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A fast dual bound for power allocation

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Abstract : In this paper, we propose a fast algorithm to compute a bound for the problem of optimizing the power of a number of users sharing a set of wireless channels. We define an equivalent problem and show how computing the dual function of this problem can be separated into a number of independent non-convex sub-problems in two variables only. We then give an analytic expression for the optimal value of the sub-problems so that the dual function can be evaluated in low polynomial time.

Keywords: Power allocation, wireless, lagrangian relaxation, non-convex optimization

Résumé : Nous proposons dans cet article un algorithme rapide pour calculer une borne supérieure au problème de la gestion de la puissance des utilisateurs d'un ensemble de canaux sans fil. Nous définissons un problème équivalent et montrons comment le calcul de la fonction duale de ce problème se décompose en sous-problèmes non convexes en deux variables. Nous calculons ensuite analytiquement la solution optimale des sous-problèmes, ce qui permet un calcul rapide de la fonction duale.

Mots clés : Contrôle de puissance, sans-fil, relaxation lagrangienne, optimisation nonconvexe

1 Introduction

An essential element in the design of communication systems is to maximize the network throughput by optimally allocating power to a number of users sharing a set of wireless transmission channels. This non-convex optimization problem has been the subject of much work, first for digital subscriber loops in the mid-2000s and later, in the context of cellular and other wireless communication networks, where it often shows up as part of a more general design problem.

While there is a number of primal algorithms available to compute approximate solutions, there has also been an interest in a dual approach to computing the optimal power. Unfortunately, the non-convexity of the problem makes it difficult to compute a dual bound and only approximate solutions have been available so far.

In this paper, we provide an exact dual bound for the power allocation problem that can be computed in low polynomial time. In order to do this, we first state in Section 2 the power allocation problem and briefly review in Section 3 the work that has been done on this topic. The core of the paper is found in Section 4 where we first define an equivalent problem. We then show how to evaluate its dual function in polynomial time and provide an analytic expression for the value of the dual functions. We discuss in Section 5 some further research directions that are now possible and we then conclude in Section 6.

2 Power allocation with interference

In this paper, we consider a generic power allocation problem where a number of users are allowed to transmit over a given set of channels. The objective is to maximize the total rate subject to a minimum rate constraint for each user and a power limit for the transmission of each user on all channels.

2.1 Definitions and notation

We use the model of [11] as a typical power allocation problem. There are K users, or mobile terminal, and N channels that these users can use to transmit.

As a rule, an upper index refers to a channel, and is denoted by n, while lower indices refer to users and are denoted by k and j whenever needed. First, define the known network parameters

K Number of users, indexed with k or j

N Number of channels, indexed with n

$G^n_{k,j}$ The channel gain between the transmitter of user j and the receiver of user k

σ^n_k The noise power for terminal k on channel n

$\overline{P}_k$ The maximum power available to user k.

$\overline{r}_k$ The minimum bit rate per Hz needed by user k.

Next, we define the decision variables and some intermediate variables that depend on them

P^n_k The transmission power of user k on channel n.

r^n_k The bit rate per Hz of user k on channel n

We can compute r^n_k, the Shannon limit for the rate received by user k on channel n

$$r^n_k = \log\left(1 + \frac{G^n_{k,k} P^n_k}{\sum_{j \neq k} G^n_{k,j} P^n_j + \sigma^n_k}\right) \tag{1}$$

where r^n_k and $\overline{r}_k$ are the actual rate and bound multiplied by a $\log(2)$ factor to simplify the notation. In the following, we will use a vector notation whenever this is more convenient to denote some subset of variables x_k^n as the case may be, e.g., $\mathbf{x}$ is the set of all x_k^n for $n = 1 \dots N$, $k = 1 \dots K$, $\mathbf{x}^n$ is the set x_k^n for $k = 1 \dots K$ and $\mathbf{x}_k$ is the set x_k^n for $n = 1 \dots N$.

2.2 Problem definition

We now define what we will call the original **Problem** $\mathcal{P}_0$, in the form of a non-convex maximization

$$\max_{\mathbf{P}} Z = \sum_{n=1}^{N} \sum_{k=1}^{K} r_k^n \left(\mathbf{P}^n\right) \tag{2}$$

$$\sum_{n=1}^{N} r_k^n \left(\mathbf{P}^n\right) \geq \overline{r}_k \qquad \forall k = 1 \dots K \tag{3}$$

$$\overline{P}_k \geq \sum_{n=1}^{N} P_k^n \qquad \forall k = 1 \dots K. \tag{4}$$

In all that follows, we assume that we can find a feasible solution for $\mathcal{P}_0$, if there is one. This is easily done with any general-purpose nonlinear solver for convex problems, which will produce a local optimum. If needed, we can try to improve this solution by giving the solver different starting points and keeping the best one.

3 Previous work

Problem (2–4) is of interest in its own right but also because it shows up as part of some more general planning problems [3, 4, 6, 7, 9, 12, 13, 14, 15, 16, 17]. Given that it is not convex, computing an exact solution in reasonable time is not possible for realistic problems. Only approximations are available and a bound is needed to evaluate their accuracy.

3.1 Dual methods

One way to reduce the difficulty of $\mathcal{P}_0$ to use Lagrangian relaxation [2]. Dualizing the constraints (3–4) with multipliers ν_k and μ_k yields the Lagrangian

$$\begin{aligned}\mathcal{L}_0(\mathbf{P}, \boldsymbol{\nu}, \boldsymbol{\mu}) = &\sum_{k,n} r_k^n \\ &+ \sum_k \nu_k \left[\sum_n r_k^n - \overline{r}_k\right] + \sum_k \mu_k \left[\overline{P}_k - \sum_n P_k^n\right] \\ = &\sum_n \mathcal{L}_0^n(\mathbf{P}^n, \boldsymbol{\nu}^n, \boldsymbol{\mu}^n)\end{aligned} \tag{5}$$

were we have defined for each n

$$\begin{aligned}\mathcal{L}_0^n(\mathbf{P}^n, \boldsymbol{\nu}^n, \boldsymbol{\mu}^n) = &\sum_k r_k^n \\ &+ \sum_k \nu_k \left[r_k^n - \overline{r}_k\right] + \sum_k \mu_k \left[\overline{P}_k - P_k^n\right].\end{aligned} \tag{6}$$

We see that the lagrange function (5) is separable in n so that the evaluation of the dual function requires the maximization of N non-convex Lagrange functions (6) in K variables each. While this is simpler that solving one non-convex problem in KN variables, there still remains the fact that the partial sub-problems remain non-convex and thus hard to solve.

The sub-problem maximization was done by exhaustive search in [2]. The evaluation of the dual function was approximated in [10] by replacing the simultaneous maximization over all k by a coordinate search method which does not guarantee an optimal solution.

3.2 Scope and contributions

From the previous discussion, given a set of multipliers, there does not currently exist a way of computing a dual bound of the problem in reasonable time for non-trivial cases, let alone minimizing the dual function. This means that Lagrangian techniques cannot be used to solve the power allocation problem. The contribution of this paper is thus to address the issue of computing a bound for a given set of multipliers, an essential element of the Lagrangian relaxation method.

The question of minimizing the dual function over the dual variables is definitely *not* in the scope of this short paper. Still, we give a short discussion of this problem in Section 5 and present some limited numerical results showing the relevance of this paper for solving the dual problem.

4 Extended problem

Because computing the dual function is intractable for the dual of $\mathcal{P}_0$, we define a different primal problem $\mathcal{P}_E$ that has two properties: 1) The primal solution of $\mathcal{P}_E$ is the same as that of $\mathcal{P}_0$ and 2) we can compute Φ_E, the Lagrange function for $\mathcal{P}_E$, very quickly. By weak duality, we know that Φ_E is a bound on the optimal value of $\mathcal{P}_E$ and thus is also a bound on the optimal value of $\mathcal{P}_0$.

First, we add a new set of *independent* variables

I_k^n the total interference power received by user k on channel n

which is given by

$$I_k^n = \sum_{j \neq k} G_{k,j}^n P_j^n. \tag{7}$$

We then re-define

$$r_k^n(P_k^n, I_k^n) = \log\left(1 + \frac{G_{k,k}^n P_k^n}{I_k^n + \sigma_k^n}\right) \tag{8}$$

explicitly as a function of both $\mathbf{P}$ and $\mathbf{I}$. Note that r_k^n now depends only on the I_k^n and P_k^n variables. It is an increasing function of P_k^n and decreasing with I_k^n with a limit of 0 at infinity. We also define the redundant bound constraints

$$P_k^n \leq \overline{P}^k \qquad \forall k = 1 \dots K \tag{9}$$

$$I_k^n \leq \overline{I}_k^n = \sum_{j \neq k} G_{j,k}^n \overline{P}_j \qquad \forall k = 1 \dots K. \tag{10}$$

We now write the extended problem **Problem** $\mathcal{P}_E$

$$\max_{\mathbf{P},\mathbf{I}} Z = \sum_{n=1}^{N} \sum_{k=1}^{K} r_k^n(P_k^n, I_k^n)$$
$$\text{subject to } (3, 4, 7, 9, 10).$$

We have added (7) as a separate constraint to take into account the fact that the I variables are not really independent but are related to the P variables. The redundant bounds (9–10) are needed for reasons explained in Appendix A. We now find a global bound for $\mathcal{P}_E$, which is by definition also a global bound for $\mathcal{P}_0$.

4.1 Lagrangian relaxation

We denote as $\mathcal{L}_E$ the Lagrangian corresponding to $\mathcal{P}_E$ and the dual function as Φ_E. We now relax constraints (3), (4) and (10) and construct the Lagrangian

$$\mathcal{L}_E(\mathbf{P}, \mathbf{I}, \boldsymbol{\nu}, \boldsymbol{\mu}, \boldsymbol{\lambda}) = \sum_{k,n} r_k^n + \sum_{k,n} \lambda_k^n \left[I_k^n - \sum_{j \neq k} G_{k,j}^n P_j^n \right]$$

$$+\sum_k \nu_k \left[\sum_n r_k^n - \overline{r}_k\right] + \sum_k \mu_k \left[\overline{P}_k - \sum_n P_k^n\right]$$

where multipliers $\boldsymbol{\mu} \geq \mathbf{0}$, $\boldsymbol{\nu} \geq \mathbf{0}$ and $\boldsymbol{\lambda}$ is of arbitrary sign. Note that the term $\sum_{j \neq k} G_{k,j}^n P_j^n$ introduces a coupling between users on each channel. We can decouple this term if we write

$$\sum_k \lambda_k^n \sum_{j \neq k} G_{k,j}^n P_j^n = \sum_k P_k^n \sum_{j \neq k} \lambda_j^n G_{j,k}^n$$

so that the Lagrangian becomes

$$\mathcal{L}_E(\mathbf{P}, \mathbf{I}, \boldsymbol{\nu}, \boldsymbol{\mu}, \boldsymbol{\lambda}) = \sum_{k,n} r_k^n + \sum_{k,n} \left[\lambda_k^n I_k^n - P_k^n \Lambda_k^n\right] \tag{11}$$

$$+\sum_k \nu_k \left[\sum_n r_k^n - \overline{r}_k\right] + \sum_k \mu_k \left[\overline{P}_k - \sum_n P_k^n\right] \tag{12}$$

where we have defined

$$\Lambda_k^n = \sum_{j \neq k} \lambda_j^n G_{j,k}^n. \tag{13}$$

Regrouping terms, we get

$$\begin{aligned}
\mathcal{L}_E(\mathbf{P}, \mathbf{I}, \boldsymbol{\nu}, \boldsymbol{\mu}, \boldsymbol{\lambda}) =& \sum_{n,k} \mathcal{L}_k^n(P_k^n, I_k^n, \boldsymbol{\nu}, \boldsymbol{\mu}, \boldsymbol{\lambda}) \\
\mathcal{L}_k^n(P_k^n, I_k^n) =& (1 + \nu_k) r_k^n(P_k^n, I_k^n) - (\mu_k + \Lambda_k^n) P_k^n \\
&+ \lambda_k^n I_k^n - \left[\nu_k \overline{r}_k - \mu_k \overline{P}_k\right].
\end{aligned} \tag{14}$$

The evaluation of the dual function for a given set of multipliers $(\boldsymbol{\nu}, \boldsymbol{\mu}, \boldsymbol{\lambda})$ is then

$$\begin{aligned}
\Phi_E(\boldsymbol{\nu}, \boldsymbol{\mu}, \boldsymbol{\lambda}) &= \max_{\mathbf{P}, \mathbf{I}} \mathcal{L}_E(\mathbf{P}, \mathbf{I}, \boldsymbol{\nu}, \boldsymbol{\mu}, \boldsymbol{\lambda}) \\
&= \max_{\mathbf{P}, \mathbf{I}} \sum_{n,k} \mathcal{L}_k^n(P_k^n, I_k^n, \boldsymbol{\nu}, \boldsymbol{\mu}, \boldsymbol{\lambda}) \\
&= \sum_{n,k} \max_{P_k^n, I_k^n} \mathcal{L}_k^n(P_k^n, I_k^n, \boldsymbol{\nu}, \boldsymbol{\mu}, \boldsymbol{\lambda})
\end{aligned}$$

subject to constraints (9–10). This separates into NK independent non convex subproblems in two variables P_k^n and I_k^n, something much simpler than solving the N subproblems in K variables each. In fact, in the present case, we can do much better than this.

4.2 Solving the sub-problem

We now consider the subproblem for a given pair n, k. To simplify the notation, we drop the indices for all variables and denote $G = G_{k,k}^n$. After dropping the terms that do not depend on P or I, the sub-problem maximization becomes

$$\begin{aligned}
\max_{I,P} f(I, P) =& (1 + \nu) \log\left(1 + \frac{GP}{I + \sigma}\right) \\
&- (\mu + \Lambda) P + \lambda I \\
&0 \leq I \leq \overline{I} \qquad 0 \leq P \leq \overline{P}.
\end{aligned} \tag{15}$$

We can now compute an analytic solution for (15). Suppose that we are given some value of P. In that case, we can show that $f(I)$ is a convex function of I so that the maximum has to be at one of the two boundaries and we need to solve (15) only at the two boundary points $I = 0$ and $I = \overline{I}$.

Based on this, consider the solution of problem (15) for a fixed I. We need to solve

$$\begin{aligned}\max_{0\le P\le \overline{P}} f(P) =&(1+\nu)\log\left(1+\frac{GP}{I+\sigma}\right)\\ &-(\mu+\Lambda)P+\lambda I. \end{aligned}\tag{16}$$

In what follows, we denote the optimal power as $P^*(I)$. Note also that f is concave in P. We consider two cases.

First, if $\mu+\Lambda \le 0$, $f(P)$ is monotone increasing so that the solution P^* is independent of I and given by

$$P^*(I) = \overline{P}. \tag{17}$$

If $\mu+\Lambda > 0$, the solution is either at one of the end points or somewhere in the interval at the KKT point, denoted P_K, given by

$$P_K(I) = \left[\frac{1+\nu}{\mu+\Lambda} - \frac{I+\sigma}{G}\right]. \tag{18}$$

The solution is then

$$P^*(I) = \begin{cases} 0 & \text{if } P_K(I)\le 0\\ \overline{P} & \text{if } P_K(I)\ge \overline{P}\\ P_K(I) & \text{otherwise.}\end{cases} \tag{19}$$

Finally, the optimal value is given by

$$f^* = \max\left\{f(P^*(I=0)), f(P^*(I=\overline{I})\right\}. \tag{20}$$

From this, we see that the interference term will always be either 0 or $\overline{I}$. The second point is that P also will be either 0, $\overline{P}$ or some intermediate value given by the third condition of (19). For this to happen, we must have $\mu+\Lambda > 0$. In addition, P_K must lie between 0 and $\overline{P}$ for the two values $I=0$ and $I=\overline{I}$. This can be written as

$$\frac{\overline{I}+\sigma}{G} < \frac{1+\nu}{\mu+\Lambda} < \overline{P} + \frac{\sigma}{G}$$

which define the region of the dual space where we can have intermediate values for P. Note that this is possible only if the upper bound is actually larger than the lower bound, i.e.,

$$\sum_{j\neq k} G_{j,k}\overline{P}_j \le G_{k,k}\overline{P}_k.$$

This stands a good chance of happening when the off-diagonal terms of the gain matrix are smaller than the diagonal ones, which will be the case in practice since users tend to be served by base stations that are not too far away. Note however that the left-hand side is the sum of the interference powers generated by *all* users other than k so that this term will get larger as K increases.

To summarize, we have shown that given some values for the multipliers, we can compute quickly the value of the dual function at that point by computing a globally optimal solution to the non-convex subproblem.

4.3 Complexity

The complexity of the dual function evaluation depends only on K and N. This is because the computation time of (17–20) is a constant independent of the problem data. In the terminology of complexity theory, this is said to be $O(1)$ complexity. This calculation has to be done KN times so that the overall complexity is $KNO(1)$.

4.4 Dual bound and no-interference solution

There is an interesting relationship between the Φ_E and the primal problem. Define $\mathcal{P}_S$ as the simplified version of problem (2–4) where we set the interference term to zero.

$$\max_{\mathbf{P}} \overline{Z}_s = \sum_{n=1}^{N}\sum_{k=1}^{K} \log\left(1 + \frac{G_{k,k}^n P_k^n}{\sigma_k^n}\right) \tag{21}$$

$$\sum_{n=1}^{N} \log\left(1 + \frac{G_{k,k}^n P_k^n}{\sigma_k^n}\right) \geq \overline{r}_k \tag{22}$$

$$\overline{P}_k \geq \sum_{n=1}^{N} P_k^n. \tag{23}$$

This problem is convex so that the optimal $\overline{Z}_s$ can easily be computed. It is unique and barring degeneracy, so are the optimal Ps. Let Φ_s be the dual function of $\mathcal{P}_s$. We can write the Lagange function

$$\begin{aligned}\mathcal{L} = &\sum_{n=1}^{N}\sum_{k=1}^{K}(1+u_k)\log\left(1 + \frac{G_{k,k}^n P_k^n}{\sigma_k^n}\right)\\ &- \sum_{n=1}^{N}\sum_{k=1}^{K} v_k P_k^n + \left(\sum_{k=1}^{K} v_k \overline{P}_k - u_k \overline{r}_k\right)\end{aligned}$$

with multipliers $u_k \geq 0$ for (22) and $v_k \geq 0$ for (23).

The evaluation of $\Phi_s(\mathbf{u},\mathbf{v})$ is given by the maximization of $\mathcal{L}$ which separates into NK independent sub-problems of the form

$$\max_{0 \leq P \leq \overline{P}} (1+u)\log\left(1 + \frac{GP}{\sigma}\right) - Pv.$$

We can see that this is precisely (15) where we have set $\lambda_k^n = 0$ for all N and K so that $\Phi_E(\boldsymbol{\nu},\boldsymbol{\mu},\boldsymbol{\lambda}=\mathbf{0})$ $= \Phi_s = Z_s$.

This last remark is of practical importance because 1) it can be used to check a numerical implementation of the dual function calculation and 2) it can be used as a starting solution for solving the dual problem.

4.5 Bound quality

We can put a limit on the accuracy of the bound by viewing the transformation from $\mathcal{P}_0$ to $\mathcal{P}_E$ as a two-step procedure. First, we construct the dual of $\mathcal{P}_0$ by dualizing (3–4). This yields the Lagrange function (5) which is the sum of n independent Lagrange functions $\mathcal{L}^n(\mathbf{P}^n, \boldsymbol{\nu}^n, \boldsymbol{\mu}^n)$ where

$$\mathcal{L}^n = \sum_k (1+\nu_k) r_k^n(\mathbf{P}^n) - \sum_k \mu_k P_k^n + \sum_k \left[\mu_k \overline{P}_k - \nu_k \overline{r}_k\right]$$

The dual function is then

$$\Phi_0 = \max_{\mathbf{P}} \sum^n \mathcal{L}^n = \sum^n \max_{\mathbf{P}^n} \mathcal{L}^n(\mathbf{P}^n) = \sum^n \Phi_0^n.$$

Define $\mathcal{P}_0^D$ as the computation of Φ_0 at some point $\boldsymbol{\mu}$, $\boldsymbol{\nu}$. This requires N global optimizations of a non-convex function $\mathcal{L}^n$ in K variables.

Because the computation of Φ_0 is too difficult, the second step of the procedure is compute an upper bound to this function. For this, we define a new problem $\mathcal{P}_1^D$ equivalent to $\mathcal{P}_0^D$ using the transformation (7–8). The evaluation of Φ_1^D, the dual function of $\mathcal{P}_1^D$, is now the maximization of a non-convex function in $K(N+2)$ variables. Given that the two problems are equivalent, we have $\Phi_1^D = \Phi_0$.

Because $\mathcal{P}_1^D$ is hard, we compute a dual bound for it by dualizing the coupling constraints (7). Call this dual function Φ_2^D. We then have $\Phi_2^D \geq \Phi_1^D$ so that algogether, we have

$$\Phi_2^D \geq \Phi_1^D = \Phi_0^D \geq Z^*.$$

In other words, the algorithm of section 4.2 computes Φ_2^D which cannot be a better bound than the actual dual Φ_0. The trade-off is the potential increase of the gap of Φ_2^D vs the large cpu time required for Φ_0.

5 Numerical results and future work

The validity of the bound (20) can be checked in two ways. One is with the no-interference solution as discussed in Section 4.4. Another way is to solve (15) exactly with a global solver such as Baron [8]. This has been done and the bound is found to be correct in both cases.

We can take advantage of this fast computation of the dual function to re-consider solution algorithms for the dual problem that were not previously possible and some potential improvements of the model.

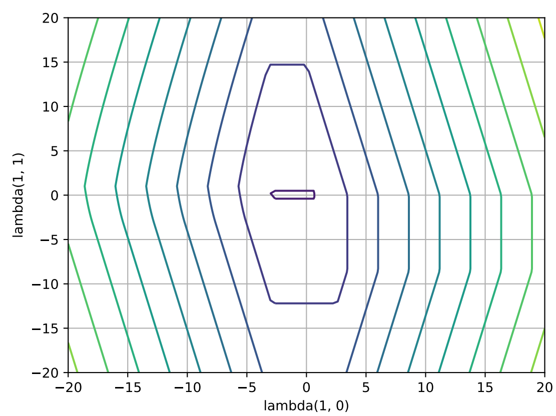

Figure 1: Dual function contours

First, we can make plots of the dual function to get an idea of the difficulty of the dual problem. An example is given on Figure 1 where we show the contours of the dual function for a small case with $K = 2$ and $N = 5$. The plot is shown in the λ_1^1 and λ_2^1 plane with all other dual variables fixed. We can see the sharp corners where the function is not differentiable but also some regions where the dual function value is independent of one of the variables.

Minimizing such a function is not trivial. The standard single-step subgradient algorithm [1] is unlikely to give good results since it is subject to jamming, is known to converge very slowly and the final solution depends on the algorithm parameters, e.g., the step size. With the fast bound, we can now use any one of a number of solution techniques for non-differentiable problems [5]. As mentioned in Section 3.2, this is outside the scope of this paper but is nonetheless an interesting avenue that is now possible for further research.

Another potential research avenue is to try and improve the extended model by adding other valid constraints in addition to (9–10) which may yield a better value of the optimal dual.

6 Conclusion

We have shown that it is possible to compute a global bound to the power allocation problem in $KNO(1)$ time. This opens up a number of research avenues both in the modeling of the extended problem and the algorithm used for solving the dual problem.

The technique presented here has been developed for a simple power allocation problem but it could be extended to more complex models, for instance, the power allocation and channel assignment problem. This kind of problem is generally solved by a decomposition method where one set of variables is kept fixed while the other is optimized. The bounds can then be computed for power optimization part even with large problems.

A Redundant constraints: an example

As an example of the usefulness of redundant constraints, consider the two upper bounds (9–10), which are clearly not needed. Suppose that we write an extended problem without these bounds. If $\lambda > 0$, the solution is at $I = \infty$ and $f = \infty$. Because we need to minmize the dual function, this value of λ cannot be an optimal solution and we need to impose an additional condition $\boldsymbol{\lambda} \leq \mathbf{0}$ on the dual variables.

Suppose now that we start the dual minmization problem with a dual solution $\boldsymbol{\lambda} = \mathbf{0}$. We need to solve the subproblem

$$\max_{I,P} f(I,P) = (1+\nu)\log\left(1 + \frac{GP}{I+\sigma}\right) - \mu P.$$

Clearly, for any given P, the optimal solution is at $I = 0$. We are then left with the problem

$$\max_{P} f(P) = (1+\nu)\log\left(1 + \frac{GP}{\sigma}\right) - \mu P$$

which is simply the power allocation with no interference with solution (18) with $\Lambda = I = 0$. We get a solution where $P \geq 0$ and $I = 0$ so that the subgradient at that point $g^{\lambda}_{n,k} \leq 0$. Because we are minimizing the dual function, we need to move in a direction $-g^{\lambda}_{n,k}$ which is positive so that λ will increase, which is impossible since we must always have $\lambda \leq 0$. In other words, without the upper bounds, the optimal solution of the dual is the no-interference solution. Only by adding the bounds can we improve on this value.

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