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An integrated location-inventory-transportation problem under demand uncertainty

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Abstract : Motivated by a real-world application at a large international pharmaceutical company, we tackle an integrated location-inventory-transportation problem under demand uncertainty. The supply chain network of this problem comprises multiple plants, distribution centers (DCs), and customers. The decision-making process involves simultaneously determining the facility locations, inventory planning, and transportation volumes. Apart from the computational complexity resulting from this integration, other practical challenges arise from the fact that the planner must determine inventory policies that account for safety stock consolidation, whereas transportation is charged based on volume-based piecewise linear costs. To this end, we propose an exact and an approximate solution framework to solve this problem. The exact approach is based on a logic-based Benders decomposition (LBBD) framework enhanced by a piecewise-linear lower-bound function and efficient logic cuts. We then improve the scalability by leveraging an approximate model with a piecewise linear approximation for safety stock computation. Finally, using the instances derived from real-world data, we empirically demonstrate the benefit of the integrated model, which yields up to 9% of potential cost savings.

Keywords : Logistics, Integrated optimization, location, inventory policies, safety stock, transportation, piecewise linearization, Benders decomposition, network design, piecewise transportation costs

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1 Introduction

Real-world logistics network planning comprises several decisions, mainly related to location, production, inventory, and transportation management. Due to their complexity, these decisions are often made in isolation or in a sequential fashion (Jalal et al. 2022b). Nevertheless, such approaches can result in a suboptimal performance of the network (Üster et al. 2008, Bouchard et al. 2017, Darvish and Coelho 2018). The integration of decisions in logistics network planning facilitates the coordination of decisions and allows the firm to substantially decrease logistics network costs (Brahimi and Khan 2014, Forouzanfar et al. 2018, Ghomi-Avili et al. 2020, Biuki et al. 2020, Jalal et al. 2022a).

Motivated by the operations of a pharmaceutical company, we present an effective solution framework for the integrated logistical network planning problem. In this context, in order to establish contracts with a third-party logistics (3PL) provider, the firm's decision-makers determine the network design (selections of suppliers and rental storage facilities), inventory management (inventory policy targets), and transportation planning (planned shipment volumes) in advance at the start of each year. More specifically, the company employs the periodic review (T, S) inventory policy in their Enterprise Resource Planning system, where the parameter T represents the review interval and the parameter S represents the target inventory level within the review interval. The period review interval T is an input parameter, but the target inventory level S is a decision, which can be different for each of the DCs and which can also vary over time since the average demand at the retailers can vary over time. As retailer demand is uncertain, safety stocks need to be maintained at DCs to provide an appropriate service level and to protect against short-term variations in retailer demand. As the demand is time-varying in this context, we also consider anticipation inventory to address the seasonal demands.

The main challenge of the integration of network design and inventory management in the supply chain is its complexity and scalability (Farahani et al. 2014). The overall supply chain performance and costs are tied to inventory management, which synchronizes manufacturing/supply and transportation planning. Nevertheless, most studies in the literature consider inventory management in isolation from the supply chain design. At the same time, most studies in network design overlook inventory decisions altogether or merely incorporate a basic and simplified approach to inventory management rather than an explicit inventory policy due to its non-linearity and computational complexity, which leads to suboptimal solutions (Chen et al. 2011, Sadjady and Davoudpour 2012, Shavandi and Bozorgi 2012). There are some studies, e.g., Üster et al. (2008), Ahmadi-Javid and Hoseinpour (2015), Wheatley et al. (2015), Jeet and Kutanoglu (2018), Candas and Kutanoglu (2020), that consider an integrated approach to supply chain network design and inventory management. Most studies, however, focus on a single-echelon network (You and Grossmann 2010) and assume linear unit transportation costs (Engebrethsen and Dauzère-Pérès 2019, Engebrethsen and Dauzère-Pérès 2023, Tamssaouet et al. 2023). As real-world transportation costs typically comprise different cost components and quantity discounts, there is a practical need to incorporate a piecewise linear cost function, which can be used to capture more complex cost structures and quantity discounts in transportation planning (Croxton et al. 2003, Engebrethsen and Dauzère-Pérès 2019, Brunaud et al. 2018). To the best of our knowledge, no paper has examined all these aspects which are simultaneously taken into account in this paper.

Our contribution is fourfold. First, we integrate important decisions in logistics network planning regarding network design, inventory management, and transportation planning. Second, we integrate features and characteristics of the real-world application in our problem, including location-based lead times, storage capacity constraints in DCs, multiple plants, multiple periods, and multiple products. Safety stock is a function of demand at each DC and the lead time from plants to DCs. Hence, safety stock calculations must be carried out simultaneously with the assignment and the location decisions. The inventory control decisions are made with a period review inventory policy, defining the amount of cycle inventory, safety stock, and anticipation inventory at open DCs. This work also addresses piecewise linear costs, which are a real feature not often considered in the literature. Third, since this integrated location-inventory-transportation model is highly complex and non-linear, we propose two distinct solution approaches. The first employs an exact method based on logic-based Benders decomposition, while the second involves a linear model with a safety stock approximation. Our computational experiments assess various logic cuts for the logic-based Benders decomposition technique. Fourth, we generate instances based on real data from the case of an international pharmaceutical company with operations in South America and carry out extensive computational experiments to analyze the performance of the decomposition framework.

The paper is organized as follows. Section 2 presents the literature review. Section 3 details the problem description, model formulation, and linearization procedure. Section 4 presents the solution methods. Section 5 presents computational results and discussion. Finally, section 6 concludes with main insights and some directions for future research.

2 Literature review

Location and inventory decisions are related since inventory decisions depend on the location of the facilities (e.g., plants, DCs, and retailers) and the assignments of retailers to DCs and DCs to plants. However, location and inventory management decisions have been commonly dealt with separately. We review studies that put forward this integration. Table 1 presents some characteristics of the relevant studies, i.e., the number of plants, type of lead time, demand sourcing, number of products, number of periods, capacity constraints, and inventory policy type. Similarly, in Table 2 we present the review of the main decisions that are related to the context of our study, i.e., location-allocation, capacity selection, safety stock, anticipation inventory, and transportation decisions, as well as the data source of instances, model type, and solution method.

Table 1: Literature review: problem characteristics

Article	#Plants	Lead times	Demand Sourcing	#Products	#Periods	Capacity Constraints	Inventory Policy
Vidyarthi et al. (2007)	Multiple	Average	Single	Multiple	Single	Cap	Min inventory cost
Park et al. (2010)	Multiple	Location-based	Single	Single	Single	Cap	*(r,Q)*
Yao et al. (2010)	Multiple	Location-based	R/C	Multiple	Single	Uncap	*(T,S)*
You and Grossmann (2010)	Multiple	Average	Single	Single	Single	Uncap	*(T,S)*
Berman et al. (2012)	Single	Location-based	Single	Single	Single	Uncap	*(T,S)*
Gzara et al. (2014)	Single	Location-based	Single	Multiple	Single	Uncap	*(S-1,S)*
Wheatley et al. (2015)	None	Average	R/C	Multiple	Single	Uncap	*(S-1,S)*
Zhang and Unnikrishnan (2016)	Single	Location-based	Single	Single	Single	Cap	*(T,S)*
Amiri-Aref et al. (2018)	Single	Location-based	Multiple	Single	Multiple	Cap	*(r,S)*
Escalona et al. (2018)	Single	Location-based	Single	Single	Single	Uncap	*(r,Q)*
Schuster Puga et al. (2019)	Single	Location-based	Single	Single	Single	Uncap	Min inventory cost
Zheng et al. (2019)	Single	Location-based	Single	Single	Single	Uncap	*(T,S)*
Tapia-Ubeda et al. (2020)	Single	Location-based	Single	Single	Single	Uncap	*(r,Q)(T,r,S)* *(S-1,S)*
Our article	Multiple	Location-based	R/C	Multiple	Multiple	Cap	*(T,S)*

R/C: Retailer per commodity

Since the company must ensure sufficient inventory and safety stocks to deal with demand uncertainty, it is necessary to define the location with minimum costs, and also to define the inventory management decisions at the DCs and inventory control policies based on a predefined service level. Under uncertain retailer demands, risk-pooling is a strategy to manage such demand uncertainty by consolidating inventory at DCs to achieve an appropriate service level. The transportation time from the plants to the DCs (lead time) is a relevant factor in the determination of the safety stock level under random retailer demands. Lead times depend on several factors, such as the physical distance and transportation mode, as well as the product type, the production technologies, etc. Nevertheless, papers in the literature incorporating the risk-pooling strategy have not considered DC-to-plant dependent lead times in the network design problems. Most papers consider a single plant or supplier

Table 2: Literature review: decisions, data source, model type, and solution method

Article	Decisions[1]					Data	Model[2]	Method
	Loc-alloc	Cap sel	SS	Ant inv	Transp			
Vidyarthi et al. (2007)	✓		✓		✓	Random data	MINLP	Heuristics
Park et al. (2010)	✓		✓			Random data	MINLP	Heuristics
Yao et al. (2010)	✓		✓			Random data	MINLP	Heuristics
You and Grossmann (2010)	✓		✓			Real data based	MINLP	Heuristics
Berman et al. (2012)	✓		✓			Random data	MINLP	Heuristics
Gzara et al. (2014)	✓		✓			Random data	MINLP	Solver
Wheatley et al. (2015)	✓		✓			Real data based	MINLP	Exact
Zhang and Unnikrishnan (2016)	✓		✓			Literature	CQMIP	Heuristics
Amiri-Aref et al. (2018)	✓		✓		✓	Generated data	MINLP	Heuristics
Escalona et al. (2018)	✓		✓			Random data	CQMIP	Solver
Schuster Puga et al. (2019)	✓		✓			Literature	CQMIP	Solver
Zheng et al. (2019)	✓		✓		✓	Real data based	CQMIP	Exact
Tapia-Ubeda et al. (2020)	✓		✓			Real data based	MINLP	Heuristics
Our article	✓	✓	✓	✓	✓	Real data based	MINLP	Exact

[1] Decisions: Location-allocation (Loc-alloc), Capacity selection (Cap sel), Safety Stock level (SS), Anticipation inventory level (Ant inv), Transportation (Transp).

[2] Model: Mixed Integer Nonlinear Programming (MINLP), Convex Quadratically Mixed-Integer Programming (CQMIP)

or source from which the DCs are supplied (Berman et al. 2012, Gzara et al. 2014, Zhang and Unnikrishnan 2016, Amiri-Aref et al. 2018, Escalona et al. 2018, Schuster Puga et al. 2019, Zheng et al. 2019, Tapia-Ubeda et al. 2020). In such a case, the lead time depends only on the DC location. Other works consider multiple plants but consider that the lead time is an average for all plant-DC pairs (Vidyarthi et al. 2007, You and Grossmann 2008). In contrast, Park et al. (2010) and Yao et al. (2010) are the only studies that consider lead times from multiple plants to DCs. This consideration results in a problem that is more difficult to solve but can still be handled using tailored heuristics. Yao et al. (2010) propose a two-phase heuristic solution algorithm based on the Lagrangian relaxation approach, and Yao et al. (2010) develop an iterative heuristic method. Both heuristics methods provide good solutions for the addressed problems.

Multiproduct problems cannot adequately be represented by single-product or single-commodity models. However, considering multiple types of products can make the problem more complex. Multi-products models enable the possibility of using multiple sources of products from plants and DCs, as demonstrated by Yao et al. (2010). Therefore, it is crucial to account for differences in lead times for production or transportation across different products, and these characteristics should be jointly considered when making decisions regarding product assignments and inventory policies at various locations. Lead times for production and transportation can vary depending on the product, and such characteristics should be considered alongside decisions about product assignment and inventory policies.

Most papers in Table 1 consider an infinite planning horizon or single-period planning that does not represent the context when the demand varies over different periods in the planning horizon. Additionally, they often overlook the diverse characteristics of products, e.g., size, weight, price, demand patterns; and requirements, e.g., environmental conditions such as temperature ranges. These considerations of a single product and period can result in suboptimal solutions (Jalal et al. 2022a, 2023).

Note that in Table 1, most papers overlook capacity constraints. However, it is an important set of constraints in practice for both DCs and plants and disregarding them can result in infeasible solutions. When capacity is fixed and demand varies, anticipatory inventory is a tactical decision to deal with capacity constraints. Thus, capacity selection becomes a crucial issue in implementing solutions, as it determines the sizes of different DCs. DC capacity can vary depending on demand assignment, allowing flexibility in accommodating different demand amounts.

The consideration of safety stocks makes the problem non-linear and much more difficult to solve. To solve this complex problem, most articles present heuristics methods, such as heuristics based on Lagrangian relaxation (Vidyarthi et al. 2007, Park et al. 2010, You and Grossmann 2010, Berman et al. 2012), Benders decomposition based on heuristics (Tapia-Ubeda et al. 2020), and approximation algorithms (Yao et al. 2010, Zhang and Unnikrishnan 2016, Amiri-Aref et al. 2018). Few papers propose exact methods. Wheatley et al. (2015) present an exact solution method using logic-based Benders decomposition and Zheng et al. (2019) propose an exact algorithm based on the Generalized Benders Decomposition method. However, such methods are applied to tackle simpler problems compared to the application considered in our case, which includes a multi-plant network and capacity constraints in a multi-period planning horizon.

Table 1 also presents the inventory policies used by the studies. A commonly used policy in practice is the periodic review and order-up-to-level *(T, S)* inventory policy, where the product is replenished up to S whereas a new ordering decision is made periodically after a review interval of T periods. In the *(r, Q)* policy, when the inventory position falls below a reorder level r, a replenishment order for Q units is placed. In the minimum/maximum *(r, S)* inventory policy, when the inventory on hand falls below a certain minimum r, a request is made for a replenishment order that will restore the on-hand inventory to a maximum number, S. The one-for-one *(S-1,S)* inventory policy, which calls for a replenishment order after each demand, is often advocated for controlling the stock levels of expensive, slow-moving items. In the *(T, r, S)* policy, the stock level is reviewed at the end of a fixed period of T time units. If the inventory level is at or below the reorder point r, a replenishment order is placed. The order quantity is determined to restore the stock level to the maximum target level S. The consideration of these policies implies the consideration of safety stock to deal with the uncertain demand.

While incorporating transportation decisions into the network design model can lead to lower supply chain costs, very few papers have considered realistic transportation cost functions. Across all papers listed in Tables 1 and 2, and in a majority of studies focusing on inventory planning, transportation costs are often oversimplified by assuming linear unit transportation costs. However, real-world transportation costs commonly involve diverse structures and discount schedules (Engebrethsen and Dauzère-Pérès 2019). Particularly, piecewise linear costs, which are frequently encountered in transportation planning (Croxton et al. 2003, Brunaud et al. 2018), are neglected in most of the related literature.

3 Problem description and modeling

3.1 Problem definition

This study addresses the integrated logistics network planning problem at the tactical level. We study a network composed of multiple plants, DCs, and retailers. The DCs are intermediate facilities between the plants and the retailers and facilitate the shipment of products in the two echelons, as shown in Figure 1. We consider the problem of defining which DCs of a 3PL provider should be selected to distribute multiple products to a set of retailers. Moreover, the problem includes selecting the capacity level for the opened DCs. The capacity levels are defined in terms of volume. The DC location costs comprise contractual fixed costs (e.g., rental space or volume in DCs). The selected DCs must remain in operation until the end of the planning horizon. Plants also have limited capacity, but this is not a decision variable within the model.

The retailers' demands are assumed to be independent and uncertain, where the uncertainty of demand (which can be estimated from forecasting errors) follows a normal distribution (Zheng et al. 2019). In addition, the expected demand at each retailer is dynamic and thus it can vary over time (as demand in this case is highly seasonal). The inventory management at the DCs is executed using a periodic review policy (T, S) presented in Figure 2. In the periodic review policy or reorder cycle policy, the stock level is only periodically observed. The parameter T represents the review interval and the parameter S is the target inventory level within the review interval, referred to as the order-

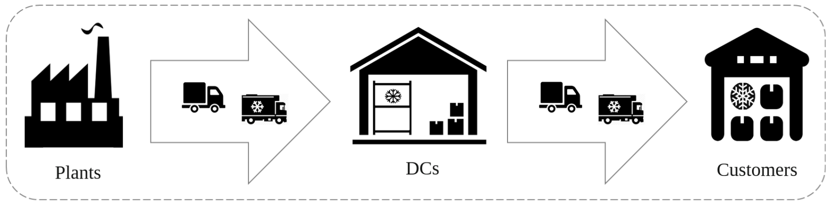

Figure 1: Logistics network

up-to-level. At each time period when the inventory is reviewed, the quantity ordered by DC from a plant $\tilde{q}$ is determined based on this order up to level S and the available inventory I' where $\tilde{q} = S - I'$. The parameter S is determined as $S = \mu(T + \ell) + \Phi_\alpha \sigma \sqrt{(T + \ell)}$, where μ is the demand mean, σ is the standard deviation, and ℓ is the lead time from plant to DC, and Φ_α is the number of standard deviations related to the service level α such that $P(Z \leq \Phi_\alpha) = \alpha$. With this definition of S, the probability that there is a stock out is up to $(1 - \alpha)$. The difference between S and the average demand in $T + \ell$ makes up the safety stock $SS = \Phi_\alpha \sigma \sqrt{(T + \ell)}$.

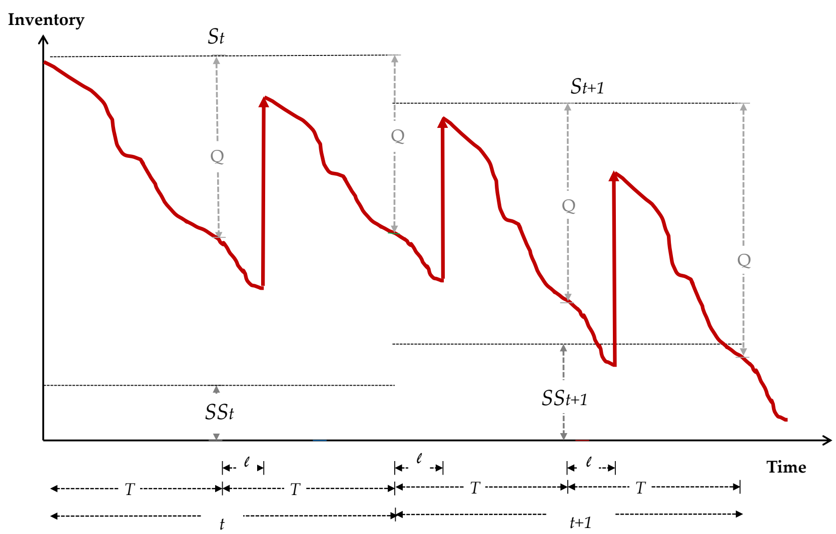

Figure 2: Periodic review policy (T, S)

The consolidation of the cycle, safety stock, and anticipation inventory at DCs are considered in this work. The review interval at the DC j for the product p, T_{jp}, is predetermined by the firm based on their inventory review schedule and assumed to be $T_{jp} \leq t$ as multiple replenishments can be carried out in each period (e.g., if t is a month, T_{jp} can be weekly or biweekly periods). The inventory decisions consist in determining the target inventory level or the order-up-to level S in each period, depending on which retailers are assigned to a DC. The target level consists of both cycle inventory and safety stock. The cycle inventory is the stock expected to be used to meet normal demand during a review interval, while safety stock is the extra stock to meet excess demand, to protect against uncertainty. We also consider the anticipation inventory that is built up to anticipate increased future retailer demands in later periods due to the limited capacity in plants (Olhager et al. 2001). The anticipation inventory for every period is determined based on the total quantity ordered by the DC from plants, the total demand allocated to the DC in the period, and the balance of safety stock. The total anticipation inventory is computed across several periods, and the average level of anticipation inventory within a period is computed as the average of the anticipation inventory at the beginning and the end of each period, as shown in Figure 3. Lastly, the total inventory cost is the sum of the costs of the target level, composed of the cycle, the safety stock, and the anticipation inventory.

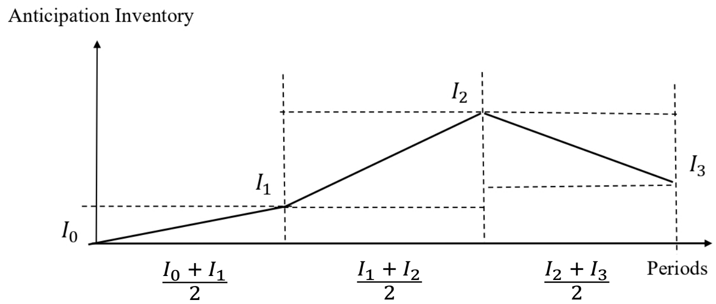

Figure 3: Anticipation inventory by period

Finally, as in Croxton et al. (2003), the transportation costs comprise different rates based on shipment volumes where each segment s has an associated fixed cost g_s and a variable cost c_s. The rate of the segment s is applied when the volume is in the range $[b_{s-1}, b_s)$. Figure 4 illustrates the transportation cost for different quantities of products: the x-axis is the load weight, thus the breakpoints are based on the weights, and the y-axis is the total transportation cost.

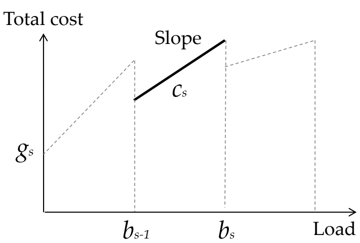

Figure 4: Transportation costs with different rates for different loads

Figure 4 also shows the impact of discounts among the segments. This transportation cost structure is applied for the transportation from the plants to DCs and from DCs to the retailers.

The carrier is responsible for the preservation of the goods from pick up to delivery. Thus, any damage that impairs the integrity of the cargo must be covered by the carrier. *Ad Valorem* is used to offset part of these costs. It is a component of the freight cost, charged to cover cargo security costs. It is a rate calculated on the value of the goods and in its composition can be considered all the measures that are taken to preserve the transported cargo, such as various insurances, investments for vehicle safety (including tracking and monitoring systems), operational costs, and security services. The *Ad Valorem* cost is explicitly modeled as part of the transportation cost in our model.

The objective of this problem is to minimize the total cost composed of DC location costs, transportation costs, and inventory holding costs.

3.2 Mathematical formulation

The notation used in the formulation is presented below.

Sets	
$i, j, k \in \mathcal{I} = \mathcal{I}_f \cup \mathcal{I}_w \cup \mathcal{I}_c$	Facilities: plants, potential DCs, and retailers
$l \in \mathcal{L}$	Capacity levels at DCs
$p \in \mathcal{P}$	Products
$s \in \mathcal{S}$	Cost segments for transportation
$t, t' \in \Theta$	Time periods
$\mathcal{A}_{fw} = \{(i,j) : (i \in \mathcal{I}_f \wedge j \in \mathcal{I}_w)\}$	Available arcs from plants to DCs
$\mathcal{A}_{wc} = \{(i,j) : (i \in \mathcal{I}_w \wedge j \in \mathcal{I}_c)\}$	Available arcs from DCs to retailers
$\mathcal{A} = \mathcal{A}_{fw} \cup \mathcal{A}_{wc}$	Available network arcs

Parameters

b_s	Breakpoint at segment s for the transportation cost
cap_{ip}	Production capacity of product p at plant i
c_{ijs}	Variable cost of the segment s to transport cargo from entity i to entity j (per unit of weight)
c'_{ij}	Variable security cost to transport cargo from entity i to entity j (per unit of value)
f_{jl}	Fixed cost for opening DC j at capacity level l
g_{ijs}	Fixed cost of the segment s to transport cargo from entity i to entity j
h_{pj}	Unitary inventory cost of product p in DC j (per period)
ℓ_{ij}	Lead time from entity i to entity j (in days)
q_l	Storage capacity at level l
T_{jp}	Prespecified review period at the DC j for the product p (in days)
η_{kt}	Number of working days at retailer k in period t
μ_{pkt}	Mean daily demand of product p at retailer k in period t
σ^2_{pkt}	Variance of daily demand of product p at retailer k in period t
ρ_p	Price of product p
υ_p	Volume of product p
ω_p	Weight of product p
Φ_α	Number of standard deviations related to the service level α such that $P(Z \leq \Phi_\alpha) = \alpha$

Continuous variables

I_{jpt}	Anticipation inventory of product p at DC j at the end of the period t
Q_{ijpt}	Total order quantity of product p from plant i to DC j in period t
S_{jpt}	Target inventory of product p at DC j in each review period within period t
SS_{jpt}	Safety stock of product p at DC j in each review period within period t
Z_{ijst}	Auxiliary variable for cargo weight transported from entity i to entity j in period t in the segment s

Integer variables

Y_{jl}	Binary variable equal to 1 if DC j is open at capacity level l; 0, otherwise
W_{ijst}	Binary variable equal to 1 if the segment s is used to transport cargo between the entities i and j in period t; 0, otherwise.
X_{jkp}	Binary variable equal to 1 if a demand of product p at retailer k is served from DC j; 0, otherwise
X'_{ijp}	Binary variable equal to 1 if the product p at DC j is sourced from plant i; 0, otherwise
$U_{ijkp} = X_{jkp}X'_{ijp}$	Binary auxiliary variable for the model linearization for product p from plant i to DC j and then to retailer k

The multi-echelon supply chain design and inventory management and transportation planning model can be formulated as a mixed-integer nonlinear program (MINLP).

The objective function (1) consists of minimizing the total cost, given by the sum of facility opening costs, inventory holding costs, and transportation costs. The first term comprises the costs related to the selection of DC location and capacity levels. The second to fourth terms correspond to the safety stock, anticipation inventory, and cycle inventory costs, respectively. The total anticipation inventory is calculated over multiple periods, with the average for each period being the mean of its initial and final levels, as shown in Figure 3. In a periodic review system, the average order quantity is equal to the daily average demand multiplied by the number of days in the review period. The average cycle inventory is approximately half of this expected demand over the review period (Nahmias and Olsen 2015). The transportation costs have two components, a variable cost associated with the cargo weight and a fixed cost associated with the segment cost corresponding to that weight, as shown in Figure 4. The fifth and sixth terms represent the variable transportation costs, while fixed transportation costs are represented by the seventh and eighth terms, for both echelons, i.e., from plants to DCs, and from DCs to retailers. Finally, the last two terms of the objective function represent the transportation security costs for both echelons.

$$
\begin{aligned}
\min \Psi \quad = \min \Bigg[& \sum_{j \in \mathcal{I}_w} \sum_{l \in \mathcal{L}} f_{jl} Y_{jl} + \sum_{j \in \mathcal{I}_w} \sum_{p \in \mathcal{P}} \sum_{t \in \Theta} h_{pj} \Big(SS_{jpt} + \frac{I_{jpt-1} + I_{jpt}}{2} + \frac{1}{2} T_{jp} \sum_{k \in \mathcal{I}_c} \mu_{pkt} X_{jkp} \Big) \\
& + \sum_{j \in \mathcal{I}_w} \sum_{s \in \mathcal{S}} \sum_{t \in \Theta} \Big(\sum_{i \in \mathcal{I}_f} c_{ijs} Z_{ijst} + \sum_{k \in \mathcal{I}_c} c_{jks} Z_{jkst} \Big) + \sum_{j \in \mathcal{I}_w} \sum_{s \in \mathcal{S}} \sum_{t \in \Theta} \Big(\sum_{i \in \mathcal{I}_f} g_{ijs} W_{ijst} + \sum_{k \in \mathcal{I}_c} g_{jks} W_{jkst} \Big)
\end{aligned}
$$

$$+ \sum_{j\in\mathcal{I}_w}\sum_{p\in\mathcal{P}}\sum_{t\in\Theta}\Big(\sum_{i\in\mathcal{I}_f} c'_{ij}\rho_p Q_{ijpt} + \sum_{k\in\mathcal{I}_c} c'_{jk}\rho_p\eta_{kt}\mu_{pkt}X_{jkp}\Big)\Bigg] \tag{1}$$

Constraints (2) to (4) define the network structure. Constraints (2) guarantee that the demand of product p at retailer k in period t is served by one DC. Constraints (3) set the relation among the two echelons, plants to DCs, and DCs to retailers. For each product separately, we guarantee that the DC sources the product from a single plant by constraints (4): if DC j is chosen, it must be served by only one plant i, else if the DC is not selected, it is not assigned to any plant.

$$\sum_{j\in\mathcal{I}_w} X_{jkp} = 1, \qquad \forall k\in\mathcal{I}_c, p\in\mathcal{P}. \tag{2}$$

$$\sum_{i\in\mathcal{I}_f} X'_{ijp} \geq X_{jkp}, \qquad \forall j\in\mathcal{I}_w, k\in\mathcal{I}_c, p\in\mathcal{P}. \tag{3}$$

$$\sum_{i\in\mathcal{I}_f} X'_{ijp} \leq \sum_{l\in\mathcal{L}} Y_{jl}, \qquad \forall j\in\mathcal{I}_w, p\in\mathcal{P}. \tag{4}$$

We consider multiple plants in the network. As a result, it becomes more difficult to incorporate the inventory management components because, for a given DC, the lead time for ordering a specific product now depends on which plant is selected to source. Thus, the actual lead times for DCs needed to determine the inventory policies depend on location decisions, rather than being parameters as commonly assumed in the literature. Using the periodic review policy (T, S), the target inventory level and the safety stock for product p at DC j in each review period within period t are defined by $\sum_{i\in\mathcal{I}_f}\sum_{k\in\mathcal{I}_c}(T_{jp}+\ell_{ij})\mu_{pkt}X'_{ijp}X_{jkp}+SS_{jpt}$ and $SS_{jpt}=\Phi_\alpha\sqrt{\sum_{i\in\mathcal{I}_f}\sum_{k\in\mathcal{I}_c}(T_{jp}+\ell_{ij})\sigma^2_{pkt}X'_{ijp}X_{jkp}}$, respectively. These equations are non-linear because of the product of two binary variables and the square root of the safety stock equation, which represents the risk pooling effect (Snyder et al. 2007, Park et al. 2010, Alenezi and Darwish 2014). To linearize the $X'_{ijp}X_{jkp}$ term, let $U_{ijkp} = X'_{ijp}X_{jkp}$. Notice that U_{ijkp} can only be non-zero if both terms in the multiplication are equal to one. Thus, $X'_{ijp} = 0$ and/or $X_{jkp} = 0$ implies that U_{ijkp} must equal zero. This is guaranteed by constraints (5) and (6). Otherwise, $U_{ijkp} = 1$ if $X'_{ijp}X_{jkp} = 1$, which only happens if both terms in the multiplication are equal to one. This is imposed by constraints (7).

$$\sum_{i\in\mathcal{I}_f} U_{ijkp} \leq X_{jkp}, \qquad \forall j\in\mathcal{I}_w, k\in\mathcal{I}_c, p\in\mathcal{P}. \tag{5}$$

$$U_{ijkp} \leq X'_{ijp}, \qquad \forall i\in\mathcal{I}_f, j\in\mathcal{I}_w, k\in\mathcal{I}_c, p\in\mathcal{P}. \tag{6}$$

$$\sum_{i\in\mathcal{I}_f} U_{ijkp} \geq \sum_{i\in\mathcal{I}_f} X'_{ijp} + X_{jkp} - 1, \qquad \forall j\in\mathcal{I}_w, k\in\mathcal{I}_c, p\in\mathcal{P}. \tag{7}$$

The target inventory level and the safety stock for product p at DC j in each review period within the period t are defined by constraints (8) and (9), respectively. S_{jpt} and SS_{jpt} are defined according to the review intervals T_{jp} within the periods t.

$$S_{jpt} = \sum_{i\in\mathcal{I}_f}\sum_{k\in\mathcal{I}_c}(T_{jp}+\ell_{ij})\mu_{pkt}U_{ijkp} + SS_{jpt}, \qquad \forall j\in\mathcal{I}_w, p\in\mathcal{P}, t\in\Theta. \tag{8}$$

$$SS_{jpt} = \Phi_\alpha\sqrt{\sum_{i\in\mathcal{I}_f}\sum_{k\in\mathcal{I}_c}(T_{jp}+\ell_{ij})\sigma^2_{pkt}U_{ijkp}}, \qquad \forall j\in\mathcal{I}_w, p\in\mathcal{P}, t\in\Theta. \tag{9}$$

Constraints (10) represent the inventory balance for every product p, at every DC j in every period t. The order quantity that arrives at DC j of each product p during a period t, originating from all plants, is determined by the total demand assigned to DC j for that product p during period t. Additionally, it includes the balance of anticipation inventory $(I_{jpt} - I_{jp,t-1})$ for the product p during the period t

and $t-1$, as well as, the safety stock for the same product $(SS_{jpt} - SS_{jp,t-1})$. Constraints (11) define the plant capacity constraints for product p at plant i in period t. Constraints (12) represent the DC capacity constraint in period t, if the DC j is chosen to be opened at level l, considering the target inventory of the periodic review policy and the anticipation inventory. This constraint puts a limit on the maximum volume in a DC. Constraints (13) ensure that only one level of capacity is selected for the DC j.

$$\sum_{i\in\mathcal{I}_f} Q_{ijpt} = \sum_{k\in\mathcal{I}_c} \eta_{kt}\mu_{pkt}X_{jkp} + I_{jpt} - I_{jpt-1} + SS_{jpt} - SS_{jp,t-1}, \qquad \forall j \in \mathcal{I}_w, p \in \mathcal{P}, t \in \Theta. \tag{10}$$

$$\sum_{j\in\mathcal{I}_w} Q_{ijpt} \le cap_{ip}, \qquad \forall i \in \mathcal{I}_f, p \in \mathcal{P}, t \in \Theta. \tag{11}$$

$$\sum_{p\in\mathcal{P}} \upsilon_p(S_{jpt} + I_{jpt}) \le \sum_{l\in\mathcal{L}} q_l Y_{jl}, \qquad \forall j \in \mathcal{I}_w, t \in \Theta. \tag{12}$$

$$\sum_{l\in\mathcal{L}} Y_{jl} \le 1, \qquad \forall j \in \mathcal{I}_w. \tag{13}$$

Constraints (14) and (15) define the total cargo weight transported in the two echelons (plant to DC, and DC to retailer) in every period. Constraints (16) guarantee that the cargo shipped between levels corresponds to one of the segments s defined by the breakpoints b_{s-1} and b_s in period t. Constraints (17) guarantee that only one segment s is chosen in each period t between the echelons.

$$\sum_{p\in\mathcal{P}} \omega_p Q_{ijpt} = \sum_{s\in\mathcal{S}} Z_{ijst}, \qquad \forall i \in \mathcal{I}_f, j \in \mathcal{I}_w, t \in \Theta. \tag{14}$$

$$\sum_{p\in\mathcal{P}} \omega_p \eta_{kt}\mu_{pkt}X_{jkp} = \sum_{s\in\mathcal{S}} Z_{jkst}, \qquad \forall j \in \mathcal{I}_w, k \in \mathcal{I}_c, t \in \Theta. \tag{15}$$

$$b_{s-1}W_{ijst} \le Z_{ijst} \le b_s W_{ijst}, \qquad \forall (i,j) \in \mathcal{A}, s \in \mathcal{S}, t \in \Theta. \tag{16}$$

$$\sum_{s\in\mathcal{S}} W_{ijst} \le 1, \qquad \forall (i,j) \in \mathcal{A}, t \in \Theta. \tag{17}$$

Finally, constraints (18) to (27) are integrality and nonnegativity constraints.

$$Y_{jl} \in \{0,1\}, \qquad \forall j \in \mathcal{I}_w, l \in \mathcal{L}. \tag{18}$$

$$X_{jkp} \in \{0,1\}, \qquad \forall j \in \mathcal{I}_w, k \in \mathcal{I}_c, p \in \mathcal{P}. \tag{19}$$

$$X'_{ijp} \in \{0,1\}, \qquad \forall i \in \mathcal{I}_f, j \in \mathcal{I}_w, p \in \mathcal{P}. \tag{20}$$

$$U_{ijkp} \in \{0,1\}, \qquad \forall i \in \mathcal{I}_f, j \in \mathcal{I}_w, k \in \mathcal{I}_c, p \in \mathcal{P}. \tag{21}$$

$$Q_{ijpt} \ge 0, \qquad \forall i \in \mathcal{I}_f, j \in \mathcal{I}_w, p \in \mathcal{P}, t \in \Theta. \tag{22}$$

$$Z_{ijst} \ge 0, \qquad \forall (i,j) \in \mathcal{A}, s \in \mathcal{S}, t \in \Theta. \tag{23}$$

$$W_{ijst} \in \{0,1\}, \qquad \forall (i,j) \in \mathcal{A}, s \in \mathcal{S}, t \in \Theta. \tag{24}$$

$$I_{jpt} \ge 0, \qquad \forall j \in \mathcal{I}_w, p \in \mathcal{P}, t \in \Theta. \tag{25}$$

$$S_{jpt} \ge 0, \qquad \forall j \in \mathcal{I}_w, p \in \mathcal{P}, t \in \Theta. \tag{26}$$

$$SS_{jpt} \ge 0, \qquad \forall j \in \mathcal{I}_w, p \in \mathcal{P}, t \in \Theta. \tag{27}$$

Note that if the network has a single plant or there is a pre-assignment of DCs to plants for the entire planning horizon, the mathematical formulation is reduced to:

$$\min \Psi \quad = \min \ (1) \tag{28}$$

$$\text{s.t.} \quad \text{Constraints}: (2), (10) - (19), (22) - (27). \tag{29}$$

$$S_{jpt} = \sum_{i\in\mathcal{I}_f}\sum_{k\in\mathcal{I}_c} (T_{jp} + \ell_{ij})u_{ijp}\mu_{pkt}X_{jkp} + SS_{jpt}, \qquad \forall j \in \mathcal{I}_w, p \in \mathcal{P}, t \in \Theta. \tag{30}$$

$$SS_{jpt} = \Phi_\alpha \sqrt{\sum_{i\in\mathcal{I}_f}\sum_{k\in\mathcal{I}_c} (T_{jp} + \ell_{ij})u_{ijp}\sigma^2_{pkt}X_{jkp}}, \qquad \forall j \in \mathcal{I}_w, p \in \mathcal{P}, t \in \Theta. \tag{31}$$

where u_{ijp} is a parameter indicating if DC j sources product p from plant i.

4 Benders reformulations

As the original MINLP is intractable, we apply the Benders decomposition (BD) method, a technique for solving large-scale problems with complicating variables that link multiple constraints, to solve this problem. In the BD method, we decompose the problem into a relaxed master problem and smaller sub-problems that are potentially more efficient to solve (Benders 1962). The BD approach allows us to systematically tackle some parts of the decisions (e.g., location) by solving multiple but smaller sub-problems (e.g., transportation) while making sure that the solution and the optimality bound converges, which can be more efficient than solving a single but very large model. The reader is referred to Rahmaniani et al. (2017) for a survey on the BD algorithm. BD method was typically applied to a partitioning of variables that leads to linear programming subproblems, whereas several studies have proposed strategies to deal with integer subproblems (Laporte and Louveaux 1993, Sherali and Fraticelli 2002, Angulo et al. 2016, Fakhri et al. 2017).

Logic-based Benders decomposition (LBBD) is an extension of the BD method, where, unlike the traditional BD approach, the generation of the Benders cuts is not based on the solution of the dual linear programs of the subproblems (Hooker and Ottosson 2003). LBBD is a versatile decomposition technique that is applied successfully to a wide variety of mixed-integer problems, especially when the Benders subproblem is not a linear program. Similar to classical BD, LBBD assigns values to the complicating variables in the master problem and finds the best solution consistent with these values. Instead of solving the dual of the LP subproblems that remain when the complicating variables take fixed values, LBBD solves an inference dual, where proof of optimality within an appropriate logical formalism is derived based on the fixed values of a subset of variables and constraints of the original problem. In most cases, Logic-based Benders cuts are devised specifically for each problem based on the structures of the inference dual and the Benders master problem. The subproblem in LBBD is a Mixed-Integer Programming (MIP) problem. As a result, standard duality theory does not apply, and we cannot derive the typical dual, feasibility, and optimality cuts based on duality. Consequently, the cuts need to be derived based on the structure of the problem. There are two common implementations of the LBBD: the original LBBD implementation, which can be seen as a cutting plane approach, and the branch–and–check implementation (B&Ch), where the cuts are generated and added during the branch–and–bound process (Roshanaei et al. 2017, Martínez et al. 2019, Martínez et al. 2022).

4.1 Basic LBBD

We decompose the problem into a master problem (MP) and a subproblem (SP). In this framework, the location Y_{jl} and allocation decisions, $X_{jkp}, X'_{ijp}, U_{ijkp}$, are part of the MP whereas the SP comprises the remaining variables $(SS_{jpt}, S_{jpt}, I_{jpt}, Q_{ijpt}, Z_{ijst}, W_{ijst})$. After decomposing the problem, we obtain an MP and an SP with integer variables and linear constraints. The MP provides a lower bound for the problem. In this basic LBBD, the master problem (MPS) is modeled as follows:

$$\min \Psi^{MPS} = \min \Bigg[\sum_{j\in\mathcal{I}_w}\sum_{l\in\mathcal{L}} f_{jl}Y_{jl} + \sum_{j\in\mathcal{I}_w}\sum_{p\in\mathcal{P}}\sum_{t\in\Theta} h_{pj}\Big(\frac{1}{2}T_{jp}\sum_{k\in\mathcal{I}_c}\mu_{pkt}X_{jkp}\Big) + \sum_{j\in\mathcal{I}_w}\sum_{p\in\mathcal{P}}\sum_{t\in\Theta}\sum_{k\in\mathcal{I}_c} c'_{jk}\rho_p\eta_{kt}\mu_{pkt}X_{jkp} + \Delta\Bigg] \tag{32}$$

$$\text{s.t.} \quad \text{Constraints}: (2)-(7), (11), (13), (18)-(22), (25). \tag{33}$$

$$\sum_{i\in\mathcal{I}_f} Q_{ijpt} = \sum_{k\in\mathcal{I}_c}\eta_{kt}\mu_{pkt}X_{jkp} + I_{jpt} - I_{jpt-1}, \qquad \forall j\in\mathcal{I}_w, p\in\mathcal{P}, t\in\Theta. \tag{34}$$

$$\sum_{p\in\mathcal{P}} \upsilon_p\Big(\sum_{i\in\mathcal{I}_f}\sum_{k\in\mathcal{I}_c}(T_{jp}+\ell_{ij})\mu_{pkt}U_{ijkp} + I_{jpt}\Big) \le \sum_{l\in\mathcal{L}} q_l Y_{jl}, \qquad \forall j\in\mathcal{I}_w, t\in\Theta. \tag{35}$$

$$\Delta \ge 0. \tag{36}$$

Constraints (34) and Constraints (35) are equivalent to constraints (10) and (12) in the original problem, but the safety stock SS_{jpt} are not considered. Finally, Constraint (36) represents the domain of the variable Δ, which retrieves the safety stock and transportation costs to the MPS, because we drop the safety stock and transportation costs and variables from the objective function (1) and from constraints (10) and (12)). Initially, the lower bound for the Δ variable is zero in the MPS and is updated as optimality cuts are added to the problem. Thus, the mathematical model (32) - (36) still lacks the feasibility and optimality cuts to be defined.

Then, the variables $\bar{Y}_{jl}, \bar{X}_{jkp}, \bar{U}_{ijkp}$ are temporarily fixed in the SP to determine the target inventory S_{jpt}, safety stock SS_{jpt}, anticipation inventory I_{jpt}, order quantity Q_{ijpt}, and segment selection Z_{ijst}, W_{ijst}. Notice that standard SP (SPS) is a mixed-integer linear model because $\bar{U}_{ijkp}$ is fixed.

$$\begin{aligned}\min \Psi^{SPS} \quad &= \min \Bigg[\sum_{j\in\mathcal{I}_w}\sum_{p\in\mathcal{P}}\sum_{t\in\Theta} h_{pj}\left(SS_{jpt} + \frac{I_{jpt-1}+I_{jpt}}{2}\right) \sum_{j\in\mathcal{I}_w}\sum_{s\in\mathcal{S}}\sum_{t\in\Theta}\Big(\sum_{i\in\mathcal{I}_f} c_{ijs}Z_{ijst} + \sum_{k\in\mathcal{I}_c} c_{jks}Z_{jkst}\Big) \\ &+ \sum_{j\in\mathcal{I}_w}\sum_{s\in\mathcal{S}}\sum_{t\in\Theta}\Big(\sum_{i\in\mathcal{I}_f} g_{ijs}W_{ijst} + \sum_{k\in\mathcal{I}_c} g_{jks}W_{jkst}\Big) + \sum_{j\in\mathcal{I}_w}\sum_{p\in\mathcal{P}}\sum_{t\in\Theta}\Big(\sum_{i\in\mathcal{I}_f} c'_{ij}\rho_p Q_{ijpt}\Big)\Bigg] \end{aligned} \tag{37}$$

$$\text{s.t.} \quad \text{Constraints}: (11), (14), (16), (17), (22) - (27). \tag{38}$$

$$S_{jpt} = \sum_{i\in\mathcal{I}_f}\sum_{k\in\mathcal{I}_c}(T_{jp}+\ell_{ij})\mu_{pkt}\bar{U}_{ijkp} + SS_{jpt}, \qquad \forall j\in\mathcal{I}_w, p\in\mathcal{P}, t\in\Theta. \tag{39}$$

$$SS_{jpt} = \Phi_\alpha\sqrt{\sum_{i\in\mathcal{I}_f}\sum_{k\in\mathcal{I}_c}(T_{jp}+\ell_{ij})\sigma^2_{pkt}\bar{U}_{ijkp}}, \qquad \forall j\in\mathcal{I}_w, p\in\mathcal{P}, t\in\Theta. \tag{40}$$

$$\sum_{i\in\mathcal{I}_f} Q_{ijpt} = \sum_{k\in\mathcal{I}_c}\eta_{kt}\mu_{pkt}\bar{X}_{jkp} + I_{jpt} - I_{jpt-1} + SS_{jpt} - SS_{jp,t-1}, \qquad \forall j\in\mathcal{I}_w, p\in\mathcal{P}, t\in\Theta. \tag{41}$$

$$\sum_{p\in\mathcal{P}} \upsilon_p(S_{jpt}+I_{jpt}) \le \sum_{l\in\mathcal{L}} q_l\bar{Y}_{jl}, \qquad \forall j\in\mathcal{I}_w, t\in\Theta. \tag{42}$$

$$\sum_{p\in\mathcal{P}} \omega_p\eta_{kt}\mu_{pkt}\bar{X}_{jkp} \le \sum_{s\in\mathcal{S}} Z_{jkst}, \qquad \forall j\in\mathcal{I}_w, k\in\mathcal{I}_c, t\in\Theta. \tag{43}$$

The objective function (37) minimizes the costs associated with safety stock and anticipation inventory costs, transportation costs, and security costs (from plants to DCs). Constraints (39), (40), (41), (42), and (43) are equivalent to constraints (8), (9), (10), (12), and (15) in the original problem, fixing the variables U_{ijkp}, X_{jkp}, and Y_j according to the MPS solution. After solving the SPS, cuts are added to the MPS to update the cost of the location-allocation decisions in the MPS or to cut off the infeasible location-allocation solutions.

4.1.1 Logic-based cuts

Given a location-allocation $(\bar{Y}, \bar{X}, \bar{U})$ that comprises vectors of the values of variables Y and X obtained by solving the MPS, let

$$\begin{aligned}\Pi(\bar{Y},\bar{X},\bar{U}) = \Bigg[&\sum_{j\in\mathcal{I}_w}\sum_{\substack{l\in\mathcal{L}:\\ \bar{Y}_{jl}=1}}(Y_{jl}-1) + \sum_{j\in\mathcal{I}_w}\sum_{k\in\mathcal{I}_c}\sum_{\substack{p\in\mathcal{P}:\\ \bar{X}_{jkp}=1}}(X_{jkp}-1) + \sum_{k\in\mathcal{I}_c}\sum_{\substack{p\in\mathcal{P}:\\ \bar{U}_{ijkp}=1}}(U_{ijkp}-1) \\ &- \sum_{j\in\mathcal{I}_w}\sum_{\substack{l\in\mathcal{L}:\\ \bar{Y}_{jl}=0}} Y_{jl} - \sum_{j\in\mathcal{I}_w}\sum_{k\in\mathcal{I}_c}\sum_{\substack{p\in\mathcal{P}:\\ \bar{X}_{jkp}=0}} X_{jkp} - \sum_{i\in\mathcal{I}_f}\sum_{j\in\mathcal{I}_w}\sum_{k\in\mathcal{I}_c}\sum_{\substack{p\in\mathcal{P}:\\ \bar{U}_{ijkp}=0}} U_{ijkp}\Bigg]\end{aligned} \tag{44}$$

Note that for a given solution $(\bar{Y},\bar{X},\bar{U})$, $\Pi(\bar{Y},\bar{X},\bar{U}) = 0$. Moreover, note that if the solution $(\bar{Y},\bar{X},\bar{U})$ changes, i.e., if at least one variable with value 1 changes to 0 or one variable with value

0 changes to 1, $\Pi(\bar{Y}, \bar{X}, \bar{U}) < 0$. Consequently, if the solution $(\bar{Y}, \bar{X}, \bar{U})$ is infeasible in the SPS, the valid feasibility cut can be added in the MPS to cut off this solution is:

$$\Pi(\bar{Y}, \bar{X}, \bar{U}) \leq -1 \tag{45}$$

Similarly, a valid optimality cut to the MPS is:

$$\Delta \ \geq \ \bar{\Psi}^{SPS^*}(\bar{Y}, \bar{X}, \bar{U}) \ + \ \bar{\Psi}^{SPS^*}(\bar{Y}, \bar{X}, \bar{U}) \ \Pi(\bar{Y}, \bar{X}, \bar{U}), \tag{46}$$

where $\Psi^{SPS^*}(\bar{Y}, \bar{X}, \bar{U})$ is the cost of the optimal solution of the subproblem SPS. Note that in this case, for solution $(\bar{Y}, \bar{X}, \bar{U})$, $\Delta \geq \bar{\Psi}^{SPS^*}(\bar{Y}, \bar{X}, \bar{U})$ $(\Pi(\bar{Y}, \bar{X}, \bar{U}) = 0)$, thus we update the cost of the solution $(\bar{Y}, \bar{X}, \bar{U})$ in the MPS according to the real cost of the solution in the subproblem SPS. If at least one variable with value 1 changes to 0 or one variable with value 0 changes to 1, $\Pi(\bar{Y}, \bar{X}, \bar{U}) < 0$ and consequently $\bar{\Psi}^{SPS^*}(\bar{Y}, \bar{X}, \bar{U}) + \bar{\Psi}^{SPS^*}(\bar{Y}, \bar{X}, \bar{U}) \ \Pi(\bar{Y}, \bar{X}, \bar{U}) \leq 0$.

4.2 Enhanced LBBD

Since a significant portion of the original model is projected to the SP, the solution can be infeasible or the lower bound obtained by the MPS can be weak. In this section, we describe several enhancements made to improve the solution process of our LBBD approach. The B&Ch algorithm is implemented using the branch-and-bound callbacks of an MIP solver, solving the linear relaxation of the MPE at each node. If the solution is infeasible or does not improve the objective, the node is pruned; otherwise, branching is performed for non-integer solutions. For integer solutions, the SPE checks constraint violations, adding cuts if necessary, and the process repeats until convergence (Moreno et al. 2019). The implementation algorithm for B&Ch is detailed in Appendix A.

4.2.1 Piecewise linear lower bound function of safety stock

The safety stock (SS_{jpt}) is a non-linear function of variance ($\sum_{i\in\mathcal{I}_f}\sum_{k\in\mathcal{I}_c}(T_{jp}+\ell_{ij})\sigma^2_{pkt}U_{ijkp}$), which itself is decision-dependent since it involves the U_{ijkp} variables. In order to estimate the safety stock values, we use a piecewise linear approximation that provides a lower bound of safety stock. This approximation involves dividing the curve of safety stock into multiple linear segments, each representing a specific range of variance values. The accuracy of the approximation is proportional to the number of segments used, and the number of segments used strongly influences the complexity of the problem. Since the segments are always under the curve to be approximated, the piecewise linear function underestimates the real values (Hamer-Lavoie and Cordeau 2006).

We define a set $\mathcal{M}$ that contains all the points marking the beginning and end of a linear segment. Every point in this set $\mathcal{M}$ corresponds to a variance value α_m on the x-axis and its corresponding safety stock value on the y-axis. Consequently, there is a total of $|\mathcal{M}|$ points and $|\mathcal{M}|-1$ segments. Every segment has an upper bound with variance value α_m and its corresponding real safety stock values $f(\alpha_m)$, as shown in Figure 5. We define the continuous variables λ_m and binary variables γ_m.

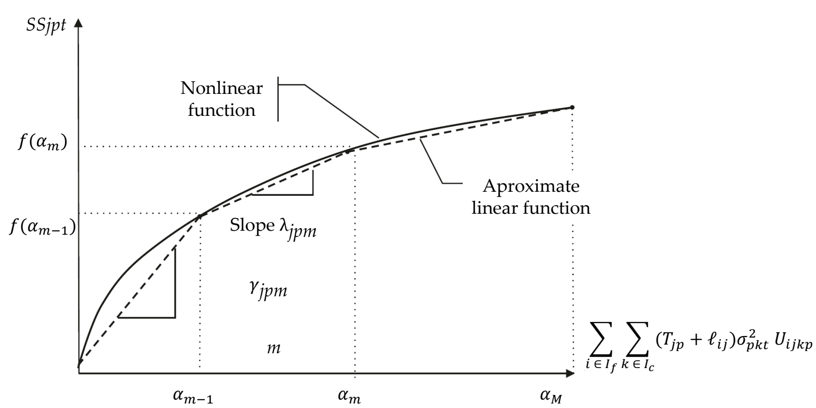

Figure 5: Piecewise linear lower bound function of safety stock

Variables λ_m associated with every point $m \in \mathcal{M}$ represent the weight that the value of the variance of this point will have in the linear approximation of the segment m bounded by the points m and $m+1$. The binary variables γ_m are associated to every segment m in the set $\{0, 1, ..., |\mathcal{M}| - 1\}$, taking the value 1 if the segment m is chosen for the linear approximation. A single γ_m, designating a segment, as well as two bounds λ_m and λ_{m+1}, designating points, must take a strictly positive value. To include the information concerning the DC, product, and period under consideration, the variables λ_{jptm} and λ_{jptm+1} are defined.

The piecewise linear lower bound function of safety stock can be expressed using the following set of constraints:

$$\sum_{i\in\mathcal{I}_f}\sum_{k\in\mathcal{I}_c}(T_{jp}+\ell_{ij})\sigma^2_{pkt}U_{ijkp} - \sum_{m\in\mathcal{M}}\alpha_m\lambda_{jptm} = 0, \quad \forall j\in\mathcal{I}_w, p\in\mathcal{P}, t\in\Theta. \tag{47}$$

$$\sum_{l\in\mathcal{L}}Y_{jl} \geq \sum_{m\in\mathcal{M}}\lambda_{jptm}, \quad \forall j\in\mathcal{I}_w, p\in\mathcal{P}, t\in\Theta. \tag{48}$$

$$X_{jkp} \leq \sum_{m\in\mathcal{M}}\lambda_{jptm}, \quad \forall j\in\mathcal{I}_w, k\in\mathcal{I}_c, p\in\mathcal{P}, t\in\Theta. \tag{49}$$

$$\sum_{l\in\mathcal{L}}Y_{jl} \geq \sum_{m\in\mathcal{M}}\gamma_{jptm}, \quad \forall j\in\mathcal{I}_w, p\in\mathcal{P}, t\in\Theta. \tag{50}$$

$$X_{jkp} \leq \sum_{m\in\mathcal{M}}\gamma_{jptm}, \quad \forall j\in\mathcal{I}_w, k\in\mathcal{I}_c, p\in\mathcal{P}, t\in\Theta. \tag{51}$$

$$\lambda_{jpt1} \leq \gamma_{jpt1}, \quad \forall j\in\mathcal{I}_w, p\in\mathcal{P}, t\in\Theta. \tag{52}$$

$$\lambda_{jptm} \leq \gamma_{jptm-1} + \gamma_{jptm}, \quad \forall j\in\mathcal{I}_w, p\in\mathcal{P}, t\in\Theta, m\in\mathcal{M}\setminus\{0, |\mathcal{M}|\}. \tag{53}$$

$$\lambda_{jpt|\mathcal{M}|} \leq \gamma_{jpt(|\mathcal{M}|-1)}, \quad \forall j\in\mathcal{I}_w, p\in\mathcal{P}, t\in\Theta. \tag{54}$$

$$\lambda_{jptm} \in [0,1], \quad \forall j\in\mathcal{I}_w, p\in\mathcal{P}, t\in\Theta, m\in\mathcal{M}. \tag{55}$$

$$\gamma_{jptm} \in \{0,1\}, \quad \forall j\in\mathcal{I}_w, p\in\mathcal{P}, t\in\Theta, m\in\mathcal{M}\setminus\{|\mathcal{M}|\}. \tag{56}$$

Constraints (47) to (49) determine the value of variables λ_{jptm} corresponding to the obtained demand variance of product p in DC j. Constraints (48) and (50) state that the variables λ_{jptm} and γ_{jptm}, respectively, are equal to zero if the warehouse j is not open. Constraints (49) and (51) force the sum of λ_{jptm} and the sum of γ_{jptm}, respectively, to be one if at least one retailer demand of product p is allocated to a warehouse j in period t. Constraints (52) to (54) link the variables γ_{jptm} and λ_{jptm}. They ensure that λ_{jptm} is strictly positive only if at least one of the adjacent segments described by variables γ_{jptm} and/or γ_{jptm-1} is active. Finally, constraints (55) to (56) represent the domain of the variables. Finally, the constraint to calculate the approximate safety stock can then be expressed as:

$$SS^{prox}_{jpt} = \Phi_\alpha \sum_{m\in\mathcal{M}}\lambda_{jptm}f(\alpha_m), \quad \forall j\in\mathcal{I}_w, p\in\mathcal{P}, t\in\Theta. \tag{57}$$

4.2.2 Enhanced master problem (MPE)

The enhanced master problem MPE incorporates the piecewise linear lower bound function of safety stock, where the complicating nonlinear constraints are replaced by linear constraints (47) to (57). The integrality condition of the γ_{jptm} variables is relaxed. Also, some variables of the subproblem are included in the MPE: auxiliary linear variables for calculating cargo weight Z_{ijst} and segment selection variables W_{ijst}. The MPE is formulated as follows:

$$\begin{aligned}\min \Psi^{MPE} \quad = \min \Bigg[&\sum_{j\in\mathcal{I}_w}\sum_{l\in\mathcal{L}}f_{jl}Y_{jl} + \sum_{j\in\mathcal{I}_w}\sum_{p\in\mathcal{P}}\sum_{t\in\Theta}h_{pj}\Big(\frac{1}{2}T_{jp}\sum_{k\in\mathcal{I}_c}\mu_{pkt}X_{jkp}\Big)\\ &+ \sum_{j\in\mathcal{I}_w}\sum_{p\in\mathcal{P}}\sum_{t\in\Theta}\sum_{k\in\mathcal{I}_c}c'_{jk}\rho_p\eta_{kt}\mu_{pkt}X_{jkp} + \Delta\Bigg]\end{aligned} \tag{58}$$

$$\text{s.t.} \quad \text{Constraints} : (2) - (7), (11), (13) - (26), (47) - (57). \tag{59}$$

$$\sum_{j\in\mathcal{I}_w}\sum_{p\in\mathcal{P}}\sum_{t\in\Theta} h_{pj}\Big(SS^{prox}_{jpt} + \frac{I_{jpt-} + I_{jpt}}{2}\Big)$$
$$+ \sum_{j\in\mathcal{I}_w}\sum_{s\in\mathcal{S}}\sum_{t\in\Theta}\Big(\sum_{i\in\mathcal{I}_f} c_{ijs}Z_{ijst} + \sum_{k\in\mathcal{I}_c} c_{ijs}Z_{ijst}\Big)$$
$$+ \sum_{j\in\mathcal{I}_w}\sum_{s\in\mathcal{S}}\sum_{t\in\Theta}\Big(\sum_{i\in\mathcal{I}_f} g_{ijs}W_{ijst} + \sum_{k\in\mathcal{I}_c} g_{jks}W_{jkst}\Big)$$
$$+ \sum_{i\in\mathcal{I}_f}\sum_{j\in\mathcal{I}_w}\sum_{p\in\mathcal{P}}\sum_{t\in\Theta} c'_{ij}\rho_p Q_{ijpt} \le \Delta. \tag{60}$$

$$S_{jpt} = \sum_{i\in\mathcal{I}_f}\sum_{k\in\mathcal{I}_c}(T_{jp} + \ell_{ij})\mu_{pkt}U_{ijkp} + SS^{prox}_{jpt}, \qquad \forall j\in\mathcal{I}_w, p\in\mathcal{P}, t\in\Theta. \tag{61}$$

$$\sum_{i\in\mathcal{I}_f} Q_{ijpt} = \sum_{k\in\mathcal{I}_c}\eta_{kt}\mu_{pkt}X_{jkp} + I_{jpt} - I_{jpt-1}$$
$$+ SS^{prox}_{jpt} - SS^{prox}_{jpt-1}, \qquad \forall j\in\mathcal{I}_w, p\in\mathcal{P}, t\in\Theta. \tag{62}$$

$$\sum_{p\in\mathcal{P}} \upsilon_p\Big(\sum_{i\in\mathcal{I}_f}\sum_{k\in\mathcal{I}_c}(T_{jp} + \ell_{ij})\mu_{pkt}U_{ijkp} + SS^{prox}_{jpt} + I_{jpt}\Big) \le \sum_{l\in\mathcal{L}} q_l Y_{jl}, \quad \forall j\in\mathcal{I}_w, t\in\Theta. \tag{63}$$

$$SS^{prox}_{jpt} \ge 0, \qquad \forall j\in\mathcal{I}_w, p\in\mathcal{P}, t\in\Theta. \tag{64}$$

$$\Delta \ge 0. \tag{65}$$

Constraints (60) allow computing the variable Δ, which defines the real cost given the decisions considered in the subproblem. The variable Δ is initially zero in the MPE and is updated as optimality cuts are added to the problem. Constraints (61), (62), and (63) are equivalent to the constraints (8), (10), and (12) in the original problem, considering the safety stock approximation. Finally, constraints (64) and (65) represent the domain of variables.

We can provide an initial solution for the master problem, as outlined in Appendix B.

4.2.3 Enhanced subproblem (SPE)

One strategy to reduce the number of variables in the subproblem is to compute a priori the value of some variables. After solving the MPE, we can obtain the value of some variables before solving the SPE as follows. First, let $\bar{S}S_{jpt}$ and $\bar{S}_{jpt}$ be the value of the variables SS_{jpt} and S_{jpt}, respectively, for the location-allocation defined by the MPE. We can calculate $\bar{S}S_{jpt} = \Phi_\alpha\sqrt{\sum_{i\in\mathcal{I}_f}\sum_{k\in\mathcal{I}_c}(T_{jp} + \ell_{ij})\sigma^2_{pkt}\bar{U}_{ijkp}}$ and sequentially $\bar{S}_{jpt} = \sum_{i\in\mathcal{I}_f}\sum_{k\in\mathcal{I}_c}(T_{jp} + \ell_{ij})$ $\mu_{pkt}\bar{U}_{ijkp} + \bar{S}S_{jpt}$, $\forall j\in\mathcal{I}_w, p\in\mathcal{P}, t\in\Theta$. Second, we can also calculate the value of the transportation variables Z_{jkst} and W_{jkst}, related to the transportation between the DC j and the retailer k. At the same time, the subproblem continues optimizing the transportation decisions for the first echelon. Let $\bar{Z}_{jkst}$ and $\bar{W}_{jkst}$ be the value of the variables Z_{jkst} and W_{jkst}, respectively, for the location-allocation solution defined by the MPE. We compute $\bar{Z}_{jkst}$ and $\bar{W}_{jkst}$ as follows:

$$\bar{Z}_{jkst} = \begin{cases} \kappa_{jkt}, s\in\mathcal{S} \text{ if } [b_{s-1} \le \kappa_{jkt} \le b_s], \\ \forall j\in\mathcal{I}_w, k\in\mathcal{I}_c, t\in\Theta. \\ 0, \text{otherwise.} \end{cases}$$

$$\bar{W}_{jkst} = \begin{cases} 1, s\in\mathcal{S} \text{ if } [b_{s-1} \le \kappa_{jkt} \le b_s], \\ \forall j\in\mathcal{I}_w, k\in\mathcal{I}_c, t\in\Theta. \\ 0, \text{otherwise.} \end{cases}$$

where κ_{jkt} is the total weight transported from DC j to retailer k in period t, defined as follows:

$$\kappa_{jkt} = \sum_{p\in\mathcal{P}} \omega_p\eta_{kt}\mu_{pkt}\bar{X}_{jkp}, \qquad \forall j\in\mathcal{I}_w, k\in\mathcal{I}_c, t\in\Theta.$$

In addition, given the single sourcing assumption (imposed separately per product) from plants to DCs, we can express variable X_{jkp} in terms of variable U_{ijkp}, i.e., $\sum_{i\in\mathcal{I}_f} U_{ijkp} = X_{jkp}$, $\forall j \in \mathcal{I}_w, k \in \mathcal{I}_c, p \in \mathcal{P}$. Thus, we only need to determine the value of the variables Y_{jl}, U_{ijkp}, Z_{jkst} and W_{jkst} in the subproblem. The enhanced SPE is modeled as follows:

$$\begin{aligned}
\min \Psi^{SPE} \quad &= \min \Bigg[\sum_{j\in\mathcal{I}_w} \sum_{p\in\mathcal{P}} \sum_{t\in\Theta} h_{pj} + \left(\bar{S}S_{jpt} + \frac{I_{jpt-1} + I_{jpt}}{2} \right) \\
&+ \sum_{j\in\mathcal{I}_w} \sum_{s\in\mathcal{S}} \sum_{t\in\Theta} \sum_{i\in\mathcal{I}_f} \left(c_{ijs} Z_{ijst} + g_{ijs} W_{ijst} \right) + \sum_{i\in\mathcal{I}_f} \sum_{j\in\mathcal{I}_w} \sum_{p\in\mathcal{P}} \sum_{t\in\Theta} c'_{ij} \rho_p Q_{ijpt} \Bigg] \qquad (66)
\end{aligned}$$

$$\begin{aligned}
\text{s.t.} \quad & \text{Constraints} : (11), (14), (16) - (17), (22) - (25), \\
& \text{fixing the variable values } \bar{Z}_{jkst} \text{ and } \bar{W}_{jkst}. \qquad (67)
\end{aligned}$$

$$I_{jpt} - I_{jpt-1} = \sum_{i\in\mathcal{I}_f} Q_{ijpt} - \left[\left(\sum_{i\in\mathcal{I}_f} \sum_{k\in\mathcal{I}_c} \eta_{kt} \mu_{pkt} \bar{U}_{ijkp} + \bar{S}S_{jpt} - \bar{S}S_{jp,t-1} \right) \right], \quad \forall j \in \mathcal{I}_w, p \in \mathcal{P}, t \in \Theta. \qquad (68)$$

$$\sum_{p\in\mathcal{P}} \upsilon_p (\bar{S}_{jpt} + I_{jpt}) \leq \sum_{l\in\mathcal{L}} q_l \bar{Y}_{jl}, \forall j \in \mathcal{I}_w, t \in \Theta. \qquad (69)$$

Constraints (68) and (69) are equivalent to the constraints (10) and (13) in the original problem, fixing the variables $U_{ijkp}, S_{jpt}, SS_{jpt}, Y_{jl}$ according to the MPE solution.

4.2.4 Logic-based cuts

Cuts (45) and (46) are enough to update the real cost of the solution and to cut off infeasible solutions. However, we also introduce additional logic–based inequalities to strengthen the bounds in MP. Let $\bar{Z}_{jkst}$ and $\bar{W}_{jkst}$ be the value of the variables Z_{jkst} and W_{jkst}, respectively, for a given solution. The additional cuts are formulated as follows:

$$Z_{jkst} \geq \bar{Z}_{jkst} - \bar{Z}_{jkst} \Bigg(\sum_{\substack{p\in\mathcal{P}: \\ \bar{X}_{jkp}=1}} (1 - X_{jkp}) + \sum_{\substack{p\in\mathcal{P}: \\ \bar{X}_{jkp}=0}} X_{jkp} \Bigg), \qquad \forall j \in \mathcal{I}_w, k \in \mathcal{I}_c, s \in \mathcal{S}, t \in \Theta. \qquad (70)$$

$$W_{jkst} \geq \bar{W}_{jkst} - \bar{W}_{jkst} \Bigg(\sum_{\substack{p\in\mathcal{P} \\ \bar{X}_{jkp}=1}} (1 - X_{jkp}) + \sum_{\substack{p\in\mathcal{P}: \\ \bar{X}_{jkp}=0}} X_{jkp} \Bigg), \qquad \forall j \in \mathcal{I}_w, k \in \mathcal{I}_c, s \in \mathcal{S}, t \in \Theta. \qquad (71)$$

Unlike cuts (45) and (46), which are single cuts for a given solution, the multiple cuts (70) and (71) apply to every DC, retailer, cost segment, and period. In cut (70), the second term of the right-hand side is equal to zero for the current solution. In this case, the cut forces the Z_{jkst} variable in the MPE to take its real value (real weight). If at least one of the allocation decisions changes, the right-hand side is less than or equal to zero. In that case, cuts (70) do not eliminate any feasible solutions to the original problem. Similarly, we add cuts (71) to strengthen the estimation of W_{jkst} based on the allocation decisions. These cuts are based only on the allocation variables X_{jkp} because this information is enough to define the transportation variables.

4.3 Linearized model APXM

We also test a mathematical model using the piecewise linear lower bound function of safety stock (APXM). This model can be initialized with part of an initial solution, the location and capacity selection, provided by the relaxed model, as presented in Section B. APXM is solved directly using CPLEX, without any tailored algorithm.

$$\begin{aligned}
\min \Psi \quad &= \min \Bigg[\sum_{j\in\mathcal{I}_w} \sum_{l\in\mathcal{L}} f_{jl} Y_{jl} + \sum_{j\in\mathcal{I}_w} \sum_{p\in\mathcal{P}} \sum_{t\in\Theta} h_{pj} \left(SS^{prox}_{jpt} + \frac{I_{jpt-1} + I_{jpt}}{2} + \frac{1}{2} T_{jp} \sum_{k\in\mathcal{I}_c} \mu_{pkt} X_{jkp} \right) \\
&+ \sum_{j\in\mathcal{I}_w} \sum_{s\in\mathcal{S}} \sum_{t\in\Theta} \left(\sum_{i\in\mathcal{I}_f} c_{ijs} Z_{ijst} + \sum_{k\in\mathcal{I}_c} c_{jks} Z_{jkst} \right) + \sum_{j\in\mathcal{I}_w} \sum_{s\in\mathcal{S}} \sum_{t\in\Theta} \left(\sum_{i\in\mathcal{I}_f} g_{ijs} W_{ijst} + \sum_{k\in\mathcal{I}_c} g_{jks} W_{jkst} \right) \\
&+ \sum_{j\in\mathcal{I}_w} \sum_{p\in\mathcal{P}} \sum_{t\in\Theta} \left(\sum_{i\in\mathcal{I}_f} c'_{ij} \rho_p Q_{ijpt} + \sum_{k\in\mathcal{I}_c} c'_{jk} \rho_p \eta_{kt} \mu_{pkt} X_{jkp} \right) \Bigg] \qquad (72)
\end{aligned}$$

s.t. Constraints (2) − (7), (11) − (25), (47) − (57).

$$S_{jpt} = \sum_{i \in \mathcal{I}_f} \sum_{k \in \mathcal{I}_c} (T_{jp} + \ell_{ij}) \mu_{pkt} U_{ijkp} + SS^{prox}_{jpt}, \qquad \forall j \in \mathcal{I}_w, p \in \mathcal{P}, t \in \Theta. \tag{73}$$

$$\sum_{i \in \mathcal{I}_f} Q_{ijpt} = \sum_{k \in \mathcal{I}_c} \eta_{kt} \mu_{pkt} X_{jkp} + I_{jpt} - I_{jpt-1} + SS^{prox}_{jpt} - SS^{prox}_{jp,t-1}, \qquad \forall j \in \mathcal{I}_w, p \in \mathcal{P}, t \in \Theta. \tag{74}$$

$$SS^{prox}_{jpt} \geq 0, \qquad \forall j \in \mathcal{I}_w, p \in \mathcal{P}, t \in \Theta. \tag{75}$$

An accurate solution can be obtained by calculating the real safety stock and its corresponding cost using the allocation demand decisions derived from the approximate model. Subsequently, we update the cost component related to the safety stock within the objective function. Lastly, we verify that the capacity constraints are still satisfied with the updated safety stock. Note that while the approximated model can yield infeasible solutions, the LBBD method always guarantees a feasible solution.

4.4 Sequential approach SQAP

The problem can be solved sequentially (which represents the existing approach employed by the company), starting with the location-allocation problem and then addressing inventory management and transportation planning. The location-allocation problem incorporates information about the original problem, such as cycle inventory, anticipation inventory, variable transportation costs, and capacity constraints, to select the location and capacity levels of DCs. Subsequently, the decisions made during location-allocation, along with those decisions derived from them (safety stock and transportation decisions from DCs to retailers), are fixed to determine the order quantity, anticipation inventory level, and cargo weight from the plant to DCs. Since the location-allocation problem doesn't consider all the constraints, the upper-level decisions may become infeasible at the lower levels. We present the formulations as follows.

Upper level

$$\begin{aligned}
\min \Psi \quad = \min \Bigg[& \sum_{j \in \mathcal{I}_w} \sum_{l \in \mathcal{L}} f_{jl} Y_{jl} + \sum_{j \in \mathcal{I}_w} \sum_{p \in \mathcal{P}} \sum_{t \in \Theta} h_{pj} \Big(\frac{I_{jpt-1} + I_{jpt}}{2} + \frac{1}{2} T_{jp} \sum_{k \in \mathcal{I}_c} \mu_{pkt} X_{jkp} \Big) \\
& + \sum_{j \in \mathcal{I}_w} \sum_{s \in \mathcal{S}} \sum_{t \in \Theta} \Big(\sum_{i \in \mathcal{I}_f} c_{ijs} Z_{ijst} + \sum_{k \in \mathcal{I}_c} c_{jks} Z_{jkst} \Big) + \sum_{j \in \mathcal{I}_w} \sum_{s \in \mathcal{S}} \sum_{t \in \Theta} \Big(\sum_{i \in \mathcal{I}_f} g_{ijs} W_{ijst} + \sum_{k \in \mathcal{I}_c} g_{jks} W_{jkst} \Big) \\
& + \sum_{j \in \mathcal{I}_w} \sum_{p \in \mathcal{P}} \sum_{t \in \Theta} \Big(\sum_{i \in \mathcal{I}_f} c'_{ij} \rho_p Q_{ijpt} + \sum_{k \in \mathcal{I}_c} c'_{jk} \rho_p \eta_{kt} \mu_{pkt} X_{jkp} \Big) \Bigg]
\end{aligned} \tag{76}$$

s.t. Constraints (2) − (7), (11) − (25). (77)

$$\sum_{i \in \mathcal{I}_f} Q_{ijpt} = \sum_{k \in \mathcal{I}_c} \eta_{kt} \mu_{pkt} X_{jkp} + I_{jpt} - I_{jpt-1}, \qquad \forall j \in \mathcal{I}_w, p \in \mathcal{P}, t \in \Theta. \tag{78}$$

Bottom level:

$$\begin{aligned}
\min \Psi \quad = \min \Bigg[& \sum_{j \in \mathcal{I}_w} \sum_{l \in \mathcal{L}} f_{jl} \bar{Y}_{jl} + \sum_{j \in \mathcal{I}_w} \sum_{p \in \mathcal{P}} \sum_{t \in \Theta} h_{pj} \Big(\bar{S}S_{jpt} + \frac{I_{jpt-1} + I_{jpt}}{2} + \frac{1}{2} T_{jp} \sum_{i \in \mathcal{I}_f} \sum_{k \in \mathcal{I}_c} \mu_{pkt} \bar{U}_{ijkp} \Big) \\
& + \sum_{j \in \mathcal{I}_w} \sum_{s \in \mathcal{S}} \sum_{t \in \Theta} \Big(\sum_{i \in \mathcal{I}_f} c_{ijs} Z_{ijst} + \sum_{k \in \mathcal{I}_c} c_{jks} \bar{Z}_{jkst} \Big) + \sum_{j \in \mathcal{I}_w} \sum_{s \in \mathcal{S}} \sum_{t \in \Theta} \Big(\sum_{i \in \mathcal{I}_f} g_{ijs} W_{ijst} + \sum_{k \in \mathcal{I}_c} g_{jks} \bar{W}_{jkst} \Big) \\
& + \sum_{i \in \mathcal{I}_f} \sum_{j \in \mathcal{I}_w} \sum_{p \in \mathcal{P}} \sum_{t \in \Theta} \Big(c'_{ij} \rho_p Q_{ijpt} + \sum_{k \in \mathcal{I}_c} c'_{jk} \rho_p \eta_{kt} \mu_{pkt} \bar{U}_{ijkp} \Big) \Bigg]
\end{aligned} \tag{79}$$

s.t. Constraints : (11), (14), (16) − (17), (22) − (25),
(68) − (69) fixing the variable values $\bar{Z}_{jkst}$ and $\bar{W}_{jkst}$. (80)

5 Computational experiments

In this section, we report the computational performance of the proposed solution method. The aim is to evaluate the efficiency of the tailored solution methods in providing good quality solutions in a reasonable running time. We first analyze and identify the most promising version of the logic-based Benders decomposition and the linearized model. Subsequently, we compare the best versions of every method. In addition, we explore the advantages of integrated planning compared to traditional sequential decision-making approaches. Lastly, we conduct a sensitivity analysis, delving into the effects of different parameter settings on the solution. Instances and detailed results can be accessed in Mendeley data (Jalal et al. 2025).

All algorithms were coded in C++ programming language and run on a PC with an Intel Gold 6148 Skylake processor with 16.0 GB of RAM and a single thread. The MIP and LP models were solved using IBM CPLEX Optimization Solver 20.1. The stopping criterion was due to the elapsed time exceeding the time limit of 12 hours or the optimality gap becoming smaller than 10^{-4}. This maximum time limit is appropriate for this problem as the company could set up a process to run the solution algorithm overnight for this tactical planning problem.

5.1 Data description

This section presents the instances created from the real-world data obtained from the pharmaceutical company. The company produces part of its commercialized products in a plant. Other products are imported from foreign plants and packed in the plant. From this plant, products are sent to DCs managed by logistics operators, from which the company fulfills the demand of retailers all over the country. The company groups the retailers according to demand areas: the capital and countryside of each state.

Aligned with the business process at company, we assume that a one-year planning horizon is appropriate to evaluate the DC location since DC rental agreements are made annually, but demand allocation and shipping decisions should be considered in shorter periods. Hence, we consider 12 months to address tactical decisions of inventory management and transportation planning. The review interval T_{jp} at all DCs is 10 days. The lead-time ℓ_{ij} was calculated considering the distance and the mean velocity of trucks on roads. We create instances by considering different subsets of products comprising up to 100 products. The product's cost ρ_p was assumed to be 40% of the product's price. The mean and variance of the daily demand, μ_{pkt} and σ^2_{pkt}, were defined according to the data provided by the company. We assume the same number of selling days at each retailer $\eta_{kt} = 30$ days. The storage capacity levels q_l were estimated based on the total demand volume, and the opening costs f_{jl} were estimated based on the fixed and operational costs of the installed DCs, i.e., inventory insurance and rental space or volume in DCs that depend on the selected capacity level. The holding costs were calculated based on the cost of \$53.45 per month to store a pallet (120x100x25 cm^3) at room temperature. From this information, the unit cost of inventory per product (h_{pj}) was calculated. Without loss of generality, the initial stocks were considered null at the beginning of the planning horizon. We assume a service level of 95%, this corresponds to $\Phi_\alpha = 1.64$. The fixed and variable costs of transportation g_{ijs} and c_{ijs}, respectively, as well as the breakpoints b_s, were defined based on the transportation tables from carriers.

To establish the piecewise linear function for safety stock, we delineate segments and determine the values of α_m based on the variance observed in retailer demand. Here, $f(\alpha_m)$ represents the square root of α_m. As different products exhibit varying demand scales and variances, we define α_m for each $j \in \mathcal{I}_w, p \in \mathcal{P}, t \in \Theta$, resulting in the parameters α_{jptm} and f_{jptm}. This definition allows adjusting the parameters to approximate the safety stock, enabling the problem to be solved with a small number of segments that more precisely represent the curve for each product, each DC, and each period. This reduces the number of variables we need to adjust the curve in the problem, subsequently reducing the time needed to achieve a good solution.

Table 3 presents the instances' characteristics, i.e., names, cardinality of sets, number of instances, and number of variables and constraints.

Table 3: Instances description

Name	Set cardinality $\lvert\mathcal{T}\rvert$	$\lvert\mathcal{I}_f\rvert$	$\lvert\mathcal{I}_w\rvert$	$\lvert\mathcal{I}_c\rvert$	$\lvert\mathcal{P}\rvert$	$\lvert\mathcal{S}\rvert$	$\lvert\mathcal{L}\rvert$	Instance number	Decision variables Binary	Continuous	Constraints
S1	12	2	3	30	30	5	3	8	14,049	11,160	26,553
S2	12	2	3	52	30	5	3	8	23,949	15,120	42,657
M1	12	2	3	52	40	5	3	8	28,689	16,920	52,327
M2	12	2	3	52	50	5	3	8	33,429	18,720	61,997
L1	12	2	3	52	75	5	3	8	45,279	23,220	86,172
L2	12	2	3	52	100	5	3	8	57,129	27,720	110,347

5.2 Performance of the solutions approaches

In this section, we discuss the numerical results obtained in terms of the performance of the proposed algorithms. The list of the different approaches compared in this section is presented in Table 4.

Table 4: Solution approaches

Name	Description
SLBBD	Basic decomposition method of Section 4.1.
ELBBD	Enhanced LBBD method of Section 4.2.
ELBBDi	ELBBD + initial solution described in Appendix B
ELBBDi+Z	ELBBD + initial solution described in Appendix B + logic cut (70)
ELBBDi+W	ELBBD + initial solution described in Appendix B + logic cut (71)
ELBBDi+ZW	ELBBD + initial solution described in Appendix B + logic cuts (70) and (71)
APXM	Approximated-safety stock model of Section 4.3.
APXMi	APXM + initial solution described in Appendix B.

5.2.1 Performance of the benders decomposition approaches

We first report the results for the small instances S1 and S2. Table 5 presents the upper bound (UB), lower bound (LB), the optimality gap computed as $Gap = 100\frac{UB-LB}{UB}$, the time and number of iterations (#iter) for the SLBBD, the ELBBD, and ELBBDi. Table 6 summarizes the performance of methods, presenting the best UB among the three methods and the gaps computed as $\%LB = 100\frac{Best\ UB-LB}{Best\ UB}$, as well as, the best LB among the three methods and the gaps computed as $\%UB = 100\frac{UB-Best\ LB}{UB}$.

From Tables 5 and 6, it is evident that ELBBDi outperforms other methods both in terms of the quantity and quality of feasible solutions. First, ELBBDi successfully solves all instances, in contrast to ELBBD which cannot cope with seven instances. Furthermore, the average lower bound (LB) achieved by ELBBDi significantly outperforms SLBBD, showcasing a remarkable 57% improvement.

We employ the ELBBDi to evaluate the performance of proposed logic cuts. Table 7 details the count of feasible solutions provided by each method within every instance size. Table 7 also summarizes the performances of the ELBBDi with different enhancements on all the instances, showcasing the average gaps computed as $\%LB = 100 \times \frac{Best\ UB-LB}{Best\ UB}$. In addition, Table 7 shows the $\%UB = 100\frac{UB-Best\ LB}{UB}$ for instances where feasible solutions are attainable through the methods. The ELBBDi+ZW method provides feasible solutions for all instances while also offering good-quality lower bounds.

Table 5: Performance of the decomposition methods for small instances (set S1 and S2)

	Instance	SLBBD UB	SLBBD LB	SLBBD Gap(%)	SLBBD #iter	ELBBD UB	ELBBD LB	ELBBD Gap(%)	ELBBD #iter	ELBBDi UB	ELBBDi LB	ELBBDi Gap(%)	ELBBDi #iter
S1	1	71,604,499	29,384,019	58.96	6,688	-	48,117,000	-	-	54,889,394	48,057,667	**12.45**	618
	2	78,567,411	24,646,425	68.63	6,081	67,061,370	60,400,189	9.93	479	67,014,451	60,886,519	**9.14**	829
	3	73,530,673	28,248,639	61.58	6,027	65,065,049	57,588,638	11.49	690	64,732,667	57,607,286	**11.01**	469
	4	61,439,203	25,574,082	58.37	6,144	-	54,624,100	-	-	64,696,561	54,778,046	**15.33**	1,245
	5	59,857,482	25,514,249	57.38	6,358	58,252,552	52,189,352	**10.41**	817	59,036,543	52,117,231	11.72	993
	6	81,431,940	21,471,740	73.63	5,344	78,238,844	73,395,081	6.19	727	78,238,838	73,588,072	**5.94**	578
	7	86,314,142	37,002,383	57.13	2,748	84,890,001	77,708,632	8.46	186	84,035,374	77,609,551	**7.65**	344
	8	37,086,026	12,267,478	66.92	6,204	36,091,292	29,985,535	16.92	308	35,136,427	29,602,610	**15.75**	696
	Average	68,728,922	25,513,627	62.83	5,699	-	56,751,066	-	-	63,472,532	56,780,873	**11.12**	722
S2	1	107,895,024	49,309,852	54.30	2,884	-	87,315,700	-	-	92,810,515	87,253,308	**5.99**	657
	2	155,551,694	38,873,533	75.01	4,420	109,508,671	102,906,786	6.03	125	109,875,341	103,375,523	**5.92**	198
	3	106,044,018	44,741,705	57.81	1,834	-	96,755,700	-	-	104,184,297	96,883,361	**7.01**	65
	4	98,720,389	40,542,769	58.93	2,745	98,342,991	92,207,776	**6.24**	164	98,343,001	92,199,239	6.25	75
	5	91,395,829	39,842,253	56.41	2,349	-	86,917,500	-	-	96,478,288	86,865,205	**9.96**	109
	6	140,191,097	31,949,676	77.21	3,352	124,780,293	122,447,515	1.87	116	125,300,732	123,021,327	**1.82**	293
	7	143,719,177	59,319,272	58.73	2,415	-	129,983,000	-	-	134,759,215	129,902,307	**3.60**	109
	8	61,192,223	16,414,777	73.18	1,368	-	47,625,300	-	-	57,457,179	47,659,956	**17.05**	178
	Average	113,088,681	40,124,230	63.95	2,671	-	95,769,910	-	-	102,401,071	95,895,028	**7.20**	211

-: No solution obtained.

Table 6: Comparison of performance of the decomposition methods for small instances (set S1 and S2)

	Instance	LB gap (%) comparison Best UB	SLBBD	ELBBD	ELBBDi	UB gap (%) comparison Best LB	SLBBD	ELBBD	ELBBDi
S1	1	54,889,394	46.47	**12.34**	12.45	48,117,000	32.8	-	**12.34**
	2	67,014,451	63.22	9.87	**9.14**	60,886,519	22.5	9.21	**9.14**
	3	64,732,667	56.36	11.04	**11.01**	57,607,286	21.66	11.46	**11.01**
	4	61,439,203	58.37	**11.09**	10.84	54,778,046	**10.84**	-	15.33
	5	58,252,552	56.20	**10.41**	10.53	52,189,352	12.81	**10.41**	11.6
	6	78,238,838	72.56	6.19	**5.94**	73,588,072	9.63	5.94	**5.94**
	7	84,035,374	55.97	**7.53**	7.65	77,708,632	9.97	8.46	**7.53**
	8	35,136,427	65.09	**14.66**	15.75	29,985,535	19.15	16.92	**14.66**
	Average	62,967,363	59.28	**10.39**	10.41	56,780,873	17.42	-	**10.94**
S2	1	92,810,515	46.87	**5.92**	5.99	87,315,700	19.07	-	**5.92**
	2	109,508,671	64.50	6.03	**5.60**	103,375,523	33.54	**5.60**	5.92
	3	104,184,297	57.06	7.13	**7.01**	96,883,361	8.64	-	**7.01**
	4	98,342,991	58.77	**6.24**	6.25	92,207,776	6.60	6.24	**6.24**
	5	91,395,829	56.41	**4.90**	4.96	86,917,500	**4.90**	-	9.91
	6	124,780,293	74.40	1.87	**1.41**	123,021,327	12.25	**1.41**	1.82
	7	134,759,215	55.98	**3.54**	3.60	129,983,000	9.56	-	**3.54**
	8	57,457,179	71.43	17.11	**17.05**	47,659,956	22.11	-	**17.05**
	Average	101,654,874	60.68	6.59	**6.48**	95,920,518	14.58	-	**7.18**

-: No solution obtained.

Table 7: Comparison of performance of proposed logic cuts

Instance	ELBBDi #feas	ELBBDi %LB	ELBBDi %UB	ELBBDi+Z #feas	ELBBDi+Z %LB	ELBBDi+Z %UB	ELBBDi+W #feas	ELBBDi+W %LB	ELBBDi+W %UB	ELBBDi+ZW #feas	ELBBDi+ZW %LB	ELBBDi+ZW %UB
S1	8	9.99	10.25	8	10.09	10.54	8	10.02	11.76	8	9.92	10.06
S2	8	4.76	6.93	8	4.78	6.24	8	4.78	8.89	8	4.73	6.94
M1	8	4.86	4.39	8	4.90	8.00	8	4.86	6.40	8	4.82	6.22
M2	8	3.58	4.74	8	3.69	6.10	8	3.59	3.79	8	3.62	4.30
L1	8	3.35	4.15	8	3.41	3.46	8	3.43	2.92	8	3.41	3.86
L2	7	3.27	3.94	7	3.27	4.92	7	3.23	4.82	8	3.23	4.95
Total	47	4.97	**5.73**	47	5.02	6.54	47	4.99	6.43	**48**	**4.96**	6.06

5.2.2 Performance based on APXM approaches

Table 8 presents, for the small instances S1 and S2, the impact of the initial solution on the APXM method. The warm-up of the APXM method with an initial solution positively affects its performance. Across most instances, APXMi achieves slightly tighter LB gaps than APXM. While APXM does not provide solutions for any instance, APXMi could obtain both the UB and LB for all the instances. The APXMi offers solutions with a piecewise linear lower bound function of safety stock. Consequently, in the APXMi, the safety stock costs are an approximation of the real safety stock costs and the objective function value is an approximation of the total cost. Thus, after solving the APXMi, we compute the real safety stock costs by using the allocation of the optimal (or last incumbent) solution provided by the solver, i.e., $\bar{U}_{ijkp}$ as $\sum_{j\in\mathcal{I}_w}\sum_{p\in\mathcal{P}}\sum_{t\in\Theta} h_{pj}\ (\Phi_\alpha\sqrt{\sum_{i\in\mathcal{I}_f}\sum_{k\in\mathcal{I}_c}(T_{jp}+\ell_{ij})\sigma^2_{pkt}\bar{U}_{ijkp}})$. Finally, we compute the real total costs which we call "Real UB" by updating the safety stock costs.

Table 8: Gap respect to the best upper bound for every instance

	Instance	APXM UB	APXM LB	APXMi Real UB	APXMi APXMi UB	APXMi LB	APXMi Gap(%)	LB gap (%) comparison APXM	LB gap (%) comparison APXMi
S1	1	-	48,660,043	64,404,156	64,024,400	48,904,827	24.07	24.45	**24.07**
	2	-	60,923,269	75,459,026	75,336,285	61,944,679	17.91	19.26	**17.91**
	3	-	57,636,766	72,533,205	72,246,483	58,128,753	19.86	20.54	**19.86**
	4	-	55,054,191	67,203,742	66,926,216	54,850,319	18.38	**18.08**	18.38
	5	-	52,506,817	67,198,894	67,060,509	52,580,726	21.75	21.86	**21.75**
	6	-	73,486,895	87,674,502	87,560,215	73,382,220	16.30	**16.18**	16.30
	7	-	77,999,643	93,955,986	93,657,057	78,517,990	16.43	16.98	**16.43**
	8	-	29,774,344	42,695,858	42,661,630	29,831,189	30.13	30.26	**30.13**
	Average	-	57,005,246	71,390,671	71,184,099	57,267,588	20.60	20.95	**20.60**
S2	1	-	87,449,567	100,131,182	99,959,459	87,344,091	12.77	**12.67**	12.77
	2	-	103,723,600	115,229,236	115,096,588	102,886,030	10.71	**9.98**	10.71
	3	-	96,885,223	108,799,828	108,672,148	96,691,491	11.13	**10.95**	11.13
	4	-	92,512,996	102,962,427	102,837,834	91,896,332	10.75	**10.15**	10.75
	5	-	87,177,007	98,120,187	98,009,932	87,277,198	11.05	11.15	**11.05**
	6	-	121,864,772	130,728,085	130,645,412	123,088,909	5.84	6.78	**5.84**
	7	-	130,082,523	142,397,547	142,234,831	129,506,560	9.05	**8.65**	9.05
	8	-	47,911,134	58,511,996	58,478,613	47,916,933	18.11	18.12	**18.11**
	Average	-	95,950,853	107,110,061	106,991,852	95,825,943	11.18	**11.06**	11.18
								16.00	**15.89**

-: No solution provided.

5.2.3 Performance of ELBBDi and APXMi

In this section, we present the best versions of the methods in the two previous sections, the ELBBDi and the APXMi. Table 9 summarizes the performance of methods for every instance size, presenting the average gaps computed as $\%LB = 100\frac{Best\ UB-LB}{Best\ UB}$, $\%UB = 100\frac{UB-Best\ LB}{UB}$, $\%GAP = 100\times\frac{UB-LB}{UB}$. As outlined in Table 9, the ELBBDi+ZW method consistently achieves the best lower bounds across all sets of instances compared to APXMi.

5.3 The complexity of the multi-plant, multi-period, and multi-product problem

Table 10 displays the average gaps and times of computational experiments comparing single-plant and multi-plant problems for the APXMi (or APXM) and ELBBDi+ZW methods. Similarly, Table 11 presents the results of computational experiments examining single-period and multi-period problems. Finally, Table 12 showcases the results of computational experiments involving single-product and multi-product problems.

The results indicate that our problem presents greater complexity compared to simpler scenarios focusing on single-plant, single-period, or single-product cases. The optimality gaps are lower for single-plant, single-period, and single-product problems in comparison to, respectively, multi-plant, multi-period, and multi-product problems for the methods.

Table 9: Average gap respect to the best upper bound

Instance size	%LB ELBBDi+ZW	%LB APXMi	%UB ELBBDi+ZW	%UB APXMi	%GAP ELBBDi+ZW	%GAP APXMi
S1	9.92	9.20	10.06	20.53	10.86	20.37
S2	4.73	4.78	6.94	10.90	7.19	11.08
M1	4.82	4.40	6.22	9.95	6.78	9.99
M2	3.62	3.38	4.30	8.51	4.77	8.62
L1	3.41	3.21	3.86	6.81	4.32	6.98
L2	3.23	3.00	4.95	6.60	5.25	6.53
Average	4.95	**4.66**	**6.06**	10.55	**6.53**	10.60

According to Table 10, the ELBBDi+ZW method demonstrates effective handling of both single-plant and multi-plant problems, consistently providing feasible solutions across all instances, with lower optimality gaps compared to the APXMi method. According to tables 11 and 12, the APXM method, even without an initial solution to initiate the process, excels in addressing problems characterized by single-period and single-product settings, providing optimal solutions across all instances, while ELBBDi+ZW yields solutions near optimality. In multi-period and multi-product scenarios, ELBBDi+ZW method performs better than the APXMi method.

Table 10: Average gaps and times for APXMi and ELBBDi+ZW considering single plant and multiplant problem

Instance						Single-plant problem				Multi-plant problem			
						APXMi		ELBBDi+ZW		APXMi		ELBBDi+ZW	
$\|\mathcal{T}\|$	$\|\mathcal{I}_w\|$	$\|\mathcal{I}_c\|$	$\|\mathcal{P}\|$	$\|\mathcal{S}\|$	$\|\mathcal{L}\|$	Time(s)	Gap(%)	Time(s)	Gap(%)	Time(s)	Gap(%)	Time(s)	Gap(%)
12	3	30	30	3	5	43,200	5.89	43,200	1.45	43,200	20.37	43,200	10.86
12	3	52	30	3	5	43,200	4.93	43,200	1.67	43,200	11.08	43,200	7.19
12	3	52	40	3	5	43,200	2.76	43,200	1.45	43,200	9.99	43,200	6.78
12	3	52	50	3	5	43,200	4.29	43,200	1.67	43,200	8.62	43,200	4.77
12	3	52	75	3	5	43,200	3.37	43,200	2.32	43,200	7.08	43,200	4.32
12	3	52	100	3	5	43,200	3.08	43,200	2.89	43,200	6.53	43,200	5.25
Average						43,200	4.05	43,200	**1.91**	43,200	10.61	43,200	**6.53**

Table 11: Average gaps and times for APXM/APXMi and ELBBDi+ZW considering single period and multi-period problem

Instance						Single-period problem				Multi-period problem			
						APXM		ELBBDi+ZW		APXMi		ELBBDi+ZW	
$\|\mathcal{I}_f\|$	$\|\mathcal{I}_w\|$	$\|\mathcal{I}_c\|$	$\|\mathcal{P}\|$	$\|\mathcal{S}\|$	$\|\mathcal{L}\|$	Time(s)	Gap(%)	Time(s)	Gap(%)	Time(s)	Gap(%)	Time(s)	Gap(%)
2	3	30	30	3	5	5	0.00	43,200	0.85	43,200	20.37	43,200	10.86
2	3	52	30	3	5	29	0.00	43,200	0.80	43,200	11.08	43,200	7.19
2	3	52	40	3	5	175	0.00	43,200	0.91	43,200	9.99	43,200	6.78
2	3	52	50	3	5	296	0.00	43,200	1.00	43,200	8.62	43,200	4.77
2	3	52	75	3	5	6,928	0.01	43,200	1.08	43,200	7.08	43,200	4.32
2	3	52	100	3	5	27,876	0.01	43,200	1.14	43,200	6.53	43,200	5.25
Average						5,885	**0.00**	43,200	0.96	43,200	10.61	43,200	**6.53**

Table 12: Average gaps and times for APXM/APXMi and ELBBDi+ZW considering single product and multiproduct problem

Instance						Single-product problem				Multi-product problem			
						APXM		ELBBDi+ZW		APXMi		ELBBDi+ZW	
$\lvert\mathcal{T}\rvert$	$\lvert\mathcal{I}_f\rvert$	$\lvert\mathcal{I}_w\rvert$	$\lvert\mathcal{I}_c\rvert$	$\lvert\mathcal{S}\rvert$	$\lvert\mathcal{L}\rvert$	Time(s)	Gap(%)	Time(s)	Gap(%)	Time(s)	Gap(%)	Time(s)	Gap(%)
12	2	3	30	3	5	122	0.00	25,698	3.16	43,200	20.37	43,200	10.86
12	2	3	52	3	5	170	0.00	20,433	0.24	43,200	11.08	43,200	7.19
12	2	3	52	3	5	100	0.00	24,264	0.18	43,200	9.99	43,200	6.78
12	2	3	52	3	5	152	0.00	23,250	0.20	43,200	8.62	43,200	4.77
12	2	3	52	3	5	133	0.00	19,306	0.24	43,200	7.08	43,200	4.32
12	2	3	52	3	5	93	0.00	34,078	0.30	43,200	6.53	43,200	5.25
Average						128	**0.00**	24,505	0.72	43,200	10.61	43,200	**6.53**

5.4 Comparison between the integrated and the sequential model

We compare sequential (SQAP) and integrated (ELBBDi) approaches in terms of cost and computational time. In the sequential approach, the location decision remains fixed, which occasionally leads to infeasible solutions for certain instances. On the other hand, the integrated approach reduces overall solution costs by an average of 0.3% to 9.3%, except in one case, where the sequential model performed 1.35% better. However, it is worth noting that the optimality gap for the integrated solution in this instance was 3.31%. While the integrated approach generally yields lower average costs, it is significantly more computationally challenging to solve, requiring all instances to take 12 hours of computation. In contrast, the sequential approach achieves a much lower average computational time of 930 seconds. The comparison results between the integrated and sequential models are provided in Appendix C.

5.5 Sensitivity analysis

We conduct a sensitivity analysis using instances M2 to evaluate the impact of variations in several network structure parameters, including the number and location of facilities, along with other planning parameters. A higher coefficient of variation increases the optimality gap and total costs due to greater safety stock requirements and inventory costs, with minor variations in transportation and location decisions. Likewise, increasing opening costs affects location decisions, resulting in changes to inventory levels and transportation costs. Fluctuations in inventory costs influence total costs and safety stock levels, with little effect on location and transportation decisions. The results of the sensitivity analysis are found in Appendix D.

6 Conclusions

In this study, we addressed key decisions about the tactical planning of logistics networks under demand variability. We have presented a MINLP model that determines the optimal network structure, transportation, and inventory levels of a multi-period, multi-echelon supply chain. Real data from a pharmaceutical supply chain was used to illustrate the applicability of the proposed model. The model determines the DC locations, shipments from plants to the DCs, and the assignment of retailers to DCs. The model considers the periodic review policy (T, S) in conjunction with anticipation inventory to control inventory in the DCs in each period. The objective is to minimize location costs, transportation costs, and inventory holding costs over the planning horizon.

To solve the problem, we present an LBBD by exploiting the structure of the problem and obtaining subproblems that preserve the characteristics of the original problem. We enhanced the master problem including information about the subproblems and used a multi-cut to accelerate the convergence of the method. We also propose a model that incorporates a piecewise linear lower bound function of safety stock. Real data was examined and used to construct realistic instances to validate the proposed approaches. The method provides good solutions for most instances.

We compare the integrated model with a sequential approach, highlighting the importance of integrating decisions within the supply chain. We empirically show that the integrated model can achieve up to 9% in potential cost savings by utilizing instances derived from real-world data. We also perform a sensitivity analysis aiming to understand how each parameter influences the supply chain design and planning problem. We find that the network design is sensitive to the coefficient of variation and the opening costs.

For future work, it is interesting to consider other inventory policies in the model and compare the implications of different policies on logistics network planning. Another extension is to address capacity planning in networking by considering decisions of closing and opening DCs or expanding or reducing capacity at DCs.

Appendix A ELBBD implementation

The B&Ch algorithm is implemented using the branch-and-bound callbacks of a MIP solver. At each node, we solve the linear relaxation of the current MPE. If it is infeasible or the objective value solution is higher than or equal to the objective value of the incumbent solution, then the node is pruned. Otherwise, integrality constraints are checked, and if the solution is not integer, then branching is performed. If the solution is integer, we solve the subproblem SPE to verify the violation of constraints (45) and (46). Constraint (45) is violated if the subproblem SPE is infeasible. If no constraint is violated, then the solution is feasible for the original LBBD and is set as the new incumbent solution. Constraints (70)–(71) are used to strengthen the bounds of the MPE. Otherwise, the MPE is modified by the addition of Benders cuts, the linear relaxation of the current MPE is resolved, and the described steps are applied again. General-purpose optimization software may additionally rely on automated cuts. The Algorithm A1 presents the B&Ch implementation.

Algorithm A1: B&Ch algorithm

1 Initialization: Initial solution; set $UB = inf$, $LB = 0$, $gap = inf$, $\epsilon = 10^{-4}$;
2 Solve the linear relaxation of MPE and obtain $LB =$ the best overall lower bound of the problem MPE ;
3 Calculate gap $= (UB - LB)/UB$;
4 **if** *gap* $\geq \epsilon$ *& an integer solution* $(\bar{Y}, \bar{U})$ *of the MPE is found* **then**
5   Go to step 9 ;
6 **else**
7   Go to step 15 ;
8 **end**
9 **if** *the solution* $(\bar{Y}, \bar{X}, \bar{U})$ *violates feasibility or optimality cuts* **then**
10   Generate and add feasibility or optimality cuts ;
11   Go to step 3 ;
12 **else**
13   Update UB ;
14 **end**
15 **if** *gap* $\leq \epsilon$ **then**
16   Stop ;
17 **end**
18 The algorithm is repeated in the next node selected by the Branch-and-bound ;

Appendix B Initial solution approach

In addition, we initialized the method with part of an initial solution: the location and capacity selection provided by solving the following reduced relaxed model:

$$\min \Psi \quad = \min \Bigg[\sum_{j \in \mathcal{I}_w} \sum_{l \in \mathcal{L}} f_{jl} Y_{jl} + \sum_{j \in \mathcal{I}_w} \sum_{p \in \mathcal{P}} \sum_{t \in \Theta} h_{pj} \Big(\frac{I_{jpt-1} + I_{jpt}}{2} + \frac{1}{2} T_{jp} \sum_{k \in \mathcal{I}_c} \mu_{pkt} X_{jkp} \Big)$$

$$
\begin{aligned}
&+ \sum_{j\in\mathcal{I}_w}\sum_{s\in\mathcal{S}}\sum_{t\in\Theta}\Big(\sum_{i\in\mathcal{I}_f} c_{ijs}Z_{ijst} + \sum_{k\in\mathcal{I}_c} c_{jks}Z_{jkst}\Big) \\
&+ \sum_{j\in\mathcal{I}_w}\sum_{s\in\mathcal{S}}\sum_{t\in\Theta}\Big(\sum_{i\in\mathcal{I}_f} g_{ijs}W_{ijst} + \sum_{k\in\mathcal{I}_c} g_{jks}W_{jkst}\Big) \\
&+ \sum_{j\in\mathcal{I}_w}\sum_{p\in\mathcal{P}}\sum_{t\in\Theta}\Big(\sum_{i\in\mathcal{I}_f} c'_{ij}\rho_p Q_{ijpt} + \sum_{k\in\mathcal{I}_c} c'_{jk}\rho_p\eta_{kt}\mu_{pkt}X_{jkp}\Big)\Bigg] \qquad \text{(B1)}
\end{aligned}
$$

$$
\text{s.t.} \quad \text{Constraints (2)} - (7), (11), (13) - (25). \qquad \text{(B2)}
$$

$$
\sum_{i\in\mathcal{I}_f} Q_{ijpt} = \sum_{k\in\mathcal{I}_c} \eta_{kt}\mu_{pkt}X_{jkp} + I_{jpt} - I_{jpt-1}, \qquad \forall j\in\mathcal{I}_w, p\in\mathcal{P}, t\in\Theta. \qquad \text{(B3)}
$$

$$
\sum_{p\in\mathcal{P}} \upsilon_p\Big(\sum_{i\in\mathcal{I}_f}\sum_{k\in\mathcal{I}_c}(T_{jp}+\ell_{ij})\mu_{pkt}U_{ijkp} + I_{jpt}\Big) \le \sum_{l\in\mathcal{L}} q_l Y_{jl}, \qquad \forall j\in\mathcal{I}_w, t\in\Theta. \qquad \text{(B4)}
$$

In this model, we drop the safety stock from the original order quantity and capacity constraints, as shown in constraints (B3) and (B4). These constraints are equivalent to constraints (10) and (13) in the original problem.

The solution provided by this model needs verification by calculating the safety stock level and assessing the capacity of the DCs. If the capacity is confirmed to be sufficient, we have an initial solution to the original problem, and the objective function value is updated accordingly. If the DC capacity is insufficient, the problem is re-evaluated by integrating the safety stock from the previous solution into the capacity constraint, followed by a reassessment of the capacity. This iterative procedure is repeated until a feasible solution is identified or a specified number of iterations is reached. If the maximum number of iterations is attained without achieving a feasible solution, the problem is addressed by considering the maximum safety stock level. This approach guarantees a feasible solution by computing the maximum safety stock for each DC, product, and period. It ensures the fulfillment of all retailer demands exclusively from that DC while accounting for the longest lead time from the plant to the DC.

Appendix C Comparison between the integrated and the sequential model

We carried out computational experiments to study the impact of integrating inventory decisions with the network design problem. We compare the sequential and integrated approaches in terms of cost and computational time. Table C1 presents, for two approaches (ELBBDi and SQAP), the costs of location, inventory, transportation, and total costs, as well as the computational time and the ratio between the total costs of the approaches that are computed as $ratio = 100\frac{SQAP-ELBBDi}{ELBBDi}$. ELBBDi represents the integrated approach, SQAP represents the sequential approach, in which the location decision is fixed in the inventory-transportation problem as described in Section 4.4.

Within the sequential approach, the location decision does not change and occasionally yields infeasible solutions for certain instances. Notice in Table C1 that the integrated approach reduces the overall solution cost by an average ranging from 0.3% to 9.3%, even when the ELBBDi+ZW method does not achieve optimality for the instances. In the case of one particular instance, namely instance 5 in L2, the sequential model exhibited superior performance compared to the integrated method, with a cost reduction of 1.35%, however, it is important to note that the optimality gap associated with the solution offered by ELBBDi+ZW for that specific instance was to 3.31%. In summary, as depicted in Table C1, the integrated problem provides overall lower average costs. Nevertheless, solving the integrated approach proves to be more difficult, as all instances report a computational time of 12 hours. In contrast, the SQAP demonstrates a significantly lower average time of 930 seconds.

Table C1: Comparison between the sequential SQAP and integrated approach ELBBDi

Instance		SQAP				ELBBDi				Ratio
		Location	Inv+Transp	Total cost	Time	Location	Inv+Transp	Total cost	Time	
S1	1	-	-	-	-	13,050,000	42,794,152	55,844,152	43,200	-
	2	-	-	-	-	12,110,000	54,112,674	66,222,674	43,200	-
	3	-	-	-	-	13,050,000	51,490,893	64,540,893	43,200	-
	4	-	-	-	-	13,050,000	47,950,866	61,000,866	43,200	-
	5	-	-	-	-	12,110,000	46,080,028	58,190,028	43,200	-
	6	-	-	-	-	13,050,000	65,927,893	78,977,893	43,200	-
	7	-	-	-	-	12,110,000	71,925,380	84,035,380	43,200	-
	8	-	-	-	-	13,100,000	22,797,985	35,897,985	43,200	-
S2	1	-	-	-	-	19,650,000	80,481,886	100,131,886	43,200	-
	2	13,100,000	103,900,060	117,000,060	637	13,100,000	93,904,598	107,004,598	43,200	9.34
	3	13,100,000	98,201,888	111,301,888	1,089	13,050,000	90,289,962	103,339,962	43,200	7.70
	4	13,100,000	93,238,702	106,338,702	885	13,050,000	85,434,290	98,484,290	43,200	7.98
	5	13,100,000	88,291,292	101,391,292	755	19,650,000	78,568,807	98,218,807	43,200	3.23
	6	13,100,000	121,675,755	134,775,755	627	13,100,000	111,680,293	124,780,293	43,200	8.01
	7	13,100,000	131,654,677	144,754,677	1,144	13,100,000	121,849,249	134,949,249	43,200	7.27
	8	-	-	-	-	13,050,000	39,924,449	52,974,449	43,200	-
M1	1	13,100,000	109,405,275	122,505,275	697	19,650,000	100,405,313	120,055,313	43,200	2.04
	2	13,100,000	152,982,022	166,082,022	707	13,100,000	142,986,560	156,086,560	43,200	6.40
	3	13,100,000	99,963,250	113,063,250	923	19,650,000	91,001,740	110,651,740	43,200	2.18
	4	13,100,000	170,882,393	183,982,393	806	13,100,000	160,875,140	173,975,140	43,200	5.75
	5	13,100,000	66,911,216	80,011,216	1,919	19,650,000	58,924,443	78,574,443	43,200	1.83
	6	13,100,000	130,159,124	143,259,124	978	13,100,000	120,163,661	133,263,661	43,200	7.50
	7	13,100,000	103,803,518	116,903,518	658	13,100,000	93,808,056	106,908,056	43,200	9.35
	8	13,100,000	123,336,993	136,436,993	739	13,100,000	113,341,531	126,441,531	43,200	7.91
M2	1	13,100,000	144,441,105	157,541,105	826	13,100,000	136,179,696	149,279,696	43,200	5.53
	2	13,100,000	150,345,052	163,445,052	896	13,100,000	140,349,589	153,449,589	43,200	6.51
	3	13,100,000	133,159,603	146,259,603	764	13,100,000	123,164,141	136,264,141	43,200	7.34
	4	13,100,000	149,490,526	162,590,526	794	17,030,000	140,350,642	157,380,642	43,200	3.31
	5	13,100,000	135,966,792	149,066,792	881	13,100,000	127,049,804	140,149,804	43,200	6.36
	6	13,100,000	130,272,372	143,372,372	780	19,650,000	121,009,641	140,659,641	43,200	1.93
	7	13,100,000	159,326,639	172,426,639	758	13,100,000	149,452,530	162,552,530	43,200	6.07
	8	13,100,000	172,681,320	185,781,320	740	13,100,000	162,685,858	175,785,858	43,200	5.69
L1	1	13,100,000	224,038,363	237,138,363	821	13,100,000	214,042,901	227,142,901	43,200	4.40
	2	13,100,000	181,211,317	194,311,317	862	13,100,000	171,215,854	184,315,854	43,200	5.42
	3	13,100,000	240,040,359	253,140,359	917	19,650,000	232,462,316	252,112,316	43,200	0.41
	4	13,100,000	214,630,327	227,730,327	828	13,100,000	204,634,865	217,734,865	43,200	4.59
	5	13,100,000	197,468,787	210,568,787	844	13,100,000	187,473,324	200,573,324	43,200	4.98
	6	13,100,000	249,097,928	262,197,928	785	13,100,000	239,102,465	252,202,465	43,200	3.96
	7	13,100,000	210,824,891	223,924,891	801	19,650,000	201,292,023	220,942,023	43,200	1.35
	8	13,100,000	166,689,557	179,789,557	1,362	13,100,000	156,694,094	169,794,094	43,200	5.89
L2	1	13,100,000	272,090,076	285,190,076	1,166	13,100,000	262,094,614	275,194,614	43,200	3.63
	2	13,100,000	259,954,050	273,054,050	1,420	19,650,000	251,071,166	270,721,166	43,200	0.86
	3	13,100,000	283,232,979	296,332,979	954	19,650,000	275,227,760	294,877,760	43,200	0.49
	4	13,100,000	269,385,423	282,485,423	967	13,100,000	259,389,961	272,489,961	43,200	3.67
	5	13,100,000	269,564,996	282,664,996	1,205	19,650,000	266,887,684	286,537,684	43,200	-1.35
	6	13,100,000	316,919,033	330,019,033	1,159	13,100,000	306,923,570	320,023,570	43,200	3.12
	7	13,100,000	264,538,137	277,638,137	1,203	13,100,000	254,542,675	267,642,675	43,200	3.73
	8	13,100,000	303,816,545	316,916,545	1,042	19,650,000	296,272,242	315,922,242	43,200	0.31

$\text{Ratio}=100\times\frac{\text{SQAP UB}-\text{ELBBDi UB}}{\text{ELBBDi UB}}$

-: Infeasible

Appendix D Sensitivity analysis

In this section, we present a sensitivity analysis using instances M2 to assess the impact of variations in several network structure parameters, such as the number and location of facilities, along with other planning parameters. We analyze the coefficient of variation and decision costs, including opening, inventory, and transportation costs. We report the average optimality gap, objective function costs, inventory levels, Key Performance Indicators (KPIs) for location decisions, and transportation decision statistics for sensitivity analysis.

For the location decision KPIs, we present the average values across instances M2. The installed capacity is defined as the selected capacity level for each opened DC, multiplied by the number of periods in the planning horizon. The percentage of used capacity is calculated as the ratio of used capacity to installed capacity. To determine used capacity, we accumulate the capacity utilization for each product at each location and for each period. This involves multiplying the product's volume by target and anticipation inventory levels. Regarding transportation statistics, we report the average values across instances M2. Thus, "Max weight plant-DC" represents the average of the highest cargo weights observed between plants and DCs over different periods across instances M2. Similarly, "Avg weight plant-DC" and "Min weight plant-DC" refer to the average and lowest cargo weights transported from any plant to a distribution center across instances M2, respectively. These statistics are also provided for transportation from DCs to retailers.

D.1 Coefficient of variation

This analysis is related to the parameter coefficient of variation, i.e., the ratio of the standard deviation to the mean demand. Table D2 presents the average gap and costs, KPIs, and statistics for different values of the coefficient of variation.

Table D2: Variation on the coefficient of variation

		Coefficient of variation			
		20%	50%	80%	100%
	Optimality gap	3.24%	3.47%	4.77%	5.15%
Objective function costs	Location costs	15,228,750	13,918,750	14,410,000	14,035,714
	Inventory costs	43,112,716	45,731,986	49,018,755	50,569,427
	Transportation costs	74,462,962	74,633,292	74,735,677	76,176,630
	Security costs	13,719,023	13,758,254	13,775,806	13,738,506
	Total costs	146,523,451	148,042,282	151,940,238	154,520,277
Inventory costs	Safety stock costs	2,553,084	5,173,621	8,459,384	10,095,059
	Anticipation inv costs	1,256	0	1,256	0
	Cycle inv costs	40,558,376	40,558,366	40,558,115	40,474,369
Inventory levels	Safety stock units	401,349	914,027	1,514,738	1,933,155
	Anticipation inv units	3	0	3	0
	Cycle inv units	7,516,518	7,516,518	7,516,518	7,834,600
	Total units	7,917,869	8,430,545	9,031,258	9,767,755
Location KPIs	Total Opened DCs	2	2	2	2
	Installed capacity	5,067	4,766	4,870	4,929
	% Used capacity	47.33%	50.80%	51.51%	52.19%
Transportation statistics	Max weight plant-DC	108,333,995	108,437,091	108,557,900	111,548,106
	Avg weight plant-DC	72,318,739	78,099,573	76,561,731	77,635,872
	Min weight plant-DC	24,172,115	27,517,408	23,979,899	29,432,119
	Max weight DC-retailer	108,251,166	108,251,166	108,251,166	111,166,690
	Avg weight DC-retailer	2,501,016	2,489,927	2,501,016	2,572,631
	Min weight DC-retailer	971,184	1,268,303	740,156	1,223,541

Notice in Table D2 that the coefficient of variation increases the average optimality gap, thus the problem seems to be more difficult to solve. The total objective cost also increases considering a higher coefficient of variation. This increase in the total cost is observed because of an increment in the inventory cost. As expected, the levels of safety stock increase, and consequently the safety stock costs increase. Notice in Table D2 that opening costs change because different DCs are opened. The percentage of used capacity varies slightly from 47% to 52%, due to the total installed capacity changes and the safety stock increases. Table D2 also shows changes in the average and minimum weight of the goods transported between plants and DCs, which suggests the use of transportation segments with different capacities. Although the retailer demand does change, the minimum weight among DCs and retailers varies. It can be explained due to the changed location decisions, so the transported cargo weight among arcs changes.

D.2 Opening costs variation

This analysis is related to the parameter of the opening costs. The model is tested for this parameter on +/- 50% of its initial value. Table D3 presents the average gap and costs, KPIs, and statistics for different values of opening costs. As expected, the variation of opening costs directly affects the total opening costs, changing the location decisions. When opening costs are high, inventory units are increased, and transportation decisions are changed, resulting in higher transportation expenses.

Table D3: Variation on Opening costs

		Variation of opening costs		
		50%	100%	150%
	Optimality gap	3.64%	4.77%	9.75%
Objective function costs	Location costs	7,368,750	14,410,000	27,018,750
	Inventory costs	49,183,323	49,018,755	50,344,468
	Transportation costs	74,063,627	74,735,677	74,851,850
	Security costs	13,735,148	13,775,806	13,726,489
	Total costs	144,350,848	151,940,238	165,941,557
Inventory costs	Safety stock costs	8,623,472	8,459,384	9,785,938
	Anticipation inv costs	1,256	1,256	0
	Cycle inv costs	40,558,595	40,558,115	40,558,529
Inventory levels	Safety stock units	1,581,670	1,514,738	1,664,895
	Anticipation inv units	3	3	0
	Cycle inv units	7,516,518	7,516,518	7,516,518
	Total units	9,098,190	9,031,258	9,181,413
Location KPIs	Total Opened DCs	2	2	3
	Installed capacity	5,316	4,870	6,158
	% Used capacity	49.70%	51.51%	40.95%
Transportation statistics	Max weight plant-DC	108,566,230	108,557,900	108,591,975
	Avg weight plant-DC	70,720,551	76,561,731	61,063,180
	Min weight plant-DC	27,537,186	23,979,899	26,332,165
	Max weight DC-retailer	108,251,166	108,251,166	108,251,166
	Avg weight DC-retailer	2,501,016	2,501,016	2,501,016
	Min weight DC-retailer	1,460,216	740,156	431,610

D.3 Inventory costs variation

This analysis is related to the parameter of the inventory costs. The model is tested for this parameter on +50% and +100% of its initial value. Table D4 presents the average gap and costs, KPIs, and statistics for different values of inventory costs. The variation of inventory costs directly affects the total inventory costs and slightly affects the location costs that increase and the transportation costs that decrease.

D.4 Transportation costs variation

This analysis is related to the parameter of the transportation costs. The model is tested for this parameter from 0% to 200% of its initial value by increments of 50%. Table D5 presents the average gap and costs, KPIs, and statistics for different values of transportation costs. The variation in transportation costs directly affects the total transportation costs. It also affects directly other decisions, such as the total inventory units which increase.

Table D4: Sensitivity analysis: variation on inventory costs

		Variation of inventory costs			
		50.00%	100.00%	150.00%	200.00%
	Optimality gap	3.23%	4.77%	4.76%	6.03%
Objective function costs	Location costs	13,918,750	14,410,000	13,918,750	14,737,500
	Inventory costs	24,494,351	49,018,755	73,455,603	98,599,936
	Transportation costs	74,277,429	74,735,677	74,402,599	74,548,397
	Security costs	13,751,342	13,775,806	13,760,303	13,744,692
	Total costs	126,441,872	151,940,238	175,537,254	201,630,525
Inventory costs	Safety stock costs	4,208,157	8,459,384	12,631,048	17,505,238
	Anticipation inv costs	0	1,256	0	0
	Cycle inv costs	20,286,194	40,558,115	60,824,555	81,094,699
Inventory levels	Safety stock units	1,468,614	1,514,738	1,469,846	1,491,236
	Anticipation inv units	0	3	0	0
	Cycle inv units	7,516,518	7,516,518	7,516,518	7,516,518
	Total units	8,985,132	9,031,258	8,986,363	9,007,753
Location KPIs	Total Opened DCs	2	2	2	2
	Installed capacity	4,859	4,870	4,859	5,052
	% Used capacity	51.84%	51.51%	51.84%	49.68%
Transportation statistics	Max weight plant-DC	108,549,021	108,557,900	108,549,256	108,555,768
	Avg weight plant-DC	77,257,843	76,561,731	76,728,013	74,113,732
	Min weight plant-DC	28,416,601	23,979,899	28,598,884	28,669,745
	Max weight DC-retailer	108,251,166	108,251,166	108,251,166	108,251,166
	Avg weight DC-retailer	2,501,016	2,501,016	2,501,016	2,501,016
	Min weight DC-retailer	1,601,528	740,156	1,351,733	1,117,602

Table D5: Sensitivity analysis: variation on transportation costs

		Variation of transportation costs				
		0%	50%	100%	150%	200%
	Optimality gap	19.23%	8.61%	4.77%	3.40%	3.00%
Objective function costs	Location costs	18,256,562	15,550,000	14,410,000	13,918,750	14,737,500
	Inventory costs	50,465,621	49,534,208	49,018,755	48,951,548	49,143,390
	Transportation costs	0	38,929,361	74,735,677	111,978,321	147,625,882
	Security costs	13,861,818	13,853,872	13,775,806	13,763,288	13,745,449
	Total costs	82,584,002	117,867,441	151,940,238	188,611,907	225,252,220
Inventory costs	Safety stock costs	9,900,911	8,975,088	8,459,384	8,393,272	8,584,649
	Anticipation inv costs	5,207	1,256	1,256	0	0
	Cycle inv costs	40,559,502	40,557,864	40,558,115	40,558,276	40,558,741
Inventory levels	Safety stock units	1,615,337	1,581,360	1,514,738	1,468,871	1,578,054
	Anticipation inv units	258	3	3	0	0
	Cycle inv units	7,516,518	7,516,518	7,516,518	7,516,518	7,516,518
	Total units	9,132,113	9,097,881	9,031,258	8,985,388	9,094,572
Location KPIs	Total Opened DCs	3	2	2	2	2
	Installed capacity	5,840	5,447	4,870	4,868	5,263
	% Used capacity	43.85%	47.41%	51.51%	51.77%	49.68%
Transportation statistics	Max weight plant-DC	108,582,925	108,570,905	108,557,900	108,549,576	108,564,541
	Avg weight plant-DC	60,241,314	67,379,417	76,561,731	76,906,998	70,826,822
	Min weight plant-DC	18,562,891	28,928,828	23,979,899	28,099,709	27,827,059
	Max weight DC-retailer	108,251,166	108,251,166	108,251,166	108,251,166	108,251,166
	Avg weight DC-retailer	2,390,389	2,497,753	2,501,016	2,497,698	2,495,322
	Min weight DC-retailer	11,954	828,281	740,156	1,263,301	1,025,019

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