Les Cahiers du GERAD ISSN: 0711–2440

Integrated lot sizing and blending problems under demand uncertainty

M. Rocha Gonçalves, R. Jans, S. A. de Araujo

G–2025–02

January 2025

La collection *Les Cahiers du GERAD* est constituée des travaux de recherche menés par nos membres. La plupart de ces documents de travail a été soumis à des revues avec comité de révision. Lorsqu'un document est accepté et publié, le pdf original est retiré si c'est nécessaire et un lien vers l'article publié est ajouté.

Citation suggérée : M. Rocha Gonçalves, R. Jans, S. A. de Araujo (Janvier 2025). Integrated lot sizing and blending problems under demand uncertainty, Rapport technique, Les Cahiers du GERAD G– 2025–02, GERAD, HEC Montréal, Canada.

Avant de citer ce rapport technique, veuillez visiter notre site Web (https://www.gerad.ca/fr/papers/G-2025-02) afin de mettre à jour vos données de référence, s'il a été publié dans une revue scientifique.

The series *Les Cahiers du GERAD* consists of working papers carried out by our members. Most of these pre-prints have been submitted to peer-reviewed journals. When accepted and published, if necessary, the original pdf is removed and a link to the published article is added.

Suggested citation: M. Rocha Gonçalves, R. Jans, S. A. de Araujo (January 2025). Integrated lot sizing and blending problems under demand uncertainty, Technical report, Les Cahiers du GERAD G–2025–02, GERAD, HEC Montréal, Canada.

Before citing this technical report, please visit our website (https://www.gerad.ca/en/papers/G-2025-02) to update your reference data, if it has been published in a scientific journal.

La publication de ces rapports de recherche est rendue possible grâce au soutien de HEC Montréal, Polytechnique Montréal, Université McGill, Université du Québec à Montréal, ainsi que du Fonds de recherche du Québec – Nature et technologies.

Dépôt légal – Bibliothèque et Archives nationales du Québec, 2025
– Bibliothèque et Archives Canada, 2025

The publication of these research reports is made possible thanks to the support of HEC Montréal, Polytechnique Montréal, McGill University, Université du Québec à Montréal, as well as the Fonds de recherche du Québec – Nature et technologies.

Legal deposit – Bibliothèque et Archives nationales du Québec, 2025
– Library and Archives Canada, 2025

GERAD HEC Montréal
3000, chemin de la Côte-Sainte-Catherine
Montréal (Québec) Canada H3T 2A7

Tél. : 514 340-6053
Téléc. : 514 340-5665
info@gerad.ca
www.gerad.ca

Integrated lot sizing and blending problems under demand uncertainty

Maurício Rocha Gonçalves [a, c]

Raf Jans [b, c]

Silvio Alexandre de Araujo [a, c]

[a] *Departamento de Matemática, UNESP, São José do Rio Preto, São Paulo, CEP 15054-000, Brazil*

[b] *Department of Logistics and Operations Management, HEC Montréal, Montréal (Qc), Canada, H3T 2A7*

[c] *GERAD, Montréal (Qc), Canada, H3T 1J4*

mauricio.rocha@unesp.br
raf.jans@hec.ca
silvio.araujo@unesp.br

January 2025
Les Cahiers du GERAD
G–2025–02

Copyright © 2025 Rocha Gonçalves, Jans, de Araujo

Les textes publiés dans la série des rapports de recherche *Les Cahiers du GERAD* n'engagent que la responsabilité de leurs auteurs. Les auteurs conservent leur droit d'auteur et leurs droits moraux sur leurs publications et les utilisateurs s'engagent à reconnaître et respecter les exigences légales associées à ces droits. Ainsi, les utilisateurs:

- Peuvent télécharger et imprimer une copie de toute publication du portail public aux fins d'étude ou de recherche privée;
- Ne peuvent pas distribuer le matériel ou l'utiliser pour une activité à but lucratif ou pour un gain commercial;
- Peuvent distribuer gratuitement l'URL identifiant la publication.

Si vous pensez que ce document enfreint le droit d'auteur, contactez-nous en fournissant des détails. Nous supprimerons immédiatement l'accès au travail et enquêterons sur votre demande.

The authors are exclusively responsible for the content of their research papers published in the series *Les Cahiers du GERAD*. Copyright and moral rights for the publications are retained by the authors and the users must commit themselves to recognize and abide the legal requirements associated with these rights. Thus, users:

- May download and print one copy of any publication from the public portal for the purpose of private study or research;
- May not further distribute the material or use it for any profit-making activity or commercial gain;
- May freely distribute the URL identifying the publication.

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

Abstract : Blending problems occur when different components are mixed to form an end product. The recipe typically has some flexibility as long as specific quality conditions are met. We consider the blending problem as a multi-period, multi-level lot sizing problem. The problem involves determining the setup periods for purchasing the components and producing the products, along with their respective quantities and blending. A two-stage stochastic programming formulation is introduced to account for demand uncertainty for the end products. In the first stage, before demand is realized, decisions are made on the setup periods for both production and purchase, and their corresponding quantities. In the second stage, after demand is revealed, the inventory and lost sales decisions are made. The goal is to minimize the expected total cost incurred by the decisions made in both stages. Heuristic approaches are proposed to solve the problem, using the expected demand through the deterministic formulation and sets of demand scenarios in the application of the sample average approximation method. Finally, numerical experiments are conducted with test instances to evaluate the modeling results for the expected cost and computational time using the proposed heuristics.

Keywords : Lot sizing problem, blending problem, demand uncertainty, stochastic optimization, sample average approximation

Acknowledgements: This work was supported by the Fundação de Amparo à Pesquisa do Estado de São Paulo (FAPESP) under Grants 2019/17295–0, 2022/05803–3, 2023/02210–4, and 2023/09205–6; and Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq) under Grants 302998/2022–5 and 402240/2023–5. The authors report there are no competing interests to declare.

1 Introduction

In contrast to discrete manufacturing, where typically a predetermined number of components is used to produce an end product, blending problems in the process industry allow for some flexibility with respect to the quantity of the inputs used. In such cases the amount used of the components may vary as long as specific quality characteristics remain within certain boundaries. Notable applications of such flexible manufacturing can be found in fertilizer (Ashayeri et al. 1994), food (Akkerman et al. 2010), petrochemical (Yang et al. 2017), and coal production (Chen et al. 2022).

Process industries determine the mix for producing the end products, while also deciding on the quantities of the components to purchase and the quantities of the end products to produce. In standard blending problems, these decisions typically aim to satisfy an external demand for the end products within a single period. However, it is sometimes possible to store quantities for later use. As a result, the planning horizon for the associated decisions can be extended to account for multiple time periods, where the inventory provides the link between consecutive periods (Pathumnakul et al. 2011).

The planning of the blending problem over multiple time periods is done at two levels. One is the production level, where recipes for the products are determined, and the setups are prepared to produce them. The other is the purchase level, where the purchase periods and quantities for the required components are determined. Clearly, there is a hierarchical relationship between these levels, which is essential for material coordination and inventory control. An integrated approach is used to manage both levels simultaneously, leading to improved planning performance in multi-level, multi-period blending problems (Fiorotto et al. 2021).

Often, the external demand for the end products is not known with certainty in advance. Because demand variability directly affects material inventory management, it is important to consider that the actual demand may be unknown when making the purchase and production decisions. However, in blending problems, the predominant assumption is still that the demand is deterministic.

The aim of this paper is to address the blending problem under demand uncertainty. The problem is considered as an integrated two-level lot sizing problem considering production flexibility. We assume that material quantity flexibility is incorporated in the choice of the proportions of the input components used to produce the output end products. Additionally, constraints related to inventory balance, machine capacity and quality requirements are also taken into account. The general problem is referred to as MCLSB, which stands for Multi-item Capacited Lot Sizing and Blending problems. In this regard, the contributions of this paper are as follows:

- Proposing a two-stage stochastic programming formulation to address the MCLSB problem under stochastic demand. In this formulation, the associated decisions for production and purchase follow the static uncertainty strategy used in the stochastic lot sizing problems;
- Developing a solution algorithm for the stochastic problem based on the Sample Average Approximation (SAA) method. We also improve the computational efficiency and solution quality through the Adjustable SAA variant (ASAA);
- Providing computational results to evaluate the expected costs and computational complexity for different heuristic solutions. The experiments include comparisons between the deterministic and SAA heuristics, as well as between the adjustable and regular SAA approaches.

The remainder of the paper is structured as follows. Section 2 reviews the literature related to the MCLSB problem. Section 3 provides the general assumptions and the deterministic Mixed-Integer Linear Programming (MILP) formulation for the MCLSB problem, followed by a two-stage stochastic programming formulation taking into account demand uncertainty. In Section 4, we describe the methodology for generating heuristic candidate solutions and evaluating them in the stochastic setting. The solution approaches for approximating the stochastic problem include the deterministic Mean Value (MV) heuristic choosing the expected value of the probabilistic demand, and the SAA and ASAA heuristics sampling sets with a number of demand realizations. Section 5 presents the computational experiments and average results conducted with randomly generated instances. Finally, Section 6 provides the conclusions and the proposals for future research.

2 Literature review

Mathematical programming models for blending problems in process industries are typically designed considering single-period planning (Ashayeri et al. 1994). In the single-period blending, a model is used to exploit material quantity flexibility to produce the demand for an end product (Rutten 1995). The concept of material quantity flexibility in blending problems enables the quantity of components in the recipe (or mix) of the end product to vary. Additionally, specific conditions in the recipe ensure that some prescribed quality requirements of the end product, derived from the quality attributes of the components used, remain within certain bounds. For example, in fuel processing involving the blending of various crude oils, chemical compound levels are monitored to control pollutant emissions and consumption rates (Singh et al. 2000), and in food production involving the blending of various flours, specific nutritional levels are established for dietary requirements (Akkerman et al. 2010). Thus, the standard single-period model for blending problems is used to determine the proportions of the components in the recipe that minimize the costs so that the end product quality specifications are met.

Unlike the single-period problem, accounting for material quantity flexibility while simultaneously considering usage in future time periods generally results in more economic planning for process industries (Williams and Redwood 1974; Reddy et al. 2004; Chen and Maravelias 2022). As such, a multi-period blending model that includes inventory considerations can play a crucial role in minimizing total costs by optimizing decisions on the purchase quantities of components and the production quantities of end products over an extended planning horizon. When also taking into account the need for setup preparations to satisfy a dynamic demand, the value provided by the inventory in the multiple-period planning can be even more beneficial (Pathumnakul et al. 2011). Moreover, industries must generally account for limitations in their planning, such as machine capacity and resource availability constraints (Kilic et al. 2013).

In the multi-period blending problem, there is a lot sizing structure at two levels: the level related to the production of the end products and the level related to the purchase plan for the components. Basically, there are two approaches to plan the coordination and optimization of these correlated activities. First, there is the sequential approach, which first optimizes the decisions at the production level to satisfy the external demand. These decisions subsequently set the limitations for the decisions at the purchase level regarding the internal demands. Second, there is the integrated approach, which covers the total problem with the use of one large model that captures the full interaction between the two levels. The early integrated blending models were usually dealt with using the sequential approach due to their prohibitive solution time and complexity drawbacks (Rutten 1993). However, the speed of computer power has increased rapidly, so nowadays new mathematical programming packages enable the implementation and solution procedure of comprehensive blending models in a reasonable amount of time. Thus, novel extensions for blending problems focus more and more on incorporating trends for cost savings, since the local optimization of compact models usually does not result in a global optimum (Fiorotto et al. 2021).

A limitation in existing approaches for process manufacturing is the reliance on deterministic assumptions for data (Mula et al. 2006). In blending problems, important industrial considerations focus on production planning procedures that utilize recipe flexibility to address uncertainty (Rutten and Bertrand 1998). One relevant source of uncertainty can arise from the supply aspect regarding the quality composition of the components, which can lead to infeasibility in meeting the requirements of the end products (Candler 1991; Yang et al. 2017; Peng et al. 2021). Another critical issue is demand uncertainty, as neglecting it can lead to stockouts and a lower service level. Whereas the deterministic approach, using the expected value of a probabilistic demand, is commonly used in blending problems to address demand uncertainty (Bertrand and Rutten 1999), the exploration of more accurate approaches is still in its early stages (Chen et al. 2022; Hilali et al. 2023). To our knowledge, planning strategies that incorporate demand uncertainty while integrating production and purchase lot sizing decisions, along with recipe flexibility, remain superficial in the literature for the blending problem. This paper aims to fill this research gap by proposing improved planning strategies through mathematical programming models.

Depending on the planning framework, different decision responses can be adopted to handle uncertainty. Within the scope of the lot sizing problem, the level of adaptability of decision-making in response to demand uncertainty is classified by Bookbinder and Tan (1988) into three uncertainty strategies: static, dynamic and static-dynamic. The static uncertainty strategy is the most stable approach as it determines that setup and production decisions are made at the beginning of the planning horizon and remain unchanged regardless of how demand is realized. In contrast, the dynamic uncertainty strategy is the most adaptable approach, as the setup and production decisions can be made after the demand is revealed in each period. The static-dynamic uncertainty strategy addresses a balance between these two previous approaches: the setup decisions are made at the beginning of the planning horizon and remain unchanged, whereas production quantities can be made after the demand is revealed. In this paper, we assume that the planning prioritizes a steady decision-making approach by employing the static uncertainty strategy. A shortcoming of limiting the production adaptability in response to the demand realization is that stock-outs can occur. In case the demand cannot be satisfied on time, a common approach is to quantify the costs of not satisfying the demand using lost sales penalties proportional to the unmet demand (Absi and Kedad-Sidhoum 2008). In lot sizing problems under demand uncertainty, lost sales can be incorporated to consider the possibility of stock-outs in some demand scenarios (Ghamari and Sahebi 2017; Quezada et al. 2020).

When demand uncertainty is considered for production and purchasing over a planning horizon, periods are naturally associated to decision stages. As a result, part of the literature focuses on a multi-stage stochastic programming approach to model the associated lot sizing problem (Tempelmeier 2013). In general, the combination of increased dimensionality and decision dependency on the possible branches for the uncertainty realization makes multi-stage stochastic problems too complex. In order to get a simpler but reasonable approximation, modeling approaches can use a two-stage stochastic programming approach (Higle 2005). In the two-stage stochastic approach, the aim is to minimize the cost incurred by decisions made before the uncertainty is revealed, called first-stage decisions, plus the expected cost for the recourse decisions after the uncertainty has been realized, called second-stage decisions. Two-stage stochastic programming approaches have been successfully used to assess the impact of demand uncertainty on planning problems, considering lost sales as a recourse mechanism (Gruson et al. 2021; Alvarez et al. 2021). Furthermore, two-stage stochastic formulations can be applied in a rolling/receding horizon fashion as a heuristic to solve multi-stage stochastic lot sizing problems under demand uncertainty (Sereshti et al. 2021, 2022).

This paper addresses a two-stage stochastic problem setting for the blending problem under demand uncertainty. The first stage consists of the planning for purchasing and production over the time periods, which includes the decision on the production quantities and blending of end products and purchase quantities of the components, along with their corresponding setup periods. The second stage involves the recourse in the form of lost sales and inventory decisions. We cannot directly solve the two-stage stochastic program using a solver due to the presence of the random data and the expectation terms in its objective function. Given this premise, a heuristic approximation to address stochastic programming problems can be implemented using sampling and scenario approximation techniques (Kaut and Stein 2003). In particular, the Sample Average Approximation (SAA) method proposed by Shapiro and Homem-de Mello (1998) is an approach utilized to tackle stochastic programs. In the SAA approach, the uncertain data is replaced by a finite number of sampled realizations, and the true expected objective value of the problem is approximated by the average cost over these realizations. Then, candidate solutions are yielded from independent replications of the sets of realizations by solving the corresponding approximated problems with a solver. The SAA method has been applied successfully for a variety of two-stage stochastic programming problems in the literature, including routing problems (Verweij et al. 2003), hub location problems (Contreras et al. 2011), supply chain design problems (Santoso et al. 2005; Schütz et al. 2009; Oliveira and Hamacher 2012), as well as stochastic lot sizing problems (Taş et al. 2019; Sereshti et al. 2024).

While larger sets intuitively provide a better approximation of the optimal solution (Kleywegt et al. 2002; Mak et al. 1999), they also increase the computational complexity, as solvers are often unable to handle large problems efficiently within reasonable time limits. As an improvement to the computational limitation of the regular SAA method, the Adjustable Sample Average Approximation (ASAA) method proposed by Agra et al. (2018) divides the generation of solutions into distinct phases. Firstly, we consider solving problems using sets with small numbers of realizations and then identify the variables that have the same value across the obtained solutions. By fixing these variables to their respective values, the complexity is reduced and a larger set of realization can be considered in further iterations. The general idea of the ASAA is to provide a good balance between solving smaller problems with less computational effort in the first phase and still getting good solutions by considering problems with a larger set of scenarios in the second phase. This technique has been successfully applied to solve two-stage stochastic lot sizing problems considering demand uncertainty (Tomazella et al. 2024).

3 Problem statement and mathematical formulations

In this section, we detail the base assumptions for the MCLSB problem and present its deterministic formulation. In Subsection 3.1, we introduce the stochastic formulation taking demand uncertainty into account.

The problem involves satisfying the demand for a set of end products over a planning horizon divided into multiple time periods. Production occurs at a blending station, where several end products are produced by mixing a range of components. Only the end products have external demand. The quantity of components used in the recipe to produce the end products can vary. The components are available to be purchased, and each component requires a different purchase setup. For each end product, a different production setup is needed when production occurs. For the final mix of the end products, specific quality parameters must be considered to ensure that all products meet the required standards. In this paper, we model the most relevant operational aspects considered in capacitated lot sizing (Jans and Degraeve 2008) and blending problems (Kilic et al. 2013). One aspect is the capacity consumption incurred for setup and production during each time period when end products are produced. This represents, for example, processing limitations or a cleaning operation that needs time to be done at the blending station. Furthermore, each quality parameter in the end products is constrained by setting limits on its proportion relative to the total amount of product produced. A linear blending is assumed to determine the composition and qualities of the end products, defined as the unit-weighted average of the qualities of the components in the final mix. The values for the quality parameters of the components are assumed to be known. Overall:

1. A single blend machine is used for blending all end products, which has a limited capacity (in units of time) in each period;
2. Setup and production consumption for end products, as well as setup costs, are sequence-independent and do not depend on the components in their blending;
3. End products and components can be held in inventory between periods incurring holding costs, starting with zero initial inventories;
4. Shortages in each time period incur a penalty cost given per unit of demand considered as lost sales.

The notation utilized in the mathematical models is:

Sets

$\mathcal{P}$: Set of end products, indexed by j.
$\mathcal{C}$: Set of components, indexed by i.
$\mathcal{Q}$: Set of quality characteristics, indexed by q.
$\mathcal{T}$: Set of time periods, indexed by t.
$\mathcal{C}_j \subset \mathcal{C}$: Set of components needed to make end product j.
$\mathcal{P}_i \subset \mathcal{P}$: Set of end products that use component i.

Parameters

M^C_{it}: Maximum purchase of component i in period t (units).
hc^C_{it}: Inventory cost of component i in period t (\$/unit).
sc^C_{it}: Setup cost of component i in period t (\$).
vc^C_{it}: Purchase cost of component i in period t (\$/unit).
M^E_{jt}: Maximum production of end product j in period t (units).
hc^E_{jt}: Inventory cost of end product j in period t (\$/unit).
sc^E_{jt}: Setup cost of end product j in period t (\$).
vc^E_{jt}: Production cost of end product j in period t (\$/unit).
pc_{jt}: Lost sale cost of end product j in period t (\$/unit).
C_t: Capacity available for production in period t (hours).
st_{jt}: Setup time for end product j in period t (hours).
vt_{jt}: Production time for end product j in period t (hours/unit).
qa_{qi}: Value of quality parameter q in component i (#/unit).
ql_{qj}: Minimum proportion for the quality parameter q in end product j (#/unit).
qu_{qj}: Maximum proportion for the quality parameter q in end product j (#/unit).
d^E_{jt}: Demand for end product j in period t (units).

Decision Variables

x^C_{it}: Amount of component i purchased in period t (units).
y^C_{it}: Binary variable indicating if a setup for purchasing component i occurs in period t (1 if setup is prepared, 0 otherwise).
s^C_{it}: Inventory of component i at the end of period t (units).
x^E_{jt}: Amount of end product j produced in period t (units).
y^E_{jt}: Binary variable indicating if a setup for producing end product j occurs in period t (1 if setup is prepared, 0 otherwise).
s^E_{jt}: Inventory of end product j at the end of period t (units).
l^E_{jt}: Lost sales of end product j in period t (units).
p^E_{ijt}: Amount of component i blended into end product j in period t (units).

The decision-making involves determining an integrated plan for the purchase of the components and the production of the end products to satisfy the dynamic demand. The primary objective is to minimize the total cost, which is incurred for the setup, purchase and inventory decisions for the components, plus the setup, production, inventory and lost sale decisions for the end products.

Blending planning models usually use deterministic demand predictions to decide on purchase and production subject to inventory balance, capacity limitations, and quality requirements constraints (Bertrand and Rutten 1999; Pathumnakul et al. 2011). Thus, the deterministic formulation for the

MCLSB problem is given by (1)–(13), which is an extension of the model presented in Fiorotto et al. (2021).

$$\min \sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{C}}(sc^C_{it}y^C_{it}+vc^C_{it}x^C_{it}+hc^C_{it}s^C_{it})+\sum_{t\in\mathcal{T}}\sum_{j\in\mathcal{P}}(sc^E_{jt}y^E_{jt}+vc^E_{jt}x^E_{jt}+hc^E_{jt}s^E_{jt}+pc_{jt}l^E_{jt}) \tag{1}$$

Subject to:

$$x^C_{it}\le M^C_{it}y^C_{it} \qquad \forall i\in\mathcal{C},t\in\mathcal{T} \tag{2}$$
$$x^E_{jt}\le M^E_{jt}y^E_{jt} \qquad \forall j\in\mathcal{P},t\in\mathcal{T} \tag{3}$$
$$s^C_{it-1}+x^C_{it}=\sum_{j\in\mathcal{P}_i}p^E_{ijt}+s^C_{it} \qquad \forall i\in\mathcal{C},t\in\mathcal{T} \tag{4}$$
$$s^E_{jt-1}+x^E_{jt}+l^E_{jt}=d^E_{jt}+s^E_{jt} \qquad \forall j\in\mathcal{P},t\in\mathcal{T} \tag{5}$$
$$l^E_{jt}\le d^E_{jt} \qquad \forall j\in\mathcal{P},t\in\mathcal{T} \tag{6}$$
$$\sum_{j\in\mathcal{P}}(st_{jt}y^E_{jt}+vt_{jt}x^E_{jt})\le C_t \qquad \forall t\in\mathcal{T} \tag{7}$$
$$\sum_{i\in\mathcal{C}_j}p^E_{ijt}=x^E_{jt} \qquad \forall j\in\mathcal{P},t\in\mathcal{T} \tag{8}$$
$$ql_{qj}x^E_{jt}\le\sum_{i\in\mathcal{C}_j}qa_{qi}p^E_{ijt} \qquad \forall j\in\mathcal{P},q\in\mathcal{Q},t\in\mathcal{T} \tag{9}$$
$$qu_{qj}x^E_{jt}\ge\sum_{i\in\mathcal{C}_j}qa_{qi}p^E_{ijt} \qquad \forall j\in\mathcal{P},q\in\mathcal{Q},t\in\mathcal{T} \tag{10}$$
$$y^E_{jt}\in\{0,1\},x^E_{jt},s^E_{jt},l^E_{jt}\ge 0 \qquad \forall j\in\mathcal{C},t\in\mathcal{T} \tag{11}$$
$$y^C_{it}\in\{0,1\},x^C_{it},s^C_{it}\ge 0 \qquad \forall i\in\mathcal{C},t\in\mathcal{T} \tag{12}$$
$$p^E_{ijt}\ge 0 \qquad \forall i\in\mathcal{C},j\in\mathcal{P},t\in\mathcal{T} \tag{13}$$

The objective (1) minimizes the total cost that is incurred by setup, purchase and inventory of components, and setup, production, inventory and lost sales of end products. Constraints (2) and (3) indicate that the purchase of components and production of end products in a period occur only if the proper setup is done. Constraints (2) and (3) use big-M constants. M^E_{jt} can be set as the maximum amount of end product $j\in\mathcal{P}$ that can be produced without violating capacity in period $t\in\mathcal{T}$, given by $M^E_{jt}=(C_t-st_{jt})/vt_{jt}$. Accordingly, for component $i\in\mathcal{C}$ in period $t\in\mathcal{T}$, M^C_{it} can be set as $M^C_{it}=\sum_{\tau=t}^{m}\sum_{j\in\mathcal{P}_i}M^E_{j\tau}$ considering the simple case of using only one component to produce these maximum amounts. Constraints (4) and (5) correspond to the inventory balance for components and end products, respectively. Constraints (6) guarantee that the lost sales of end products in each period are at most the realized demand. Constraints (7) impose that the total time for setup and production in each period is limited to the time capacity available for the blending machine. Constraints (8) define the final blending of components into end products. More specifically, the total amount of an end product made in a period is equal to the sum of the amounts of all the components that go into that end product. Constraints (9) and (10) ensure that the final blends meet all the required qualities. Finally, constraints (11), (12) and (13) define the domain of the decision variables.

3.1 A stochastic programming formulation

In the MCLSB problem under demand uncertainty, we consider that demand for each end product $j\in\mathcal{P}$ in period $t\in\mathcal{T}$ is a random parameter denoted by $\tilde{d}^E_{jt}$. The uncertain demand is caracterized by a specific probability distribution considering the corresponding mean value $\mathbb{E}[\tilde{d}^E_{jt}]$, and standard deviation $\sigma[\tilde{d}^E_{jt}]$ that is represented as a fraction of this mean (Tunc et al. 2018; Gruson et al. 2021).

To address the randomness of the demand realizations in the planning, we assume the decision-making process is divided into two stages according to the uncertainty realization. In the first stage, we assume that only the joint probability distribution for the random demand vector $\tilde{d}^E = (\tilde{d}^E_{jt})_{j\in\mathcal{P},t\in\mathcal{T}}$ and its support Ω underlying the potential realizations are available. In the second stage, the demand over the whole planning horizon is revealed, where each $\omega \in \Omega$ corresponds to a specific realization $d^E(\omega)$ of the stochastic demand, which is referred to as a demand scenario.

A stochastic formulation considering a static uncertainty strategy

In the two-stage stochastic setting, the first-stage decisions are made before the uncertain parameters are revealed. In our model, we assume that the quantity and setup decisions related to the production of the end products and the purchase of the components must be made before the demand is realized. These are hence our first-stage decisions. Then, in the second stage when the actual demand information is revealed, the remaining decisions (i.e., inventory and lost-sale variables) are the recourse decisions, which permit reacting to the demand scenario that is observed. In this sense, uncertainty must be absorbed through the proper dimensioning of the lot sizes for the production and purchase plans. The approach proposed is equivalent to using a frozen schedule, which aligns with the Static Uncertainty Strategy (Bookbinder and Tan 1988). The application of the static uncertainty strategy is particularly justified in cases where it is important to avoid production nervousness and define a capacity-feasible plan in order to organize the production resources in advance.

Since the purchase quantities for the components and the production quantities for the end products are first-stage variables, blending decisions can also be considered as first-stage variables. This is because blending decisions depend directly on the quantities of the components and the products (e.g., defining ratios or recipes), which have been determined in the first stage, and there are no costs associated with these blending decisions. Thus, treating $p^E_\omega = p^E$ as fixed across scenarios $\omega \in \Omega$ has no impact on the recourse mechanism. Moreover, this also implies that inventory for components in the second stage are directly determined. Indeed, observe that the component inventory variables $s^C_\omega = s^C$ results from constraints (4), which are rewritten as $s^C_{it\omega} = \sum_{\tau=1}^{t}\left(x^C_{i\tau} - \sum_{j\in\mathcal{P}_i} p^E_{ij\tau}\right) + s^C_{i0}$ for each period $t \in \mathcal{T}$ and component $i \in \mathcal{C}$ over all scenarios $\omega \in \Omega$.

In the first stage, the decision $z^{\mathrm{F}} = (x^C, y^C, s^C, p^E, x^E, y^E)$ is initially made without knowing the demand scenario that will be realized, incurring a fixed first-stage cost. Subsequently, full information is revealed on the realization of the demand scenario $d^E(\omega)$. The second stage decision $z^{\mathrm{S}}_\omega = (s^E_\omega, l^E_\omega)$, is then made to react to the realized demand, which incurs a second stage cost $R_{d^E(\omega)}(x^C, y^C, s^C, p^E, x^E, y^E)$. This cost is computed as the optimal value of the second stage problem (14)–(17). The timing of events for the Static Uncertainty strategy applied to the MCLSB under demand uncertainty (Static MCLSB problem) can be visualized in Figure 1. The two-stage stochastic programming formulation for the MCLSB problem is stated as (18)–(28).

$$R_{d^E(\omega)}(x^C, y^C, s^C, p^E, x^E, y^E) = \min \sum_{t\in\mathcal{T}}\sum_{j\in\mathcal{P}} \left(hc^E_{jt}s^E_{jt} + pc_{jt}l^E_{jt}\right) \tag{14}$$

Subject to:

$$s^E_{jt-1} + x^E_{jt} + l^E_{jt} = d^E_{jt}(\omega) + s^E_{jt} \qquad \forall j\in\mathcal{P}, t\in\mathcal{T} \tag{15}$$

$$l^E_{jt} \le d^E_{jt}(\omega) \qquad \forall j\in\mathcal{P}, t\in\mathcal{T} \tag{16}$$

$$s^E_{jt}, l^E_{jt} \ge 0 \qquad \forall j\in\mathcal{P}, t\in\mathcal{T} \tag{17}$$

$$\min \sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{C}} (sc^C_{it}y^C_{it} + vc^C_{it}x^C_{it} + hc^C_{it}s^C_{it}) + \sum_{t\in\mathcal{T}}\sum_{j\in\mathcal{P}} (sc^E_{jt}y^E_{jt} + vc^E_{jt}x^E_{jt}) + \mathbb{E}_\Omega[R_{\tilde{d}^E}(x^C, y^C, s^C, p^E, x^E, y^E)] \tag{18}$$

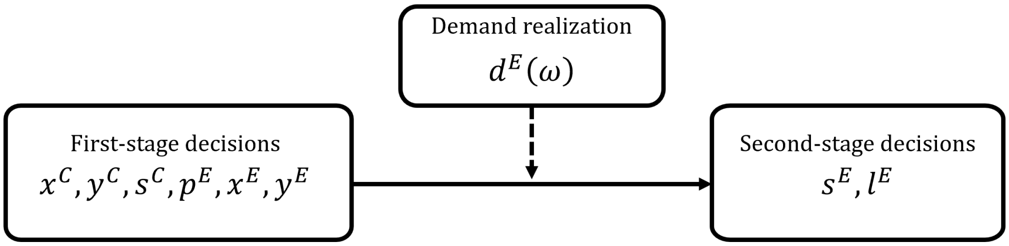

Figure 1: Timing of events for the Static MCLSB problem.

Subject to:

$$x_{it}^C \leq M_{it}^C y_{it}^C \qquad \forall i \in \mathcal{C}, t \in \mathcal{T} \tag{19}$$

$$x_{jt}^E \leq M_{jt}^E y_{jt}^E \qquad \forall j \in \mathcal{P}, t \in \mathcal{T} \tag{20}$$

$$s_{it-1}^C + x_{it}^C = \sum_{j \in \mathcal{P}_i} p_{ijt}^E + s_{it}^C \qquad \forall i \in \mathcal{C}, t \in \mathcal{T} \tag{21}$$

$$\sum_{j \in \mathcal{P}} (st_{jt} y_{jt}^E + vt_{jt} x_{jt}^E) \leq C_t \qquad \forall t \in \mathcal{T} \tag{22}$$

$$\sum_{i \in \mathcal{C}_j} p_{ijt}^E = x_{jt}^E \qquad \forall j \in \mathcal{P}, t \in \mathcal{T} \tag{23}$$

$$ql_{qj} x_{jt}^E \leq \sum_{i \in \mathcal{C}_j} qa_{qi} p_{ijt}^E \qquad \forall j \in \mathcal{P}, k \in \mathcal{Q}, t \in \mathcal{T} \tag{24}$$

$$qu_{qj} x_{jt}^E \geq \sum_{i \in \mathcal{C}_j} qa_{qi} p_{ijt}^E \qquad \forall j \in \mathcal{P}, q \in \mathcal{Q}, t \in \mathcal{T} \tag{25}$$

$$y_{jt}^E \in \{0,1\}, x_{jt}^E \geq 0 \qquad \forall j \in \mathcal{P}, t \in \mathcal{T} \tag{26}$$

$$y_{it}^C \in \{0,1\}, x_{it}^C, s_{it}^C \geq 0 \qquad \forall i \in \mathcal{C}, t \in \mathcal{T} \tag{27}$$

$$p_{ijt}^E \geq 0 \qquad \forall i \in \mathcal{C}, j \in \mathcal{P}, t \in \mathcal{T} \tag{28}$$

The objective cost (18) minimizes the first-stage cost for deciding production and purchase in advance of the demand realization, while accounting for the expected lost sale and inventory costs resulting from the second-stage decisions. For first-stage decisions z^{F} satisfying constraints (19)–(28), the second-stage problem can be solved for any arbitrary demand scenario $d^E(\omega)$ with a solver. However, solving the two-stage stochastic problem as a whole, considering all realizations, is prohibitive due to the computation of the expected value term, which depends on the number of realizations in Ω for the random demand vector. To make the problem numerically tractable in order to find the optimal objective value, discretizing the distribution of the random demand $\tilde{d}^E$ is typically done (Kaut and Stein 2003). In this approach, sample approximation is an effective method, where the original support Ω is approximated based on finite sets of randomly generated realizations.

4 Solution methodology

In this section, we present the approaches proposed to obtain and evaluate candidate solutions for the two-stage stochastic MCLSB problem. The deterministic approximation (MV) heuristic is first discussed in Subsection 4.1, and the application of the Sample Average Approximation (SAA) heuristic is described in Subsection 4.2. Then, the procedure for deriving statistical bounds and indicators regarding the optimality is discussed in Subsection 4.3. Lastly, we describe a variant of the SAA in Subsection 4.4, called the Adjustable Sample Average Approximation (ASAA) heuristic.

4.1 Deterministic approximation heuristic

A simple procedure to obtain a first-stage solution to the stochastic problem consists of disregarding uncertain values and solve the deterministic approximation problem that uses expected demand values instead. The solution derived from this approach provides a first-stage solution, which must be evaluated over a large number of demand realizations to compute the incurred costs in the stochastic setting, as will be explained. This counterpart is referred to as the mean value formulation (MV) or expected value problem, which is a common baseline heuristic in the literature (Verweij et al. 2003; Santoso et al. 2005; Contreras et al. 2011; Alvarez et al. 2021).

We obtain the MV formulation by replacing the values for the demands d^E in the deterministic mathematical formulation (1)–(13) with their expected values $\mathbb{E}[\tilde{d}^E]$. Thus, in the MV formulation, the new constraints (29) and (30) are considered instead of (5) and (6). Since the resulting formulation is a MILP model, the first-stage decision $z^{\mathrm{F}}_{\mathrm{EV}}$ can be obtained with available commercial solvers.

$$s^E_{jt-1} + x^E_{jt} + l^E_{jt} = \mathbb{E}[\tilde{d}^E_{jt}] + s^E_{jt} \qquad \forall j \in \mathcal{P}, t \in \mathcal{T} \tag{29}$$

$$l^E_{jt} \leq \mathbb{E}[\tilde{d}^E_{jt}] \qquad \forall j \in \mathcal{P}, t \in \mathcal{T} \tag{30}$$

4.2 Sample average approximation heuristic

To better approximate uncertainty, a finite number of specific realizations is randomly sampled. These realizations then serve in the two-stage stochastic problem as a set of scenarios to approximate the expected second-stage cost term. An approximate problem for the stochastic problem is then solved, from which a first-stage solution can be derived using a solver. In the literature, this heuristic approach is referred to as the Sample Average Approximation (SAA) method (Shapiro and Homem-de Mello 1998; Kleywegt et al. 2002). Since the yielded first-stage solution is based on an approximation, an evaluation over an independent larger set of scenarios is required.

Sampling approximation in the two-stage stochastic MCLSB problem is applied considering a set $\Omega_K \subset \Omega$ of K realizations of the random demand $\tilde{d}^E$ corresponding to possible demand scenarios $\{d^E(\omega_1), \cdots, d^E(\omega_K)\}$. The associated sample average approximation problem is (31) subject to (19)–(28) in conjunction with (32)–(34), where the objective is to minimize the sum of the first-stage cost $cost(z^{\mathrm{F}})$ (related to the decisions on setup, purchase and inventory of components, and setup and production of end products) and the associated expected recourse cost $(1/K)\sum_{s=1}^{K} R_{d^E(\omega_s)}(z^{\mathrm{F}})$ (associated with the decisions on lost sales and inventory of products over all demand scenarios in Ω_K). The formulation is a MILP model, which is solved using a solver. The procedure is then repeated in M iterations using independent sets $(\Omega_K)_1, \cdots, (\Omega_K)_M \subset \Omega$ of K demand scenarios each, providing the associated objective function values $(Z_K)_1, \cdots, (Z_K)_M$ and first-stage decisions $z^{\mathrm{F}}_1, \ldots, z^{\mathrm{F}}_M$.

$$\min \sum_{t\in\mathcal{T}} \sum_{i\in\mathcal{C}} (sc^C_{it} y^C_{it} + vc^C_{it} x^C_{it} + hc^C_{it} s^C_{it}) + \sum_{j\in\mathcal{P}} \sum_{t\in\mathcal{T}} (sc^E_{jt} y^E_{jt} + vc^E_{jt} x^E_{jt}) + \frac{1}{K} \sum_{\omega\in\Omega_K} \sum_{j\in\mathcal{P}} \sum_{t\in\mathcal{T}} \left(hc^E_{jt} s^E_{jt\omega} + pc_{jt} l^E_{jt\omega} \right) \tag{31}$$

$$s^E_{jt-1\omega} + x^E_{jt} + l^E_{jt} = d^E_{jt}(\omega) + s^E_{jt\omega} \qquad \forall j \in \mathcal{P}, t \in \mathcal{T}, \omega \in \Omega_K \tag{32}$$

$$l^E_{jt\omega} \leq d^E_{jt}(\omega) \qquad \forall j \in \mathcal{P}, t \in \mathcal{T}, \omega \in \Omega_K \tag{33}$$

$$s^E_{jt\omega}, l^E_{jt\omega} \geq 0 \qquad \forall j \in \mathcal{P}, t \in \mathcal{T}, \omega \in \Omega_K \tag{34}$$

4.3 Bounding procedure

At this point, a feasible first-stage decision z^{F} is available using either the sample approximation or the deterministic heuristics. We now discuss the ideas for the bounding technique in order to estimate the quality of candidate solutions compared to the optimum one, as outlined in Mak et al. (1999).

Upper bounds

The expected second-stage term needs to be estimated in the evaluation of a given feasible first-stage solution in the stochastic setting. To mitigate the inherent error coming from the individual samples, the estimation process requires computing the second-stage value over a set of realizations that is much larger than the number of scenarios used for the approximation problems. This process consequently derives an upper bound for the true optimal stochastic program problem value based on Monte Carlo simulation.

The evaluation process requires computing the second-stage cost $R_{d^E(\omega)}(z^{\mathrm{F}})$. For a given first-stage decision z^{F} and demand scenario $d^E(\omega)$, the value of $R_{d^E(\omega)}(z^{\mathrm{F}})$ is obtained by solving a pure linear programming problem on the second-stage variables, i.e., (14)–(17). In the estimation of the expected second-stage cost for a given first-stage decision z^{F}, the set of scenarios $\Omega_{K'}$ containing a total $K^{'} >> K$ of demand scenarios is employed. Thus, the value (35) provides an upper-bound estimate in the stochastic setting.

$$UB_{K'}(z^{\mathrm{F}}) = cost(z^{\mathrm{F}}) + \frac{1}{K'}\sum_{s=1}^{K^{'}} R_{d^E(\omega_s)}(z^{\mathrm{F}}) \tag{35}$$

Lower bounds

In the SAA method, solving M problems using independent sets of scenarios yields a set of optimal objective values. The average of these values allows us to estimate the expected objective value of the SAA problems, which serves as a statistical lower bound for the true optimal value of the stochastic problem (Bidhandi and Patrick (2017)).

More specifically, to derive a lower bound for the optimal value of the two-stage stochastic MCLSB problem, we need the optimal objective values $(Z_K)_1, \dots, (Z_K)_M$ obtained from solving M SAA problems considering sets $(\Omega_K)_1, \dots, (\Omega_K)_M$ of K demand scenarios. Given the difficulty in solving these problems to optimality, if sub-optimal solutions are returned due to stopping criteria, the average of the final lower bounds $\left(Z_K^{LB}\right)_1, \dots, \left(Z_K^{LB}\right)_M$ hence provides a conservative estimate. Consequently, the value in (36) is a lower bound in the stochastic setting.

$$LB_{K,M} = \frac{1}{M}\sum_{r=1}^{M}\left(Z_K^{LB}\right)_r \tag{36}$$

Gap and confidence

The lower and upper bounds are used to compute the estimated optimality gap of a first-stage decision in (37). It is also possible to calculate the confidence interval for the optimality gap, assuming it is normally distributed (Contreras et al. 2011).

$$Gap_{K,M,K'}(z^{\mathrm{F}}) = UB_{K'}(z^{\mathrm{F}}) - LB_{K,M} \tag{37}$$

The sample variance of the estimated expected second-stage cost in (38) is used to calculate the variance of the upper bound. Similarly, the variance of the optimal objective values from the SAA problems in (39) provides the variance of the statistical lower bound. Then, the estimated optimality gap variance can be calculated as in (40). For the level of confidence 95%, the extremes of the confidence interval for the relative optimality gap are computed as in (41).

$$\sigma^2_{UB_{K'}(z^{\mathrm{F}})} = \frac{1}{K'(K'-1)}\sum_{r=1}^{K^{'}}\left(R_{d^E(\omega_r)}(z^{\mathrm{F}}) - \frac{1}{K'}\sum_{s=1}^{K} R_{d^E(\omega_s)}(z^{\mathrm{F}})\right)^2 \tag{38}$$

$$\sigma^2_{LB_{K,M}} = \frac{1}{M(M-1)} \sum_{r=1}^{M} \left(Z(\Omega_K)_r - LB_{K,M}\right)^2 \tag{39}$$

$$\sigma^2_{Gap(z^{\mathrm{F}})} = \sigma^2_{UB_{K'}(z^{\mathrm{F}})} + \sigma^2_{LB_{K,M}} \tag{40}$$

$$CI(95\%; Gap_{K,M,K'}) = (Gap_{K,M,K'}) \pm 1.96 \times (\sigma_{Gap_{K,M,K'}}) \tag{41}$$

4.4 Adjustable sample average approximation heuristic

While larger scenario sets of scenarios in the SAA method generally lead to better approximations of the optimal solution, they also increase MILP problem size, which makes it more difficult to yield good solutions. To address this scalability issue, the Adjustable Sample Average Approximation (ASAA) method proposed by Agra et al. (2018) divides the process into two phases to balance computational effort and solution quality.

Phase 1: Solving smaller problems and identifying consistent variable values

In the ASAA approach for the two-stage stochastic MCLSB problem, the goal of the first phase is to identify and fix the setup decisions in y^C and y^E for those variables that consistently have the same values. More specifically, in Phase 1, the first-stage decisions $z^{\mathrm{F}}_1, \cdots, z^{\mathrm{F}}_{\hat{M}}$ are initially obtained using $\hat{M}$ independent sets $(\Omega_{\hat{K}})_1, \ldots, (\Omega_{\hat{K}})_{\hat{M}}$ of $\hat{K}$ demand scenarios. Then, for each setup variables y, a weighted average, $\sum_{s=1}^{\hat{M}} \mathrm{w}_s y_s$, is computed to assess how consistent each setup value is across the $\hat{M}$ iterations. In this weighted average, the weight formula is $\mathrm{w}_s = (UB^{\max} + LD - UB(z^{\mathrm{F}}_{SAA_s}))/\sum_{r=1}^{\hat{M}} \left(UB^{\max} + LD - UB(z^{\mathrm{F}}_{SAA_r})\right)$, where the two normalizing factors $UB^{\max} = \max\{UB(z^{\mathrm{F}}_{SAA_s});\ s = 1, \cdots, \hat{M}\}$ and $LD = \min\{UB^{\max} - UB(z^{\mathrm{F}}_{SAA_s});\ s = 1, \cdots, \hat{M};\ UB^{\max} \neq UB(z^{\mathrm{F}}_{SAA_s})\}$ assign higher weights to the decisions with the lower evaluated cost $UB(z^{\mathrm{F}}_{SAA_s})$. Therefore, the evaluation of the initial first-stage decisions is necessary in Phase 1. Based on this weighted value, the rule to either fix the setup at a value or leave it free for further iterations is set according to two thresholds γ_1 and γ_2 as:

$$y = \begin{cases} 0, & \text{if } \sum_{s=1}^{\hat{M}} \mathrm{w}_s y_s \leq \gamma_1; \\ 1, & \text{if } \sum_{s=1}^{\hat{M}} \mathrm{w}_s y_s \geq \gamma_2; \\ \text{not fixed}, & \text{otherwise.} \end{cases}$$

Phase 2: Fixing variable values and completing the partial solution

Once the y^C and y^E variables having the most consistent values across the $\hat{M}$ iterations have been identified, a partial first-stage decision $\hat{\mathfrak{z}}^{\mathrm{F}}$ is created. Fixing these consistent variables in $\hat{\mathfrak{z}}^{\mathrm{F}}$ simplifies the MILP problem, allowing subsequent iterations to focus on solving SAA problems with a larger set of scenarios. In Phase 2, the simplified SAA problems uses M independent sets $(\Omega_K)_1, \ldots, (\Omega_K)_M$ of $K' > K > \hat{K}$ demand scenarios. Thus, Phase 2 yields a set of first-stage decisions $z^{\mathrm{F}}_1, \ldots, z^{\mathrm{F}}_M$. As in the regular SAA, the final evaluation of these solutions is performed over the set $\Omega_{K'}$ of K' demand scenarios.

Because certain variables are fixed in Phase 2, the solutions from these Phase 2 replications cannot be used to calculate the statistical lower bound, as they do not fully represent the entire stochastic problem. Therefore, the statistical lower bound is calculated using only the solutions of Phase 1. Finally, the workflow of the SAA and ASAA for the two-stage stochastic MCLSB problem is given in Figures 2 and 3, respectively.

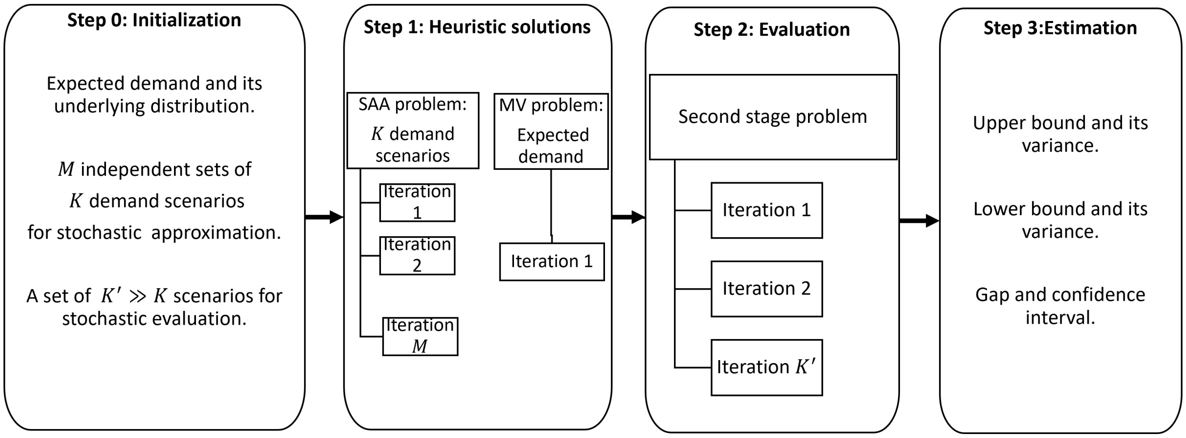

Figure 2: Schematic diagram for the regular SAA solution procedure.

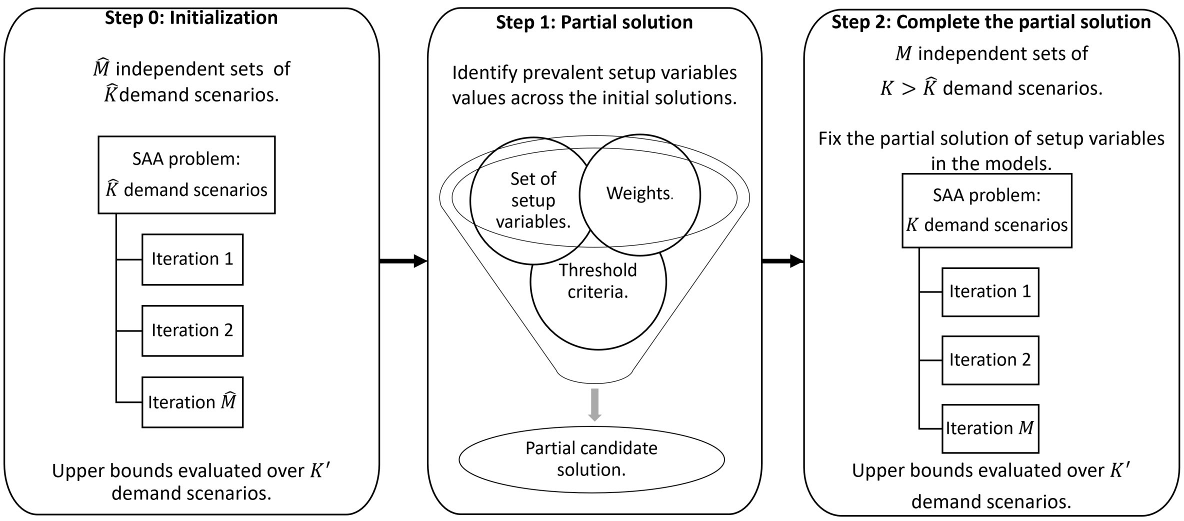

Figure 3: Schematic diagram for the ASAA solution procedure.

5 Computational experiments

The Python programming language was used to implement the heuristics. The MILP and LP models used to solve the problem were coded and solved using the Gurobi 11.0.0 solver. The stopping criteria for the solver are the time limit of 3600 seconds and the MIP gap tolerance of 0.01%. All experiments were performed on an Intel(R) Core i7(R) CPU-1255U with 1.70 GHz and 16 GB of RAM. The computational times provided in the results are expressed in seconds.

Since there is no particular database for the MCLSB problem, two blending problems adapted from the literature and one set of generated instances are used to define the set of instances. The main test problems and their parameters defining the set of instances are as follows: Problem A is a fuel production problem involving $|\mathcal{P}| = 3$ products, $|\mathcal{C}| = 3$ components and $|\mathcal{Q}| = 2$ qualities; Problem B is a feed production problem involving $|\mathcal{P}| = 2$ products, $|\mathcal{C}| = 4$ components and $|\mathcal{Q}| = 2$ qualities; Problem C is a random problem involving $|\mathcal{P}| = 6$ products, $|\mathcal{C}| = 6$ components and $|\mathcal{Q}| = 4$ qualities. For all problems, $|\mathcal{T}| = 15$ time periods are considered.

For each main problem, six instances were generated changing the parameters for components availability M^C and production capacity C. For each main problem, we consider three levels for the component availability parameter (M^C, $2 \times M^C$ and $M_{it}^{C'} = \sum_{\tau=t}^{m} \sum_{j \in \mathcal{P}_i} M_{j\tau}^{E}$), and two levels for the production capacity (C and $1.3 \times C$). Six instances were hence generated for each main problem, giving a total of 18 instances.

During the experimentation, the value of the stochastic model is assessed by comparing the solution from the SAA method to the mean value counterpart. The expected demand $\mathbb{E}[\tilde{d}^E]$ is taken to define the mean value (MV) problem, and to represent the uncertainty, demand scenarios $d^E(\omega)$ are generated following a uniform distribution: $\tilde{d}_{jt}^E \sim U\left[0.8 \times \mathbb{E}[\tilde{d}_{jt}^E], 1.2 \times \mathbb{E}[\tilde{d}_{jt}^E]\right]$. In the evaluation step, a set of demand scenarios $\Omega_{K'} \subset \Omega$ consisting of $K^{'} = 1 \times 10^4$ demand scenarios is used for the evaluation of the solutions.

The link bit.ly/4h3M1oI includes the input and output data used in the experiments, and the appendices, which provide the procedure utilized for both the data generation and definition of the instance data sets (Section A of the online supplements), as well as extended computational results (Section B of the online supplements).

5.1 Complexity and evaluation of the heuristics in the recourse problem

We present an initial assessment of the SAA and MV heuristics for the two-stage stochastic MCLSB formulation to analyze how hard it is to solve the approximation models, as well as to calculate the value for their candidate solutions. For this initial phase of the computational experiments, we solve $M = 1$ replication for sets of demand scenarios Ω_K with $|\Omega_K| =$ 10, 50, 100 and 500 scenarios using the SAA heuristic.

Table 1 presents the average results of the experiments for the 18 instances. We report the corresponding key indicators obtained for the different models: the number of instances solved to optimality (#Optimal), the average final relative optimality gap for the solutions found by the solver (MIP Gap $= 1 - Z^{LB}/Z^{UB}$), the average computational time taken to optimize the corresponding instances (Solution Time), and the average computational time to evaluate the upper bound for the given candidate solutions (Evaluation Time). More detailed computational results are presented in Tables B16 to B22 in Section B of the online supplements for the same experiment, which are grouped by: instance problem, capacity factor for production and availability for components' purchase.

Table 1 also reports the associated costs for the stochastic solutions: the upper bound (UB) obtained by the evaluation over the set of demand scenarios $\Omega_{K'}$, and the values corresponding to the fixed first stage cost and the expected recourse cost (i.e., FSC= $cost(z^{\mathrm{F}})$ and SSC= $(1/K^{'}) \sum_{s=1}^{K^{'}} R_{d^E(\omega_s)}(z^{\mathrm{F}})$). In Tables B20 to B22 in Section B of the online supplements, these results are grouped by instances sharing the same characteristics.

Table 1: General results for the MV and SAA heuristics considering $M = 1$ iteration.

Model	#Opt.	MIP Gap	Sol. Time	Eva. Time	UB	FSC	SSC
MV, $\mathbb{E}[\tilde{d}^E]$	12	0.17%	1,431.9	119.9	11,259,603	9,823,867	1,435,736
SAA, $\|\Omega_K\| = 10$	11	0.16%	1,657.1	127.9	11,168,465	9,799,713	1,368,752
SAA, $\|\Omega_K\| = 50$	7	0.34%	2,221.5	126.9	11,142,801	9,802,922	1,339,879
SAA, $\|\Omega_K\| = 100$	7	0.44%	2,244.2	118.3	11,146,004	9,780,545	1,365,460
SAA, $\|\Omega_K\| = 500$	6	1.03%	2,561.5	109.8	11,182,981	9,827,151	1,355,830

The general results in Table 1 for the two-stage stochastic MCLSB problem show that the majority of the instances are solved to optimality using the MV model, while this value drops to less than half using the SAA with $|\Omega_K| = 500$ scenarios. Notice also that the computational time needed to find the candidate solutions is considerably higher than the computational time necessary to evaluate the solutions over the large evaluation set. Moreover, while the time to find the candidate solutions grows with the number of demand scenarios, the evaluation time remains consistent since we always use the same number of scenarios for the evaluation. The grouped results (Tables B18 and B19 in Section B of the online supplements) show that the candidate solutions for the instances derived from Problem C are more difficult to be found and evaluated, given that they comprise the largest instances in terms of the number of variables in both the first and second stage. In the sensitivity analysis of the parameters, a noticeable increase in the solution time is observed when the availability for components' purchase is loose, while a slight impact on the difficulty of the solution process is observed when the tight capacity for production is considered.

Overall, the SAA model with $|\Omega_K| = 50$ demand scenarios provides the candidate solutions with the smallest average upper bound. The candidate solutions using $|\Omega_K| = 10$ demand scenarios or the MV approach yield high upper bounds for the total cost in the evaluation. Conversely, while increasing the number of demand scenarios initially enhances the solution quality, the computational complexity and MIP optimality gaps increase significantly with a very large number of scenarios (e.g., the SAA solutions with $|\Omega_K| = 500$). Indeed, these stochastic approximation problems with too many demand scenarios can lead to less efficient solutions compared to all other numbers of demand scenarios and the MV heuristic, as observed in Problem C (Table B20), suggesting that there might be diminishing returns or even negative effects with too many scenarios.

Examining the total cost structure, the SAA heuristic's advantage is more pronounced in the second stage cost reduction. Analyzing the candidate solutions derived from the SAA approach, the results suggest that these solutions are able to generate a more resilient production plan in response to the variability in demand. However, the deterministic baseline solution is less capable of coping with such fluctuations, making the solution more susceptible to experiencing lost sales.

5.2 Generating multiple solutions

In this part of the experiments, we investigate the performance of the SAA heuristic for the two-stage stochastic MCLSB problem for a different number of replications. We consider solving up to $M = 5$ or 10 replications for sets Ω_K with $|\Omega_K| = 10$, 50 or 100 demand scenarios using the SAA heuristic. From the first-stage decisions $z_1^{\mathrm{F}}, \cdots, z_M^{\mathrm{F}}$ across the M candidate solutions obtained in the SAA heuristic with multiple replications, the upper bound of the first-stage decision z_{SAA}^{F} returning the best evaluated value as given in (42) is selected. Considering the first-stage variables $z_{\mathrm{EV}}^{\mathrm{F}}$ obtained by solving the mean value formulation, the upper bound value for the deterministic baseline heuristic is given by (43). This value was already reported in Table 1, but we use it here for measuring the improvement of the solution provided by SAA compared to the solution provided by the deterministic baseline heuristic (Contreras et al. 2011; Alvarez et al. 2021; Agra et al. 2018) computed as in (44).

$$RP = \min\left\{UB_{K'}(z_1^{\mathrm{F}}), \cdots, UB_{K'}(z_M^{\mathrm{F}})\right\} \tag{42}$$

$$EEV = UB_{K'}(z_{\mathrm{EV}}^{\mathrm{F}}) \tag{43}$$

$$\Delta = \frac{EEV - RP}{EEV} \tag{44}$$

Table 2 reports the average results for the 18 instances using the SAA method with multiple iterations. The second column presents the best upper bound value RP. The third column LB represents the statistical lower bound for the two-stage stochastic programming problem. The fourth column provides the estimated relative stochastic gap (Gap/RP). The last column gives the 95% confidence interval using the standard deviation of the optimality gap. Since the variance of the statistical lower bound cannot be computed when $M = 1$ in the SAA heuristic, the presence of a '-' for the corresponding confidence interval indicates that the estimations for the optimality gap is not possible.

In general, the SAA heuristic considering $M = 10$ replications with sets of $|\Omega_K| = 50$ demand scenarios achieves the best results for the minimum upper bound value. With an average optimality gap of 0.4%, this configuration provides high quality stochastic solutions. The tighter confidence intervals also indicate the higher reliability in solving the two-stage stochastic MCLSB problem in this configuration for the SAA heuristic, compared to $|\Omega_K| = 10$.

Next, we will discuss the improvement of the various versions of the SAA heuristic, compared the baseline Mean Value heuristic, as defined by (44) and indicted in Table 2. In addition, we also provide the computational time required to find the M solutions, the time to evaluate all of them over the larger set of demand scenarios and the corresponding total time.

For a given number of scenarios, increasing the number of replications leads to a higher improvement, but we observe some differences. Multiple replications show a greater improvement in the upper bounds considering $|\Omega_K| = 10$, because the small number of demand scenarios used for the approximation problems does not consistently capture demand uncertainty. However, increasing the number of replications for $|\Omega_K| = 50$ and $|\Omega_K| = 100$ causes only a slight improvement in the upper bounds, showing that fewer runs are required until the uncertainty is adequately captured for these sizes for sets of demand scenarios. However, taking into account the total time, iterating from $M = 5$ to $M = 10$ replications with problems of $|\Omega_K| = 50$ or $|\Omega_K| = 100$ demand scenarios does not provide substantial improvements, since the upper bounds are similar while the setting with $M = 10$ takes approximately double the time.

As noted in the experiments for the SAA with $M = 1$ iteration, solving the large problem with $|\Omega_K| = 500$ is computationally complex, resulting in a solution of poor quality during the evaluation step. Consequently, we do not provide results for multiple iterations in this configuration. We report that the optimality gap is 1.06%, the improvement over the MV approach is 1.52%, and the total computational time is 2,671.3 seconds when $M = 1$ and $|\Omega_K| = 500$.

Table 2: General results for the SAA method considering $M = 1, 5$ and 10 iterations.

Parameters	SAA							
$(M, \|\Omega_K\|)$	*RP*	*LB*	*Gap*	$CI(95\%; Gap)$	Δ	Sol. Time	Eva. Time	Tot. Time
(1, 10)	11,168,465	11,021,200	1.22%	-	1.40%	1,657.1	127.9	1,784.9
(5, 10)	11,156,256	11,046,350	0.84%	(0.14%, 1.55%)	1.63%	8,886.6	598.8	9,485.4
(10, 10)	11,153,007	11,050,983	0.80%	(0.33%, 1.26%)	1.66%	18,576.9	1,176.5	19,753.4
(1, 50)	11,142,801	11,048,395	0.72%	-	1.70%	2,221.5	126.9	2,348.4
(5, 50)	11,137,859	11,068,296	0.51%	(0.27%, 0.76%)	1.77%	11,100.3	598.1	11,698.4
(10, 50)	11,135,677	11,079,441	0.40%	(0.16%, 0.64%)	1.79%	22,032.9	1,174.7	23,207.5
(1, 100)	11,146,004	11,069,843	0.63%	-	1.72%	2,244.2	118.3	2,362.5
(5, 100)	11,137,772	11,059,285	0.59%	(0.37%, 0.81%)	1.77%	11,570.0	606.6	12,176.6
(10, 100)	11,137,170	11,058,822	0.56%	(0.37%, 0.74%)	1.78%	23,641.5	1,222.2	24,863.6

5.3 Combining multiple solutions in the adjustable approach

In this part of the computational experiments, we assess the performance of the Adjustable variant of the SAA heuristic (ASAA) to solve the two-stage stochastic MCLSB problem. During Phase 1 of the ASAA approach, we consider that the initial solutions obtained using sets of demand scenarios $\Omega_{\hat{K}}$ with $|\Omega_{\hat{K}}| = 10$ are used to define the weights and a partial first-stage solution (by fixing the setup decision variables that have common values in most initial solutions). For this analysis, we consider doing $\hat{M} = 5$ or $\hat{M} = 10$ replications in Phase 1. We use $\gamma_1 \in \{0, 0.1, 0.2, 0.3, 0.4\}$ for $\hat{M} = 5$ replications, and $\gamma_1 \in \{0, 0.1, 0.2, 0.3\}$ for $\hat{M} = 10$ replications. In both cases, we set $\gamma_2 = 1 - \gamma_1$. Based on these thresholds, we obtain the partial solution that fixes some of the first-stage variables in Phase 2 of the ASAA approach. In Phase 2, we consider obtaining $M = 5$ or $M = 10$ final candidate solutions using sets of demand scenarios Ω_K with $|\Omega_K| = 50$ or $|\Omega_K| = 100$ using the partial solution. Furthermore, we consider solving $M = 1$ large SAA problem with $|\Omega_K| = 500$ demand scenarios. The solutions $z_1^{\mathrm{F}}, \cdots, z_M^{\mathrm{F}}$ obtained from Phase 2 are evaluated over the evaluation set of demand scenarios $\Omega_{K'}$ to provide the cost in the recourse problem, where the upper bound of the first-stage decisions z_{ASAA}^{F} with best evaluated value is selected as in (42).

Table 3 gives some numbers on the % fixed of setup variables in the partial solution at the level of the components, end products, and overall, using different settings in Phase 1 of the ASAA method. The corresponding total time to obtain and evaluate the set of initial candidate solutions used to determine the partial solution in Phase 1, and the lower bounds used next in Phase 2, are also reported. These last values correspond to the values reported in Table 2 for the settings $M = 5$, $|\Omega_K| = 10$ and $M = 10$, $|\Omega_K| = 10$. Table 4 provides the upper bound provided by the best evaluated solution in Phase 2, the optimality gap (using the statistical lower bound from Phase 1), as well as the improvement compared to the Mean Value approach. Table 5 reports the information on the computational time needed to obtain and evaluate the candidate solutions in Phase 2, and the corresponding sum of these times for the total time in Phase 2. The sum of the time spent for Phases 1 and 2 in the ASAA heuristic is also reported.

A comparison of the results in Table 4 for Phase 2 of the adjustable SAA approach with the results in Table 2 for the regular SAA approach shows that the quality of the solutions is generally higher for the ASAA heuristic. In Phase 2 of the ASAA approach with $|\Omega_K| = 100$ demand scenarios, no significant difference in the best upper bound value *RP* and relative optimality gap *Gap* can be observed by increasing the replications from $M = 5$ to $M = 10$. For the ASAA approach with $|\Omega_K| = 50$ demand scenarios, the increase in the number of replications from $M = 5$ to $M = 10$ provides a slight improvement of the best upper bound value *RP* and relative optimality gap *Gap*. In general, a positive effect on performance in terms of the best upper bound is achieved using a moderate threshold to fix the setup variables. This enhancement observed in the adjustable approach over the regular SAA method partially lies in the better values for the (first-stage) decision variables related to the production and purchase quantities obtained in the second phase of the algorithm, since we can solve the problem now with a large number of scenarios due to the fixing of some of the setup variables. Indeed, the comparison between the results in Tables 1 and 4 with respect to the upper bounds demonstrates that the proposed adjustable strategy performs better than the regular SAA heuristic to handle the large problem with $|\Omega_K| = 500$.

Increasing the value of γ from zero to a positive value leads to more fixed setup variables, and hence problems that are easier to solve, as clearly shown by the solution times provided in Table 5. As a result of this fixing, in the ASAA approach, the total time spent to solve one large problem in Phase 2 with $|\Omega_K| = 500$ demand scenarios, as well as the problems considering $|\Omega_K| = 50$ or $|\Omega_K| = 100$ demand scenarios with $M = 5$ and 10 replications, is significantly reduced compared to the corresponding times reported in Tables 1 and 2. The increase in γ initially also has a beneficial effect on the quality of the solution. However, the threshold value used should not be too large, since the quality of the solution in the ASAA approach can become worse when too many variables are fixed a priori, as it can be observed for $\hat{M} = 5$ with $\gamma = 0.4$ and $\hat{M} = 10$ with $\gamma = 0.3$ in Phase 1.

Next, we will discuss the performance of the regular and adjustable SAA heuristics considering the results for the best upper bound value and the overall time taken (in Phase 1 plus in Phase 2) to obtain the final set of candidate solutions. The results demonstrate how both time saving and enhancement of the solution quality can be obtained with the adjustable SAA heuristic. In fact, taking into account the best upper bound obtained with the regular SAA approach in the configuration with $M = 10$ replications and $|\Omega_K| = 50$ demand scenarios, a reduction in this best upper bound value can be achieved with Phase 2 of the ASAA approach by generating $\hat{M} = 5$ solutions with demand scenarios $\hat{K} = 10$ and using the threshold value of $\gamma = 0.1$ during Phase 1. Although this reduction is marginal, we observe that the total time for Phases 1 and 2 for the ASAA heuristic compared to the regular SAA heuristic is significantly shorter. Thus, the ASAA approach can produce a small improvement in the upper bound by carefully configuring Phase 1 to solve smaller problems in advance rather than solving SAA problems directly as in the regular SAA heuristic. For large problems, when Phase 1 of the ASAA approach comprises fewer replications considering $\hat{M} = 5$ with $\gamma = 0.1$, 0.2 and 0.3, the improvement for the quality of the solution and the total time is even more substantial. Specifically, in Phase 1 of the adjustable SAA approach with $\hat{M} = 5$ replications comprising $|\Omega_{\hat{K}}| = 10$) demand scenarios and fixing at $\gamma = 0.1$ and 0.2, the simplification results a less complex model in Phase 2 considering $|\Omega_K| = 500$ demand scenarios, which also yields the best results. In Tables B28 to B33 in Section B of the online supplement, the results indicate that ASAA generally outperforms SAA across most instance groups as well.

Table 3: General results for Phase 1 of the ASAA method.

Parameters	Phase 1 of ASAA						
	% variables fixed						
$(\hat{M}, \lvert\Omega_{\hat{K}}\rvert, \gamma)$	y^C	y^E	$y^C + y^E$	Sol. Time	Eva. Time	Time Phase 1	*LB*
(5, 10, 0)	0.75	0.57	0.68	8,886.6	598.8	9,485.4	11,046,350
(5, 10, 0.1)	0.88	0.72	0.81	8,886.6	598.8	9,485.4	11,046,350
(5, 10, 0.2)	0.94	0.79	0.87	8,886.6	598.8	9,485.4	11,046,350
(5, 10, 0.3)	0.96	0.88	0.92	8,886.6	598.8	9,485.4	11,046,350
(5, 10, 0.4)	0.97	0.92	0.95	8,886.6	598.8	9,485.4	11,046,350
(10, 10, 0)	0.71	0.42	0.58	18,576.9	1,176.5	19,753.4	11,050,983
(10, 10, 0.1)	0.83	0.62	0.74	18,576.9	1,176.5	19,753.4	11,050,983
(10, 10, 0.2)	0.90	0.74	0.82	18,576.9	1,176.5	19,753.4	11,050,983
(10, 10, 0.3)	0.94	0.85	0.90	18,576.9	1,176.5	19,753.4	11,050,983

Table 4: General solution quality results for Phase 2 of the ASAA method.

Phase 1 parameters	Phase 2 of ASAA					
	$M = 5$, $\lvert\Omega_K\rvert = 50$			$M = 10$, $\lvert\Omega_K\rvert = 50$		
$(\hat{M},\lvert\Omega_{\hat{K}}\rvert,\gamma)$	*RP*	*Gap*	Δ	*RP*	*Gap*	Δ
(5,10,0)	11,136,770	0.70%	1.78%	11,135,673	0.69%	1.79%
(5,10,0.1)	11,136,728	0.70%	1.78%	11,135,650	0.69%	1.79%
(5,10,0.2)	11,136,753	0.70%	1.78%	11,135,676	0.69%	1.79%
(5,10,0.3)	11,138,414	0.71%	1.79%	11,137,074	0.70%	1.78%
(5,10,0.4)	11,256,869	1.45%	1.78%	11,256,030	1.44%	1.00%
(10,10,0)	11,136,818	0.68%	1.78%	11,135,455	0.67%	1.79%
(10,10,0.1)	11,136,832	0.68%	1.78%	11,135,801	0.67%	1.79%
(10,10,0.2)	11,137,724	0.69%	1.77%	11,136,637	0.68%	1.78%
(10,10,0.3)	11,146,569	0.76%	1.70%	11,145,203	0.75%	1.71%

Phase 1 parameters	Phase 2 of ASAA					
	$M = 5$, $\lvert\Omega_K\rvert = 100$			$M = 10$, $\lvert\Omega_K\rvert = 100$		
$(\hat{M},\lvert\Omega_{\hat{K}}\rvert,\gamma)$	*RP*	*Gap*	Δ	*RP*	*Gap*	Δ
(5,10,0)	11,134,809	0.68%	1.79%	11,134,583	0.68%	1.79%
(5,10,0.1)	11,134,741	0.68%	1.79%	11,134,551	0.68%	1.79%
(5,10,0.2)	11,134,734	0.68%	1.79%	11,134,551	0.68%	1.79%
(5,10,0.3)	11,136,212	0.69%	1.78%	11,136,027	0.69%	1.78%
(5,10,0.4)	11,254,985	1.43%	1.01%	11,254,726	1.43%	1.01%
(10,10,0)	11,134,537	0.66%	1.80%	11,134,262	0.66%	1.80%
(10,10,0.1)	11,134,604	0.66%	1.80%	11,134,392	0.66%	1.80%
(10,10,0.2)	11,135,572	0.67%	1.79%	11,135,343	0.67%	1.79%
(10,10,0.3)	11,144,282	0.74%	1.72%	11,144,038	0.74%	1.72%

Phase 1 parameters	Phase 2 of ASAA		
	$M = 1$, $\lvert\Omega_K\rvert = 500$		
$(\hat{M},\lvert\Omega_{\hat{K}}\rvert,\gamma)$	*RP*	*Gap*	Δ
(5,10,0)	11,133,861	0.67%	1.80%
(5,10,0.1)	11,132,943	0.67%	1.81%
(5,10,0.2)	11,132,945	0.67%	1.81%
(5,10,0.3)	11,134,528	0.68%	1.79%
(5,10,0.4)	11,253,505	1.42%	1.02%
(10,10,0)	11,140,579	0.71%	1.80%
(10,10,0.1)	11,134,576	0.67%	1.79%
(10,10,0.2)	11,133,853	0.66%	1.80%
(10,10,0.3)	11,142,642	0.73%	1.73%

Table 5: General solution time results for Phase 2 of the ASAA method.

Phase 1 parameters	Phase 2 of ASAA							
	$M = 5$, $\lvert\Omega_K\rvert = 50$				$M = 10$, $\lvert\Omega_K\rvert = 50$			
($\hat{M}$,$\lvert\Omega_{\hat{K}}\rvert$,$\gamma$)	Sol.	Eva.	Phase 2	Phase 1&2	Sol.	Eva.	Phase 2	Phase 1&2
(5, 10, 0)	2,752.6	570.9	3,323.4	12,808.8	5,393.3	1,143.1	6,536.4	16,021.8
(5, 10, 0.1)	1,814.4	566.2	2,380.6	11,866.0	3,620.2	1,132.0	4,752.2	16,021.8
(5, 10, 0.2)	5.9	546.2	552.1	10,037.5	12.0	1,095.6	1,107.6	10,593.0
(5, 10, 0.3)	2.5	552.1	554.6	10,037.5	5.1	1,103.0	1,108.1	10,593.5
(5, 10, 0.4)	1.4	571.7	573.1	10,058.5	2.8	1,123.2	1,126.0	10,611.4
(10, 10, 0)	3,819.4	590.3	4,409.6	24,163.0	8,124.3	1,162.9	9,287.3	29,040.6
(10, 10, 0.1)	2,872.1	633.8	3,505.8	23,259.2	5,772.3	1,324.9	7,097.1	26,850.5
(10, 10, 0.2)	180.6	710.8	891.4	20,644.8	354.5	1,404.5	1,759.0	21,512.4
(10, 10, 0.3)	5.2	686.3	691.5	20,444.8	10.2	1,367.7	1,377.8	21,131.2
Phase 1 parameters	Phase 2 of ASAA							
	$M = 5$, $\lvert\Omega_K\rvert = 100$				$M = 10$, $\lvert\Omega_K\rvert = 100$			
($\hat{M}$,$\lvert\Omega_{\hat{K}}\rvert$,$\gamma$)	Sol.	Eva.	Phase 2	Phases 1&2	Sol.	Eva.	Phase 2	Phases 1&2
(5, 10, 0)	3,488.4	649.1	4,137.5	13,622.9	6,325.0	1,216.8	7,541.7	17,027.1
(5, 10, 0.1)	1,947.9	590.4	2,538.2	12,023.6	3,727.5	1,153.0	4,880.5	14,365.9
(5, 10, 0.2)	19.7	576.2	595.9	10,081.3	39.1	1,140.3	1,179.4	10,664.8
(5, 10, 0.3)	6.6	547.0	553.6	10,039.0	14.3	1,106.2	1,120.6	10,606.0
(5, 10, 0.4)	3.6	570.4	574.0	10,059.4	7.3	1,154.5	1,161.9	10,647.3
(10, 10, 0)	4,838.3	580.6	5,418.8	25,172.2	9,130.8	1,159.0	10,289.8	30,043.2
(10, 10, 0.1)	3,030.0	581.6	3,611.6	23,365.	6,157.1	1,158.6	7,315.7	27,069.1
(10, 10, 0.2)	456.6	576.3	1,032.9	23,365.0	870.1	1,139.8	2,009.9	21,763.3
(10, 10, 0.3)	11.4	667.9	679.3	20,432.7	19.7	1,256.4	1,276.1	21,029.5
Phase 1 parameters	Phase 2 of ASAA							
	$M = 1$, $\lvert\Omega_K\rvert = 500$							
($\hat{M}$, $\lvert\Omega_{\hat{K}}\rvert$, γ)	Sol.	Eva.	Phase 2	Phases 1&2				
(5, 10, 0)	1,103.8	125.1	1,228.9	10,714.3				
(5, 10, 0.1)	443.9	115.4	559.2	10,044.6				
(5, 10, 0.2)	51.2	109.6	160.8	9,646.2				
(5, 10, 0.3)	14.6	108.5	123.0	9,608.4				
(5, 10, 0.4)	5.4	116.0	121.5	9,606.9				
(10, 10, 0)	1,103.8	125.1	1,228.9	21,276.6				
(10, 10, 0.1)	952.5	122.3	1,074.9	20,828.3				
(10, 10, 0.2)	506.6	119.7	626.3	20,379.7				
(10, 10, 0.3)	20.5	108.9	129.5	19,882.9				

6 Conclusions

In this paper, we address a blending problem that integrates purchasing and production decisions over multiple time periods. We develop a stochastic programming formulation to consider demand uncertainty. The stochastic programming formulation we have proposed aims to minimize the minimum expected overall costs, considering a combination of two-stage stochastic programming modeling (Higle 2005) and the static uncertainty strategy discussed for stochastic lot sizing problems (Bookbinder and Tan 1988). In this formulation, demand uncertainty is taken into account with determining production and purchasing decisions upfront to demand realization. The Sample Average Approximation (SAA) method is used as a heuristic scheme to obtain candidate solutions (Shapiro and Homem-de Mello 1998) and statistical bounds (Mak et al. 1999). Furthermore, we implemented an adjustable variant of this heuristic (ASAA) method (Agra et al. 2018) to enhance the efficiency of the solution approach by using information from the solutions generated in the process.

Computational experiments were conducted on randomly generated instances and examples to provide an evaluation of the benefits and shortcomings of the proposed stochastic approach and its solutions. The computational results demonstrated that the SAA heuristic outperforms the deterministic approach, resulting in substantially lower total costs when the number of demand scenarios was appropriately selected. Additionally, the results demonstrate that the ASAA approach can lead to faster computation times while improving solution quality of the solutions obtained by the regular SAA approach. In general, our findings contribute to a better understanding of blending planning under demand uncertainty, highlighting the advantages and limitations of stochastic programming methods in achieving cost effectiveness for production and purchase plans.

The data that support the findings of this study are openly available in Google Drive at `bit.ly/4h3M1oI`, reference number Online Supplement – MCLSB Problem.

Further research can be focused on the investigation of the different configurations in which integrated blending and lot sizing problems may appear in practice under demand uncertainty. Other extensions could be proposed such as a multi-stage stochastic programming problem considering a rolling horizon fashion. An interesting formulation could be based on the static-dynamic uncertainty strategy, which would consider only setup variables as first-stage decisions for the lot sizing problem associated with the production of end products. The resulting dynamic stochastic framework would make the problem inherently difficult to solve exactly, requiring some approximation approaches. In this sense, we could explore the approximated solutions in the new approach obtained from the two-stage stochastic MCLSB formulation in a multi-stage stochastic context. Similar implementations were investigated by Sereshti et al. (2024). Finally, a comparison between the different strategies could then provide insights into steady-versus-adaptable approaches through different recourse mechanisms to address demand uncertainty. Similar approaches were investigated by Tomazella et al. (2024).

A Instances set

In this section, we describe the primary instances utilized for the implementation of the proposed method and the formulations discussed for the MCLSB problem. These instances cover two specific applications as well as a procedure used to generate random instances. The formulas and ideas used to define part of the parameters in the test problems were adapted from Fiorotto et al. (2021); Absi et al. (2013).

A.1 Random instances and data generation

We consider that parameters are invariant across time periods, unless stated otherwise. The parameters listed below are taken from a uniform distribution considering the indicated intervals.

- Production time for end products ($vt_{jt} = vt_j$): $[1, 5]$;
- Setup time for end products ($st_{jt} = st_j$): $[10, 50]$;
- Production cost for components ($vc^C_i = vc^C_i$): $[50, 150]$;
- Expected demands of end products ($\mathbb{E}[\tilde{d}^E_{jt}] = \mathbb{E}[\tilde{d}^E_j]$): $[500, 1500]$.

Dataset generation for blending

To design the blending problem, it is necessary to determine the relation between the components and end products, as well as their quality conditions.

The relation between components is generated to entail a product structure based on a Bill-of-Materials (BOM), which is typically present in the production line of companies that integrate procurement and production activities. Thus, $\mathcal{C}_j$ for each end product $j \in \mathcal{P}$ is determined by random and distinct subsets of components of size $|\mathcal{C}_j| = 0.5 \times |\mathcal{C}|$, $\forall j \in \mathcal{P}$.

For all components $i \in \mathcal{C}$, the generation of quality attributes $qa_i = (qa_{iq})_{\forall q \in \mathcal{Q}}$ takes place in two steps, as follows.

1. Initialization: The entries of qa_i are initialized using a uniform distribution in $[0,1]$ to define its entries representing the rate for the qualities;
2. Adjustment: Then, qa_i is altered to turn $qa_{q_0 i} = 0$ for a random quality index $q_0 \in \mathcal{Q}$.

The second step is predicated to recognize that some components typically contain negligible quality attributes in practical blending problems.

To define the quality requirements across all end products, the average quality content of the components that are linked through the BOM are utilized. In particular, the quality requirement is a range centered on qr in which the extremes are defined by ql and qu, where:

$$
\begin{aligned}
qr_{qj} &= \sum_{i \in \mathcal{C}_j} \frac{qa_{qi}}{|\mathcal{C}_j|}, \forall q \in \mathcal{Q}, \forall j \in \mathcal{P}, \\
qu_{qj} &= 1.1 \times qr_{qj}, \forall j \in \mathcal{P}, \\
ql_{qj} &= 0.9 \times qr_{qj}, \forall j \in \mathcal{P}.
\end{aligned}
$$

Overall, this generation of quality attributes determines that all products need at least two components in their blend in order to meet the feasibility for quality requirements. This condition is verified analyzing the solutions of a single-period blending model involving the expected demand and the quality constraints for the end products. In this step, it is also verified that solutions in the integrated problem do not need for adjustment via lost sales decisions to compensate the nonexistence of a feasible blending.

Dataset generation for production setting

For all end products $j \in \mathcal{P}$ in the random instances, production cost is zero $vc^E_{jt} = 0$ and the setup cost is defined as $sc^E_{jt} = sc^E_j = 1000 \times \sum_{i \in \mathcal{C}_j} vc^C_i / |\mathcal{C}_j|$.

We set the inventory cost of components proportional to its corresponding purchasing cost, which is $hc^C_{it} = hc^C_i = 0.003 \times vc^C_i$. The storage costs of the end products are considered as a fraction of the average cost for components in the BOM $hc^E_{jt} = hc^E_j = 0.0045 \times \sum_{i \in \mathcal{C}_j} vc^C_i / |\mathcal{C}_j|$. The factors in these expressions derives from the typical average weekly rate for inventory cost.

The setup cost of components are defined consistently with the other costs based on the EOQ cost approximation. The formula gives $sc^C_{it} = sc^C_i = \left(hc^C_i/2\right) \cdot \left(TBO^C_i\right)^2 \cdot \mathbb{E}[\tilde{d}^C_i]$, where TBO^C_i stands for the Time Between Orders and $\mathbb{E}[\tilde{d}^C_i]$ represents the expected demand for the component $i \in \mathcal{C}$. Since these two terms are not given beforehand, the missing information is approximated based on the BOM size and the maximum Time Between Orders for the expected demand of end products that are linked through the BOM. In this approach, $TBO^C_i = \max_{j \in \mathcal{P}_i} \{TBO^E_j\}$ and $\mathbb{E}[\tilde{d}^C_i] = \sum_{j \in \mathcal{P}_i} \mathbb{E}[\tilde{d}^E_j] / |\mathcal{C}_j|$, where $TBO^E_j = \left(\sqrt{2sc^E_j \mathbb{E}[\tilde{d}^E_j] / hc^E_j}\right) / \mathbb{E}[\tilde{d}^E_j]$ is considered the time between orders of the end product $j \in \mathcal{P}$.

Since the lost sale cost of end products is considered as a penalty cost, the estimated total production cost for the expected demand is used to set this cost to avoid lost sales when production is possible. The lost sale cost is given by $pc_{jt} = pc_j = sc^E_j / \mathbb{E}[\tilde{d}^E_j] + \sum_{i \in \mathcal{C}_j} sc^C_i / \mathbb{E}[\tilde{d}^E_j] + \sum_{i \in \mathcal{C}_j} vc^C_i / |\mathcal{C}_j| + vc^E_j$.

Capacity is defined as $C_t = C = \sum_{j \in \mathcal{P}} \left(vt_j \mathbb{E}[\tilde{d}^E_j] + st_j\right)$ for all time periods $t \in \mathcal{T}$. For the case where the big M constants M^C and M^E are not specified for the setup constraints (2) and (3), the value of M^E_{jt} can be determined by the maximum amount of end product $j \in \mathcal{P}$ that can be produced without violating capacity constraints in period $t \in \mathcal{T}$, which is $M^E_{jt} = \left(C_t - st_{jt}\right) / vt_{jt}$. Consequently, considering the trivial BOM of selecting only one component to produce these maximum amounts, a valid value for M^C is derived summing up the limits M^E_{jt} of all end products in $\mathcal{P}$ for periods $\{t, \cdots, m\}$, which is $M^C_{it} = \sum_{\tau=t}^{m} \sum_{j \in \mathcal{P}_i} M^E_{j\tau}$ for the component $i \in \mathcal{C}$ in period $t \in \mathcal{T}$. However, in regard to the purchase of the components, the constants used in the setup constraints are defined as $M^C_{it} = M^C_i = \mathbb{E}[\tilde{d}^C_i]$ unless stated otherwise.

During our analysis in the computational experimentation, we are restricted to a generated random instance problem with $|\mathcal{P}| = |\mathcal{C}| = 6$ end products and components under the condition of $|\mathcal{Q}| = 4$ required quality attributes. Tables A1 to A5 list all the parameters and costs for the generated random example.

Table A1: Qualities specification for products (ql and qu) in # per unit.

	$j1$		$j2$		$j3$		$j4$		$j5$		$j6$	
$q1$	0.28	0.35	0.28	0.35	0	0	0.08	0.09	0.53	0.64	0.24	0.30
$q2$	0.44	0.54	0.43	0.53	0.33	0.40	0.59	0.73	0.41	0.51	0.57	0.69
$q3$	0.30	0.37	0.37	0.45	0.42	0.52	0.48	0.58	0.51	0.62	0.67	0.81
$q4$	0.33	0.40	0.29	0.35	0.41	0.51	0.33	0.40	0.09	0.11	0.29	0.36

Table A2: Qualities content of components (qa) in # per unit.

	$i1$	$i2$	$i3$	$i4$	$i5$	$i6$
$q1$	0.94	0	0	0	0.55	0.26
$q2$	0.44	0.95	0.05	0.09	0	0.94
$q3$	0.10	0.62	0.51	0.28	0.91	0.68
$q4$	0	0.67	0.29	0.42	0.31	0

Table A3: Components information — price for purchasing, setup and inventory \$ per unit and setup and bound for purchase in units.

	vc^C	sc^C	hc^C	M^C
$i1$	68	48,035	0.203	1,059
$i2$	108	177,615	0.324	1,595
$i3$	128	79,471	0.383	603
$i4$	87	81,823	0.26	916
$i5$	95	45,579	0.284	631
$i6$	122	93,345	0.367	1,000

Table A4: Products information — cost for production setup, inventory and lost sales \$ per unit and setup; expected demand in units; production and setup consumption utilization per unit and bound for production in units.

	vc^E	sc^E	hc^E	pc	$\mathbb{E}[\tilde{d}^E]$	vt	st	M^E
$j1$	0	87,440	0.394	485	994	2	36	9,232
$j2$	0	101,127	0.455	450	1,164	5	19	3,649
$j3$	0	107,487	0.484	798	647	2	46	9,079
$j4$	0	105,653	0.475	520	1,107	1	48	13,801
$j5$	0	94,803	0.427	371	1,021	4	18	3,821
$j6$	0	108,283	0.487	595	873	3	22	6,008

Table A5: Production information — bill-of-materials of products ($\mathcal{C}_j$) and total capacity.

$j1$	$j2$	$j3$	$j4$	$j5$	$j6$	C
$i4, i1, i2$	$i3, i2, i1$	$i2, i3, i4$	$i4, i2, i6$	$i1, i5, i6$	$i5, i6, i2$	17,023

A.2 Animal feed production

Blending models are particularly prevalent in the feed and food industries, where the blending of cattle feed, flour, and other edible products requires precise formulation to ensure nutritional value, taste, and cost-effectiveness. Notable papers demonstrating the versatility and effectiveness of blending models in addressing challenges for study-case problems in these industrial contexts include: (Akkerman et al. 2010) on optimizing food production processes, (Kilic et al. 2013) on intermediate flour products blending, and Pathumnakul et al. (2011) on strategic formulations for feed mills. The data used in our experiments is sourced from Arenales et al. (2015).

The base example problem is contextualized for a manufacturer in which animal feed products $\mathcal{P} = \{Feed\ A, Feed\ B\}$ are produced using components: $\mathcal{C} = \{Corn, Bone, Soy, Fish\}$. Furthermore, the blending of animal feed must meet the specifications regarding the minimum and maximum percentages of qualities in its final composition: $\mathcal{Q} = \{Protein, Calcium\}$. All values reported to the purchase, production, and blending data for the various products and components are given per tons. Tables A6 to A10 list all the parameters and costs for the example.

Table A6: Qualities information — nutritional requirements of animal feeds (ql and qu) in % per ton.

	Feed A		*Feed B*	
Protein	0.4	0.5	0.3	0.8
Calcium	0.3	0.6	0.5	0.8

Table A7: Qualities information — nutritional content of components (qa) in % per ton.

	Corn	*Bone*	*Soy*	*Fish*
Protein	0.2	0.4	0.5	0.8
Calcium	0.6	0.4	0.4	0.1

Table A8: Components information — price for purchasing, setup and inventory $ per ton and setup and bounds in tons for ordering of components.

	vc^C	sc^C	hc^C	M^C
Corn	27	15,500	0.081	10
Bone	35	7,788	0.105	10
Soy	51	58,556	0.153	14
Fish	41	9,111	0.123	12

Table A9: Production information — cost for production setup, inventory and lost sales $ per ton; expected demand in tons; production time necessary per ton and setup and bound for production of end products in ton.

	vc^E	sc^E	hc^E	pc	$\mathbb{E}[\tilde{d}^E]$	vt	st	M^E
Feed A	0	39,667	0.178	6,505	19	1	10	61
Feed B	0	37,667	0.170	9,996	12	3	6	22

Table A10: Production information — bill-of-materials of products ($\mathcal{C}_|$) and total capacity time available.

Feed A	*Feed B*	C
Corn, Fish, Soy	*Soy, Corn, Bone*	71

A.3 Petroleum company manufacturer

Blending models are essential in the petroleum industry, particularly in the optimization of fuel processing, where blends must meet specific rating requirements influencing the efficiency and profitability of refinery operations. Research in line with this domain has focused on advanced mathematical programming techniques to address blending challenges, in which significant contributions include: (Yang et al. 2017) exploring chance-constrained optimization for blending planning, (Peng et al. 2021) developing robust optimization bounds for blending operations. Understanding the critical role of blending models in the petroleum industry, we have adapted a similar example problem from CPLEX 22.1.0 library (IBM 2022).

The example contextualizes an oil company that manufactures different types of gasoline $\mathcal{P} = \{Super, Regular, Diesel\}$. Being each type of gasoline the blending obtained using alternative types of crude oil: $\mathcal{C} = \{Crude\ 1, Crude\ 2, Crude\ 3\}$, the end products are required to contain an index rate of quality conditions: $\mathcal{Q} = \{Lead, Octane\}$. All values considered for the purchase, production and blending data for the various products and components are given per unit of a barrel (bbl). Tables A11 to A15 list all the parameters and costs for the example.

Table A11: Qualities information — specifications for chemicals in gasoline products (ql and qu) given in rate per bbl.

	Super		*Regular*		*Diesel*	
Octane	10	12	8	12	6	12
Lead	0.5	1	0.5	2	0.5	1

Table A12: Qualities information — chemical content of crude oils (qa) given in rate per bbl.

	Crude 1	*Crude* 2	*Crude* 3
Octane	12	6	8
Lead	0.5	2	3

Table A13: Crude oils information: price for purchasing, setup and inventory in \$ per bbl and setup; and bounds for ordering of components in bbl.

	vc^C	sc^C	hc^C	M^C
Crude 1	45	540,000	0.135	5,000
Crude 2	35	105,000	0.105	5,000
Crude 3	25	75,000	0.075	5,000

Table A14: Gasoline products information — cost for production setup, inventory, and lost sales in \$ per bbl and setup; expected demand in bbl; production and setup consumption; and bound for production of gasolines in bbl.

	vc^E	sc^E	hc^E	pc	$\mathbb{E}[\tilde{d}^E]$	vt	st	M^E
Super	4	35,000	0.158	357	3,000	1	0	14,000
Regular	4	35,000	0.158	174	9,500	1	0	14,000
Diesel	4	35,000	0.158	840	1,000	1	0	14,000

Table A15: Production information — bill-of-materials of gasoline ($\mathcal{C}_j$) and total available processing capacity.

Super	*Regular*	*Diesel*	C
Crude 1, 2, 3	*Crude* 1, 2, 3	*Crude* 1, 2, 3	14,000

B Complementary results

Table B16: Grouped results for the number of optimal solutions using the MV and SAA heuristics for the 18 instances problems considering one replication.

Instance	MV, $\mathbb{E}[\tilde{d}^E]$	SAA, $\lvert\Omega_K\rvert = 10$	SAA, $\lvert\Omega_K\rvert = 50$	SAA, $\lvert\Omega_K\rvert = 100$	SAA, $\lvert\Omega_K\rvert = 500$
Prob.A	5	4	4	4	4
Prob.B	4	4	2	2	2
Prob.C	3	3	1	1	0
$C\times1$	6	5	3	3	3
$C\times1.3$	6	6	4	4	3
$M^C\times1$	5	5	2	2	2
$M^C\times2$	1	0	0	0	0
$M^C\times$UNL	6	6	5	5	4

Table B17: Grouped results for the MIP Gap for the solutions using the MV and SAA heuristics for the 18 instances problems considering one replication.

Instance	MV, $\mathbb{E}[\tilde{d}^E]$	SAA, $\lvert\Omega_K\rvert = 10$	SAA, $\lvert\Omega_K\rvert = 50$	SAA, $\lvert\Omega_K\rvert = 100$	SAA, $\lvert\Omega_K\rvert = 500$
Prob.A	0.09%	0.10%	0.13%	0.13%	0.53%
Prob.B	0.15%	0.11%	0.23%	0.26%	0.26%
Prob.C	0.27%	0.27%	0.67%	0.93%	2.29%
C×1	0.17%	0.20%	0.46%	0.58%	1.17%
C×1.3	0.18%	0.11%	0.22%	0.30%	0.89%
M^C×1	0.10%	0.09%	0.27%	0.38%	0.88%
M^C×2	0.42%	0.38%	0.59%	0.72%	1.66%
M^C×UNL	0.00%	0.00%	0.17%	0.22%	0.54%

Table B18: Grouped results for the CPU times to solve the MIP models using the MV and SAA heuristics for the 18 instances problems considering one replication.

Instance	MV, $\mathbb{E}[\tilde{d}^E]$	SAA, $\lvert\Omega_K\rvert = 10$	SAA, $\lvert\Omega_K\rvert = 50$	SAA, $\lvert\Omega_K\rvert = 100$	SAA, $\lvert\Omega_K\rvert = 500$
Prob.A	641.5	1,203.0	1,226.0	1,255.7	1,672.3
Prob.B	1,211.4	1,247.3	2,401.6	2,401.5	2,411.6
Prob.C	2,442.7	2,521.0	3,036.8	3,075.5	3,600.7
C×1	1,575.8	1,938.2	2,404.7	2,412.7	2,500.9
C×1.3	1,287.9	1,375.9	2,038.2	2,075.7	2,622.2
M^C×1	735.7	877.2	2,423.5	2,443.5	2,764.3
M^C×2	3,038.5	3,600.7	3,602.5	3,604.6	3,601.3
M^C×UNL	521.4	493.3	638.3	684.6	1,319.0

Table B19: Grouped results for the CPU times to evaluate the solutions of the MV and SAA heuristics for the 18 instances problems considering one replication.

Instance	MV, $\mathbb{E}[\tilde{d}^E]$	SAA, $\lvert\Omega_K\rvert = 10$	SAA, $\lvert\Omega_K\rvert = 50$	SAA, $\lvert\Omega_K\rvert = 100$	SAA, $\lvert\Omega_K\rvert = 500$
Prob.A	123.9	130.2	129.4	99.9	91.7
Prob.B	75.6	86.8	90.7	90.6	84.7
Prob.C	160.1	166.6	160.7	164.4	153.0
C×1	116.8	129.6	121.2	121.9	110.3
C×1.3	123.0	126.1	132.6	114.8	109.3
M^C×1	114.3	136.7	140.9	119.3	109.5
M^C×2	129.3	126.7	129.3	127.9	110.2
M^C×UNL	116.1	120.2	110.6	107.8	109.8

Table B20: Grouped results for the first stage cost term ($cost(z^F)$) of the MV and SAA solutions for the 18 instances problems considering one replication.

Instance	MV, $\mathbb{E}[\tilde{d}^E]$	SAA, $\lvert\Omega_K\rvert = 10$	SAA, $\lvert\Omega_K\rvert = 50$	SAA, $\lvert\Omega_K\rvert = 100$	SAA, $\lvert\Omega_K\rvert = 500$
Prob.A	13,967,680	13,899,906	13,889,649	13,856,832	13,885,526
Prob.B	948,556	956,223	956,676	949,964	949,961
Prob.C	14,555,365	14,543,010	14,562,442	14,534,838	14,645,965
C×1	9,739,903	9,743,803	9,758,155	9,738,090	9,777,203
C×1.3	9,907,831	9,855,623	9,847,690	9,822,999	9,877,098
M^C×1	12,617,293	12,562,621	12,535,932	12,548,566	12,617,326
M^C×2	9,730,400	9,657,258	9,670,229	9,636,850	9,710,256
M^C×UNL	7,123,908	7,179,260	7,202,606	7,156,217	7,153,870

Table B21: Grouped results for the estimated expected second stage cost term $\left((1/K') \sum_{s=1}^{K'} R_{d^E(\omega_s)}(z^F)\right)$ of the MV and SAA solutions for the 18 instances problems considering one replication.

Instance	MV, $\mathbb{E}[\tilde{d}^E]$	SAA, $\lvert\Omega_K\rvert = 10$	SAA, $\lvert\Omega_K\rvert = 50$	SAA, $\lvert\Omega_K\rvert = 100$	SAA, $\lvert\Omega_K\rvert = 500$
Prob.A	1,377,059	1,236,125	1,213,013	1,243,355	1,210,885
Prob.B	191,601	162,906	158,509	164,374	164,268
Prob.C	2,738,547	2,707,225	2,648,116	2,688,651	2,692,338
$C\times1$	1,706,389	1,637,336	1,599,687	1,624,917	1,631,215
$C\times1.3$	1,165,082	1,100,168	1,080,071	1,106,002	1,080,445
$M^C\times1$	2,252,525	2,238,291	2,251,193	2,244,825	2,213,764
$M^C\times2$	1,092,321	1,073,024	1,024,321	1,060,329	1,049,431
$M^C\times$UNL	962,360	794,940	744,124	791,225	804,296

Table B22: Grouped results for the evaluated cost in the two-stage stochastic MCLSB problem (UB) of the MV and SAA solutions for the 18 instances considering one replication.

Instance	MV, $\mathbb{E}[\tilde{d}^E]$	SAA, $\lvert\Omega_K\rvert = 10$	SAA, $\lvert\Omega_K\rvert = 50$	SAA, $\lvert\Omega_K\rvert = 100$	SAA, $\lvert\Omega_K\rvert = 500$
Prob.A	15,344,739	15,136,030	15,102,662	15,100,186	15,096,410
Prob.B	1,140,157	1,119,129	1,115,185	1,114,338	1,114,229
Prob.C	17,293,912	17,250,234	17,210,557	17,223,488	17,338,303
$C\times1$	11,446,292	11,381,138	11,357,842	11,363,007	11,408,418
$C\times1.3$	11,072,913	10,955,791	10,927,761	10,929,001	10,957,544
$M^C\times1$	14,869,818	14,800,911	14,787,125	14,793,391	14,831,090
$M^C\times2$	10,822,721	10,730,282	10,694,550	10,697,179	10,759,687
$M^C\times$UNL	8,086,269	7,974,200	7,946,729	7,947,442	7,958,165

Table B23: Grouped results for the best upper bound (RP) for the 18 instances of two-stage stochastic MCLSB problem using the SAA method with $\lvert\Omega_K\rvert$ demand scenarios and M replications.

	SAA setting (M,$\lvert\Omega_K\rvert$)					
Instance	(5,10)	(5,50)	(5,100)	(10,10)	(10,50)	(10,100)
Prob.A	15,128,525	15,100,961	15,097,529	15,122,631	15,099,126	15,097,263
Prob.B	1,114,815	1,114,327	1,114,233	1,114,590	1,114,287	1,114,215
Prob.C	17,225,428	17,198,289	17,201,553	17,221,802	17,193,617	17,200,033
$C\times1$	11,368,893	11,350,925	11,352,449	11,365,602	11,348,185	11,351,974
$C\times1.3$	10,943,619	10,924,793	10,923,094	10,940,412	10,923,169	10,922,366
$M^C\times1$	14,798,729	14,782,769	14,781,312	14,797,171	14,780,869	14,780,503
$M^C\times2$	10,704,980	10,690,009	10,689,702	10,701,682	10,687,013	10,689,544
$M^C\times$UNL	7,965,059	7,940,799	7,942,301	7,960,170	7,939,149	7,941,464

Table B24: Grouped results for the lower bound (LB) for the 18 instances of the two-stage stochastic MCLSB problem using the SAA method with $\lvert\Omega_K\rvert$ demand scenarios and M replications.

	SAA setting (M,$\lvert\Omega_K\rvert$)					
Instance	(5,10)	(5,50)	(5,100)	(10,10)	(10,50)	(10,100)
Prob.A	14,990,537	15,038,060	15,054,387	14,992,607	15,059,901	15,060,685
Prob.B	1,109,988	1,111,220	1,110,111	1,109,193	1,112,070	1,110,942
Prob.C	17,038,523	17,055,609	17,013,357	17,051,149	17,066,351	17,004,841
$C\times1$	11,250,233	11,265,595	11,251,775	11,254,753	11,277,357	11,251,729
$C\times1.3$	10,842,466	10,870,998	10,866,796	10,847,213	10,881,524	10,865,916
$M^C\times1$	14,678,697	14,696,931	14,687,123	14,682,483	14,711,735	14,689,821
$M^C\times2$	10,587,448	10,608,379	10,594,534	10,591,329	10,620,080	10,588,593
$M^C\times$UNL	7,872,904	7,899,580	7,896,199	7,879,137	7,906,507	7,898,053

Table B25: Grouped results for the stochastic optimality gap (CI(95%) for stochastic Gap) for the 18 instances of the two-stage stochastic MCLSB problem using the SAA method with $|\Omega_K|$ demand scenarios and M replications.

	SAA setting $(M,|\Omega_K|)$					
Instance	(5,10)	(5,50)	(5,100)	(10,10)	(10,50)	(10,100)
Prob.A	(0.10% , 1.90%)	(0.20% , 0.67%)	(-0.01% , 0.61%)	(0.44% , 1.41%)	(0.06% , 0.50%)	(0.02% , 0.49%)
Prob.B	(-0.07% , 0.96%)	(0.06% , 0.55%)	(0.17% , 0.65%)	(0.03% , 0.93%)	(-0.12% , 0.55%)	(0.11% , 0.54%)
Prob.C	(0.39% , 1.78%)	(0.54% , 1.05%)	(0.95% , 1.16%)	(0.52% , 1.46%)	(0.55% , 0.86%)	(0.99% , 1.20%)
$C\times1$	(0.11% , 1.70%)	(0.40% , 0.86%)	(0.50% , 0.94%)	(0.35% , 1.32%)	(0.28% , 0.73%)	(0.53% , 0.87%)
$C\times1.3$	(0.16% , 1.40%)	(0.14% , 0.65%)	(0.23% , 0.68%)	(0.31% , 1.21%)	(0.05% , 0.55%)	(0.22% , 0.62%)
$M^C\times1$	(-0.04% , 1.38%)	(0.21% , 0.66%)	(0.27% , 0.69%)	(0.19% , 1.17%)	(0.10% , 0.55%)	(0.28% , 0.61%)
$M^C\times2$	(0.33% , 1.53%)	(0.48% , 0.99%)	(0.63% , 1.07%)	(0.48% , 1.28%)	(0.39% , 0.86%)	(0.66% , 1.07%)
$M^C\times$UNL	(0.12% , 1.74%)	(0.11% , 0.62%)	(0.20% , 0.66%)	(0.32% , 1.34%)	(0.00% , 0.50%)	(0.17% , 0.55%)

Table B26: Grouped results for the lower bound on the value of the stochastic solution ($\triangle$) for the 18 instances of the two-stage stochastic MCLSB problem using the SAA method with $|\Omega_K|$ demand scenarios and M replications.

	SAA setting $(M,|\Omega_K|)$					
Instance	(5,10)	(5,50)	(5,100)	(10,10)	(10,50)	(10,100)
Prob.A	1.58%	1.79%	1.82%	1.63%	1.80%	1.82%
Prob.B	2.86%	2.90%	2.91%	2.88%	2.91%	2.91%
Prob.C	0.45%	0.62%	0.59%	0.47%	0.65%	0.60%
$C\times1$	1.71%	1.85%	1.85%	1.75%	1.87%	1.85%
$C\times1.3$	1.55%	1.69%	1.70%	1.58%	1.70%	1.71%
$M^C\times1$	0.57%	0.65%	0.66%	0.58%	0.66%	0.67%
$M^C\times2$	1.64%	1.76%	1.76%	1.68%	1.78%	1.76%
$M^C\times$UNL	2.68%	2.90%	2.90%	2.73%	2.92%	2.91%

Table B27: Grouped results for the totaling CPU times (solution plus evaluation) for the 18 instances of the two-stage stochastic MCLSB problem using the SAA method with $|\Omega_K|$ demand scenarios and M replications.

	SAA setting $(M,|\Omega_K|)$					
Instance	(5,10)	(5,50)	(5,100)	(10,10)	(10,50)	(10,100)
Prob.A	6,631.3	6,622.3	6,776.8	13,183.6	13,234.0	13,829.2
Prob.B	7,761.5	12,459.9	12,455.7	17,149.4	24,404.7	24,922.8
Prob.C	14,063.3	16,013.1	17,297.3	28,927.1	31,983.9	35,838.8
$C\times1$	10,525.7	12,620.2	12,689.8	21,869.4	24,992.9	25,415.9
$C\times1.3$	8,445.1	10,776.6	11,663.4	17,637.4	21,422.2	24,311.3
$M^C\times1$	7,132.1	12,697.7	12,853.9	16,070.5	24,895.2	25,843.7
$M^C\times2$	18,547.0	18,643.1	18,654.6	37,064.9	37,270.9	37,293.5
$M^C\times$UNL	2,777.1	3,754.4	5,021.2	6,124.8	7,456.5	11,453.6

Table B28: Grouped results for the 18 instances of the two-stage stochastic MCLSB problem with candidate solutions in the initial phase of the ASAA method considering $|\Omega_{\hat{K}}| = 10$ demand scenarios and $\hat{M} = 5$ or $\hat{M} = 10$ replications with different threshold γ values.

	$\hat{M} = 5, |\Omega_{\hat{K}}| = 10$					$\hat{M} = 10, |\Omega_{\hat{K}}| = 10$			
Instance	$\gamma = 0$	$\gamma = 0.1$	$\gamma = 0.2$	$\gamma = 0.3$	$\gamma = 0.4$	$\gamma = 0$	$\gamma = 0.1$	$\gamma = 0.2$	$\gamma = 0.3$
Prob.A	0.68	0.73	0.81	0.88	0.93	0.55	0.64	0.72	0.82
Prob.B	0.66	0.83	0.90	0.95	0.95	0.51	0.81	0.91	0.95
Prob.C	0.69	0.88	0.91	0.94	0.97	0.67	0.77	0.85	0.93
$C\times1$	0.74	0.83	0.86	0.91	0.94	0.56	0.73	0.80	0.89
$C\times1.3$	0.61	0.80	0.89	0.94	0.96	0.59	0.74	0.85	0.91
$M^C\times1$	0.57	0.87	0.90	0.91	0.95	0.56	0.78	0.82	0.90
$M^C\times2$	0.63	0.70	0.83	0.91	0.94	0.47	0.61	0.78	0.87
$M^C\times$UNL	0.83	0.87	0.89	0.95	0.97	0.70	0.82	0.87	0.92

Table B29: Grouped results for the improvement over the deterministic solution ($\triangle$) and the overall CPU times for the 18 instances of the two-stage stochastic MCLSB problem using the regular and adjustable SAA methods to solve $M = 10$ SAA problems with $|\Omega_K| = 50$ demand scenarios.

		ASAA (Phase 1 $\hat{M} = 10$, $\lvert\Omega_{\hat{K}}\rvert = 10$)				ASAA (Phase 1 $\hat{M} = 5$, $\lvert\Omega_{\hat{K}}\rvert = 10$)				
Instance	SAA	$\gamma = 0$	$\gamma = 0.1$	$\gamma = 0.2$	$\gamma = 0.3$	$\gamma = 0$	$\gamma = 0.1$	$\gamma = 0.2$	$\gamma = 0.3$	$\gamma = 0.4$
Prob.A	1.82%	1.80%	1.80%	1.80%	1.78%	1.80%	1.80%	1.80%	1.80%	0.75%
Prob.B	2.91%	2.91%	2.91%	2.91%	2.89%	2.91%	2.91%	2.91%	2.90%	2.90%
Prob.C	0.60%	1.87%	1.87%	1.86%	1.73%	0.65%	0.65%	0.65%	0.63%	-0.65%
$C\times1$	1.85%	a%	b%	c%	d%	1.87%	1.87%	1.87%	1.86%	0.90%
$C\times1.3$	1.71%	1.70%	1.70%	1.70%	1.69%	1.70%	1.70%	1.70%	1.69%	1.10%
$M^C\times1$	0.67%	0.66%	0.66%	0.66%	0.64%	0.66%	0.66%	0.66%	0.66%	0.31%
$M^C\times2$	1.76%	1.78%	1.78%	1.77%	1.76%	1.78%	1.78%	1.78%	1.77%	0.89%
$M^C\times$UNL	2.91%	2.92%	2.92%	2.90%	2.73%	2.92%	2.92%	2.92%	2.90%	1.81%
Average	1.78%	1.79%	1.79%	1.78%	1.71%	1.79%	1.79%	1.79%	1.78%	1.00%

		ASAA (Phase 1: $\hat{M} = 10$, $\lvert\Omega_{\hat{K}}\rvert = 10$)				ASAA (Phase 1: $\hat{M} = 5$, $\lvert\Omega_{\hat{K}}\rvert = 10$)				
Instance	SAA	$\gamma = 0$	$\gamma = 0.1$	$\gamma = 0.2$	$\gamma = 0.3$	$\gamma = 0$	$\gamma = 0.1$	$\gamma = 0.2$	$\gamma = 0.3$	$\gamma = 0.4$
Prob.A	13,829.2	20,334.6	20,428.4	14,805.4	14,374.5	13,655.5	13,599.5	7,600.4	7,596.7	7,592.1
Prob.B	24,922.8	21,728.7	17,907.7	18,168.4	18,072.4	9,800.8	8,502.3	8,486.9	8,503.8	8,533.1
Prob.C	35,838.8	45,058.7	42,215.6	31,563.4	30,946.9	10,545.7	6,547.7	1,628.2	1,616.5	1,645.6
$C\times1$	25,415.9	30,971.4	27,567.8	23,710.6	23,249.5	15,604.8	14,919.4	11,637.6	11,635.0	11,654.5
$C\times1.3$	24,311.3	27,109.9	26,133.2	19,314.2	19,012.9	16,438.8	13,555.8	9,548.3	9,551.9	9,568.2
$M^C\times1$	25,843.7	20,872.6	17,386.3	17,499.4	17,438.2	9,552.9	8,266.3	8,234.1	8,242.1	8,271.6
$M^C\times2$	37,293.5	55,035.1	55,308.0	39,475.9	38,458.3	34,485.8	30,548.0	19,663.1	19,653.6	19,677.3
$M^C\times$UNL	11,453.6	11,214.3	7,857.4	7,562.0	7,497.3	4,026.6	3,898.6	3,881.7	3,884.7	3,885.2
Average	24,863.6	29,040.6	26,850.5	21,512.4	21,512.4	16,021.8	14,237.6	10,592.9	10,593.4	10,611.4

Table B30: Grouped results for the improvement over the deterministic solution ($\triangle$) and the overall CPU times for the 18 instances of the two-stage stochastic MCLSB problem using the regular and adjustable SAA methods to solve $M = 5$ SAA problems with $|\Omega_K| = 50$ demand scenarios.

Instance	SAA	ASAA (Phase 1 $\hat{M} = 10$, $\lvert\Omega_{\hat{K}}\rvert = 10$)				ASAA (Phase 1 $\hat{M} = 5$, $\lvert\Omega_{\hat{K}}\rvert = 10$)				
		$\gamma = 0$	$\gamma = 0.1$	$\gamma = 0.2$	$\gamma = 0.3$	$\gamma = 0$	$\gamma = 0.1$	$\gamma = 0.2$	$\gamma = 0.3$	$\gamma = 0.4$
Prob.A	1.82%	1.79%	1.79%	1.79%	1.77%	1.79%	1.79%	1.79%	1.79%	0.74%
Prob.B	2.91%	2.90%	2.90%	2.90%	2.88%	2.90%	2.90%	2.90%	2.90%	2.90%
Prob.C	0.59%	0.64%	0.64%	0.62%	0.45%	0.64%	0.64%	0.64%	0.61%	-0.65%
$C\times1$	1.85%	1.86%	1.86%	1.85%	1.72%	1.86%	1.86%	1.86%	1.85%	0.89%
$C\times1.3$	1.70%	1.69%	1.69%	1.69%	1.69%	1.69%	1.69%	1.69%	1.68%	1.10%
$M^C\times1$	0.66%	0.65%	0.65%	0.65%	0.64%	0.65%	0.65%	0.65%	0.65%	0.31%
$M^C\times2$	1.76%	1.77%	1.77%	1.77%	1.76%	1.77%	1.77%	1.77%	1.76%	0.89%
$M^C\times$UNL	2.90%	2.91%	2.91%	2.89%	2.71%	2.91%	2.91%	2.91%	2.88%	1.79%
Average	1.77%	1.78%	1.78%	1.77%	1.70%	1.78%	1.78%	1.78%	1.77%	1.00%

Instance	SAA	ASAA (Phase 1: $\hat{M} = 10$, $\lvert\Omega_{\hat{K}}\rvert = 10$)				ASAA (Phase 1: $\hat{M} = 5$, $\lvert\Omega_{\hat{K}}\rvert = 10$)				
		$\gamma = 0$	$\gamma = 0.1$	$\gamma = 0.2$	$\gamma = 0.3$	$\gamma = 0$	$\gamma = 0.1$	$\gamma = 0.2$	$\gamma = 0.3$	$\gamma = 0.4$
Prob.A	6,776.8	16,765.4	16,741.2	14,015.6	13,780.4	10,343.1	10,142.4	7,181.7	7,113.4	7,110.7
Prob.B	12,455.7	18,870.0	17,535.3	17,698.3	17,613.0	8,812.9	8,135.6	8,317.8	8,134.3	8,160.7
Prob.C	17,297.3	36,853.8	35,501.2	30,220.4	29,941.1	5,401.8	3,256.7	14,612.9	14,872.4	14,904.0
$C\times1$	12,689.8	26,277.7	25,426.9	22,795.2	22,561.8	13,849.1	12,906.3	11,079.6	11,079.7	11,100.6
$C\times1.3$	11,663.4	22,048.4	18,023.3	18,494.3	18,327.8	10,825.7	8,999.0	8,995.3	9,000.4	9,016.4
$M^C\times1$	12,853.9	17,880.9	16,692.6	16,792.4	16,758.6	8,372.1	7,698.3	7,682.4	7,690.6	7,716.0
$M^C\times2$	18,654.6	45,909.1	46,084.2	38,289.4	37,767.3	26,646.7	24,561.3	19,103.3	19,100.1	19,124.4
$M^C\times$UNL	5,021.2	8,699.0	7,000.8	6,852.5	6,808.6	3,407.7	3,338.4	3,326.6	3,329.4	3,335.0
Average	12,176.6	22,128.0	21,449.6	20,644.8	20,444.8	12,808.8	11,866.0	10,037.4	10,040.0	10,058.5

Table B31: Grouped results for the improvement over the deterministic solution ($\triangle$) and the overall CPU times for the 18 instances of the two-stage stochastic MCLSB problem using the regular and adjustable SAA methods to solve $M = 10$ SAA problems with $|\Omega_K| = 100$ demand scenarios.

		ASAA (Phase 1 $\hat{M} = 10$, $\lvert\Omega_{\hat{K}}\rvert = 10$)				ASAA (Phase 1 $\hat{M} = 5$, $\lvert\Omega_{\hat{K}}\rvert = 10$)				
Instance	SAA	$\gamma = 0$	$\gamma = 0.1$	$\gamma = 0.2$	$\gamma = 0.3$	$\gamma = 0$	$\gamma = 0.1$	$\gamma = 0.2$	$\gamma = 0.3$	$\gamma = 0.4$
Prob.A	1.82%	1.82%	1.82%	1.81%	1.80%	1.81%	1.81%	1.81%	1.81%	0.76%
Prob.B	2.91%	2.91%	2.91%	2.91%	2.89%	2.91%	2.91%	2.91%	2.91%	2.91%
Prob.C	0.60%	0.66%	0.66%	0.65%	0.48%	0.66%	0.66%	0.66%	0.63%	-0.63%
$C\times1$	1.85%	1.89%	1.89%	1.87%	1.74%	1.88%	1.88%	1.88%	1.87%	0.91%
$C\times1.3$	1.71%	1.71%	1.71%	1.71%	1.70%	1.71%	1.71%	1.71%	1.70%	1.11%
$M^C\times1$	0.67%	0.67%	0.67%	0.67%	0.65%	0.67%	0.67%	0.67%	0.67%	0.32%
$M^C\times2$	1.76%	1.78%	1.78%	1.78%	1.77%	1.78%	1.78%	1.78%	1.77%	0.90%
$M^C\times$UNL	2.91%	2.94%	2.94%	2.92%	2.74%	2.93%	2.93%	2.93%	2.91%	1.82%
Average	1.78%	1.80%	1.80%	1.79%	1.72%	1.79%	1.79%	1.79%	1.78%	1.01%

		ASAA (Phase 1: $\hat{M} = 10$, $\lvert\Omega_{\hat{K}}\rvert = 10$)				ASAA (Phase 1: $\hat{M} = 5$, $\lvert\Omega_{\hat{K}}\rvert = 10$)				
Instance	SAA	$\gamma = 0$	$\gamma = 0.1$	$\gamma = 0.2$	$\gamma = 0.3$	$\gamma = 0$	$\gamma = 0.1$	$\gamma = 0.2$	$\gamma = 0.3$	$\gamma = 0.4$
Prob.A	13,829.2	20,586.7	20,536.6	15,905.9	14,447.1	13,855.8	13,768.1	7,743.7	7,614.8	7,636.0
Prob.B	24,922.8	22,234.6	17,941.1	17,922.7	17,987.5	10,398.5	8,515.0	8,549.2	8,512.1	8,561.4
Prob.C	35,838.8	47,308.2	42,729.7	31,461.1	30,654.0	26,827.1	20,814.7	15,701.3	15,691.0	15,744.4
$C\times1$	25,415.9	32,849.8	28,014.9	23,577.1	23,144.2	16,216.6	15,107.8	11,710.0	11,650.6	11,688.7
$C\times1.3$	24,311.3	27,236.5	26,123.3	19,949.4	18,914.9	17,837.6	13,624.0	9,619.6	9,561.3	9,605.8
$M^C\times1$	25,843.7	21,170.5	17,269.9	17,275.0	17,327.1	10,215.2	8,278.8	8,291.2	8,245.6	8,300.3
$M^C\times2$	37,293.5	55,648.8	55,497.6	40,641.6	38,358.7	36,411.9	30,881.1	19,755.0	19,684.7	19,712.9
$M^C\times$UNL	11,453.6	13,310.1	8,439.9	7,373.1	7,402.7	4,454.2	3,937.9	3,948.0	3,887.6	3,928.6
Average	24,863.6	30,043.2	27,069.1	21,763.2	21,029.5	17,027.1	14,365.9	10,664.8	9,427.5	10,647.3

Table B32: Grouped results for the improvement over the deterministic solution (△) and the overall CPU times for the 18 instances of the two-stage stochastic MCLSB problem using the regular and adjustable SAA methods to solve $M = 5$ SAA problems with $|\Omega_K| = 100$ demand scenarios.

		ASAA (Phase 1 $\hat{M} = 10$, $\lvert\Omega_{\hat{K}}\rvert = 10$)				ASAA (Phase 1 $\hat{M} = 5$, $\lvert\Omega_{\hat{K}}\rvert = 10$)				
Instance	SAA	$\gamma = 0$	$\gamma = 0.1$	$\gamma = 0.2$	$\gamma = 0.3$	$\gamma = 0$	$\gamma = 0.1$	$\gamma = 0.2$	$\gamma = 0.3$	$\gamma = 0.4$
Prob.A	1.82%	1.82%	1.82%	1.81%	1.80%	1.81%	1.81%	1.81%	1.81%	0.76%
Prob.B	2.91%	2.91%	2.91%	2.91%	2.89%	2.91%	2.91%	2.91%	2.91%	2.91%
Prob.C	0.59%	0.66%	0.66%	0.64%	0.47%	0.66%	0.66%	0.66%	0.63%	-0.64%
$C\times1$	1.85%	1.89%	1.89%	1.87%	1.74%	1.88%	1.88%	1.88%	1.87%	0.91%
$C\times1.3$	1.70%	1.71%	1.71%	1.70%	1.70%	1.71%	1.71%	1.71%	1.70%	1.11%
$M^C\times1$	0.66%	0.67%	0.67%	0.67%	0.65%	0.67%	0.67%	0.67%	0.67%	0.32%
$M^C\times2$	1.76%	1.78%	1.78%	1.78%	1.77%	1.78%	1.78%	1.78%	1.77%	0.89%
$M^C\times$UNL	2.90%	2.94%	2.94%	2.92%	2.74%	2.93%	2.93%	2.93%	2.91%	1.82%
Average	1.77%	1.80%	1.80%	1.79%	1.72%	1.79%	1.79%	1.79%	1.78%	1.01%

		ASAA (Phase 1: $\hat{M} = 10$, $\lvert\Omega_{\hat{K}}\rvert = 10$)				ASAA (Phase 1: $\hat{M} = 5$, $\lvert\Omega_{\hat{K}}\rvert = 10$)				
Instance	SAA	$\gamma = 0$	$\gamma = 0.1$	$\gamma = 0.2$	$\gamma = 0.3$	$\gamma = 0$	$\gamma = 0.1$	$\gamma = 0.2$	$\gamma = 0.3$	$\gamma = 0.4$
Prob.A	6,776.8	16,877.0	16,846.3	14,624.9	13,919.0	10,343.1	10,249.9	7,208.4	7,141.3	7,135.2
Prob.B	12,455.7	20,654.0	17,547.6	17,551.2	17,579.2	9,473.8	8,141.5	8,143.8	8,129.2	8,161.4
Prob.C	17,297.3	37,985.7	35,701.0	30,182.8	29,799.8	21,051.8	17,679.4	14,891.6	14,846.5	14,881.6
$C\times1$	12,689.8	27,786.7	24,975.0	22,707.5	22,545.7	13,415.3	12,992.7	11,124.9	11,079.8	11,100.7
$C\times1.3$	11,663.4	22,557.7	21,754.9	18,865.1	18,319.6	13,830.5	11,054.5	9,037.7	8,998.2	9,018.1
$M^C\times1$	12,853.9	19,552.1	16,669.8	16,687.4	16,730.7	9,112.8	7,723.3	7,713.9	7,684.8	7,714.2
$M^C\times2$	18,654.6	46,235.2	46,093.5	38,921.4	37,759.2	28,134.5	24,992.2	19,162.4	19,106.2	19,118.6
$M^C\times$UNL	5,021.2	9,729.3	7,331.6	6,750.2	6,808.1	3,621.4	3,355.3	3,367.6	3,326.1	3,345.4
Average	12,176.6	25,172.2	23,365.0	20,786.3	20,432.7	13,622.9	12,023.6	10,081.3	10,039.0	10,059.4

Table B33: Grouped results for the improvement over the deterministic solution ($\triangle$) and the overall CPU times for the 18 instances of the two-stage stochastic MCLSB problem using the regular and adjustable SAA methods to solve $M = 1$ SAA problems with $|\Omega_K| = 500$ demand scenarios.

		ASAA (Phase 1 $\hat{M} = 10$, $\lvert\Omega_{\hat{K}}\rvert = 10$)				ASAA (Phase 1 $\hat{M} = 5$, $\lvert\Omega_{\hat{K}}\rvert = 10$)				
Instance	SAA	$\gamma = 0$	$\gamma = 0.1$	$\gamma = 0.2$	$\gamma = 0.3$	$\gamma = 0$	$\gamma = 0.1$	$\gamma = 0.2$	$\gamma = 0.3$	$\gamma = 0.4$
Prob.A	1.83%	1.83%	1.83%	1.83%	1.81%	1.83%	1.83%	1.83%	1.83%	0.78%
Prob.B	2.91%	2.91%	2.91%	2.91%	2.89%	2.91%	2.91%	2.91%	2.91%	2.91%
Prob.C	-0.18%	0.52%	0.64%	0.66%	0.49%	0.65%	0.67%	0.67%	0.65%	-0.62%
$C\times1$	1.54%	1.79%	1.87%	1.88%	1.75%	1.88%	1.89%	1.89%	1.88%	0.92%
$C\times1.3$	1.50%	1.71%	1.71%	1.71%	1.71%	1.72%	1.72%	1.72%	1.71%	1.12%
$M^C\times1$	0.44%	0.68%	0.68%	0.68%	0.66%	0.68%	0.68%	0.68%	0.68%	0.33%
$M^C\times2$	1.34%	1.75%	1.79%	1.79%	1.77%	1.79%	1.79%	1.79%	1.78%	0.90%
$M^C\times$UNL	2.78%	2.82%	2.91%	2.93%	2.75%	2.93%	2.95%	2.95%	2.92%	1.83%
Average	1.52%	1.75%	1.79%	1.80%	1.73%	1.80%	1.81%	1.81%	1.79%	1.02%

		ASAA (Phase 1: $\hat{M} = 10$, $\lvert\Omega_{\hat{K}}\rvert = 10$)				ASAA (Phase 1: $\hat{M} = 5$, $\lvert\Omega_{\hat{K}}\rvert = 10$)				
Instance	SAA	$\gamma = 0$	$\gamma = 0.1$	$\gamma = 0.2$	$\gamma = 0.3$	$\gamma = 0$	$\gamma = 0.1$	$\gamma = 0.2$	$\gamma = 0.3$	$\gamma = 0.4$
Prob.A	1,764.0	14,534.3	14,307.8	14,010.4	13,305.6	7,400.6	7,378.9	6,782.4	6,740.3	6,734.5
Prob.B	2,496.2	18,341.9	17,242.7	17,246.8	17,225.0	8,619.0	7,842.5	7,835.5	7,838.3	6,734.5 7,837.7
Prob.C	3,753.8	30,953.8	30,934.4	29,881.9	29,118.0	16,123.2	14,912.5	14,320.8	14,246.7	14,248.4
$C\times1$	2,611.2	23,528.4	23,060.7	22,574.3	22,004.1	11,632.3	11,109.2	10,708.7	10,655.4	10,650.9
$C\times1.3$	2,731.5	19,024.8	18,595.9	18,185.1	17,761.6	9,796.3	8,980.1	8,583.8	8,561.4	8,562.8
$M^C\times1$	2,873.8	17,272.7	16,280.6	16,263.7	16,192.1	8,123.3	7,260.3	7,261.8	7,252.8	7,255.8
$M^C\times2$	3,711.4	39,694.6	39,346.4	38,490.7	37,202.9	20,518.7	19,909.4	18,716.8	18,668.0	18,669.3
$M^C\times$UNL	1,428.8	6,862.6	6,857.9	6,384.6	6,254.2	3,500.8	2,964.1	2,960.0	2,904.4	2,895.5
Average	2,671.3	21,276.6	20,828.3	20,379.7	19,882.8	10,714.3	10,044.6	9,646.2	9,608.4	9,606.9

References

Nabil Absi and Safia Kedad-Sidhoum. The multi-item capacitated lot-sizing problem with setup times and shortage costs. European Journal of Operational Research, 185(3):1351–1374, 2008.

Nabil Absi, Boris Detienne, and Stéphane Dauzère-Pérès. Heuristics for the multi-item capacitated lot-sizing problem with lost sales. Computers & Operations Research, 40(1):264–272, 2013.

Agostinho Agra, Cristina Requejo, and Filipe Rodrigues. An adjustable sample average approximation algorithm for the stochastic production-inventory-routing problem. Networks, 72(1):5–24, 2018.

Renzo Akkerman, Dirk Van Der Meer, and Dirk Pieter Van Donk. Make to stock and mix to order: Choosing intermediate products in the food-processing industry. International Journal of Production Research, 48 (12):3475–3492, 2010.

Aldair Alvarez, Jean-François Cordeau, Raf Jans, Pedro Munari, and Reinaldo Morabito. Inventory routing under stochastic supply and demand. Omega, 102:102304, 2021.

M. Arenales, R. Morabito, V. Armentano, and H. Yanasse. Pesquisa Operacional: Para cursos de engenharia. Elsevier Brasil, 2015.

J Ashayeri, AGM Van Eijs, and P Nederstigt. Blending modelling in a process manufacturing: A case study. European Journal of Operational Research, 72(3):460–468, 1994.

J Will M Bertrand and Werner GMM Rutten. Evaluation of three production planning procedures for the use of recipe flexibility. European Journal of Operational Research, 115(1):179–194, 1999.

Hadi Mohammadi Bidhandi and Jonathan Patrick. Accelerated sample average approximation method for two-stage stochastic programming with binary first-stage variables. Applied Mathematical Modelling, 41: 582–595, 2017.

James H Bookbinder and Jin-Yan Tan. Strategies for the probabilistic lot-sizing problem with service-level constraints. Management Science, 34(9):1096–1108, 1988.

Wilfred Candler. Coal blending—with acceptance sampling. Computers & Operations Research, 18(7):591–596, 1991.

Qi Chen, Izak Duenyas, and Stefanus Jasin. Optimal use and replenishment of two substitutable raw materials in a stochastic capacitated make-to-order production system. Manufacturing & Service Operations Management, 2022.

Yifu Chen and Christos T Maravelias. Variable bound tightening and valid constraints for multiperiod blending. INFORMS Journal on Computing, 2022.

Ivan Contreras, Jean-François Cordeau, and Gilbert Laporte. Stochastic uncapacitated hub location. European Journal of Operational Research, 212(3):518–528, 2011.

Diego Jacinto Fiorotto, Raf Jans, and Silvio Alexandre de Araujo. Integrated lot sizing and blending problems. Computers & Operations Research, 131:105255, 2021.

Ali Ghamari and Hadi Sahebi. The stochastic lot-sizing problem with lost sales: a chemical-petrochemical case study. Journal of Manufacturing Systems, 44:53–64, 2017.

Matthieu Gruson, Jean-François Cordeau, and Raf Jans. Benders decomposition for a stochastic three-level lot sizing and replenishment problem with a distribution structure. European Journal of Operational Research, 291(1):206–217, 2021.

Julia L Higle. Stochastic programming: optimization when uncertainty matters. In Emerging theory, methods, and applications, pages 30–53. Informs, 2005.

Hajar Hilali, Vincent Hovelaque, and Vincent Giard. Integrated scheduling of a multi-site mining supply chain with blending, alternative routings and co-production. International Journal of Production Research, 61(6): 1829–1848, 2023.

IBM. Blending problem. https://www.ibm.com/docs/en/icos/22.1.0?topic=programming-blending-problems, 2022. Accessed: 2024-05-03.

Raf Jans and Zeger Degraeve. Modeling industrial lot sizing problems: a review. International Journal of Production Research, 46(6):1619–1643, 2008.

Michal Kaut and W Stein. Evaluation of scenario-generation methods for stochastic programming. Humboldt-Universität zu Berlin, Mathematisch-Naturwissenschaftliche Fakultät II, Institut für Mathematik, 2003.

Onur A Kilic, Renzo Akkerman, Dirk Pieter Van Donk, and Martin Grunow. Intermediate product selection and blending in the food processing industry. International Journal of Production Research, 51(1):26–42, 2013.

Anton J Kleywegt, Alexander Shapiro, and Tito Homem-de Mello. The sample average approximation method for stochastic discrete optimization. SIAM Journal on Optimization, 12(2):479–502, 2002.

Wai-Kei Mak, David P Morton, and R Kevin Wood. Monte carlo bounding techniques for determining solution quality in stochastic programs. Operations Research Letters, 24(1-2):47–56, 1999.

Josefa Mula, Raul Poler, Jose P García-Sabater, and Francisco Cruz Lario. Models for production planning under uncertainty: A review. International Journal of Production Economics, 103(1):271–285, 2006.

Fabrício Oliveira and Silvio Hamacher. Optimization of the petroleum product supply chain under uncertainty: A case study in northern brazil. Industrial & Engineering Chemistry Research, 51(11):4279–4287, 2012.

Supachai Pathumnakul, Mongkon Ittiphalin, Kullapapruk Piewthongngam, and Somkiet Rujikietkumjorn. Should feed mills go beyond traditional least cost formulation? Computers and Electronics in Agriculture, 75(2):243–249, 2011.

Shen Peng, Francesca Maggioni, and Abdel Lisser. Bounds for probabilistic programming with application to a blend planning problem. European Journal of Operational Research, 2021.

Franco Quezada, Céline Gicquel, Safia Kedad-Sidhoum, and Dong Quan Vu. A multi-stage stochastic integer programming approach for a multi-echelon lot-sizing problem with returns and lost sales. Computers & Operations Research, 116:104865, 2020.

P Chandra Prakash Reddy, IA Karimi, and R Srinivasan. Novel solution approach for optimizing crude oil operations. AIChE Journal, 50(6):1177–1197, 2004.

WGMM Rutten. Hierarchical mathematical programming for operational planning in a process industry. European Journal of Operational Research, 64(3):363–369, 1993.

W.G.M.M. Rutten. The use of recipe flexibility in production planning and inventory control. Phd thesis 1 (research tu/e / graduation tu/e), Industrial Engineering and Innovation Sciences, 1995.

Wilhelmus Gerardus Martinus Maria Rutten and Jan Willem Marie Bertrand. Balancing stocks, flexible recipe costs and high service level requirements in a batch process industry: A study of a small scale model. European Journal of Operational Research, 110(3):626–642, 1998.

Tjendera Santoso, Shabbir Ahmed, Marc Goetschalckx, and Alexander Shapiro. A stochastic programming approach for supply chain network design under uncertainty. European Journal of Operational Research, 167(1):96–115, 2005.

Peter Schütz, Asgeir Tomasgard, and Shabbir Ahmed. Supply chain design under uncertainty using sample average approximation and dual decomposition. European Journal of Operational Research, 199(2):409–419, 2009.

Narges Sereshti, Yossiri Adulyasak, and Raf Jans. The value of aggregate service levels in stochastic lot sizing problems. Omega, 102:102335, 2021.

Narges Sereshti, Merve Bodur, and James R Luedtke. Stochastic dynamic lot-sizing with supplier-driven substitution and service level constraints. arXiv preprint arXiv:2301.00258, 2022.

Narges Sereshti, Yossiri Adulyasak, and Raf Jans. Managing flexibility in stochastic multi-level lot sizing problem with service level constraints. Omega, 122:102957, 2024.

Alexander Shapiro and Tito Homem-de Mello. A simulation-based approach to two-stage stochastic programming with recourse. Mathematical Programming, 81(3):301–325, 1998.

A Singh, JF Forbes, PJ Vermeer, and SS Woo. Model-based real-time optimization of automotive gasoline blending operations. Journal of Process Control, 10(1):43–58, 2000.

Duygu Taş, Michel Gendreau, Ola Jabali, and Raf Jans. A capacitated lot sizing problem with stochastic setup times and overtime. European Journal of Operational Research, 273(1):146–159, 2019.

Horst Tempelmeier. Stochastic lot sizing problems. In Handbook of stochastic models and analysis of manufacturing system operations, pages 313–344. Springer, 2013.

C.P. Tomazella, M.O. Santos, D. Alem, and R. Jans. Managing flexibility in stochastic procurement and production problems using adjustable sample average approximation. Journal Name, Volume Number:Page Numbers, 2024.

Huseyin Tunc, Onur A Kilic, S Armagan Tarim, and Roberto Rossi. An extended mixed-integer programming formulation and dynamic cut generation approach for the stochastic lot-sizing problem. INFORMS Journal on Computing, 30(3):492–506, 2018.

Bram Verweij, Shabbir Ahmed, Anton J Kleywegt, George Nemhauser, and Alexander Shapiro. The sample average approximation method applied to stochastic routing problems: a computational study. Computational Optimization and Applications, 24(2):289–333, 2003.

H Paul Williams and AC Redwood. A structured linear programming model in the food industry. Journal of the Operational Research Society, 25(4):517–527, 1974.

Yu Yang, Phebe Vayanos, and Paul I Barton. Chance-constrained optimization for refinery blend planning under uncertainty. Industrial & Engineering Chemistry Research, 56(42):12139–12150, 2017.