}}\right)=\\
&\qquad\begin{cases}
\sum_{i=1}^{5}\left(x_i^{\text{cont}}\right)^2-10 & \text{if } x^{\text{cat}}=\text{A}\\
x_2^{\text{cont}}x_3^{\text{cont}}-5x_4^{\text{cont}}x_5^{\text{cont}} & \text{if } x^{\text{cat}}=\text{B}\\
\left(x_1^{\text{cont}}\right)^3+\left(x_2^{\text{cont}}\right)^3+1 & \text{if } x^{\text{cat}}=\text{C}\\
\frac{1}{2}\left(x_2^{\text{cont}}x_3^{\text{cont}}-5x_4^{\text{cont}}x_5^{\text{cont}}+\left(x_1^{\text{cont}}\right)^3+\left(x_2^{\text{cont}}\right)^3+1\right) & \text{if } x^{\text{cat}}=\text{D}
\end{cases}
\end{aligned}
$$

- $n^{\text{cat}}=1$ and $x^{\text{cat}} \in \{\text{A}, \text{B}, \text{C}, \text{D}\}$;
- $n^{\text{int}}=1$ and $x^{\text{int}} \in \{0,1,2,3,4,5\}$;
- $n^{\text{cont}}=5$ and $x_i^{\text{cont}} \in [-2,2]$ for $i \in I^{\text{cont}}$.

3.9 Cat-9: modified Rastragin [12]

$$
f\left(x_1^{\text{cat}}, x_2^{\text{cat}}, x_1^{\text{int}}, x_2^{\text{int}}, \boldsymbol{x}^{\text{cont}}\right)=10s_1\left(x_1^{\text{cat}}, x_1^{\text{int}}, \boldsymbol{x}^{\text{cont}}\right)+\sum_{i=1}^{n^{\text{cont}}}\left(\left(x_i^{\text{cont}}\right)^2-s_2\left(x_2^{\text{cat}}, x_2^{\text{int}}\right)+\cos\left(x_1^{\text{cont}}\right)\right)
$$

where

$$s_1\left(x_1^{\text{cat}}, x_1^{\text{int}}, \boldsymbol{x}^{\text{cont}}\right) = \frac{1}{n^{\text{cont}}} \begin{cases} \sum_{i=1}^{n^{\text{cont}}} \max\left\{0, x_1^{\text{int}} + x_i^{\text{cont}}\right\}^2 & \text{if } x_1^{\text{cat}} = \text{A} \\ \frac{1}{2} \sum_{i=1}^{n^{\text{cont}}} \left|x_1^{\text{int}} + x_i^{\text{cont}}\right| & \text{if } x_1^{\text{cat}} = \text{B} \\ \sum_{i=1}^{n^{\text{cont}}} \max\left\{x_1^{\text{int}} + x_i^{\text{cont}}, 0\right\} \left|x_i^{\text{cont}}\right| & \text{if } x_1^{\text{cat}} = \text{C} \end{cases}$$

and

$$s_2\left(x_2^{\text{cat}}, x_2^{\text{int}}\right) = \begin{cases} 1 + 0.1 x_2^{\text{int}} & \text{if } x_2^{\text{cat}} = \text{A} \\ \frac{1}{1+0.1x_2^{\text{int}}} & \text{if } x_2^{\text{cat}} = \text{B} \\ 1 + 0.05\left(x_2^{\text{int}}\right)^2 & \text{if } x_2^{\text{cat}} = \text{C} \end{cases}$$

- $n^{\text{cat}} = 2$ and $x_1^{\text{cat}}, x_2^{\text{cat}} \in \{\text{A}, \text{B}, \text{C}\}$;
- $n^{\text{int}} = 2$ and $x_1^{\text{int}} \in \{-5, -4, \dots, 4, 5\}$, $x_2^{\text{int}} \in \{-2, -1, 0, 1, 2\}$;
- $n^{\text{cont}} \in \mathbb{N}$ and $x_i^{\text{cont}} \in [-5.12, 5.12]$ for $i \in I^{\text{cont}}$.

3.10 Cat-10: modified Rosenbrock [12]

$$\begin{aligned} f\left(x_1^{\text{cat}}, x_2^{\text{cat}}, x_1^{\text{int}}, x_2^{\text{int}}, \boldsymbol{x}^{\text{cont}}\right) = \left|x_1^{\text{int}}\right| + s\left(x_2^{\text{cat}}, \boldsymbol{x}^{\text{cont}}\right) + & \\ \begin{cases} \sum\limits_{i=1}^{n^{\text{cont}}-1} 100\left(x_{i+1}^{\text{cont}} - \left(x_i^{\text{cont}}\right)^2\right)^2 + x_2^{\text{int}}(x_i^{\text{cont}} - 1)^2 & \text{if } x_1^{\text{cat}} = \text{A} \\ \sum\limits_{i=1}^{n^{\text{cont}}-1} 100\left|x_{i+1}^{\text{cont}} - \left(x_i^{\text{cont}}\right)^2\right| + 5x_2^{\text{int}}\left|x_i^{\text{cont}} - 1\right| & \text{if } x_1^{\text{cat}} = \text{B} \end{cases} & \end{aligned}$$

where

$$s(x_2^{\text{cat}}, \boldsymbol{x}^{\text{cont}}) = \begin{cases} \frac{1}{n^{\text{cont}}} \sum_{i=1}^{n^{\text{cont}}} 1.1 \max\{0, x_i^{\text{cont}}\} & \text{if } x_2^{\text{cat}} = \text{A} \\ \frac{1}{n^{\text{cont}}} \sum_{i=1}^{n^{\text{cont}}} -0.9 \min\{0, x_i^{\text{cont}}\} & \text{if } x_2^{\text{cat}} = \text{B} \\ \frac{1}{n^{\text{cont}}} \sum_{i=1}^{n^{\text{cont}}} \left|x_i^{\text{cont}}\right| & \text{if } x_2^{\text{cat}} = \text{C} \end{cases}$$

- $n^{\text{cat}} = 2$ and $x_1^{\text{cat}} \in \{\text{A}, \text{B}\}$, $x_2^{\text{cat}} \in \{\text{A}, \text{B}, \text{C}\}$;
- $n^{\text{int}} = 2$ and $x_1^{\text{int}} \in \{-2, -1, 0, 1, 2\}$, $x_2^{\text{int}} \in \{-5, -4, \dots, 4, 5\}$;
- $n^{\text{cont}} \in \mathbb{N}$ and $x_i^{\text{cont}} \in [-10, 10]$ for $i \in I^{\text{cont}}$.

3.11 Cat-11: modified Rosen-Suzuki [13]

$$\begin{aligned} f(x^{\text{cat}}, x^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}, x_4^{\text{cont}}) &= s(x^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}, x_4^{\text{cont}}) \\ &+ \begin{cases} 0 & \text{if } x^{\text{cat}} = \text{A} \\ 10\left(\left(x_1^{\text{cont}}\right)^2 + \left(x_2^{\text{cont}}\right)^2 + \left(x_3^{\text{cont}}\right)^2 + \left(x_4^{\text{cont}}\right)^2 + x_1^{\text{cont}} - x_2^{\text{cont}} + x_3^{\text{cont}} - x_4^{\text{cont}} - x^{\text{int}} - 8\right) & \text{if } x^{\text{cat}} = \text{B} \\ 10\left(\left(x_1^{\text{cont}}\right)^2 + 2\left(x_2^{\text{cont}}\right)^2 + \left(x_3^{\text{cont}}\right)^2 + 2\left(x_4^{\text{cont}}\right)^2 - x_1^{\text{cont}} - x_4^{\text{cont}} - 2x^{\text{int}} - 10\right) & \text{if } x^{\text{cat}} = \text{C} \\ 10\left(2\left(x_1^{\text{cont}}\right)^2 + \left(x_2^{\text{cont}}\right)^2 + \left(x_3^{\text{cont}}\right)^2 + 2\left(x_4^{\text{cont}}\right)^2 - x_1^{\text{cont}} - x_2^{\text{cont}} - 3x^{\text{int}} - 5\right) & \text{if } x^{\text{cat}} = \text{D} \end{cases} \end{aligned}$$

where

$$\begin{aligned} s(x^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}, x_4^{\text{cont}}) = \left(x_1^{\text{cont}}\right)^2 + \left(x_2^{\text{cont}}\right)^2 + 2\left(x_3^{\text{cont}}\right)^2 + \left(x_4^{\text{cont}}\right)^2 & \\ - 5x_1^{\text{cont}} - 5x_2^{\text{cont}} - 21x_3^{\text{cont}} + 7x_4^{\text{cont}} + x^{\text{int}} x_3^{\text{cont}} & \end{aligned}$$

- $n^{\text{cat}} = 1$ and $x^{\text{cat}} \in \{\text{A}, \text{B}, \text{C}, \text{D}\}$;
- $n^{\text{int}} = 1$ and $x^{\text{int}} \in \{-5, -4, \dots, 4, 5\}$;
- $n^{\text{cont}} = 4$ and $x_i^{\text{cont}} \in [-5, 5]$ for $i \in I^{\text{cont}}$.

3.12 Cat-12: modified Styblinski-Tang [12]

$$f\left(x^{\mathrm{cat}}, \boldsymbol{x}^{\mathrm{int}}, \boldsymbol{x}^{\mathrm{cont}}\right) = \frac{1}{2}\sum_{i=1}^{n^{\mathrm{cont}}}\left(\left(x_i^{\mathrm{cont}}\right)^4 - 16\left(x_i^{\mathrm{cont}}\right)^2 + 5x_i^{\mathrm{int}} + 8s\left(x^{\mathrm{cat}}, x_i^{\mathrm{cont}}\right)\right)$$

where $n^{\mathrm{cont}} = n^{\mathrm{int}}$ and

$$s\left(x_1^{\mathrm{cat}}, x_i^{\mathrm{cont}}\right) = \begin{cases} \left|x_i^{\mathrm{cont}} - 1\right| & \text{if } x^{\mathrm{cat}} = \mathrm{A} \\ \frac{\left(\left(x_i^{\mathrm{cont}}\right)^2 + 1\right)}{2} & \text{if } x^{\mathrm{cat}} = \mathrm{B} \\ \exp\left(\left|x_i^{\mathrm{cont}} + 1\right|\right) - 1 & \text{if } x^{\mathrm{cat}} = \mathrm{C} \\ \frac{\left(x_i^{\mathrm{cont}}\right)^2}{\left(1 + \left|x_i^{\mathrm{cont}}\right|\right)} & \text{if } x^{\mathrm{cat}} = \mathrm{D} \\ 1 - \exp\left(-\left(x_i^{\mathrm{cont}}\right)^2\right) & \text{if } x^{\mathrm{cat}} = \mathrm{E} \end{cases}$$

- $n^{\mathrm{cat}} = 1$ and $x^{\mathrm{cat}} \in \{\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}, \mathrm{E}\}$;
- $n^{\mathrm{int}} \in \mathbb{N}$ and $x_i^{\mathrm{int}} \in \{-5, -4, \ldots, 9, 10\}$ for $i \in I^{\mathrm{int}}$;
- $n^{\mathrm{cont}} \in \mathbb{N}$ and $x_j^{\mathrm{cont}} \in [-5, 10]$ for $i \in I^{\mathrm{cont}}$.

3.13 Cat-13: modified Toy [14]

$$f\left(x^{\mathrm{cat}}, x_1^{\mathrm{cont}}, x_2^{\mathrm{cont}}, x_3^{\mathrm{cont}}, x_4^{\mathrm{cont}}\right) = 2 + \begin{cases} \cos\left(3.6\pi\left(x_1^{\mathrm{cont}} - 2\right) + x_2^{\mathrm{cont}}\right) + x_3^{\mathrm{cont}} - 1 + \left(x_4^{\mathrm{cont}}\right)^2 & \text{if } x^{\mathrm{cat}} = \mathrm{A}, \\ 2\cos\left(1.1\pi\exp\left(x_1^{\mathrm{cont}}\right)\right) - \frac{x_2^{\mathrm{cont}}}{2} + \left(x_3^{\mathrm{cont}}\right)^2 + 2\log\left(1 + \left(x_4^{\mathrm{cont}}\right)^2\right) & \text{if } x^{\mathrm{cat}} = \mathrm{B}, \\ \cos\left(2\pi x_1^{\mathrm{cont}}\right) + \frac{x_2^{\mathrm{cont}}}{2} + x_3^{\mathrm{cont}} x_4^{\mathrm{cont}} & \text{if } x^{\mathrm{cat}} = \mathrm{C}, \\ x_1\cos\left(3.4\pi\left(x_1^{\mathrm{cont}} - 1\right)\right) - x_2^{\mathrm{cont}} - 1 + x_3^{\mathrm{cont}} + \left(x_4^{\mathrm{cont}}\right)^3 & \text{if } x^{\mathrm{cat}} = \mathrm{D}, \\ -\frac{\left(x_1^{\mathrm{cont}}\right)^2}{2} + \log\left(1 + \left(x_2^{\mathrm{cont}}\right)^2\right) + \left(x_3^{\mathrm{cont}}\right)^2 + x_4^{\mathrm{cont}} & \text{if } x^{\mathrm{cat}} = \mathrm{E}, \\ 2\cos\left(\frac{\pi}{4}\exp\left(-\left(x_1^{\mathrm{cont}}\right)^4\right)\right)^2 - \frac{x_2^{\mathrm{cont}}}{2} + x_3^{\mathrm{cont}} x_4^{\mathrm{cont}} + 1 & \text{if } x^{\mathrm{cat}} = \mathrm{F}, \\ x_1\cos\left(3.4x_1^{\mathrm{cont}}\right) - \frac{x_2^{\mathrm{cont}}}{2} + x_3^{\mathrm{cont}} + \left(x_4^{\mathrm{cont}}\right)^3 + 1 & \text{if } x^{\mathrm{cat}} = \mathrm{G}, \\ x_1^{\mathrm{cont}}\left(-\cos\left(\frac{7}{2\pi}\right)\frac{x_2^{\mathrm{cont}}}{2}\right) + x_3^{\mathrm{cont}} + x_4^{\mathrm{cont}} + 2 & \text{if } x^{\mathrm{cat}} = \mathrm{H}, \\ -\frac{\left(x_1^{\mathrm{cont}}\right)^3}{2} + \left(x_2^{\mathrm{cont}}\right)^2 + x_3^{\mathrm{cont}} x_4^{\mathrm{cont}} + 1 & \text{if } x^{\mathrm{cat}} = \mathrm{I}, \\ -\cos\left(5\pi x_1^{\mathrm{cont}}\right)^2\sqrt{x_1^{\mathrm{cont}}} - \frac{-\log\left(x_2^{\mathrm{cont}} + x_3^{\mathrm{cont}} + 0.5\right)}{2} + \left(x_4^{\mathrm{cont}}\right)^3 - 1.3 & \text{if } x^{\mathrm{cat}} = \mathrm{J}. \end{cases}$$

- $n^{\mathrm{cat}} = 1$ and $x^{\mathrm{cat}} \in \{\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}, \mathrm{E}, \mathrm{F}, \mathrm{G}, \mathrm{H}, \mathrm{I}, \mathrm{J}\}$;
- $n^{\mathrm{int}} = 0$;
- $n^{\mathrm{cont}} = 4$ and $x_i^{\mathrm{cont}} \in [0, 1]$ for $i \in I^{\mathrm{cont}}$.

3.14 Cat-14: modified Toy [14]

$$f(x^{\text{cat}}, x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}, x_4^{\text{cont}}, x_5^{\text{cont}}, x_6^{\text{cont}}, x_7^{\text{cont}}, x_8^{\text{cont}}) = 2 +
\begin{cases}
\cos(3.6\pi(x_1^{\text{cont}} + x_2^{\text{cont}} - 2)) + |x_3^{\text{cont}}| + \lfloor x_4^{\text{cont}} \rfloor - 0.5 & \text{if } x^{\text{cat}} = \text{A}, \\
2\cos(1.1\pi e^{x_1^{\text{cont}} + x_5^{\text{cont}}}) - \frac{|x_2^{\text{cont}} + x_6^{\text{cont}}|}{2} + |x_3^{\text{cont}} - x_4^{\text{cont}}| + 2 & \text{if } x^{\text{cat}} = \text{B}, \\
\cos(2\pi(x_1^{\text{cont}} + x_2^{\text{cont}} + x_3^{\text{cont}})) + \frac{|x_4^{\text{cont}} + x_5^{\text{cont}}|}{2} - \lfloor x_6^{\text{cont}} \rfloor & \text{if } x^{\text{cat}} = \text{C}, \\
x_1^{\text{cont}} x_2^{\text{cont}} \left(\cos(3.4\pi(x_3^{\text{cont}} - 1)) - \frac{|x_4^{\text{cont}} + x_5^{\text{cont}}|}{2} \right) & \text{if } x^{\text{cat}} = \text{D}, \\
-\frac{|x_1^{\text{cont}} x_6^{\text{cont}}|^2}{2} + x_3^{\text{cont}} + |x_4^{\text{cont}} - x_5^{\text{cont}}| & \text{if } x^{\text{cat}} = \text{E}, \\
2\cos^2\left(\frac{\pi}{4} e^{-(x_3^{\text{cont}} x_5^{\text{cont}})^4} \right) - \frac{x_6^{\text{cont}} + x_7^{\text{cont}}}{2} + |x_8^{\text{cont}}| + 1 & \text{if } x^{\text{cat}} = \text{F}, \\
x_2 \cos(3.4\pi(x_4^{\text{cont}} + x_5^{\text{cont}})) - \frac{x_6^{\text{cont}}}{2} + |x_1^{\text{cont}} - x_7^{\text{cont}}| + 0.25 & \text{if } x^{\text{cat}} = \text{G}, \\
x_1^{\text{cont}} x_8^{\text{cont}} \left(-\cos\left(\frac{7\pi}{2}(x_2^{\text{cont}} + x_3^{\text{cont}}) \right) - \frac{x_6^{\text{cont}}}{2} + 2 \right) & \text{if } x^{\text{cat}} = \text{H}, \\
-\frac{|x_1^{\text{cont}} x_2^{\text{cont}} x_3^{\text{cont}}|^3}{2} + |x_4^{\text{cont}}| + |x_5^{\text{cont}}| & \text{if } x^{\text{cat}} = \text{I}, \\
-\cos^2\left(5\pi(x_1^{\text{cont}} x_2^{\text{cont}} + x_3^{\text{cont}}) \right) \sqrt{|x_4^{\text{cont}}|} - \frac{\log(|x_5^{\text{cont}}| + 0.5)}{2} + |x_6^{\text{cont}}| - 0.6 & \text{if } x^{\text{cat}} = \text{J}.
\end{cases}$$

- $n^{\text{cat}} = 1$ and $x^{\text{cat}} \in \{\text{A, B, C, D, E, F, G, H, I, J}\}$;
- $n^{\text{int}} = 0$;
- $n^{\text{cont}} = 8$ and $x_i^{\text{cont}} \in [0, 1]$ for $i \in I^{\text{cont}}$.

3.15 Cat-15: modified Wong-1 [13]

$$f\left(x^{\text{cat}}, x_1^{\text{int}}, x_2^{\text{int}}, x_3^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}, x_4^{\text{cont}}\right) = s(x_1^{\text{int}}, x_2^{\text{int}}, x_3^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}, x_4^{\text{cont}})
+ \begin{cases}
0 & \text{if } x^{\text{cat}} = \text{A} \\
10\left(2\left(x_1^{\text{cont}}\right)^2 + 3\left(x_1^{\text{int}}\right)^4 + x_2^{\text{cont}} + \left(x_2^{\text{int}}\right)^2 + 5x_3^{\text{cont}} - 127\right) & \text{if } x^{\text{cat}} = \text{B} \\
10\left(7x_1^{\text{cont}} + 3x_1^{\text{int}} + 10\left(x_2^{\text{cont}}\right)^3 + x_2^{\text{int}} - x_3^{\text{cont}} - 282\right) & \text{if } x^{\text{cat}} = \text{C} \\
10\left(23x_1^{\text{cont}} + \left(x_1^{\text{int}}\right)^2 + 6x_3^{\text{int}} - 8x_4^{\text{cont}} - 196\right) & \text{if } x^{\text{cat}} = \text{D} \\
10\left(4\left(x_1^{\text{cont}}\right)^2 + \left(x_1^{\text{int}}\right) - 3x_1^{\text{cont}} x_1^{\text{int}} + 2\left(x_2^{\text{cont}}\right)^2 + 5x_3^{\text{int}} - 11x_4^{\text{cont}}\right) & \text{if } x^{\text{cat}} = \text{E}
\end{cases}$$

where

$$s(x_1^{\text{int}}, x_2^{\text{int}}, x_3^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}, x_4^{\text{cont}}) = \left(x_1^{\text{cont}} - 10\right)^2 + 5\left(x_1^{\text{int}} - 12\right)^2 + \left(x_2^{\text{cont}}\right)^4 + 3\left(x_2^{\text{int}} - 11\right)^2
+ 10\left(x_3^{\text{cont}}\right)^6 + 7\left(x_3^{\text{int}}\right)^2 + \left(x_4^{\text{cont}}\right)^4 - 4x_3^{\text{int}} x_4^{\text{cont}} - 10x_3^{\text{int}} - 8x_4^{\text{cont}}$$

- $n^{\text{cat}} = 1$ and $x^{\text{cat}} \in \{\text{A, B, C, D, E}\}$;
- $n^{\text{int}} = 3$ and $x_i^{\text{int}} \in \{-1, 0, 1\}$ for $i \in I^{\text{int}}$;
- $n^{\text{cont}} = 4$ and $x_j^{\text{cont}} \in [-1, 1]$ for $i \in I^{\text{cont}}$.

3.16 Cat-16: modified Zakharov [12]

$$f\left(x_1^{\text{cat}}, x_2^{\text{cat}}, x_1^{\text{int}}, x_2^{\text{int}}, \boldsymbol{x}^{\text{cont}}\right) = 1 + \sum_{i=1}^{n^{\text{cont}}} \left(x_i^{\text{cont}}\right)^2 + \left(\sum_{i=1}^{n^{\text{cont}}} 0.5i \left(x_i^{\text{cont}} + s_1\left(x_1^{\text{cat}}, x_1^{\text{int}}, x_2^{\text{int}}, x_i^{\text{cont}}\right)\right)\right)^2 + \left(\sum_{i=1}^{n^{\text{cont}}} 0.5i \left(x_i^{\text{cont}} + s_2\left(x_2^{\text{cat}}, x_1^{\text{int}}, x_2^{\text{int}}, x_i^{\text{cont}}\right)\right)\right)^4$$

where

$$s_1\left(x_1^{\text{cat}}, x_1^{\text{int}}, x_2^{\text{int}}, x_i^{\text{cont}}\right) = \begin{cases} 0.1\left(x_1^{\text{int}} - x_2^{\text{int}} + x_i^{\text{cont}}\right) & \text{if } x_1^{\text{cat}} = \text{A} \\ 0.05\lfloor x_1^{\text{int}} + x_2^{\text{int}} + x_i^{\text{cont}} \rfloor & \text{if } x_1^{\text{cat}} = \text{B} \\ -0.1\sqrt{x_i^{\text{cont}} + 5}\left(x_1^{\text{int}} + x_2^{\text{int}}\right) & \text{if } x_1^{\text{cat}} = \text{C} \end{cases}$$

and

$$s_2\left(x_2^{\text{cat}}, x_1^{\text{int}}, x_2^{\text{int}}, x_i^{\text{cont}}\right) = \begin{cases} 0.1\left(-x_1^{\text{int}} + x_2^{\text{int}} - x_i^{\text{cont}}\right) & \text{if } x_2^{\text{cat}} = \text{A} \\ 0.05\lfloor -x_1^{\text{int}} - x_2^{\text{int}} - x_i^{\text{cont}} \rfloor & \text{if } x_2^{\text{cat}} = \text{B} \\ 0.1\sqrt{x_i^{\text{cont}} + 5}\left(x_1^{\text{int}} + x_2^{\text{int}}\right) & \text{if } x_2^{\text{cat}} = \text{C} \end{cases}$$

- $n^{\text{cat}} = 2$ and $x_1^{\text{cat}}, x_2^{\text{cat}} \in \{\text{A}, \text{B}, \text{C}\}$;
- $n^{\text{int}} = 2$ and $x_1^{\text{int}}, x_2^{\text{int}} \in \{-3, -2, \ldots, 2, 3\}$;
- $n^{\text{cont}} \in \mathbb{N}$ and $x_i^{\text{cont}} \in [-5, 5]$ for $i \in I^{\text{cont}}$.

4 Description of the constrained problems

The following subsections details the constrained problems.

4.1 Cat-cstrs-1: modified Beale [12]

$$f\left(x_1^{\text{cat}}, x_2^{\text{cat}}, x_1^{\text{int}}, x_2^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}\right) = \left(1.5 - s_1\left(x_1^{\text{cat}}, x_1^{\text{cont}}\right) + x_1^{\text{int}}\left(1 - s_2(x_2^{\text{cat}}, x_2^{\text{cont}})\right)\right)^2 + \left(2.25 - s_1\left(x_1^{\text{cat}}, x_1^{\text{cont}}\right) + x_2^{\text{int}}\left(1 - s_2(x_2^{\text{cat}}, x_2^{\text{cont}})\right)^2\right)^2 + \left(2.625 - s_1\left(x_1^{\text{cat}}, x_1^{\text{cont}}\right) + x_3^{\text{cont}}\left(1 - s_2(x_2^{\text{cat}}, x_2^{\text{cont}})\right)^3\right)^2$$

where

$$s_1(x_1^{\text{cat}}, x_1^{\text{cont}}) = \begin{cases} \lfloor x_1^{\text{cont}} \rfloor & \text{if } x_1^{\text{cat}} = \text{A} \\ x_1^{\text{cont}} & \text{if } x_1^{\text{cat}} = \text{B} \\ \exp\left(\frac{x_1^{\text{cont}}}{2}\right) & \text{if } x_1^{\text{cat}} = \text{C} \end{cases} \quad \text{and} \quad s_2(x_2^{\text{cat}}, x_2^{\text{cont}}) = \begin{cases} \sqrt{|x_2^{\text{cont}}| + 1} & \text{if } x_2^{\text{cat}} = \text{A} \\ |x_2^{\text{cont}}| & \text{if } x_2^{\text{cat}} = \text{B} \\ \left(x_2^{\text{cont}}\right)^2 - 2 & \text{if } x_2^{\text{cat}} = \text{C} \end{cases}$$

with constraints

$$\begin{aligned} g_1\left(x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}\right) &= 3\left(x_1^{\text{cont}} - 2\right)^2 + 4\left(x_2^{\text{cont}} - 3\right)^2 + 2\left(x_3^{\text{cont}}\right) - 100 \leq 0 \\ g_2\left(x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}\right) &= 5\left(x_1^{\text{cont}}\right)^2 + 8x_2^{\text{cont}} + \left(x_3^{\text{cont}} - 6\right)^2 - 30 \leq 0 \\ g_3\left(x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}\right) &= \left(x_1^{\text{cont}}\right)^2 + 2\left(x_2^{\text{cont}} - 2\right)^2 - 2x_1^{\text{cont}}x_2^{\text{cont}} - 6x_3^{\text{cont}} \leq 0 \end{aligned}$$

- $n^{\text{cat}} = 2$ and $x_1^{\text{cat}}, x_2^{\text{cat}} \in \{\text{A}, \text{B}, \text{C}\}$;
- $n^{\text{int}} = 2$ and $x_1^{\text{int}}, x_2^{\text{int}} \in \{-2, -1, 0, 1, 2\}$;
- $n^{\text{cont}} = 3$ and $x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}} \in [-4.5, 4.5]$.

4.2 Cat-cstrs-2: modified Branin [15]

$$f\left(x_1^{\text{cat}}, x_2^{\text{cat}}, x_1^{\text{int}}, x_2^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}\right) = \begin{cases} s\left(x_1^{\text{int}}, x_2^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}\right) & \text{if } x_1^{\text{cat}} = \text{A and } x_2^{\text{cat}} = \text{A} \\ 0.4s\left(x_1^{\text{int}}, x_2^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}\right) & \text{if } x_1^{\text{cat}} = \text{A and } x_2^{\text{cat}} = \text{B} \\ -0.75s\left(x_1^{\text{int}}, x_2^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}\right) & \text{if } x_1^{\text{cat}} = \text{B and } x_2^{\text{cat}} = \text{A} \\ -0.5s\left(x_1^{\text{int}}, x_2^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}\right) & \text{if } x_1^{\text{cat}} = \text{B and } x_2^{\text{cat}} = \text{B} \end{cases}$$

where

$$s\left(x_1^{\text{int}}, x_2^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}\right) =$$
$$\frac{1}{a}\left[\left(\left(15x_2^{\text{cont}} - \frac{x_1^{\text{int}}}{4\pi^2}\left(15x_1^{\text{cont}} - x_2^{\text{int}}\right)^2 + \frac{x_1^{\text{int}}}{\pi}\left(15x_1^{\text{cont}} - x_2^{\text{int}}\right)\right)^2 + \left(x_1^{\text{int}} - \frac{1}{8\pi}\right)\cos\left(15x_1^{\text{cont}} - 5\right) + x_1^{\text{int}}\right) - b\right]$$

with $a = 51.9496$, $b = 54.8104$ and constraint

$$g_1\left(x_1^{\text{cat}}, x_2^{\text{cat}}, x_1^{\text{int}}, x_2^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}\right) = \begin{cases} -x_1^{\text{cont}}x_2^{\text{cont}} + \frac{x_1^{\text{int}}}{10} & \text{if } x_1^{\text{cat}} = 0 \text{ and } x_2^{\text{cat}} = 0 \\ -1.5x_1^{\text{cont}}x_2^{\text{cont}} + \frac{x_2^{\text{int}}}{10} & \text{if } x_1^{\text{cat}} = 0 \text{ and } x_2^{\text{cat}} = 1 \\ -1.5x_1^{\text{cont}}x_2^{\text{cont}} + \frac{x_1^{\text{int}}}{5} & \text{if } x_1^{\text{cat}} = 1 \text{ and } x_2^{\text{cat}} = 0 \\ -1.2x_1^{\text{cont}}x_2^{\text{cont}} + \frac{x_2^{\text{int}}}{7.5} & \text{if } x_1^{\text{cat}} = 1 \text{ and } x_2^{\text{cat}} = 1 \end{cases}$$

- $n^{\text{cat}} = 2$ and $x_1^{\text{cat}}, x_2^{\text{cat}} \in \{\text{A}, \text{B}\}$;
- $n^{\text{int}} = 2$ and $x_1^{\text{int}}, x_2^{\text{int}} \in \{1, 2, 3, 4, 5\}$;
- $n^{\text{cont}} = 2$ and $x_1^{\text{cont}}, x_2^{\text{cont}} \in [0, 1]$.

4.3 Cat-cstrs-3: modified Bukin6 [12]

$$f\left(x_1^{\text{cat}}, x_2^{\text{cat}}, x_1^{\text{int}}, x_2^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}, x_4^{\text{cont}}\right) =$$
$$100\sqrt{\left|s_1\left(x_1^{\text{cat}}, x_2^{\text{cont}}, x_3^{\text{cont}}\right) - 0.01 s_2\left(x_2^{\text{cat}}, x_1^{\text{int}}, x_1^{\text{cont}}, x_4^{\text{cont}}\right)\right|} + 0.01\left|s_2\left(x_2^{\text{cat}}, x_1^{\text{int}}, x_1^{\text{cont}}, x_4^{\text{cont}}\right) + x_2^{\text{int}}\right|$$

where

$$s_1(x_1^{\text{cat}}, x_2^{\text{cont}}, x_3^{\text{cont}}) = \begin{cases} \sqrt{\left|x_2^{\text{cont}} + x_3^{\text{cont}}\right| + 2} & \text{if } x_1^{\text{cat}} = \text{A} \\ \left|x_2^{\text{cont}} + x_3^{\text{cont}}\right| & \text{if } x_1^{\text{cat}} = \text{B} \\ \frac{\left(x_2^{\text{cont}} + x_3^{\text{cont}}\right)^2}{1.25} + 1 & \text{if } x_1^{\text{cat}} = \text{C} \end{cases}$$

and

$$s_2(x_2^{\text{cat}}, x_1^{\text{int}}, x_1^{\text{cont}}, x_4^{\text{cont}}) = \begin{cases} \sqrt{\left|x_1^{\text{int}} + x_1^{\text{cont}} + x_4^{\text{cont}}\right| + 1.5} & \text{if } x_2^{\text{cat}} = \text{A} \\ \left|x_1^{\text{int}} + x_1^{\text{cont}} + x_4^{\text{cont}}\right| & \text{if } x_2^{\text{cat}} = \text{B} \\ \frac{x_1^{\text{int}} + \left(x_1^{\text{cont}} + x_4^{\text{cont}}\right)^2}{1.25} + 1 & \text{if } x_2^{\text{cat}} = \text{C} \end{cases}$$

with constraints

$$g_1(x_1^{\text{cat}}, x_1^{\text{int}}, x_1^{\text{cont}}, x_4^{\text{cont}}) = \begin{cases} 1.5\sin\left(\frac{x_1^{\text{cont}} + x_4^{\text{cont}}}{5}\right) + 0.5x_1^{\text{int}} - 0.1\left(x_1^{\text{cont}}\right)^2 - 2 & \text{if } x_1^{\text{cat}} = \text{A}, \\ 1.5\sin\left(\frac{x_1^{\text{cont}} - x_4^{\text{cont}}}{5}\right) + 0.3x_1^{\text{int}} - 0.1\left(x_4^{\text{cont}}\right)^2 - 2.5 & \text{if } x_1^{\text{cat}} = \text{B}, \\ \exp\left(0.2x_1^{\text{cont}} + 0.1x_4^{\text{cont}}\right) + 0.1x_1^{\text{int}} - 4 & \text{if } x_1^{\text{cat}} = \text{C}. \end{cases}$$

$$g_2(x_2^{\text{cat}}, x_2^{\text{int}}, x_2^{\text{cont}}, x_3^{\text{cont}}) = \begin{cases} 1.5\cos\left(\frac{x_2^{\text{cont}} + x_3^{\text{cont}}}{5}\right) + 0.4x_2^{\text{int}} - 0.2\left(x_2^{\text{cont}}\right)^2 - 1.5 & \text{if } x_2^{\text{cat}} = \text{A}, \\ 1.5\cos\left(\frac{x_2^{\text{cont}} - x_3^{\text{cont}}}{5}\right) + 0.6x_2^{\text{int}} - 0.2\left(x_3^{\text{cont}}\right)^2 - 2 & \text{if } x_2^{\text{cat}} = \text{B}, \\ \log\left(1 + \left(x_2^{\text{cont}} + x_3^{\text{cont}}\right)^2\right) + 0.2x_2^{\text{int}} - 3 & \text{if } x_2^{\text{cat}} = \text{C}. \end{cases}$$

- $n^{\text{cat}} = 2$ and $x_1^{\text{cat}}, x_2^{\text{cat}} \in \{\text{A}, \text{B}, \text{C}\}$;
- $n^{\text{int}} = 2$ and $x_1^{\text{int}}, x_2^{\text{int}} \in \{-5, -4, \dots, 4, 5\}$;
- $n^{\text{cont}} = 4$ and $\left(x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}, x_4^{\text{cont}}\right) \in [-15, 5] \times [-3, 3] \times [-15, 5] \times [-3, 3]$.

4.4 Cat-cstrs-4: modified Dembo-5 [13]

$$\begin{aligned} f\left(x^{\text{cat}}, x_1^{\text{int}}, x_2^{\text{int}}, x_3^{\text{int}}, x_4^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}, x_4^{\text{cont}}\right) &= -\left(x_1^{\text{cont}} + x_2^{\text{cont}} + x_3^{\text{cont}}\right) \\ &+ \begin{cases} 0 & \text{if } x^{\text{cat}} = \text{A} \\ -10^5\left(\frac{833.33252 x_4^{\text{cont}}}{x_1^{\text{cont}} x_2^{\text{int}}} + \frac{100}{x_2^{\text{int}}} - \frac{83333.333}{x_1^{\text{cont}} x_2^{\text{int}}} - 1\right) & \text{if } x^{\text{cat}} = \text{B} \\ -10^5\left(\frac{x_4^{\text{cont}}}{x_3^{\text{int}}} + \frac{1250\left(x_1^{\text{int}} - x_4^{\text{cont}}\right)}{x_2^{\text{cont}} x_3^{\text{int}}} - 1\right) & \text{if } x^{\text{cat}} = \text{C} \\ -10^5\left(\frac{1250000}{x_3^{\text{cont}} x_4^{\text{int}}} + \frac{x_1^{\text{int}}}{x_4^{\text{int}}} - \frac{2500 x_1^{\text{int}}}{x_3^{\text{cont}} x_4^{\text{int}}} - 1\right) & \text{if } x^{\text{cat}} = \text{D} \end{cases} \end{aligned}$$

with constraints

$$\begin{aligned} g_1\left(x_2^{\text{int}}, x_4^{\text{cont}}\right) &= 0.0025\left(x_2^{\text{int}} + x_4^{\text{cont}}\right) - 1 \\ g_2\left(x_1^{\text{int}}, x_3^{\text{int}}, x_4^{\text{cont}}\right) &= 0.0025\left(x_1^{\text{int}}, x_3^{\text{int}}, x_4^{\text{cont}}\right) - 1 \\ g_3\left(x_1^{\text{int}}, x_4^{\text{int}}\right) &= 0.01\left(x_4^{\text{int}} - x_1^{\text{int}}\right) - 1 \end{aligned}$$

- $n^{\text{cat}} = 1$ and $x^{\text{cat}} \in \{\text{A}, \text{B}, \text{C}, \text{D}\}$;
- $n^{\text{int}} = 4$ and $x_1^{\text{int}}, x_2^{\text{int}}, x_3^{\text{int}}, x_4^{\text{int}} \in \{10, 11, \dots, 1000\}$;
- $n^{\text{cont}} = 4$ and $x_1^{\text{cont}} \in [100, 10000]$, and $x_2^{\text{cont}} \in [1000, 10000]$, and $x_3^{\text{cont}} \in [1000, 10000]$, and $x_4^{\text{cont}} \in [10, 1000]$

4.5 Cat-cstrs-5: modified EVD52 [13]

$$f(x^{\text{cat}}, x^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}) = -1 \times \begin{cases} \left(x_1^{\text{cont}}\right)^2 + \left(x_2^{\text{cont}}\right)^2 + \left(x_3^{\text{cont}}\right)^2 - 1 - \frac{x^{\text{int}}}{50} & \text{if } x^{\text{cat}} = \text{A} \\ \left(x_1^{\text{cont}}\right)^2 + \left(x_2^{\text{cont}}\right)^2 + \left(x_3^{\text{cont}} - 2\right)^2 - \frac{x^{\text{int}}}{50} & \text{if } x^{\text{cat}} = \text{B} \\ x_1^{\text{cont}} + x_2^{\text{cont}} + x_3^{\text{cont}} - 1 - \frac{x^{\text{int}}}{50} & \text{if } x^{\text{cat}} = \text{C} \\ x_1^{\text{cont}} + x_2^{\text{cont}} - x_3^{\text{cont}} + 1 + \frac{x^{\text{int}}}{50} & \text{if } x^{\text{cat}} = \text{D} \\ 2\left(x_1^{\text{cont}}\right)^3 + 6\left(x_2^{\text{cont}}\right)^2 + 2\left(5x_3^{\text{cont}} - x_1^{\text{cont}} + 1\right)^2 + \frac{x^{\text{int}}}{50} & \text{if } x^{\text{cat}} = \text{E} \\ \left(x_1^{\text{cont}}\right)^2 - 9x_3^{\text{cont}} + \frac{x^{\text{int}}}{50} & \text{if } x^{\text{cat}} = \text{F} \end{cases}$$

with constraint

$$g(x^{\text{cat}}, x^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}) = \begin{cases} \left|x_1^{\text{cont}} + x_2^{\text{cont}} - 5\right|^{1.5} + \left(x_3^{\text{cont}} + 1\right)^2 + \frac{|x^{\text{int}}|}{10} - 30 & \text{if } x^{\text{cat}} = \text{A}, \\ \left(x_1^{\text{cont}} \cdot x_2^{\text{cont}}\right)^2 + \left|x_3^{\text{cont}} - 1\right| + \frac{|x^{\text{int}} - 5|}{10} - 30 & \text{if } x^{\text{cat}} = \text{B}, \\ \left|x_1^{\text{cont}} - x_2^{\text{cont}}\right|^3 + \left(x_3^{\text{cont}}\right)^2 + \frac{|x^{\text{int}} + 5|}{10} - 30 & \text{if } x^{\text{cat}} = \text{C}, \\ \left(x_1^{\text{cont}} + x_2^{\text{cont}}\right)^2 + \left|x_3^{\text{cont}} - 0.5\right|^{1.5} + \frac{|x^{\text{int}} - 10|}{10} - 30 & \text{if } x^{\text{cat}} = \text{D}, \\ \left(x_1^{\text{cont}} - 2x_2^{\text{cont}}\right)^2 + \left(x_3^{\text{cont}} - 1.5\right) + \frac{|x^{\text{int}} + 10|}{10} - 30 & \text{if } x^{\text{cat}} = \text{E}, \\ \left(x_1^{\text{cont}} \cdot x_3^{\text{cont}}\right)^2 + \left|x_2^{\text{cont}} + 1.5\right| + \frac{|x^{\text{int}} - 15|}{10} - 30 & \text{if } x^{\text{cat}} = \text{F}. \end{cases}$$

- $n^{\text{cat}} = 1$ and $x^{\text{cat}} \in \{\text{A}, \text{B}, \text{C}, \text{D}, \text{E}, \text{F}\}$;
- $n^{\text{int}} = 1$ and $x^{\text{int}} \in \{-25, -24, \dots, 24, 25\}$;
- $n^{\text{cont}} = 3$ and $x_i^{\text{cont}} \in [-25, 25]$ for $i \in I^{\text{cont}}$.

4.6 Cat-cstrs-6: modified G-09 [6]

$$f\left(x_1^{\text{cat}}, x_2^{\text{cat}}, x_1^{\text{int}}, x_2^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}\right) = \left(x_1^{\text{cont}} - 10\right)^2 + 5\left(x_2^{\text{cont}} - 12\right)^2 + \left(x_3^{\text{cont}}\right)^4 + 3\left(x_1^{\text{int}} - 11\right)^2 + 10\left(x_2^{\text{int}}\right)^6$$
$$+ 7\left(s_1\left(x_1^{\text{cat}}, x_2^{\text{cat}}\right)\right)^2 + \left(s_2\left(x_1^{\text{cat}}, x_2^{\text{cat}}\right)\right)^2 - 4s_1\left(x_1^{\text{cat}}, x_2^{\text{cat}}\right)s_2\left(x_1^{\text{cat}}, x_2^{\text{cat}}\right) - 10s_1\left(x_1^{\text{cat}}, x_2^{\text{cat}}\right) - 8s_2\left(x_1^{\text{cat}}, x_2^{\text{cat}}\right)$$

where

Table 3: Images of $s_1\left(x_1^{\text{cat}}, x_2^{\text{cat}}\right)$.

	$x_1^{\text{cat}} = \text{A}$	$x_1^{\text{cat}} = \text{B}$	$x_1^{\text{cat}} = \text{C}$
$x_2^{\text{cat}} = \text{A}$	-5	2.5	7.5
$x_2^{\text{cat}} = \text{B}$	-5	3.5	8.5
$x_2^{\text{cat}} = \text{C}$	-4	4.5	10

Table 4: Images of $s_2\left(x_1^{\text{cat}}, x_2^{\text{cat}}\right)$.

	$x_1^{\text{cat}} = \text{A}$	$x_1^{\text{cat}} = \text{B}$	$x_1^{\text{cat}} = \text{C}$
$x_2^{\text{cat}} = \text{A}$	-5	-5	-4
$x_2^{\text{cat}} = \text{B}$	2.5	4.5	3
$x_2^{\text{cat}} = \text{C}$	1	6.5	9

with constraints

$$g_1\left(x_1^{\text{int}}, x_2^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}\right) = \frac{2\left(x_1^{\text{cont}}\right)^2 + 3\left(x_2^{\text{cont}}\right)^4 + x_3^{\text{cont}} + 4\left(x_1^{\text{int}}\right)^2 + 5x_2^{\text{int}}}{2} - 127$$
$$g_2\left(x_1^{\text{int}}, x_2^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}\right) = \frac{7x_1^{\text{cont}} + 3x_2^{\text{cont}} + 10\left(x_3^{\text{cont}}\right)^2 + x_1^{\text{int}} - x_2^{\text{int}}}{2} - 282$$
$$g_3\left(x_1^{\text{cat}}, x_2^{\text{cat}}, x_1^{\text{cont}}, x_2^{\text{cont}}\right) = 23x_1^{\text{cont}} + \left(x_2^{\text{cont}}\right)^2 + \left(s_1\left(x_1^{\text{cat}}, x_2^{\text{cat}}\right)\right) - 8s_2\left(x_1^{\text{cat}}, x_2^{\text{cat}}\right) - 196$$
$$g_4\left(x_1^{\text{cat}}, x_2^{\text{cat}}, x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}\right) = 4\left(x_1^{\text{cont}}\right)^2 + \left(x_2^{\text{cont}}\right)^2 - 3x_1^{\text{cont}}x_2^{\text{cont}} + 2\left(x_3^{\text{cont}}\right)^2 + 5s_1\left(x_1^{\text{cat}}, x_2^{\text{cat}}\right) - 11s_2\left(x_1^{\text{cat}}, x_2^{\text{cat}}\right)$$

- $n^{\text{cat}} = 2$ and $x_1^{\text{cat}}, x_2^{\text{cat}} \in \{\text{A}, \text{B}, \text{C}\}$;
- $n^{\text{int}} = 2$ and $x_1^{\text{int}}, x_2^{\text{int}} \in \{-10, -9, \ldots, 10\}$;
- $n^{\text{cont}} = 3$ and $x_1^{\text{cont}}, x_2^{\text{cont}} \in [-10, 10]$.

4.7 Cat-cstrs-7: Goldstein taken directly from [15]

$$\begin{aligned}
f\left(x_1^{\text{cat}}, x_2^{\text{cat}}, x_1^{\text{cont}}, x_2^{\text{cont}}\right) = \\
& 53.3108 + 0.184901x_1^{\text{cont}} \\
& - 5.02914\left(x_1^{\text{cont}}\right)^3 \times 10^{-6} + 7.72522\left(x_1^{\text{cont}}\right)^4 \times 10^{-8} \\
& - 0.0870775x_2^{\text{cont}} - 0.106959s_1(x_1^{\text{cat}}) \\
& + 7.98772\left(s_1(x_1^{\text{cat}})\right)^3 \times 10^{-6} + 0.00242482s_2(x_2^{\text{cat}}) \\
& + 1.32851\left(s_2\left(x_2^{\text{cat}}\right)\right)^3 \times 10^{-6} - 0.00146393x_1^{\text{cont}}x_2^{\text{cont}} \\
& - 0.00301588x_1^{\text{cont}}s_1(x_1^{\text{cat}}) - 0.00272291x_1^{\text{cont}}s_2(x_2^{\text{cat}}) \\
& + 0.0017004x_2^{\text{cont}}s_1(x_1^{\text{cat}}) + 0.0038428x_2^{\text{cont}}s_2(x_2^{\text{cat}}) \\
& - 0.000198969s_1(x_1^{\text{cat}})s_2(x_2^{\text{cat}}) \\
& + 1.86025x_1^{\text{cont}}x_2^{\text{cont}}s_1(x_1^{\text{cat}}) \times 10^{-5} \\
& - 1.88719x_1^{\text{cont}}x_2^{\text{cont}}s_2(x_2^{\text{cat}}) \times 10^{-6} \\
& + 2.50923x_1^{\text{cont}}s_1(x_1^{\text{cat}})s_2(x_2^{\text{cat}}) \times 10^{-5} \\
& - 5.62199x_2^{\text{cont}}s_1(x_1^{\text{cat}})s_2(x_2^{\text{cat}}) \times 10^{-5}
\end{aligned}$$

where

$$s_1\left(x_1^{\text{cat}}\right) = \begin{cases} 20 & \text{if } x_1^{\text{cat}} = \text{A}, \\ 50 & \text{if } x_1^{\text{cat}} = \text{B}, \\ 80 & \text{if } x_1^{\text{cat}} = \text{C}, \end{cases} \quad \text{and} \quad s_2\left(x_2^{\text{cat}}\right) = \begin{cases} 20 & \text{if } x_2^{\text{cat}} = \text{A}, \\ 50 & \text{if } x_2^{\text{cat}} = \text{B}, \\ 80 & \text{if } x_2^{\text{cat}} = \text{C}. \end{cases}$$

with constraints

$$g_1\left(x_1^{\text{cat}}, x_2^{\text{cat}}, x_1^{\text{cont}}, x_2^{\text{cont}}\right) = s_1(x_1^{\text{cat}}) \sin\left(\frac{x_1^{\text{cont}}}{100}\right)^3 + s_2(x_2^{\text{cat}}) \sin\left(\frac{x_2^{\text{cont}}}{10}\right)^3$$

- $n^{\text{cat}} = 2$ and $x_1^{\text{cat}}, x_2^{\text{cat}} \in \{\text{A}, \text{B}, \text{C}\}$;
- $n^{\text{int}} = 0$;
- $n^{\text{cont}} = 2$ and $x_1^{\text{cont}}, x_2^{\text{cont}} \in [0, 100]$.

4.8 Cat-cstrs-8: modified Himmelblau [12]

$$f\left(x_1^{\text{int}}, x_2^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}\right) = \left(\left(x_1^{\text{cont}}\right)^2 + x_2^{\text{cont}} - 6 - x_1^{\text{int}}\right)^2 + \left(x_1^{\text{cont}} + \left(x_2^{\text{cont}}\right)^2 - 2 - x_2^{\text{int}}\right)^2 + 10$$

with constraints

$$\begin{aligned} g_1\left(x_1^{\text{cat}}, x_1^{\text{cont}}, x_2^{\text{cont}}\right) &= -s_1\left(x_1^{\text{cat}}\right) x_1^{\text{cont}} - x_2^{\text{cont}} + 0.5 \\ g_2\left(x_2^{\text{cat}}, x_1^{\text{cont}}, x_2^{\text{cont}}\right) &= 3x_1^{\text{cont}} - s_2\left(x_2^{\text{cat}}\right) x_2^{\text{cont}} + 1 \end{aligned}$$

where

$$s_1\left(x_1^{\text{cat}}\right) = \begin{cases} 0.75 & \text{if } x_1^{\text{cat}} = \text{A} \\ 1.5 & \text{if } x_1^{\text{cat}} = \text{B} \\ 2.25 & \text{if } x_1^{\text{cat}} = \text{C} \\ 3 & \text{if } x_1^{\text{cat}} = \text{D} \\ 3.75 & \text{if } x_1^{\text{cat}} = \text{E} \end{cases} \qquad \text{and} \qquad s_2\left(x_2^{\text{cat}}\right) = \begin{cases} -1.25 & \text{if } x_2^{\text{cat}} = \text{A} \\ -0.5 & \text{if } x_2^{\text{cat}} = \text{B} \\ 0.25 & \text{if } x_2^{\text{cat}} = \text{C} \\ 1 & \text{if } x_2^{\text{cat}} = \text{D} \\ 1.75 & \text{if } x_2^{\text{cat}} = \text{E} \end{cases}$$

- $n^{\text{cat}} = 2$ and $x_1^{\text{cat}}, x_2^{\text{cat}} \in \{\text{A}, \text{B}, \text{C}, \text{D}, \text{E}\}$;
- $n^{\text{int}} = 2$ and $x_1^{\text{int}}, x_2^{\text{int}} \in \{0, 1, \ldots, 5\}$
- $n^{\text{cont}} = 2$ and $x_1^{\text{cont}}, x_2^{\text{cont}} \in [-5, 5]$.

4.9 Cat-cstrs-9: modified HS144 [13]

$$\begin{aligned} f&\left(x_1^{\text{cat}}, x_2^{\text{cat}}, x_1^{\text{int}}, x_2^{\text{int}}, x_3^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}, x_4^{\text{cont}}, x_5^{\text{cont}}\right) = \\ &- 5.04x_1^{\text{cont}} - 0.035x_2^{\text{cont}} - 10x_3^{\text{cont}} - 3.36x_5^{\text{cont}} + 0.063x_2^{\text{int}} x_4^{\text{cont}} \\ &+ s_1\left(x_1^{\text{cat}}, x_3^{\text{int}}, x_1^{\text{cont}}, x_4^{\text{cont}}\right) + s_2\left(x_2^{\text{cat}}, x_1^{\text{int}}, x_2^{\text{int}}, x_3^{\text{int}}\right) \end{aligned}$$

where

$$s_1\left(x_1^{\text{cat}}, x_3^{\text{int}}, x_1^{\text{cont}}, x_4^{\text{cont}}\right) = \begin{cases} -500\left(1.12x_1^{\text{cont}} + 0.13167x_3^{\text{int}} x_1^{\text{cont}} - 0.00667\left(x_3^{\text{int}}\right)^2 x_1^{\text{cont}} - \dfrac{1}{0.99} x_4^{\text{cont}}\right) & \text{if } x_1^{\text{cat}} = \text{A} \\ 500\left(1.12x_1^{\text{cont}} + 0.13167x_3^{\text{int}} x_1^{\text{cont}} - 0.00667\left(x_3^{\text{int}}\right)^2 x_1^{\text{cont}} - \dfrac{1}{0.99} x_4^{\text{cont}}\right) & \text{if } x_1^{\text{cat}} = \text{B} \end{cases}$$

and

$$s_2\left(x_2^{\text{cat}}, x_1^{\text{int}}, x_2^{\text{int}}, x_3^{\text{int}}\right) = \begin{cases} -500\left(1.098x_3^{\text{int}} - 0.038\left(x_3^{\text{int}}\right)^2 + 0.325x_1^{\text{int}} - \dfrac{1}{0.99} x_2^{\text{int}} + 57.425\right) & \text{if } x_2^{\text{cat}} = \text{A} \\ 500\left(1.098x_3^{\text{int}} - 0.038\left(x_3^{\text{int}}\right)^2 + 0.325x_1^{\text{int}} - \dfrac{1}{0.99} x_2^{\text{int}} + 57.425\right) & \text{if } x_2^{\text{cat}} = \text{B} \end{cases}$$

with constraints

$$\begin{aligned} g_1\left(x_2^{\text{cont}}, x_5^{\text{cont}}\right) &= 0.02x_2^{\text{cont}} + 0.1x_5^{\text{cont}} - 100 \\ g_2\left(x_2^{\text{cont}}, x_5^{\text{cont}}\right) &= -0.1x_2^{\text{cont}} - 0.5x_5^{\text{cont}} + 100 \\ g_3\left(x_1^{\text{cont}}, x_4^{\text{cont}}\right) &= -0.5x_4^{\text{cont}} + 0.1x_1^{\text{cont}} + 500 \\ g_4\left(x_1^{\text{cont}}, x_4^{\text{cont}}\right) &= 0.2x_4^{\text{cont}} - 0.2x_1^{\text{cont}} - 500 \end{aligned}$$

- $n^{\text{cat}} = 2$ and $x_1^{\text{cat}}, x_2^{\text{cat}} \in \{\text{A}, \text{B}\}$;
- $n^{\text{int}} = 3$ and $x_1^{\text{int}} \in \{83, 84, \ldots, 93\}$, and $x_2^{\text{int}} \in \{90, 91, \ldots, 95\}$, and $x_3^{\text{int}} \in \{3, 4, \ldots, 12\}$.
- $n^{\text{cont}} = 5$ and $x_1^{\text{cont}}, x_5^{\text{cont}} \in [10^{-5}, 2000]$. and $x_2^{\text{cont}} \in [10^{-5}, 16000]$, and $x_3^{\text{cont}} \in [10^{-5}, 120]$ and $x_4^{\text{cont}} \in [10^{-5}, 5000]$.

4.10 Cat-cstrs-10: modified Pentagon [13]

$$f\left(x^{\text{cat}}, x_1^{\text{int}}, x_2^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}, x_4^{\text{cont}}\right) = \begin{cases} \sqrt{(x_1^{\text{cont}} - x_1^{\text{int}})^2 + (x_2^{\text{cont}} - x_2^{\text{int}})^2} & \text{if } x^{\text{cat}} = \text{A} \\ \sqrt{(x_1^{\text{int}} - x_3^{\text{cont}})^2 + (x_2^{\text{int}} - x_4^{\text{cont}})^2} & \text{if } x^{\text{cat}} = \text{B} \\ \sqrt{(x_3^{\text{cont}} - x_1^{\text{cont}})^2 + (x_4^{\text{cont}} - x_2^{\text{cont}})^2} & \text{if } x^{\text{cat}} = \text{C} \end{cases}$$

with constraints

$$\begin{aligned}
g_1\left(x_1^{\text{int}}, x_2^{\text{int}}\right) &= x_1^{\text{int}} \cos\left(\frac{2\pi}{5}\right) + x_2^{\text{int}} \sin\left(\frac{2\pi}{5}\right) - 1 \\
g_2\left(x_1^{\text{int}}, x_2^{\text{int}}\right) &= x_1^{\text{int}} \cos\left(\frac{8\pi}{5}\right) + x_2^{\text{int}} \sin\left(\frac{8\pi}{5}\right) - 1 \\
g_3\left(x_1^{\text{cont}}, x_2^{\text{cont}}\right) &= x_1^{\text{cont}} \cos\left(\frac{2\pi}{5}\right) + x_2^{\text{cont}} \sin\left(\frac{2\pi}{5}\right) - 1 \\
g_4\left(x_1^{\text{cont}}, x_2^{\text{cont}}\right) &= x_1^{\text{cont}} \cos\left(\frac{8\pi}{5}\right) + x_2^{\text{cont}} \sin\left(\frac{8\pi}{5}\right) - 1 \\
g_5\left(x_3^{\text{cont}}, x_4^{\text{cont}}\right) &= x_3^{\text{cont}} \cos\left(\frac{2\pi}{5}\right) + x_4^{\text{cont}} \sin\left(\frac{2\pi}{5}\right) - 1 \\
g_6\left(x_3^{\text{cont}}, x_4^{\text{cont}}\right) &= x_3^{\text{cont}} \cos\left(\frac{8\pi}{5}\right) + x_4^{\text{cont}} \sin\left(\frac{8\pi}{5}\right) - 1
\end{aligned}$$

- $n^{\text{cat}} = 1$ and $x^{\text{cat}} \in \{\text{A}, \text{B}, \text{C}\}$;
- $n^{\text{int}} = 2$ and $x_1^{\text{int}}, x_2^{\text{int}} \in \{-3, -2, \ldots, 3\}$
- $n^{\text{cont}} = 4$ and $x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}, x_4^{\text{cont}} \in [-\pi, \pi]$.

4.11 Cat-cstrs-11: modified Pressure-vessel [6]

$$\begin{aligned}
f\left(x^{\text{cat}}, x_1^{\text{int}}, x_2^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}\right) &= 0.6224 x_1^{\text{int}} x_1^{\text{cont}} s\left(x^{\text{cat}}, x_2^{\text{cont}}\right) + \\
&\quad 1.7781 x_2^{\text{int}} \left(s\left(x^{\text{cat}}, x_2^{\text{cont}}\right)\right)^2 + 3.1661 \left(x_1^{\text{int}}\right)^2 x_1^{\text{cont}} + 19.84 \left(x_1^{\text{int}}\right)^2 s\left(x^{\text{cat}}, x_2^{\text{cont}}\right)
\end{aligned}$$

where

$$s\left(x_1^{\text{cat}}, x_2^{\text{cont}}\right) = \begin{cases}
20 + 0.8 x_2^{\text{cont}} + 15 \sin\left(0.02 x_2^{\text{cont}}\right) & \text{if } x^{\text{cat}} = \text{A} \\
50 + 0.85 x_2^{\text{cont}} + 18 \sin\left(0.025 x_2^{\text{cont}} + 0.3\right) & \text{if } x^{\text{cat}} = \text{B} \\
10 + 0.75 x_2^{\text{cont}} + 12 \sin\left(0.018 x_2^{\text{cont}} - 0.4\right) & \text{if } x^{\text{cat}} = \text{C} \\
40 + 0.82 x_2^{\text{cont}} + 20 \sin\left(0.022 x_2^{\text{cont}} + 0.5\right) & \text{if } x^{\text{cat}} = \text{D} \\
200 - 0.8 x_2^{\text{cont}} - 15 \sin\left(0.02 x_2^{\text{cont}}\right) & \text{if } x^{\text{cat}} = \text{E} \\
160 - 0.85 x_2^{\text{cont}} - 18 \sin\left(0.025 x_2^{\text{cont}} + 0.3\right) & \text{if } x^{\text{cat}} = \text{F} \\
220 - 0.75 x_2^{\text{cont}} - 12 \sin\left(0.018 x_2^{\text{cont}} - 0.4\right) & \text{if } x^{\text{cat}} = \text{G} \\
180 - 0.82 x_2^{\text{cont}} - 20 \sin\left(0.022 x_2^{\text{cont}} + 0.5\right) & \text{if } x^{\text{cat}} = \text{H}
\end{cases}$$

with constraints

$$\begin{aligned}
g_1\left(x^{\text{cat}}, x_1^{\text{int}}, x_2^{\text{cont}}\right) &= -x_1^{\text{int}} + 0.0193 s\left(x^{\text{cat}}, x_2^{\text{cont}}\right) \\
g_2\left(x^{\text{cat}}, x_2^{\text{int}}, x_2^{\text{cont}}\right) &= -x_2^{\text{int}} + 0.00954 s\left(x^{\text{cat}}, x_2^{\text{cont}}\right) \\
g_3\left(x^{\text{cat}}, x_1^{\text{cont}}, x_2^{\text{cont}}\right) &= -\pi x_1^{\text{cont}} \left(s\left(x^{\text{cat}}, x_2^{\text{cont}}\right)\right)^2 - \frac{4\pi}{3} \left(s\left(x^{\text{cat}}, x_2^{\text{cont}}\right)\right)^3 + 1296000
\end{aligned}$$

- $n^{\text{cat}} = 1$ and $x^{\text{cat}} \in \{\text{A}, \text{B}, \text{C}, \text{D}, \text{E}, \text{F}, \text{G}, \text{H}\}$;
- $n^{\text{int}} = 2$ and $x_1^{\text{int}} = 0.0625a_1,\ x_2^{\text{int}} = 0.0625a_2$ with $a_1, a_2 \in \{1, 2, \dots, 99\}$
- $n^{\text{cont}} = 2$ and $x_1^{\text{cont}}, x_2^{\text{cont}} \in [10, 200]$.

4.12 Cat-cstrs-12: modified Reinforced-concrete-beam [6]

$$f\left(x_1^{\text{cat}}, x_2^{\text{cat}}, x^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}\right) = 29.4s\left(x_1^{\text{cat}}, x_2^{\text{cat}}\right) + 0.6x^{\text{int}}\left(x_1^{\text{cont}} + x_2^{\text{cont}}\right)$$

where

Table 5: Images of $s\left(x_1^{\text{cat}}, x_2^{\text{cat}}\right)$.

	$x_1^{\text{cat}} = \text{A}$	$x_1^{\text{cat}} = \text{B}$	$x_1^{\text{cat}} = \text{C}$	$x_1^{\text{cat}} = \text{D}$	$x_1^{\text{cat}} = \text{E}$
$x_2^{\text{cat}} = \text{A}$	0.2	0.52	0.83	1.13	1.45
$x_2^{\text{cat}} = \text{B}$	0.27	0.58	0.87	1.19	1.49
$x_2^{\text{cat}} = \text{C}$	0.33	0.63	0.91	1.24	1.54
$x_2^{\text{cat}} = \text{D}$	0.38	0.68	0.96	1.3	1.62
$x_2^{\text{cat}} = \text{E}$	0.42	0.73	1.01	1.35	1.66

with (modified) constraints

$$g_1\left(x_1^{\text{cat}}, x_2^{\text{cat}}, x^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}\right) = 5\left(x_1^{\text{cont}} + x_2^{\text{cont}}\right) - 4x^{\text{int}} + 2s\left(x_1^{\text{cat}}, x_2^{\text{cat}}\right) - 2.5$$

$$g_2\left(x_1^{\text{cat}}, x_2^{\text{cat}}, x^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}\right) = 2.5\left(s\left(x_1^{\text{cat}}, x_2^{\text{cat}}\right)\right)^2 + 25x^{\text{int}} - s\left(x_1^{\text{cat}}, x_2^{\text{cat}}\right)x^{\text{int}}\left(x_1^{\text{cont}} + x_2^{\text{cont}}\right) - 2.5$$

- $n^{\text{cat}} = 2$ and $x_1^{\text{cat}}, x_2^{\text{cat}} \in \{\text{A}, \text{B}, \text{C}, \text{D}, \text{E}\}$;
- $n^{\text{int}} = 1$ and $x^{\text{int}} \in \{28, 29, \dots, 40\}$;
- $n^{\text{cont}} = 2$ and $x_1^{\text{cont}}, x_2^{\text{cont}} \in [5, 10]$.

4.13 Cat-cstrs-13: modified Rosenbrock [12]

$$f\left(x_1^{\text{cat}}, x_2^{\text{cat}}, x_1^{\text{int}}, x_2^{\text{int}}, \boldsymbol{x}^{\text{cont}}\right) = \left|x_1^{\text{int}}\right| + s\left(x_2^{\text{cat}}, \boldsymbol{x}^{\text{cont}}\right) + \begin{cases} \displaystyle\sum_{i=1}^{n^{\text{cont}}-1} 100\left(x_{i+1}^{\text{cont}} - \left(x_i^{\text{cont}}\right)^2\right)^2 + x_2^{\text{int}}(x_i^{\text{cont}} - 1)^2 & \text{if } x_1^{\text{cat}} = \text{A} \\ \displaystyle\sum_{i=1}^{n^{\text{cont}}-1} 100\left|x_{i+1}^{\text{cont}} - \left(x_i^{\text{cont}}\right)^2\right| + 5x_2^{\text{int}}\left|x_i^{\text{cont}} - 1\right| & \text{if } x_1^{\text{cat}} = \text{B} \end{cases}$$

where

$$s(x_2^{\text{cat}}, \boldsymbol{x}^{\text{cont}}) = \begin{cases} \frac{1}{n^{\text{cont}}} \sum_{i=1}^{n^{\text{cont}}} 1.1\max\{0, x_i^{\text{cont}}\} & \text{if } x_2^{\text{cat}} = \text{A} \\ \frac{1}{n^{\text{cont}}} \sum_{i=1}^{n^{\text{cont}}} -0.9\min\{0, x_i^{\text{cont}}\} & \text{if } x_2^{\text{cat}} = \text{B} \\ \frac{1}{n^{\text{cont}}} \sum_{i=1}^{n^{\text{cont}}} \left|x_i^{\text{cont}}\right| & \text{if } x_2^{\text{cat}} = \text{C} \end{cases}$$

with constraint

$$g_1\left(x_2^{\text{cat}}, x_1^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}, x_4^{\text{cont}}\right) = -\sqrt{(x_1^{\text{cont}})^2 + (x_2^{\text{cont}})^2 + (x_3^{\text{cont}})^2 + (x_4^{\text{cont}})^2} + \left(\frac{x_1^{\text{int}}}{2}\right)^2 + \begin{cases} 4.25^2 & \text{if } x_2^{\text{cat}} = \text{A} \\ 5.5^2 & \text{if } x_2^{\text{cat}} = \text{B} \\ 8^2 & \text{if } x_2^{\text{cat}} = \text{C} \end{cases}$$

- $n^{\text{cat}} = 2$ and $x_1^{\text{cat}} \in \{\text{A}, \text{B}\},\ x_2^{\text{cat}} \in \{\text{A}, \text{B}, \text{C}\}$;
- $n^{\text{int}} = 2$ and $x_1^{\text{int}} \in \{-2, -1, 0, 1, 2\},\ x_2^{\text{int}} \in \{-5, -4, \dots, 4, 5\}$;
- $n^{\text{cont}} \in \mathbb{N}$ and $x_i^{\text{cont}} \in [-10, 10]$ for $i \in I^{\text{cont}}$.

4.14 Cat-cstrs-14: modified Styblinski-Tang [12]

$$f\left(x^{\text{cat}}, \boldsymbol{x}^{\text{int}}, \boldsymbol{x}^{\text{cont}}\right) = \frac{1}{2}\sum_{i=1}^{n^{\text{cont}}} \left(\left(x_i^{\text{cont}}\right)^4 - 16\left(x_i^{\text{cont}}\right)^2 + 5x_i^{\text{int}} + 8s\left(x^{\text{cat}}, x_i^{\text{cont}}\right)\right)$$

where $n^{\text{cont}} = n^{\text{int}}$ and

$$s\left(x^{\text{cat}}, x_i^{\text{cont}}\right) = \begin{cases} \left|x_i^{\text{cont}} - 1\right| & \text{if } x^{\text{cat}} = \text{A} \\ \frac{\left(\left(x_i^{\text{cont}}\right)^2 + 1\right)}{2} & \text{if } x^{\text{cat}} = \text{B} \\ \exp\left(\left|x_i^{\text{cont}} + 1\right|\right) - 1 & \text{if } x^{\text{cat}} = \text{C} \\ \frac{\left(x_i^{\text{cont}}\right)^2}{\left(1 + \left|x_i^{\text{cont}}\right|\right)} & \text{if } x^{\text{cat}} = \text{D} \\ 1 - \exp\left(-\left(x_i^{\text{cont}}\right)^2\right) & \text{if } x^{\text{cat}} = \text{E} \end{cases}$$

with constraints

$$g_1\left(x^{\text{cat}}, x_1^{\text{cont}}, x_2^{\text{cont}}\right) = \begin{cases} \exp\left(x_1^{\text{cont}} + x_2^{\text{cont}}\right) - 10 & \text{if } x^{\text{cat}} = \text{A} \\ \exp\left(x_1^{\text{cont}} + 2x_2^{\text{cont}}\right) - 18 & \text{if } x^{\text{cat}} = \text{B} \\ \exp\left(2x_1^{\text{cont}} + x_2^{\text{cont}}\right) - 18 & \text{if } x^{\text{cat}} = \text{C} \\ \exp\left(x_1^{\text{cont}} - x_2^{\text{cont}}\right) + \log\left(1 + \left|x_2^{\text{cont}}\right|\right) - 12 & \text{if } x^{\text{cat}} = \text{D} \\ \exp\left(x_1^{\text{cont}} - 0.5x_2^{\text{cont}}\right) + \log\left(1 + \left|x_2^{\text{cont}}\right|\right) - 11 & \text{if } x^{\text{cat}} = \text{E} \end{cases}$$

$$g_2\left(x^{\text{cat}}, x_1^{\text{cont}}, x_2^{\text{cont}}\right) = \begin{cases} \left(x_1^{\text{cont}} + 2\right)^3 + \left(x_2^{\text{cont}} - 1\right)^2 + 0.1\left(x_1^{\text{int}} - 1\right)^2 - 50 & \text{if } x^{\text{cat}} = \text{A} \\ \left(x_1^{\text{cont}} - 1\right)^3 + \left(x_2^{\text{cont}} + 2\right)^2 + 0.1\left(x_1^{\text{int}} + 2\right)^2 - 40 & \text{if } x^{\text{cat}} = \text{B} \\ \left(x_1^{\text{cont}}\right)^3 + \left(x_2^{\text{cont}} - 2\right)^2 + 0.1\left(x_1^{\text{int}} - 3\right)^2 - 45 & \text{if } x^{\text{cat}} = \text{C} \\ \sin\left(\frac{x_1^{\text{cont}} + x_2^{\text{cont}}}{10}\right) + \left(x_1^{\text{cont}}\right)^2 + 0.2\left|x_1^{\text{int}} - 1\right| - 3 & \text{if } x^{\text{cat}} = \text{D} \\ \sin\left(\frac{x_1^{\text{cont}} + x_2^{\text{cont}}}{12.5}\right) + \left(x_1^{\text{cont}}\right)^2 + 0.2\left|x_1^{\text{int}} - 2\right| - 4 & \text{if } x^{\text{cat}} = \text{E} \end{cases}$$

- $n^{\text{cat}} = 1$ and $x^{\text{cat}} \in \{\text{A}, \text{B}, \text{C}, \text{D}, \text{E}\}$
- $n^{\text{int}} \in \mathbb{N}$ and $x_i^{\text{int}} \in \{-5, 10\}$ for $i \in I^{\text{int}}$.
- $n^{\text{cont}} \in \mathbb{N}$ and $x_j^{\text{cont}} \in [-5, 10]$ for $j \in I^{\text{cont}}$.

4.15 Cat-cstrs-15: modified Toy [14]

$$f\left(x^{\text{cat}}, x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}, x_4^{\text{cont}}\right) = 5+$$
$$\begin{cases} \cos\left(3.6\pi\left(x_1^{\text{cont}}-2\right)+x_2^{\text{cont}}\right)+x_3^{\text{cont}}-1+\left(x_4^{\text{cont}}\right)^2 & \text{if } x^{\text{cat}}=\text{A}, \\ 2\cos\left(1.1\pi\exp\left(x_1^{\text{cont}}\right)\right)-\frac{x_2^{\text{cont}}}{2}+\left(x_3^{\text{cont}}\right)^2+2\log\left(1+\left(x_4^{\text{cont}}\right)^2\right) & \text{if } x^{\text{cat}}=\text{B}, \\ \cos\left(2\pi x_1^{\text{cont}}\right)+\frac{x_2^{\text{cont}}}{2}+x_3^{\text{cont}}x_4^{\text{cont}} & \text{if } x^{\text{cat}}=\text{C}, \\ x_1\cos\left(3.4\pi\left(x_1^{\text{cont}}-1\right)\right)-x_2^{\text{cont}}-1+x_3^{\text{cont}}+\left(x_4^{\text{cont}}\right)^3 & \text{if } x^{\text{cat}}=\text{D}, \\ -\frac{\left(x_1^{\text{cont}}\right)^2}{2}+\log\left(1+\left(x_2^{\text{cont}}\right)^2\right)+\left(x_3^{\text{cont}}\right)^2+x_4^{\text{cont}} & \text{if } x^{\text{cat}}=\text{E}, \\ 2\cos\left(\frac{\pi}{4}\exp\left(-\left(x_1^{\text{cont}}\right)^4\right)\right)^2-\frac{x_2^{\text{cont}}}{2}+x_3^{\text{cont}}x_4^{\text{cont}}+1 & \text{if } x^{\text{cat}}=\text{F}, \\ x_1\cos\left(3.4x_1^{\text{cont}}\right)-\frac{x_2^{\text{cont}}}{2}+x_3^{\text{cont}}+\left(x_4^{\text{cont}}\right)^3+1 & \text{if } x^{\text{cat}}=\text{G}, \\ x_1^{\text{cont}}\left(-\cos\left(\frac{7}{2\pi}\right)\frac{x_2^{\text{cont}}}{2}\right)+x_3^{\text{cont}}+x_4^{\text{cont}}+2 & \text{if } x^{\text{cat}}=\text{H}, \\ -\frac{\left(x_1^{\text{cont}}\right)^3}{2}+\left(x_2^{\text{cont}}\right)^2+x_3^{\text{cont}}x_4^{\text{cont}}+1 & \text{if } x^{\text{cat}}=\text{I}, \\ -\cos\left(5\pi x_1^{\text{cont}}\right)^2\sqrt{x_1^{\text{cont}}}-\frac{-\log\left(x_2^{\text{cont}}+x_3^{\text{cont}}+0.5\right)}{2}+\left(x_4^{\text{cont}}\right)^3-1.3 & \text{if } x^{\text{cat}}=\text{J}. \end{cases}$$

with constraints

$$g_1\left(x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}, x_4^{\text{cont}}\right) = \sqrt{(x_1^{\text{cont}})^2+(x_2^{\text{cont}})^2+(x_3^{\text{cont}})^2+(x_4^{\text{cont}})^2}-0.25^2$$
$$g_2\left(x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}, x_4^{\text{cont}}\right) = -\sqrt{(x_1^{\text{cont}})^2+(x_2^{\text{cont}})^2+(x_3^{\text{cont}})^2+(x_4^{\text{cont}})^2}+0.1^2$$

- $n^{\text{cat}} = 1$ and $x^{\text{cat}} \in \{\text{A}, \text{B}, \text{C}, \text{D}, \text{E}, \text{F}, \text{G}, \text{H}, \text{I}, \text{J}\}$;
- $n^{\text{int}} = 0$;
- $n^{\text{cont}} = 4$ and $x_i^{\text{cont}} \in [0, 1]$ for $i \in I^{\text{cont}}$.

4.16 Cat-cstrs-16: modified Wong-2 [13]

$$f\Big(x^{\text{cat}}, x_1^{\text{int}}, x_2^{\text{int}}, x_3^{\text{int}}, x_4^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}, x_4^{\text{cont}}, x_5^{\text{cont}}, x_6^{\text{cont}}\Big) =$$
$$-s\Big(x_1^{\text{int}}, x_2^{\text{int}}, x_3^{\text{int}}, x_4^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}, x_4^{\text{cont}}, x_5^{\text{cont}}, x_6^{\text{cont}}\Big)$$
$$+\begin{cases} 0 & \text{if } x^{\text{cat}}=\text{A} \\ 10\left(3\left(x_1^{\text{cont}}-2\right)^2+4\left(x_1^{\text{int}}-3\right)^2+2\left(x_2^{\text{cont}}\right)^2-7\left(x_2^{\text{int}}\right)^2-120\right) & \text{if } x^{\text{cat}}=\text{B} \\ 10\left(5\left(x_1^{\text{cont}}\right)^2+8x_1^{\text{int}}+6\left(x_2^{\text{cont}}-6\right)^2-2x_2^{\text{int}}-40\right) & \text{if } x^{\text{cat}}=\text{C} \\ 10\left(0.5\left(x_1^{\text{cont}}-8\right)^2+2\left(x_1^{\text{int}}-4\right)^2+3\left(x_3^{\text{cont}}\right)^2-x_3^{\text{int}}-30\right) & \text{if } x^{\text{cat}}=\text{D} \\ 10\left(\left(x_1^{\text{cont}}\right)^2+2\left(x_1^{\text{int}}-2\right)^2-2x_1^{\text{cont}}x_1^{\text{int}}+14x_3^{\text{cont}}-6x_3^{\text{int}}\right) & \text{if } x^{\text{cat}}=\text{E} \\ 10\left(-3x_1^{\text{cont}}+6x_1^{\text{int}}+12\left(x_5^{\text{cont}}-8\right)^2-7x_6^{\text{cont}}\right) & \text{if } x^{\text{cat}}=\text{F} \end{cases}$$

where

$$s(x_1^{\text{int}}, x_2^{\text{int}}, x_3^{\text{int}}, x_4^{\text{int}}, x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}, x_4^{\text{cont}}, x_5^{\text{cont}}, x_6^{\text{cont}}) =$$
$$\left(x_1^{\text{cont}}\right)^2+\left(x_1^{\text{int}}\right)^2+x_1^{\text{cont}}x_1^{\text{int}}-14x_1^{\text{cont}}-16x_1^{\text{int}}+\left(x_2^{\text{cont}}-10\right)^2+4\left(x_2^{\text{int}}-5\right)^2+\left(x_3^{\text{cont}}-3\right)^2$$
$$+2\left(x_3^{\text{int}}-1\right)^2+5\left(x_4^{\text{cont}}\right)+7\left(x_4^{\text{int}}-11\right)^2+2\left(x_5^{\text{cont}}-10\right)^2+\left(x_6^{\text{cont}}-7\right)^2+45$$

with constraints

$$g_1\left(x_1^{\text{int}}, x_4^{\text{int}}, x_1^{\text{cont}}, x_4^{\text{cont}}\right) = \begin{cases} 4x_1^{\text{cont}} + 5x_1^{\text{int}} - 3x_4^{\text{cont}} + 9x_4^{\text{int}} - 105 \le 0 & \text{if } x_1^{\text{cat}} = \mathrm{A} \\ 3x_1^{\text{cont}} + 6x_1^{\text{int}} - 3x_4^{\text{cont}} + 9x_4^{\text{int}} - 105 \le 0 & \text{if } x_1^{\text{cat}} = \mathrm{B} \\ 4x_1^{\text{cont}} + 4x_1^{\text{int}} - 2x_4^{\text{cont}} + 9x_4^{\text{int}} - 105 \le 0 & \text{if } x_1^{\text{cat}} = \mathrm{C} \\ 4x_1^{\text{cont}} + 5x_1^{\text{int}} - 4x_4^{\text{cont}} + 10x_4^{\text{int}} - 105 \le 0 & \text{if } x_1^{\text{cat}} = \mathrm{D} \\ 5x_1^{\text{cont}} + 5x_1^{\text{int}} - 3x_4^{\text{cont}} + 8x_4^{\text{int}} - 105 \le 0 & \text{if } x_1^{\text{cat}} = \mathrm{E} \\ 3x_1^{\text{cont}} + 4x_1^{\text{int}} - 1x_4^{\text{cont}} + 8x_4^{\text{int}} - 95 \le 0 & \text{if } x_1^{\text{cat}} = \mathrm{F} \end{cases}$$

$$g_2\left(x_1^{\text{int}}, x_4^{\text{int}}, x_1^{\text{cont}}, x_4^{\text{cont}}\right) = \begin{cases} 10x_1^{\text{cont}} - 8x_1^{\text{int}} - 17x_4^{\text{cont}} + 2x_4^{\text{int}} \le 0 & \text{if } x_1^{\text{cat}} = \mathrm{A} \\ 8x_1^{\text{cont}} - 6x_1^{\text{int}} - 17x_4^{\text{cont}} + 2x_4^{\text{int}} \le 0 & \text{if } x_1^{\text{cat}} = \mathrm{B} \\ 10x_1^{\text{cont}} - 10x_1^{\text{int}} - 15x_4^{\text{cont}} + 2x_4^{\text{int}} \le 0 & \text{if } x_1^{\text{cat}} = \mathrm{C} \\ 10x_1^{\text{cont}} - 8x_1^{\text{int}} - 19x_4^{\text{cont}} + 4x_4^{\text{int}} \le 0 & \text{if } x_1^{\text{cat}} = \mathrm{D} \\ 12x_1^{\text{cont}} - 8x_1^{\text{int}} - 19x_4^{\text{cont}} + 1x_4^{\text{int}} \le 0 & \text{if } x_1^{\text{cat}} = \mathrm{E} \\ 9x_1^{\text{cont}} - 9x_1^{\text{int}} - 18x_4^{\text{cont}} + 1x_4^{\text{int}} + 10 \le 0 & \text{if } x_1^{\text{cat}} = \mathrm{F} \end{cases}$$

$$g_3\left(x_1^{\text{int}}, x_1^{\text{cont}}, x_5^{\text{cont}}, x_6^{\text{cont}}\right) = \begin{cases} -8x_1^{\text{cont}} + 2x_1^{\text{int}} + 5x_5^{\text{cont}} - 2x_6^{\text{int}} \le 0 & \text{if } x_1^{\text{cat}} = \mathrm{A} \\ -4x_1^{\text{cont}} + 4x_1^{\text{int}} + 5x_5^{\text{cont}} - 2x_6^{\text{int}} \le 0 & \text{if } x_1^{\text{cat}} = \mathrm{B} \\ -8x_1^{\text{cont}} + 1x_1^{\text{int}} + 10x_5^{\text{cont}} - 2x_6^{\text{int}} \le 0 & \text{if } x_1^{\text{cat}} = \mathrm{C} \\ -8x_1^{\text{cont}} + 2x_1^{\text{int}} + 2.5x_5^{\text{cont}} - 4x_6^{\text{int}} \le 0 & \text{if } x_1^{\text{cat}} = \mathrm{D} \\ -16x_1^{\text{cont}} + 2x_1^{\text{int}} + 5x_5^{\text{cont}} - 1x_6^{\text{int}} \le 0 & \text{if } x_1^{\text{cat}} = \mathrm{E} \\ -4x_1^{\text{cont}} + 1x_1^{\text{int}} + 10x_5^{\text{cont}} - 4x_6^{\text{int}} + 10 \le 0 & \text{if } x_1^{\text{cat}} = \mathrm{F} \end{cases}$$

- $n^{\text{cat}} = 1$ and $x^{\text{cat}} \in \{\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}, \mathrm{E}, \mathrm{F}\}$;
- $n^{\text{int}} = 4$ and $x_1^{\text{int}}, x_2^{\text{int}}, x_3^{\text{int}}, x_4^{\text{int}} \in \{0, 1, \ldots, 10\}$
- $n^{\text{cont}} = 6$ and $x_1^{\text{cont}}, x_2^{\text{cont}}, x_3^{\text{cont}}, x_4^{\text{cont}}, x_5^{\text{cont}}, x_6^{\text{cont}} \in [0, 10]$.

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