Les Cahiers du GERAD ISSN: 0711–2440

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G–2025–05

Janvier 2025

La publication de ces rapports de recherche est rendue possible grâce au soutien de HEC Montréal, Polytechnique Montréal, Université McGill, Université du Québec à Montréal, ainsi que du Fonds de recherche du Québec – Nature et technologies.

Dépôt légal – Bibliothèque et Archives nationales du Québec, 2025
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The publication of these research reports is made possible thanks to the support of HEC Montréal, Polytechnique Montréal, McGill University, Université du Québec à Montréal, as well as the Fonds de recherche du Québec – Nature et technologies.

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A globally convergent observer for estimating sphere structures with inertial measurements

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Abstract : In this paper, we address the problem of online estimation of spherical features in the field of camera for robotics. Specifically, we consider a mobile robot equipped with inertial measurement units (IMUs) – providing linear acceleration and rotational velocity measurements in the body-fixed frame – and a pinhole camera that projects 3D points onto the image plane. To tackle this problem, we adopt the parameter estimation-based observer (PEBO) approach on manifolds to design a feature observer. Under a sufficient excitation condition, our design guarantees a globally exponentially convergent estimate of both the radius and the center coordinates of spherical targets. Simulation results validate the theoretical analysis and demonstrate the performance of the proposed feature observer.

Keywords : Observers Design, nonlinear systems, visual estimation, robotics

Acknowledgements: This work was supported in part by the Natural Sciences and Engineering Research Council of Canada (NSERC) and the Programme PIED of Polytechnique Montréal.

1 Introduction

Accurate online estimation of visual features from images taken by a camera on a mobile robot is a fundamental topic in robotics, computer vision, and control engineering. In particular, consistent estimation of visual features, such as pose, depth, and size of objects in the environment, is critical for applications like navigation [18], object tracking [13], and simultaneous localization and mapping (SLAM) [25].

The existing approaches to this type of estimation problem can be broadly classified into smoothing and filtering methods. In smoothing methods, online state estimation is formulated as a batch optimization, which generally provides higher accuracy by considering the entire set of historical measurements [5]. However, this approach comes with significant scalability issues, as it requires substantial memory and computational resources. In contrast, filtering approaches solve the estimation problem recursively, which remain very popular in many robotic applications [16].

In this paper, we follow the second category of approaches to estimate 3D features. A large body of literature has explored the use of various Kalman filters to achieve efficient online estimation [8]. These methods, particularly extended Kalman filters (EKFs), have been widely adopted due to their ability to recursively process sensor measurements and provide real-time state estimates. However, EKFs often suffer from a key limitation: they only provide locally convergent solutions and thus require an accurate initial guess. This is due to the highly nonlinear nature of the output function of a planar camera. In recent years, there has been significant interest within the control community in addressing this challenge, leading to the development of various globally convergent observers, e.g. [3, 4, 6, 14, 21]. In [17], the authors provide a general feature observer framework to three types of 3D structures, including points, spheres, and cylinders.

In most existing works, it is typically assumed that the robot has direct access to its linear and rotational velocities in the body-fixed frame. However, in practice, a mobile robot is usually equipped with inertial measurement units (IMUs), which only provide inertial measurements, such as linear accelerations and rotational velocities, rather than the velocities themselves. As pointed out in [21], the main technical challenge in this case arises from the unknown orientation matrix, which lives on the Lie group $SO(3)$, and its appearance in the dynamics. Our previous work [21] presented the first solution in the literature to such a problem for feature points. In this paper, we aim to extend this approach to tackle spherical visual features.

Our design is based on the recently proposed parameter estimation-based observer (PEBO) [9, 10], and its extension to matrix Lie groups [21, 22, 24]. The fundamental idea behind PEBO is to design a dynamical extension that transforms the estimation of time-varying system states into the estimation of constant parameters. This approach has shown great success in solving many open problems in nonlinear observer design. The interested reader may refer to [20] for its geometric interpretation, and to [23] for its connections to the widely popular IMU preintegration approach in robotics [7]. The main contribution of this paper is to extend our previous approach [21] from feature points to 3D sphere targets with only inertial measurements and images, leading to a simple spherical feature observer design. Under certain persistency of excitation (PE) conditions, the proposed design ensures global exponential convergence.

Notation. Throughout the paper, the arguments and subscripts of functions or signals are omitted when clear from context. We use generally ϵ_t to represent exponentially decaying terms with proper dimensions. $0_n \in \mathbb{R}^n$ and $0_{n\times m} \in \mathbb{R}^{n\times m}$ denote the zero column vector of dimension n and the zero matrix of dimension $n \times m$, respectively. We use p to represent the differential operator $p := \frac{d}{dt}[\cdot]$, and $|\cdot|$ as the Euclidean norm of a vector. We use $SO(3) = \{R \in \mathbb{R}^{3\times 3} | R^\top R = I_3,\ \det(R) = 1\}$ to represent the special orthogonal group, and $\mathfrak{so}(3)$ is its Lie algebra. Given $a \in \mathbb{R}^3$, we define the operator $(\cdot)_\times$ as $a_\times := \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix} \in \mathfrak{so}(3)$.

Nomenclature

$\{\mathcal{I}\}, \{B\}$	Inertial and body-fixed frames
$x \in \mathbb{R}^3$	Robot's position in the inertial frame $\{\mathcal{I}\}$
$v \in \mathbb{R}^3$	Linear velocity in the body frame $\{B\}$
$a \in \mathbb{R}^n$	Apparent linear acceleration in $\{B\}$
$\omega \in \mathbb{R}^3$	Rotational velocity in $\{B\}$
$R \in SO(3)$	Robot's attitude
$g \in \mathbb{R}^3$	Gravity with the value $[0, 0, 9.8]^\top$ m/s^2
$q_0, {}^I q_0, \in \mathbb{R}^3$	Structure center coordinates in $\{B\}$ and $\{\mathcal{I}\}$
$r \in \mathbb{R}_{>0}$	Radius of the spherical target
$y \in \mathbb{R}^3$	Measurable output from the camera
$(\hat{\cdot})$	Estimate of a variable or signal $(\cdot)$
$\mathcal{L}$	Planar limb surface in the camera view
$d \in \mathbb{R}_{>0}$	Distance from the camera to $\mathcal{L}$

2 Problem formulation

2.1 Model

The paper extends the results on visual feature structure estimation in [21] from points to spheres. In this section, we introduce the dynamical model of a mobile robot and the output function of the camera.

Dynamical Model and Inputs. We consider a mobile robot moving in three-dimensional space, with its kinematics given by

$$\begin{aligned} \dot{x} &= Rv \\ \dot{R} &= R\omega_\times, \end{aligned} \tag{1}$$

and the dynamics is

$$\dot{v} = -\omega_\times v + a + R^\top g, \tag{2}$$

where $x \in \mathbb{R}^3$ is the coordinate of the robot's central point, and $R \in SO(3)$ represents the robot's orientation in three-dimensional space. The mobile robot is equipped with IMUs, and thus the "system inputs" – including the linear apparent acceleration $a \in \mathbb{R}^3$ and the rotational velocity $\omega \in \mathbb{R}^3$ – are available. However, we do not have the information of the attitude R, the position x, or the linear velocity v. See [21] for additional details and, without loss of generality, we assume the following.

Assumption 1. The system input (ω, a) is bounded such that all the system states in the model (1)–(2) are bounded over time.

Spherical Target and Output Function. We consider a spherical target appearing within the camera's field of view, adopting the output model in [17]. For ease of presentation, we assume that the camera is positioned at the robot's central point, meaning that the camera's frame of reference coincides with the robot's coordinate system.

The spherical target is assumed with radius $r > 0$ and center located at the coordinate ${}^I q_0 \in \mathbb{R}^3$ in the inertial frame $\{\mathcal{I}\}$, as shown in Fig. 1. Equivalently, the center coordinate q_0 in the body frame $\{B\}$ is

$$q_0 = R^\top ({}^I q_0 - x) =: \begin{bmatrix} q_{0,x} & q_{0,y} & q_{0,z} \end{bmatrix}^\top. \tag{3}$$

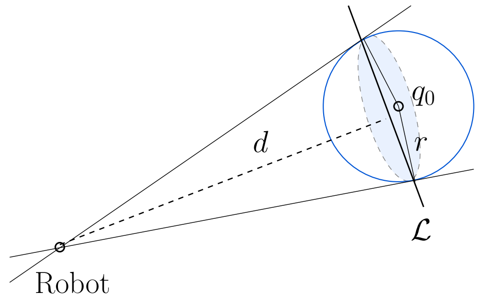

Figure 1: Spherical target at the center q_0

Note that ${}^I q_0$ is constant in $\{\mathcal{I}\}$, but q_0 is a time-varying signal in the body-fixed frame. The planar limb surface $\mathcal{L}$ in the camera view is characterized as

$$\mathcal{L} = \{\mathbf{x} \in \mathbb{R}^3 : n_0^\top \mathbf{x} + d = 0\},$$

where $d \in \mathbb{R}$ is the plane distance to the camera and $n_0 \in \mathbb{S}^2$ is the plane unit normal vector.

We assume the following for the robot's trajectory and the visual feature.

Assumption 2. The robot never coincides with the spherical target over time, i.e.

$$|x(t) - {}^I q_0| = |q_0(t)| \geq r + \epsilon, \qquad \forall t \geq 0,$$

for some $\epsilon > 0$.

Consider a calibrated pinhole camera with the planar projection model. Namely, for a 3D point $q = \mathrm{col}(q_x, q_y, q_z)$ in the body frame $\{B\}$, its perspective projection onto the image plane of the camera is given by

$$ {}^c q = \left[\tfrac{q_x}{q_z}, \tfrac{q_y}{q_z}, 1\right]^\top \tag{4}$$

In order to estimate spherical features, including the radius r and the center coordinates q_0, [17] consider the output $y = h(q_0, r)$ with the function

$$\begin{aligned} h(q_0, r) &= \begin{bmatrix} \frac{q_{0,x} q_{0,z}}{(q_{0,z}^2 - r^2) h_3(\cdot) a_1^2} \\ \frac{q_{0,y} q_{0,z}}{(q_{0,z}^2 - r^2) h_3(\cdot) a_1^2} \\ \sqrt{\frac{1+a_1^2}{a_1^2}} \end{bmatrix}, \\ a_1^2 &= \frac{r^2}{q_{0,z}^2 - r^2}, \end{aligned} \tag{5}$$

where $h_3(\cdot)$ represents the third element of the function h. Indeed, the output is a function of the barycenter and normalized centered moments of order 2 measured from the elliptical projection of the sphere on the image plane. It can be calculated from image quantities in real-time; see [2, 17] for the calculation and additional details.

2.2 Problem set

We consider a mobile robot satisfying the model introduced in the above subsection, with the image output $y = h(q_0, r)$ defined in (5), and the "inputs" (a, ω) obtained from the IMU. The objective is to design an observer in the form of

$$\begin{aligned} \dot{\xi} &= N(\xi, a, \omega, y) \\ \begin{bmatrix} \hat{q}_0 \\ \hat{r} \end{bmatrix} &= M(\xi, a, \omega, y), \end{aligned}$$

with the observer state ξ on manifolds, the observer output $(\hat{q}_0, \hat{r}) \in \mathbb{R}^3 \times \mathbb{R}$, and some functions N and M to be constructed, such that it achieves asymptotically convergent estimates, i.e.

$$\lim_{t\to\infty} \left| \begin{bmatrix} \hat{r}(t) - r \\ \hat{q}_0(t) - q_0(t) \end{bmatrix} \right| = 0, \tag{6}$$

when the robot trajectory satisfies certain excitation condition.

Remark 1. [17] proposes a novel solution with the measurements of velocities (v, ω) in the body-fixed frame $\{B\}$. In contrast, in this paper we address a more challenging scenario, relying only on the inertial measurements (a, ω). As figured out in [21], the main difficulty lies in the fact that the dynamics of the unknown linear velocity v depends on the unknown attitude $R \in SO(3)$.

3 Main results

3.1 Observer design

In this section, we introduce a novel adaptive observer designed for the online estimation of sphere structures. Building on the PEBO approach, we propose the following observer, whose properties are subsequently analyzed and discussed in detail throughout the paper.

Observer Design:

- Observer dynamics:

$$\begin{aligned} \dot{Q} &= Q\omega_\times \\ \dot{\eta} &= A(t)\eta + u_a(t), & \eta(0) = 0_6 \\ \dot{\Phi} &= A(t)\Phi, & \Phi(0) = I_6 \\ \dot{\hat{\theta}} &= \gamma\phi(t)[Y(t) - \phi(t)^\top \hat{\theta}] \end{aligned} \tag{7}$$

  with $Q \in SO(3)$, and the other variables in Euclidean space.

- Observer output:[1]

$$\begin{aligned} \hat{r} &= \hat{\theta}_1^{-1} \\ \hat{q}_0 &= y\hat{r} \end{aligned} \tag{8}$$

- Intermediate signals:

$$\begin{aligned} A(t) &= \begin{bmatrix} -\omega_\times(t) & Q^\top(t) \\ 0 & 0 \end{bmatrix} \\ u_a(t) &= \begin{bmatrix} a(t) \\ 0 \end{bmatrix} \\ Y(t) &= p^2 H(p)[y] - pH(p)[y_\times \omega] \\ \phi(t) &= H(p)[\psi^\top] \\ \psi(t) &= \begin{bmatrix} c(t) & \begin{bmatrix} \omega_\times & -Q^\top \end{bmatrix} \Phi \end{bmatrix} \\ c(t) &= \begin{bmatrix} \omega_\times & -Q^\top \end{bmatrix} \eta - a \end{aligned} \tag{9}$$

  with the differential operator $p := \frac{d}{dt}$ and the stable filter

$$H(p)[\cdot] := \frac{1}{(p + \lambda_1)(p + \lambda_2)}$$

- Gains: the adaptation gain $\gamma > 0$, and the filtering gains $\lambda_1, \lambda_2 > 0$

□

[1] $\hat{\theta}_1$ is the first element of $\hat{\theta}$.

Proposition 1. *Consider the dynamical model in Section 2 and the sphere feature observer (7) under Assumptions 1–2. If the mobile robot trajectory satisfies the persistency of excitation (PE) condition for some $T, \delta > 0$*

$$\int_t^{t+T} \phi(s)\phi^\top(s)ds \geq \delta I, \qquad \forall t \geq 0 \tag{10}$$

where ϕ is defined above in (9), then the proposed observer achieves the convergence (6) globally and exponentially, and all the internal variables in the observer are bounded over time.

Proof. Following [2, 17] and invoking (1)–(2), the output y and the variables (v, R) satisfy the dynamics

$$\begin{cases} \dot{R} = R\omega_\times \\ \dot{v} = -\omega_\times v + a + R^\top g \\ \dot{y} = -\dfrac{1}{r}v + y_\times \omega \end{cases} \tag{11}$$

Define the error between Q and R on the group $SO(3)$ as

$$E(R, Q) := RQ^\top,$$

and we have

$$\dot{E} = \dot{R}Q^\top - RQ^{-1}\dot{Q}Q^{-1} = 0,$$

thus there exists a constant matrix $Q_c \in SO(3)$ such that the attitude variable R can be parameterized as

$$R(t) = Q_c Q(t), \qquad \forall t \geq 0.$$

Note that the signal $Q(t)$ is available. Similarly to [21], we have

$$\begin{aligned} R(t)^\top g &= [Q_c Q(t)]^\top g \\ &=: Q(t)^\top g_c \end{aligned} \tag{12}$$

in which we have defined a constant (unknown) vector $g_c := Q_c^\top g$.

Now, we may rewrite the dynamics of v as

$$\begin{aligned} \begin{bmatrix} \dot{v} \\ g_c \end{bmatrix} &= \begin{bmatrix} -\omega_\times(t) & Q(t)^\top \\ 0 & 0 \end{bmatrix} \begin{bmatrix} v \\ g_c \end{bmatrix} + \begin{bmatrix} a(t) \\ 0 \end{bmatrix} \\ &= A(t)\chi_v + u_a(t), \end{aligned} \tag{13}$$

and for simplicity, we have defined

$$\chi_v := \mathrm{col}(v, g_c) \in \mathbb{R}^3.$$

Following the idea of parameter estimation-based observer [10] but for this reduced-order dynamics, it is straightforward to get the dynamics of $(\chi_v - \eta)$ as

$$\dot{\overbrace{\chi_v - \eta}} \;=\; A(t)\big[\chi_v - \eta\big].$$

It can be viewed a linear time-varying (LTV) system, and Φ is its corresponding fundamental matrix. Therefore, we have

$$\begin{aligned} \chi_v - \eta = \Phi\big(\chi_v(0) - \eta(0)\big) & \\ \implies \quad & \chi_v(t) = \eta(t) + \Phi(t)\chi_v(0) \\ \implies \quad & \chi_v(t) = \eta(t) + \Phi(t)\beta, \end{aligned} \tag{14}$$

where in the last implication we have defined the constant vector

$$\beta := \begin{bmatrix} v(0) \\ g_c \end{bmatrix} \in \mathbb{R}^6$$

that is *unknown*. This leads to the parameterization to v and g_c:

$$\begin{bmatrix} v(t) \\ g_c \end{bmatrix} = \eta(t) + \Phi(t)\beta.$$

The next step is to get a linear regression model from the dynamics of y. By taking its second-order time derivative, we have

$$\begin{aligned} \ddot{y} - \dot{\overbrace{y_\times \omega}} &= -\frac{1}{r}\dot{v} \\ &= \frac{1}{r}\left(\omega_\times v - a - Q^\top g_c\right) \\ &= \frac{1}{r}\left(\underbrace{\begin{bmatrix}\omega_\times & -Q^\top\end{bmatrix}\eta - a}_{:=c(t)} + \begin{bmatrix}\omega_\times & -Q^\top\end{bmatrix}\Phi\beta\right) \\ &= \underbrace{\begin{bmatrix} c(t) & \begin{bmatrix}\omega_\times & -Q^\top\end{bmatrix}\Phi\end{bmatrix}}_{\psi(t)} \underbrace{\begin{bmatrix}\frac{1}{r} \\ \frac{\beta}{r}\end{bmatrix}}_{\theta}. \end{aligned} \tag{15}$$

We apply the stable LTI filter

$$H(p) := \frac{1}{(p+\lambda_1)(p+\lambda_2)}$$

to both sides, then yielding

$$p^2 H(p)[y] - pH(p)[y_\times \omega] = H(p)[\psi]\theta + \epsilon_t,$$

or equivalently

$$Y(t) = \phi(t)^\top \theta + \epsilon_t, \tag{16}$$

in which ϵ_t is an exponentially decaying term stemming from the initial conditions of these stable LTI filters. Since it does not affect the stability analysis under sufficient excitation, we omit it in the sequel of the proof.

The estimation error $\tilde{\theta} := \hat{\theta} - \theta$ admits the following LTV dynamics

$$\dot{\tilde{\theta}} = -\gamma\phi(t)\phi(t)^\top\tilde{\theta},$$

for which the zero equilibrium is globally exponentially stable in terms of the PE condition (10) [15]. It is straightforward to verify the boundedness of the observer. On the other hand, since the first element of θ is $\frac{1}{r}$ and we also have the algebraic relationship $q_0 = yr$, it completes the proof.

□

3.2 Discussions

Some remarks to the proposed adaptive observer are in order.

Remark 2. When numerically implementing the observer, we need to use the state-space realizations for $H(p)[\psi]$, $p^2H(p)[y]$ and $pH(p)[y_\times\omega]$. For convenience, we define $k_1 = \lambda_1 + \lambda_2$ and $k_2 = \lambda_1\lambda_2$ thus $H(p) = \frac{1}{p^2+k_1p+k_2}$. These realizations are given by

- $H(p)[\psi]$

$$\begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &= -k_1x_2 - k_2x_1 + \psi\\ H(p)[\psi] &= x_1\end{aligned}$$

- $pH(p)[y_\times\omega] = \frac{p}{p^2+k_1p+k_2}[y_\times\omega]$

$$\begin{aligned}\dot{x}_3 &= x_4\\ \dot{x}_3 &= -k_1x_4 - k_2x_3 + y_\times\omega\\ pH(p)[y_\times\omega] &= x_4\end{aligned}$$

- $p^2H(p)[y] := \frac{p^2}{p^2+k_1p+k_2}[y]$

$$\begin{aligned}\dot{x}_5 &= x_6\\ \dot{x}_5 &= -k_1x_6 - k_2x_5 + y\\ p^2H(p)[y] &= -k_1x_6 - k_2x_5 + y.\end{aligned}$$

Since all the dynamics are stable, there are no specific requirements for the initial conditions, and we may simply choose zero initial conditions.

Remark 3. In practice, the measured acceleration a often includes a bias $b_a \in \mathbb{R}^3$ that needs to be calibrated. The proposed approach can be easily adapted to account for this case, where the dynamics of the linear velocity v becomes

$$\dot{v} = -\omega_\times v + a + b_a + R^\top g.$$

The unknown bias b_a approximately assumed constant, can be included into the model the χ_v. To be precise, the vector $\chi_v = \mathrm{col}(v, g_c)$ is extended to $\chi_v = \mathrm{col}(v, g_c, b_a)$. The subsequent steps in the observer design proceed *mutatis mutandis*.

Remark 4. The calculation of the inverse in (8) may encouter singularity issues. To address this, the first equation can be modified as follows:

$$\hat{r} = \begin{cases}\hat{\theta}_1^{-1} & \text{if } |\hat{\theta}_1| \geq \epsilon_0\\ \epsilon_0^{-1} & \text{otherwise}\end{cases}$$

where $\epsilon_0 > 0$ is a small positive constant chosen to avoid singularity. Alternatively, the equation $\hat{\theta}_1(t)r = 1$ can be viewed as a linear regression model, and this allows the use of standard gradient descent to estimate $\hat{r}$.

Remark 5. For ease of presentation, we adopt the standard gradient descent estimator to identify the unknown constant vector θ. Its convergence relies on the well-known persistency of excitation (PE) condition. It can be significantly relaxed by using advanced techniques, e.g. dynamic regressor extension and mixing (DREM) [1, 19] and composite learning [12].

Remark 6. The main technical challenge in using only inertial measurements is the presence of the attitude variable $R \in SO(3)$ in the dynamics of v. This can be effectively solved using the PEBO on manifolds [21, 23, 24]. Specifically, in our context, the dynamic extension $\dot{Q} = Q\omega_\times$ transforms the daunting term $R(t)^\top g$ into $Q(t)^\top g_c$, where $Q(t) \in SO(3)$ is an available signal, and g_c is an unknown but constant vector.

4 Simulations

In this section, we present some simulations results to validate our theoretical results.

The simulations were performed in Matlab/Simulink. The robot's trajectory $x(t)$ and the spherical target are shown in Fig. 2 with $r = 0.5$ m, where we assumed that the target always appeared in the camera's field of view. Regarding our proposed observer, we selected the following initial conditions:

$$Q(0) = I_3,\ \eta(0) = 0_6,\ \Phi_0 = I_6,$$

with the initial parameter guess $\hat{\theta}(0) = 0_7$, ensuring that the observer starts with neutral assumptions about the system's dynamics and parameters. The observer gains, crucial for regulating the convergence rate and stability of the estimation, were selected as $\lambda_1 = 7.65$, $\lambda_2 = 11.45$, and $\gamma = 0.154$.

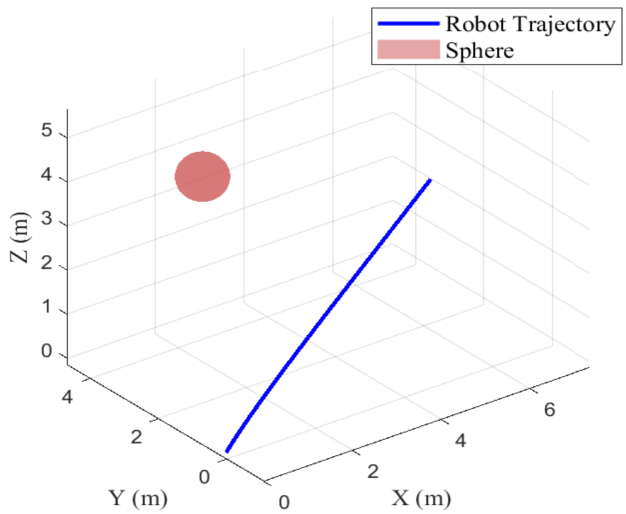

Figure 2: Robot's trajectory and the spherical target

Fig. 3 compares the estimated radius $\hat{r}$ and the real radius r of the spherical target. As shown, the estimated radius $\hat{r}$ rapidly converges to the true value r, demonstrating the accuracy of the observer in estimating this parameter over time. Similarly, Fig. 4 shows the estimated sphere center coordinate, $\hat{q_0}(t)$, alongside its real coordinate, $q_0(t)$, both expressed in the body-fixed frame $\{B\}$. The results indicate that the observer successfully converges to the true position of the sphere's center.

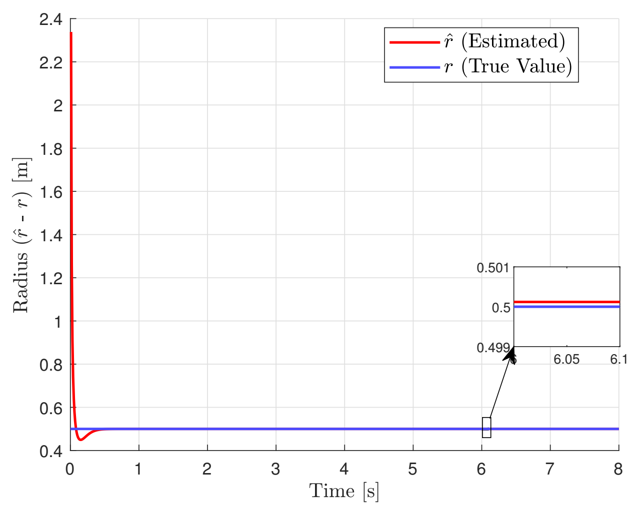

Figure 3: Estimated and real radius

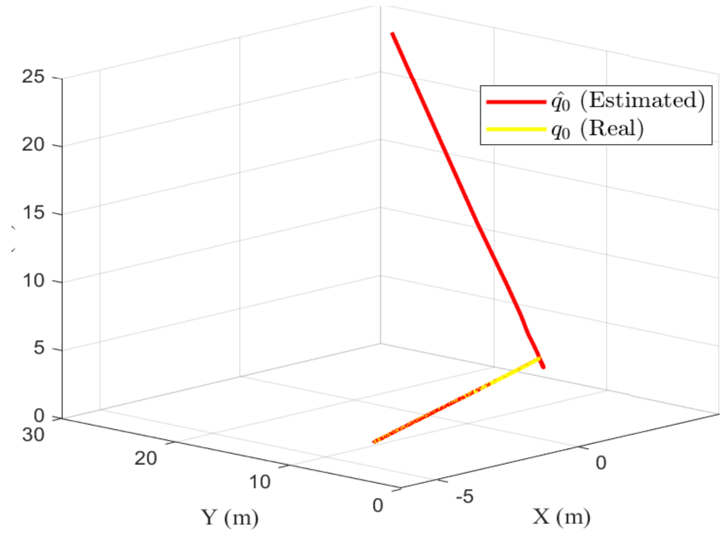

Figure 4: The estimated sphere center coordinate $q_0(t)$ and its real value $\hat{q}_0(t)$ over time in the body-fixed coordinate

5 Concluding remarks

In this paper, we have proposed a novel observer design to online estimate spherical features using only the information of IMUs and a pinhole camera. By employing the PEBO approach on the matrix Lie group $SO(3)$, we are able to reparameterize the unknown term $R(t)^\top g$ in the dynamics of the unavailable linear velocity v as $Q(t)^\top g_c$. This reparameterization significantly simplifies the problem, enabling a robust and efficient observer design. Building on this formulation, we constructed a linear regression model on an extended parameter vector that includes the inverse of the unknown radius $r > 0$ of the spherical feature. By carefully utilizing the output function, we demonstrated that the proposed approach enables the online estimation of this parameter in a globally exponentially convergent way under a PE condition.

Some future research is underway under the following directions:

- Addressing the overparameterization issue in the proposed approach to potentially relax excitation requirements and improve estimation performance
- Further elaborating the proposed approach to handle more complex features and geometries;
- Cascading the proposed observer to other control-related tasks, e.g. vision-based control;
- Extending the framework to handle time-varying target features in the camera field, such as moving or deforming objects;
- Implementing and testing the proposed observer in real-world robotic data sets to validate its performance and scalability.

References

[1] A. Bobtsov, B. Yi, R. Ortega and A. Astolfi, Generation of new exciting regressors for consistent online estimation of unknown constant parameters, IEEE Trans. Autom. Control, vol. 67, no. 9, pp. 4746–4753.

[2] R. T. Fomena and F. Chaumette, Improvements on visual servoing from spherical targets using a spherical projection model, IEEE Trans. Robot., vol. 25, no. 4, pp. 874–886, 2009.

[3] T. Hamel and C. Samson, Position estimation from direction or range measurements, Automatica vol. 82, pp. 137–144, 2017.

[4] D. Karagiannis and A. Astolfi, A new solution to the problem of range identification in perspective vision systems, IEEE Trans. Autom. Control, vol. 50, no. 12, pp. 2074–2077, 2005.

[5] F. Lu and E. Milios, Globally consistent range scan alignment for environment mapping, Autonom. Robot., vol. 4, pp. 333–349, 1997.

[6] A. de Luca, G. Oriolo and P. R. Giordano, Feature depth observation for image-based visual servoing: Theory and experiments, Int. J. Robot. Res., vo. 27, no. 11, pp. 1093–1116, 2008.

[7] T. Lupton and S. Sukkarieh, Visual-inertial-aided navigation for high-dynamic motion in built environments without initial conditions, IEEE Trans. Robot., vol. 28, no. 1, pp. 61–76, 2011.

[8] L. Matthies, T. Kanade and R. Szeliski, Kalman filter-based algorithms for estimating depth from image sequences, Int. J. Comput. Vis., vol. 3, no. 3, pp. 209–238, 1989.

[9] R. Ortega, A. Bobtsov, A. Pyrkin and S. Aranovskiy, A parameter estimation approach to state observation of nonlinear systems, Syst. Control Lett., vol. 85, pp. 84–94, 2015.

[10] R. Ortega, A. Bobtsov, N. Nikolaev, J. Schiffer, and D. Dochain, Generalized parameter estimation-based observers: Application to power systems and chemical-biological reactors, Automatica, vol. 129, Art. no. 109635, 2021.

[11] R. Ortega, L. Praly, S. Aranovskiy, B. Yi and W. Zhang, On dynamic regressor extension and mixing parameter estimators: Two Luenberger observers interpretations, Automatica, vol. 95, pp. 548–551, 2018.

[12] Y. Pan and H. Yu, Composite learning robot control with guaranteed parameter convergence, Automatica, vol. 89, pp. 398–406, 2018.

[13] A. Parikh, R. Kamalapurkar and W. E. Dixon, Target tracking in the presence of intermittent measurements via motion model learning, IEEE Trans. Robot., vol. 34, no. 3, pp. 805–819, 2018.

[14] M. Sassano, D. Carnevale and A. Astolfi, Observer design for range and orientation identification, Automatica, vol. 46, no. 8, pp. 1369–1375, 2010.

[15] S. Sastry and M. Bodson, Adaptive Control: Stability, Convergence and Robustness, Courier Corporation, 2011.

[16] H. Strasdat, J. M. Montiel and A. J. Davison, Visual SLAM: Why filter?, Image Vis. Comput., vol. 30, pp. 65–77, 2012.

[17] R. Spica, P.R. Giordano and F. Chaumette, Active structure from motion: Application to point, sphere, and cylinder, IEEE Trans. Robot., vol. 30, no. 6, pp. 1499–1513, 2014.

[18] P. van Goor, P. Wickramasinghe, M. Hampsey and R. Mahony, Constructive synchronous observer design for inertial navigation with delayed GNSS measurements, Eur. J. Control, vol. 18, Art. no. 101047, 2024.

[19] L. Wang, R. Ortega, A. Bobtsov, J. G. Romero and B. Yi, Identifiability implies robust, globally exponentially convergent on-line parameter estimation, Int. J. Control, vol. 97, pp. 1967–1983, 2024.

[20] B. Yi, R. Ortega and W. Zhang, On state observers for nonlinear systems: A new design and a unifying framework, IEEE Trans. Autom. Control, vol. 64, no. 3, pp. 1193–1200, 2018.

[21] B. Yi, C. Jin and I. R. Manchester, Globally convergent visual-feature range estimation with biased inertial measurements, Automatica, vol. 146, Art. no. 110639, 2022.

[22] B. Yi, C. Jin, L. Wang, G. Shi, V. Ila and I. R. Manchester, PEBO-SLAM: Observer design for visual inertial SLAM with convergence guarantees, IEEE Trans. Autom. Control, pp. 1–16, 2024. (In press)

[23] B. Yi and I. R. Manchester, On IMU preintegration: A nonlinear observer viewpoint and its applications, Syst. Control Lett., vol. 193, Art. no. 105933, 2024.

[24] B. Yi, L. Wang and I. R. Manchester, Attitude estimation from vector measurements: Necessary and sufficient conditions and convergent observer design, IEEE Trans. Autom. Control, vol. 69, no. 1, pp. 511–518, 2023.

[25] D. E. Zlotnik and J. R. Forbes, Gradient-based observer for simultaneous localization and mapping, IEEE Trans. Autom. Control, vol. 63, no. 12, pp. 4338–4344, 2018.