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Optimizing profile block bids in short-term hydropower scheduling: A two-phase model for the day-ahead market

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Abstract : This paper proposes a two-phase optimization framework for short-term hydropower scheduling in the day-ahead electricity market using profile block bids grouped in exclusive sets. The first phase solves a nonlinear deterministic model that generates a diverse and operationally feasible set of production blocks by accounting for startup costs, opportunity costs, and hydrological constraints. In the second phase, a two-stage stochastic program is used to select a subset of blocks for market submission under price uncertainty. The proposed approach captures a wide range of production scenarios while ensuring compliance with market design rules. By decomposing the problem and relaxing binary variables, the framework significantly reduces computational complexity and achieves fast solution times. Numerical experiments based on a real hydropower system demonstrate the model's ability to produce effective bidding strategies, comparable to the hourly bidding methods.

Keywords: Short-term hydropower optimization, day-ahead electricity market, profile block bids, exclusive groups, stochastic programming.

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Notation

$t \in \{1, 2, \ldots, T\}$	Index of periods
$c \in \{1, 2, 3, \ldots, C\}$	Index of hydropower plants
$b \in \{1, 2, 3, \ldots, B_c\}$	Index of block for plant c
$s \in \{1, 2, \ldots, S\}$	Index of scenarios
$r \in \{1, 2, \ldots, R^c\}$	Index of plants upstream of plant c
$j \in \{1, 2, \ldots, J_t^c\}$	Index of power production surface j for plant c at period t

Parameters

ζ	Conversion factor from (m^3/s) to (Mm^3/h)
q^c_{min}	Minimum water discharge at plant c (m^3/s)
q^c_{max}	Maximum water discharge at plant $c (m^3/s)$
v^c_{min}	Minimal volume of plant c reservoir (Mm^3)
v^c_{max}	Maximum volume of plant c reservoir (Mm^3)
$v^c_{Initial}$	Initial volume of reservoir c (Mm^3)
$P_{t,b}$	Price of profile b at hour t (Phase 1)
ϵ^c	Penalty for the start-up of the turbines for plant c
δ^c_t	Inflow in period t (Mm^3)
$\rho_{s,t}$	Market price for scenario s at hours t
γ	Penalty for each additional selected profile beyond the first one
π^s	Probability of scenario s
$\tau^{r\to c}$	Time delay (h) for water transfer from upstream reservoir r
R_{bs}	Revenue from using profile b in scenario s
C_s	Cost of not selecting any profile in scenario s

Variables

q^c_t	Water discharge at period t (m^3/s)
v^c_t	Reservoir volume at period t (Mm^3/h)
g^c_t	Water spillage at plant c and period $t (m^3/s)$
$\alpha^c_t(v^c_t)$	Opportunity cost associated with water usage at plant c during period t.
$\chi^c_{j,t}$	Power production for surface j (MW)
oc^c	Lost opportunity costs associated with using water from the plant c at period t
z_j	$\begin{cases} 1, & \text{if surface j is chosen,} \\ 0, & \text{otherwise.} \end{cases}$
x_{bs}	$\begin{cases} 1, & \text{profile } b \text{ is in the exclusive group in scenario } s, \\ 0, & \text{otherwise.} \end{cases}$
w_b	$\begin{cases} 1, & \text{1 if profile } b \text{ is selected,} \\ 0, & \text{otherwise.} \end{cases}$
u_s	$\begin{cases} 1, & \text{1 if no profile is selected in scenario } s, \\ 0, & \text{otherwise.} \end{cases}$

1 Introduction

Bids can be structured in various formats in the electricity market, such as hourly bids, flexible hourly bids, and block bids that group multiple hours together. Block bids are used in systems with intertemporal dependencies between reservoirs, where water flow delays can cause mismatches between production and market prices. Block bids contribute to stable production over longer time periods and are particularly suitable in situations where there are conflicts between upstream and downstream hydropower plants [1]. Block bidding provides producers in electricity markets with an organized method to manage operational restrictions and cost considerations effectively. Block bidding offers a structured approach to managing the complexities of power markets by taking into account operational limitations and cost considerations. These bids facilitate conditional and time-linked power delivery, ensuring better alignment between production schedules and market demand. Block bids increase stability and reduce inefficiencies, especially under complex conditions [2].

Block bids, characterized by an all-or-nothing acceptance condition, enable conditional and intertemporal power delivery. These bids address market and operational needs through various types. Regular block bids deliver constant power over a specific period, profile block bids enable variable energy profiles, and linked block bids establish parent-child relationships for conditional acceptance [2, 3, 4]. Profile block orders, unlike regular block orders, allow producers to offer variable energy quantities across multiple periods, aligning delivery with market price fluctuations. Their clearing relies on comparing the offered price with the weighted average market clearing price over the selected intervals [5].

Several studies have explored the integration of block bids into the offering strategies of hydropower producers. For instance, [6] investigates the use of regular block orders in hourly offering problems, focusing on their role in addressing operational constraints like startup and shutdown costs. Similarly, [7] examine day-ahead bidding strategies, incorporating both hourly and block bids to optimize production while managing price uncertainties and balancing market dynamics. Alnæs et al. [8] further provide an empirical analysis of Norwegian hydropower producers, highlighting the interplay between block and hourly bids and their effectiveness in addressing marginal water values and operational efficiencies. In [9], a day-ahead planning model integrates stochastic programming and recurrent neural networks, addressing hourly and block bids with price uncertainties for hydropower producers in the Nordic market. In [10], block bids are optimized for combined heat and power units using stochastic programming, highlighting their role in managing price uncertainties and operational constraints. Karasavvidis et al. [2] extend previous research by developing an optimization framework for hydrothermal systems that incorporates advanced bidding structures, such as profile-based and linked block bids. These approaches address operational challenges, improve production flexibility, and optimize profits under varying price and regulatory conditions. Recent developments in electricity market design have highlighted the role of exclusive groups of block bids, also known as mutually exclusive block bids, in addressing intertemporal constraints and enabling more realistic representations of production capabilities. These bid formats allow participants to submit multiple alternative power profiles while ensuring that at most one of them is accepted, thus preventing overcommitment and supporting better operational alignment. In this context, [11] present an in-depth analysis of package bidding mechanisms used in European electricity auctions. Their work is particularly relevant to our study, as the two-phase modeling approach we adopt, especially the second-phase optimization problem characterized by total unimodularity, is inspired by their formulation. They show how the limitation on the number of allowable block bids can lead to welfare losses and propose algorithmic solutions to optimize bid selection under these constraints. Their insights form a theoretical foundation for our stochastic profile selection model.

As highlighted in the literature, most existing studies on block bidding focus on regular block bids, while fewer works address the profiled block bids. In this paper, we present a two-phase model for profiled block bidding for participating in the Norwegian day-ahead market. In the first phase, a short-term hydropower optimization problem is solved, incorporating operational constraints, water usage costs, and startup costs. Since the model can handle various price conditions and sequences of consecutive hours, it is capable of generating diverse and realistic bidding blocks—an important feature for dealing with complex systems. The second phase is a two-stage linear stochastic program that selects the most suitable blocks to offer to the market from the block set generated in the first phase, based on the price scenarios. The two-phase structure of the approach causes a reduction in computational time, and the model converges to the solution within a short period.

This paper is organized as follows: Section 2 presents the methodology and mathematical formulation of the two-phase model, including the profile generation model developed in Phase 1 and the two-stage stochastic profile selection model of Phase 2. Section 3 introduces a case study and the hydropower system. The results are reported in Section 4, and the performance of the model is compared to the hourly bidding strategies to evaluate its effectiveness in Section 5. Finally, Section 6 presents the conclusion and directions for future research.

2 Methodology

Short-term hydropower scheduling involves determining the optimal hourly production plan for one or more hydro units over a short planning horizon, typically ranging from one to several days. This planning must consider operational constraints such as reservoir storage bounds, turbine operating limits, and startup costs. Power production depends on technical factors including reservoir volume, discharge, and net head. The main objective is to allocate water resources in a way that ensures feasible operation while responding effectively to market price variations [12, 13]. Electricity trading is commonly organized into three main markets: the day-ahead market, the intraday market, and the balancing (real-time) market. The day-ahead market main role where most electricity transactions are conducted. In this market, producers and consumers submit their bids for the following day, typically before noon. After collecting all offers, the market operator performs market clearing and publishes the hourly market prices and committed quantities around 1 p.m. [1, 14]. The intraday market allows participants to update their positions closer to delivery, offering increased flexibility in response to new forecasts or unexpected changes. Finally, the balancing market, operated by the Transmission System Operator (TSO), is used to resolve real-time imbalances and ensure system stability. In this market, producers can offer flexible ramping capacity or make adjustments based on the actual system conditions [1]. Producers can submit different types of bids to participate in these markets. In addition to hourly bids, market designs increasingly allow for more structures such as block bids, which consist of fixed quantities over multiple hours with an all-or-nothing acceptance rule. These include regular block bids (constant quantity over time), profile block bids (varying quantity), and linked block bids that define conditional relationships between bids. Some market designs allow producers to submit several alternative block bids as part of an exclusive group, with the rule that only one of them can be accepted. This structure prevents overcommitment and allows producers to adapt their bidding strategy to different possible operating conditions [11].

This paper proposes a two-phase optimization framework to identify an optimal profile block bidding strategy for short-term hydropower scheduling in the day-ahead electricity market under price uncertainty. The methodology is designed to ensure operational feasibility while maximizing market-based profitability. In the first phase, a deterministic optimization model is solved using forecasted electricity prices and inflow data. This model incorporates key operational features such as opportunity costs, startup costs, and hydrological constraints including reservoir balance and turbine limits. The goal is to generate a diverse and feasible set of production profiles (blocks) that are compliant with the rules of block bidding in electricity markets. Each block represents a continuous production period with durations ranging from a minimum of 3 consecutive hours up to 24 hours. This diversity allows the model to accommodate various operational conditions and prepares it to respond flexibly to a wide range of market scenarios.

The feasible blocks generated in this phase, along with their associated production costs, opportunity costs, and startup costs, are passed to the second phase. In this phase, a two-stage stochastic programming model is used to select the most profitable subset of blocks for market participation. Price uncertainty is modeled through a set of price scenarios that become available close to the bidding deadline. Based on these scenarios, the model evaluates the expected economic performance of each block and selects a fixed number (e.g., 15 blocks) that maximize the overall expected profit. This selection process reflects actual market design, where producers are typically allowed to submit a limited number of block bids grouped into exclusive sets. In such exclusive groups, only one block can be accepted per scenario. The objective accounts for all relevant costs, including production, opportunity, and startup costs. The proposed formulation ensures computational efficiency, even for large-scale instances, and provides a structured and scenario-driven approach to support informed bidding decisions under uncertainty.

2.1 Phase 1: Profile generation

Phase 1 of the proposed methodology focuses on generating a set of profile block bids that define potential operational schedules for the hydropower plant over a given time horizon. To achieve this, a nonlinear deterministic optimization model is formulated, in which market prices and inflows are considered as parameters. The objective is to maximize revenue while accounting for key operational costs, including water usage, opportunity costs, and turbine startup expenses. the model ensures efficient water resource allocation while respecting hydrological and operational constraints.

Hydropower optimization is inherently nonlinear, as power production depends on water discharge, reservoir volume, and turbine efficiency. The net water head, which directly influences power generation, is determined by the forebay and tailrace elevations, as well as penstock losses. Additionally, turbine efficiency varies across units, meaning that even under similar water discharge and head conditions, different turbines may yield different power outputs. Instead of explicitly modeling each turbine configuration, the model employs the maximum power output surface, which approximates the nonlinear relationship between water discharge and reservoir volume and power production using polynomial regression. These power output surfaces, derived from a combination of feasible turbine operations, provide a computationally efficient way to capture the complexities of turbine efficiency and head variations. Instead of modeling each turbine individually and considering all possible configurations, the model utilizes the maximum power output surface, which simplifies the representation of power generation while maintaining accuracy.

The inclusion of power output surfaces introduces binary variables, leading to a Mixed-Integer Nonlinear Programming (MINLP) formulation. While MINLP models provide precise solutions, they can be computationally expensive, particularly in large-scale hydropower systems. To improve tractability, the model is formulated in a way that ensures total unimodularity in the constraint matrix. As a result, even when binary variables are relaxed, the problem still yields integer solutions.

In this case, the matrix of the coefficients of the constraints is totally unimodular and therefore meets these three criterias to be defined so: 1) All submatrices have elements in the set {-1, 0, 1}. 2) Each column has at most two nonzero elements. 3) There exists a partition of rows such that every column with two nonzero elements satisfies this partition. If these conditions are met, the binary selection problem can be solved as a continuous nonlinear problem while still yielding integer solutions. The optimization model aims to maximize total revenue by selecting the most efficient production profiles while accounting for key operational costs. The objective function Eq (1) maximizes the total profit, where P_t denotes the market price at time t, and $\chi^c_{j,t}(q^c_t, v^c_t)$ represents the power output as a function of water discharge and reservoir volume. An essential component of the model is the inclusion of water usage costs or opportunity costs. These costs are modeled using a linear function that depends on both reservoir volume and water discharge. The intuition behind this is straightforward: when the reservoir is near full capacity, the opportunity cost of using water is low, as there is little risk of scarcity. However, as the water level drops, the opportunity cost increases, reflecting the growing value of conserving water for future use. This dynamic encourages more strategic water allocation, especially during periods of low storage. In addition, startup costs are included for each production block. These costs are determined by solving a unit commitment problem, following the methodology described in [12]. The mathematical formulation of Phase 1 is presented as follows.

$$\max \sum_{c\in C}\sum_{t\in T}\sum_{j\in J} P_t \times \chi^c_{j,t}(q^c_t, v^c_t) \times z^c_{j,t} - \sum_{c\in C}\sum_{t\in T} \alpha^c_t(q^c_t, v^c_t) \tag{1}$$

$$\begin{aligned}
\text{Subject to: } v^c_{t+1} =& v^c_t - \zeta \times w_t \times (q^c_t + g^c_t) + \zeta \times \delta^c_t \\
&+ \sum_{r\in R} \zeta \times w_{t-\tau^{r\to c}} \times (q^r_{t-\tau^{r\to c}} + g^r_{t-\tau^{r\to c}}), && \forall t \in T, c \in C, && (2)\\
\sum_{j\in J} z^c_{j,t} \leq& 1, && \forall t \in T, c \in C, && (3)\\
v^c_1 =& v^c_{Initial}, && \forall c \in C, && (4)\\
q^c_{min} \leq& q^c_t \leq q^c_{max}, && \forall t \in T, c \in C, && (5)\\
v^c_{min} \leq& v^c_t \leq v^c_{max}, && \forall t \in T, c \in C, && (6)\\
v^c_t \geq& 0, q^c_t \geq 0, && \forall t \in T, c \in C, && (7)\\
z^c_{j,t} \in& \{0, 1\}, && \forall t \in T, j \in J, c \in C. && (8)
\end{aligned}$$

Equation (2) defines the water balance for each reservoir in the system. It ensures that the volume of water stored in reservoir c at time $t + 1$, denoted by v^c_{t+1}, is equal to the volume at time t, v^c_t, minus the water released for power production and spillage, $w_t(q^c_t + g^c_t)$, plus the natural inflow δ^c_t, all scaled by the conversion factor ζ, which converts discharge from m^3/s to Mm^3/h. Additionally, the equation accounts for water inflow from upstream reservoirs that are hydraulically connected to reservoir c. These contributions are modeled with a delay $\tau^{r\to c}$, representing the travel time of water from an upstream reservoir r to reservoir c. This formulation provides a realistic representation of reservoir interactions, especially in systems where water released from upstream plants does not immediately reach downstream reservoirs. Equation (3) guarantees that, for each unit c and every time period t, exactly one production surface is selected from the available set. Equation 4 sets the initial reservoir volume to a predefined value $v_1^c = v_{Initial}^c$, ensuring a known starting condition. Equations 5 and 6 impose operational constraints on water discharge and reservoir volume, restricting them within their respective minimum and maximum limits to maintain system feasibility. Equation 7 enforces non-negativity constraints on reservoir volume and water discharge to ensure physically meaningful solutions. Finally, Equation 8 defines the binary nature of $z_{j,t}^c$.

2.2 Phase 2: Two stage stochastic profile selection optimization

The objective of Phase 2, a two-stage stochastic mixed-integer linear programming model, is to select the optimal blocks based on new price scenarios from among the block set generated in Phase 1. Thus, the optimal power production values and associated block costs calculated in Phase 1 are considered as inputs for Phase 2. Additionally, the scenarios incorporate the uncertainties of day-ahead market prices. The objective function in Phase 2 includes revenue from each block under different price scenarios, deducts associated costs—such as opportunity and startup costs—and incorporates a penalty term to prevent the selection of blocks that do not contribute to improving the objective function value. The mathematical formulation of the second phase is as follows:

$$\max \sum_{b\in B}\sum_{s\in S}\pi^s R_{bs}x_{bs} - \sum_{s\in S}\pi^s C_s u_s - \gamma\left(\sum_{b\in B} w_b - 1\right) \tag{9}$$

$$\sum_{b\in B} w_b \le N_{blocks}, \tag{10}$$

$$\sum_{b\in B} x_{bs} + u_s = 1, \qquad \forall s\in S, \tag{11}$$

$$x_{bs} \le w_b, \qquad \forall b\in B, \forall s\in S, \tag{12}$$

$$w_b \in \{0,1\}, \qquad \forall b\in B, \tag{13}$$

$$x_{bs} \in \{0,1\}, \qquad \forall b\in B, \forall s\in S. \tag{14}$$

Equation (10) ensures that the total number of selected profiles does not exceed the predefined limit N_{blocks}, controlling the maximum number of bids submitted to the market. Equation (11) enforces that for each scenario, exactly one decision is made—either one of the available profiles is selected, or no profile is chosen, which is indicated by u_s. This guarantees that the sum of selections per scenario equals one. Equation (12) ensures that a profile b can only be selected in scenario s if it has already been included in the bidding set. This maintains logical consistency between the profile selection variable w_b and its scenario-dependent selection x_{bs}. Equation (13) specifies that each profile is either included in the bidding set or not, ensuring that no partial profile selections occur. Equation (14) enforces a binary decision on whether a profile is selected in a specific scenario, maintaining the discrete nature of the problem.

3 Case study

The two-phase model has been evaluated in a case study of a hydropower system in Norway, which includes multiple reservoirs and power plants. This system consists of six interconnected reservoirs (Sverjesjoen, Falningsjoen, Innerdalsvannet, Storfossdammen, Granasjoen, and Bjorsetdammen) that supply water to five hydroelectric power plants: Ulset, Litjfossen, Brattset, Grana, and Svorkmo. Each plant has specific generation capacities and water discharge constraints. The installed capacity varies across plants, with Brattset (88 MW), Grana (82.5 MW), and Litjfossen (84 MW) having the highest output potential, while Ulset (40 MW) and Svorkmo (57.7 MW) provide additional flexibility. The reservoirs also differ significantly in volume: Innerdalsvannet (153.4 Mm^3), Falningsjoen (125.2 Mm^3), and Granasjoen (138.8 Mm^3) offer substantial storage capacity, whereas smaller reservoirs such as Bjorsetdammen (0.02 Mm^3) and Storfossdammen (1.69 Mm^3) are primarily used for short-term regulation and discharge routing. The system's topology incorporates a network of bypass channels and spillways, which regulate water flow between reservoirs, enhancing operational flexibility and stability.

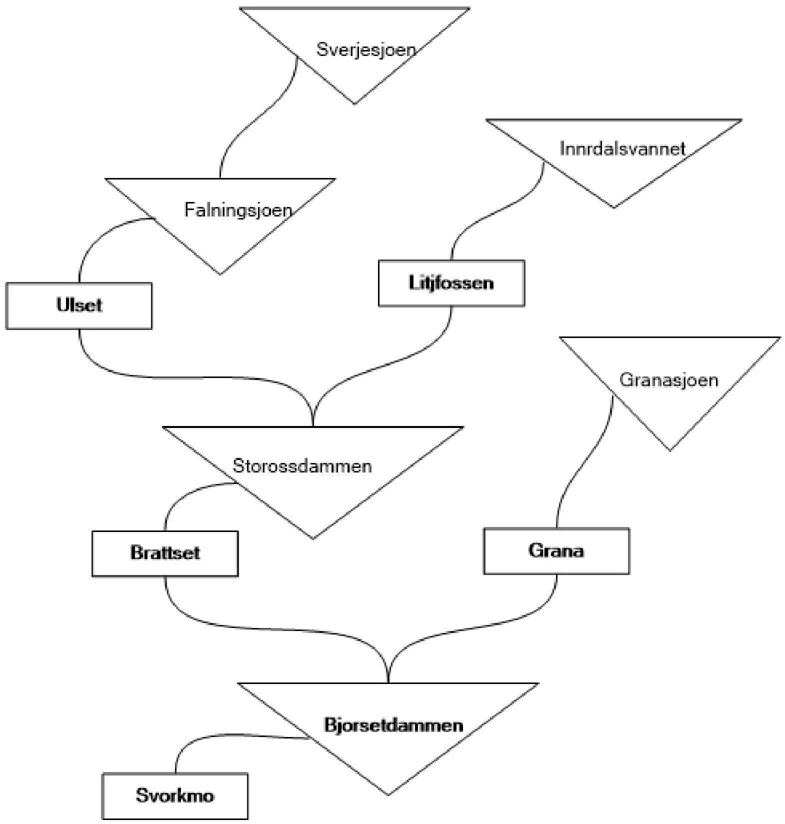

Figure 1: System topology

The two-phase model involves solving two optimization problems with different structures. Phase 1 solves a short-term hydropower scheduling problem formulated as a mixed-integer nonlinear program (MINLP), aiming to generate feasible production profiles that account for water use, opportunity costs, and startup costs. This problem is solved using the Ipopt solver [15].

Phase 2 uses a two-stage stochastic linear program to select the most profitable subset of blocks based on multiple day-ahead price scenarios. This model is solved with the CLP solver [16]. To validate the Phase 1 results, the BONMIN solver [17], which handles nonlinear problems with binary variables, is also used. The entire implementation is done in Julia [18], and the experiments are conducted on a system with an Intel Core i5 processor and 8 GB of RAM.

4 Results

Since Phase 1 of the problem is solved deterministically, this section investigates the impact of the number of candidate blocks generated in Phase 1 on the performance of Phase 2. Specifically, we analyze how increasing the number of blocks in Phase 1 affects the objective value and the quality of the solution obtained in Phase 2.

To evaluate this, 5 representative days were randomly selected from the dataset. For each case, the stochastic second phase was solved using 30 day-ahead price scenarios. The corresponding results are summarized in Table 1, which reports both the objective function values and the computation times for different numbers of blocks.

Phase 1 was executed for various numbers of blocks: 25, 50, 100, 250, 500, 750, 1000, and 1500. The input parameters for Phase 1 include electricity prices and initial reservoir volumes. Each block is subject to predefined feasibility criteria, such as a minimum duration of 3 consecutive hours and a maximum of 24 hours. Any block that does not satisfy these criteria is excluded from the candidate set. The feasible blocks, along with their associated opportunity and startup costs, are then passed to Phase 2. In this stage, a two-stage stochastic programming model is used to select the most profitable combination of blocks to be offered in the day-ahead electricity market. The price scenarios reflect real market conditions close to the bidding time.

As shown in Table 1, increasing the number of candidate blocks in Phase 1 generally leads to better objective function values in Phase 2. This indicates that having access to a richer set of block options improves bidding decisions under uncertainty.

Table 1: Objective function value and computation time (in seconds) for different numbers of candidate blocks.

		Number of blocks							
Case	**Measure**	**25**	**50**	**100**	**250**	**500**	**750**	**1000**	**1500**
Case 1	Time (s)	0.01	0.01	0.01	0.04	0.06	0.12	0.14	0.22
	Obj. Value	33479	34082	34608	36475	38521	38529	38540	38548
Case 2	Time (s)	0.01	0.00	0.01	0.02	0.03	0.05	0.14	0.15
	Obj. Value	59205.1	60458.5	61632.7	63676.3	65304	65843.9	65871.3	65871.3
Case 3	Time (s)	0.00	0.01	0.01	0.02	0.06	0.11	0.14	0.22
	Obj. Value	95381.2	97592	101592	103629	103629	104701	104701	104720
Case 4	Time (s)	0.01	0.01	0.01	0.02	0.04	0.09	0.10	0.11
	Obj. Value	13237.7	13237.7	13991.4	14308.8	15003.7	15043.7	15049.3	15079.7
Case 5	Time (s)	0.01	0.01	0.02	0.02	0.02	0.04	0.05	0.09
	Obj. Value	18178.3	18329.1	18784	19525.3	20468	20597.8	20617.7	20629

Figure 2 visualizes the normalized performance of Phase 2 based on the objective values presented in Table 1. The x-axis represents the number of candidate blocks used in Phase 1, while the y-axis shows the corresponding objective value expressed as a percentage of the maximum value (i.e., the value obtained with 1500 blocks, set to 100%).

These percentages were calculated by dividing the objective function value for each block size by the maximum value observed across all tested sizes, and then multiplying by 100. Formally, for a given number of blocks n, the normalized value is computed as:

$$\text{Percentage}_n = \left(\frac{Obj_n}{Obj_{\max}}\right) \times 100 \tag{15}$$

where Obj_n is the objective function value for n blocks, and $Obj_{\max}$ is the maximum value obtained (in this case, with 1500 blocks). As shown in the Figure 2, increasing the number of candidate blocks leads to improvements in the objective function value. However, beyond 500 blocks, the improvements occur at a slower rate, indicating that the marginal benefit of adding more blocks diminishes.

Although the solution time increases slightly with the number of candidate blocks, Phase 2 remains computationally efficient. Even with up to 1500 blocks and 30 price scenarios, the problem can be solved in less than a second, which demonstrates the scalability of the proposed method.

Figure 3 illustrates the average solution time across five representative cases for different numbers of candidate blocks. As shown in the figure, the computational time grows gradually as more candidate blocks are introduced, but remains consistently low—well below one second—even for the largest problem sizes considered. This further confirms that the relaxation of binary variables, supported by the total unimodularity of the constraint matrices, enables fast and scalable optimization in the second phase.

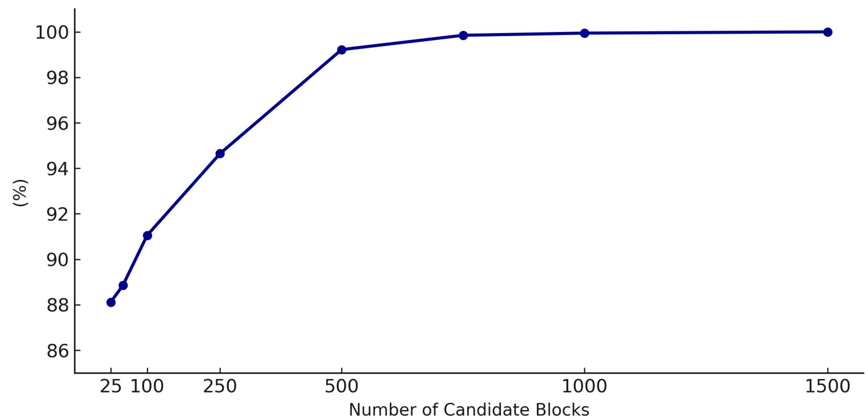

Figure 2: Normalized objective function value for different numbers of candidate blocks.

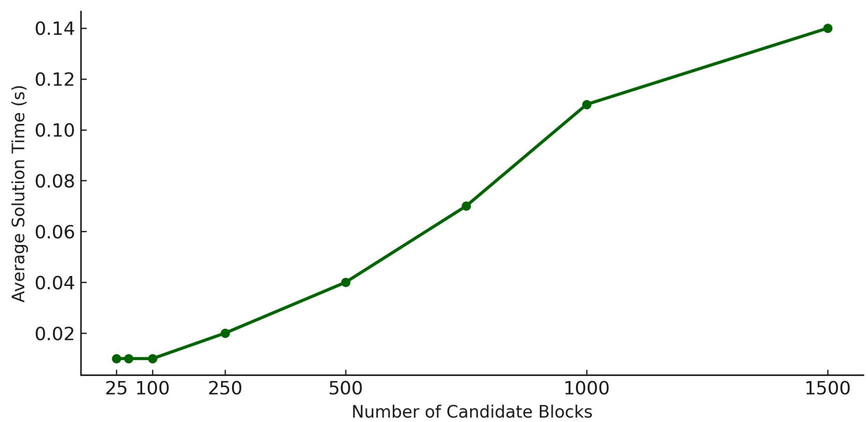

Figure 3: R2: Average solution time (S) for different numbers of candidate blocks.

5 Model evaluation

To evaluate the proposed two-phase model, we compare it with a model introduced in [19], which formulates the day-ahead hourly bidding problem as a two-stage stochastic mixed-integer nonlinear programming model. In this reference framework, first-stage decisions determine the bid volumes, while second-stage decisions reflect the actual hourly dispatch under different price scenarios. In the hourly bidding model, the imbalance between committed and realized volumes is explicitly considered, with corresponding rewards and penalties determined based on participation in the balancing market. For comparison, all input parameters such as initial reservoir volumes, inflows, and operational constraints are considered identical in both models. Given that the hourly bidding model requires separate offers for each hour of the day, each block generated in our proposed approach is also designed to span a full 24-hour period. Moreover, the water usage cost has been added to the objective function of the hourly bidding model. The hourly bidding model also follows market rules, requiring offer curves to be non-decreasing with respect to price levels. Likewise, committed volumes are determined based on market-clearing prices using linear interpolation. Additionally, the same set of price scenarios used in the second phase of the block bidding model is applied to the hourly model to ensure that both methods are evaluated under identical market uncertainty.

After the market is cleared, the profit from each submitted block and the profit from the hourly bidding model are calculated for comparison purposes. In the profiled block bidding model, since at most one block can be accepted, the selected block will be the one with the highest offered price that does not exceed the market price. This ensures compatibility with market rules. Given that the hourly bidding model is a two-stage stochastic mixed-integer nonlinear program and that the case study involves five hydropower plants and six reservoirs, solving the model under a large number of scenarios presents computational challenges. Therefore, to enable a meaningful and tractable comparison between the two models, five representative price scenarios are considered. Due to the complexity of the hourly bidding formulation, startup costs could not be incorporated in that model; hence, for consistency, startup costs were also excluded from the block bidding model in this part of the evaluation.

The comparison results are summarized in Table 2, which includes multiple test cases evaluated with varying inputs and price scenarios on different days.

Table 2: Comparison of hourly bidding and selected block profit.

Case	Number of Blocks	Number of Scenarios	Hourly Bidding Profit	Selected Block Profit
Case 1	750	5	70,369	69,327
Case 2	750	5	144 ,060	147,883
Case 3	750	5	89 ,783	89,594
Case 4	750	5	54 ,069	55,380
Case 5	750	5	71 ,586	71,890

As shown in Table 2, the proposed method provides better results or results that are very close compared to the hourly bidding model. Block bids are particularly valuable for hydropower producers because they allow for offering energy over multiple consecutive hours with operationally feasible patterns. This is beneficial in systems with reservoir dependencies, startup costs, or limited flexibility. Moreover, block bids contribute to more stable production schedules and better alignment with market prices under uncertainty. The model presented in this paper benefits from relaxing the binary variables in both phases, resulting in very short solution times. Therefore, it can be highly efficient and can be applied to complex and large-scale problems.

6 Conclusion

This paper presented a two-phase optimization framework for hydropower producers participating in the day-ahead electricity market using profile block bids organized in exclusive groups. The first phase generates a diverse set of feasible production blocks through a deterministic MINLP model that incorporates operational constraints, startup costs, and opportunity costs. The second phase employs a two-stage stochastic program to select the most profitable combination of these blocks based on a set of market price scenarios, while respecting market design rules such as exclusivity within block groups. Computational results confirmed the model's effectiveness in producing high-quality bidding strategies with relatively low solution times. Additionally, a comparison with an hourly bidding strategy showed that the proposed method can achieve similar or improved profits while significantly reducing complexity. For future work, the opportunity cost formulation could be refined by incorporating more detailed seasonal patterns and inflow variability, allowing for a more accurate representation of water value dynamics over time. Additionally, while the current experiments were limited to five price scenarios due to the computational burden of the MINLP, evaluating the framework using a broader set of scenarios and a wider range of test cases would strengthen the robustness of the results. A possible extension would be to develop a model that integrates both hourly and profiled block bids for the day-ahead market, providing producers with a unified strategy to participate more effectively in electricity markets. Finally, further evaluation of the model under different system conditions—including reservoir connectivity, inflow uncertainty, and price volatility—would provide deeper insights into the practical benefits and limitations of profile-based bidding strategies.

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