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Scalable multi-stage stochastic optimization for freight procurement in transportation-inventory systems

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Abstract : The procurement of freight services is an important element for the supply chain management of a shipper (i.e., a manufacturer or retailer) that sources transportation services from the third-party logistics market. Motivated by a practical freight procurement problem faced by shippers, we provide a holistic approach to designing freight procurement strategies for transportation-inventory systems that captures the interconnections between freight procurement, transportation, and inventory management. In view of the supply and demand uncertainties, we consider the problem in a multi-stage decision process that complies with the revealing process of the uncertain data. To handle instances of realistic size, we propose an enhanced stochastic dual dynamic programming solution approach. We conduct extensive numerical experiments to test the performance of the approach. The results demonstrate that our approach scales to huge instances with up to 50^{18} scenarios and that the proposed enhancement strategies significantly improve its performance. Compared to methods commonly adopted to solve similar problems, our approach could potentially help reduce the total cost for shippers by 7.5% to 47.2% based on our generated instances from real-world data and simulations.

Keywords : Freight procurement, transportation-inventory management, multi-stage stochastic optimization, stochastic dual dynamic programming

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1 Introduction

Effective distribution and storage of commodities are crucial for any company acting as a *shipper* in a supply chain. While some have their own capabilities, many shippers rely on the transportation market (i.e., third-party logistics, 3PL) for distributing commodities. 3PL services play a vital part in global trade and the 3PL market is valued at more than one trillion US dollars (USD) worldwide (Allied Market Research 2019).

In the transportation market, shippers procure freight services from *carriers* (i.e., 3PL service providers). The basic elements in freight service procurement are *lanes*, and a lane is an origin-destination pair with transportation demand over a period. Freight service procurement on a lane is an auction process with shippers acting as auctioneers and carriers acting as bidders (Lim et al. 2008). Results in the auction on a lane are *capacity contracts* negotiated between the shipper and carriers who win the auction. A capacity contract specifies the number and schedule of shipments to be performed by the carrier on the lane, the capacity of each shipment, as well as the freight rate payable by the shipper. On top of capacity contracts, freight services for single shipments can also be acquired through non-contractual freight rates. For serving the same lane, the freight rates of capacity contracts are typically lower than the non-contractual freight rates.

Normally, the service period of a capacity contract on a lane ranges from several months to two years (Sheffi 2004). Considering its long service period, typically, a capacity contract has to be determined by a shipper without fully knowing the transportation demand on the lane. Therefore, when determining the capacity contracts on a lane, the shipper should consider the uncertainty in transportation demand and the necessary adjustments under different demand scenarios.

This paper introduces a joint freight procurement and transportation-inventory management problem (FPTMP) under supply and demand uncertainty faced by a shipper. The supply and demand information is assumed to be gradually disclosed to the shipper at different time periods during the planning horizon. We, therefore, consider the resulting stochastic FPTMP (SFPTMP) in a multi-stage process. In the first stage, freight service procurement decisions are made such that the capacity contracts between the shipper and the carriers must be determined. The loading quantity for each shipment in the capacity contracts, the volume to be transported through a non-contractual rate on each lane in each period, and the inventory and backlog levels at various sites are decided in the subsequent stages after the supplies and demands in these stages become known. The objective of the problem is to minimize the expected total cost incurred by procuring freight services, distributing the commodity, holding inventories, and backlogging supplies and demands.

1.1 Background

This study is motivated by the transportation and inventory management of iron ore at a large Chinese steel manufacturer. The manufacturer imports iron ore from two loading ports in Australia and Brazil to its two unloading ports in China (i.e., there are four physical shipping lanes in total). Iron ore is stored at both the mine yards and the steel plants.

The manufacturer buys freight services from bulk shipping companies, mainly through contracts of affreightment (COAs) and voyage charters (VCs). COAs are capacity contracts in bulk shipping. The COAs for shipping iron ore are typically signed or renewed at the beginning of a year and a COA typically covers a service period of a year. Supply and demand information for an entire year is not fully known to the manufacturer when signing the COAs. Nevertheless, historical supply and demand data for estimating the distribution is available. In a COA, the shipping company is required to perform multiple shipments between a pair of loading and unloading ports during the service period. For each shipment in a COA, the shipping company is required to provide a ship with the same (or similar) capacity. In practice, loading times of shipments in a COA will be evenly spread over the service period.

Unlike COAs, VCs are for single shipments. They are more flexible and can be obtained whenever shipping demands arise. Both COAs and VCs stipulate freight rates payable by the shippers. For shipping cargoes between the same pair of ports, the freight rate in a COA is typically much lower than that in a VC. For concrete examples of COAs and VCs, we refer to the template contracts for bulk cargo COAs (i.e., VOLCOA and GENCOA) and VCs (e.g., GENCON and NUVOY) and to the explanatory notes provided by BIMCO (2024), which is the largest international shipping association representing shipowners.

The manufacturer thus faces an SFPTMP for arranging its iron ore transportation. This problem typically involves a one-year planning horizon where COAs are determined at the beginning and arrangements of non-contractual shipments (i.e., VCs) are made at the subsequent periods when the relevant demand and supply information becomes known.

1.2 Contributions

Our study makes four main contributions:

1. We study a joint freight procurement, transportation, and inventory control problem under supply and demand uncertainty motivated by a real-world application. Such a problem commonly arises in the supply chain management of a manufacturer or a retailer. We prove that this problem is NP-hard.
2. We develop a solution approach based on stochastic dual dynamic programming (SDDP) that decomposes the problem into stage-wise subproblems and captures the complex connections between different stages. We propose novel feasibility inequalities for these subproblems to enhance the computational efficiency of the method.
3. We propose two additional computational enhancements for the SDDP approach: optimality inequalities formulated based on smaller scenario trees, and a primal-dual lower bound lifting procedure based on stage-wise convergence.
4. We conduct extensive experiments using instances generated from an existing benchmark suite. The results show that the approach scales to instances with up to 18 stages and 50 scenarios per stage (i.e., 50^{18} scenarios) and demonstrate that multi-stage stochastic optimization can reduce the overall costs of a transportation-inventory system by more than 30%.

1.3 Outline

The remainder of this paper is structured as follows. Section 2 reviews relevant literature. We formally describe the SFPTMP in Section 3 and formulate it as a compact mixed-integer linear programming (MILP) model in Section 4. The SDDP approach for solving the problem as well as the enhancement strategies are described in Section 5. Computational experiments are reported in Section 6, followed by conclusions in Section 7. We provide all mathematical proofs in EC.3 of the electronic companion.

2 Literature review

In this section, we first review studies focusing on freight procurement problems that are related to the SFPTMP considered in this paper. Subsequently, we review the applications of SDDP in the literature.

2.1 Review of studies on freight procurement

The SFPTMP is related to the transportation procurement problem (TPP), especially the TPP under uncertainty. The TPP is also referred to as the winner determination problem (WDP) of freight services (Jones and Koehler 2005). The problem was introduced by Caplice and Sheffi (2003). The decisions considered in the problem are to procure freight services (or select bids) from carriers to match the given transportation plan at the lowest cost. The TPP with uncertain lane demands was considered by Ma et al. (2010), Zhang et al. (2014), and Meng et al. (2015) under the assumption that the distribution of the uncertain parameters is known and by Remli and Rekik (2013), Zhang et al. (2015), Remli et al. (2019), and Lee et al. (2021) under the assumption that the distribution information is not fully available.

The most closely related studies in the literature are Bertazzi et al. (2015) and Boujemaa et al. (2022). In the problem considered by Bertazzi et al. (2015), a single commodity is shipped from a single supplier to multiple customers with stochastic demands in a finite discrete-time planning horizon. Transportation between the supplier and the customers is outsourced such that a 3PL carrier performs all deliveries on all lanes (with an unlimited capacity on each lane) in one period at a fixed cost. Boujemaa et al. (2022) considered a shipment assignment problem on a distribution network in a finite discrete-time planning horizon. The commodities are distributed by a set of (contracted) core carriers under guaranteed capacities and freight rates and a set of spot carriers. It is assumed that both the selection of the core carriers and their service conditions have been decided. Demands are stochastic and are dynamically revealed in the planning horizon. The objective is to formulate the best shipment strategy that minimizes the expected total cost incurred by inventories, backlogs, and shipments.

The SFPTMP is different from the TPP and the problems considered by Bertazzi et al. (2015) and Boujemaa et al. (2022), as it integrates and co-optimizes transportation decisions (including commodity delivery and inventory and backlog management) with freight service procurement.

2.2 Review of studies on SDDP

We solve the SFPTMP through SDDP. The approach was introduced by Pereira and Pinto (1991) and is an extension of the nested Benders or L-shaped decomposition method proposed by Birge (1985). SDDP has been widely adopted in energy operations, especially the hydrothermal generation scheduling problem (HGSP), which manages power generation across hydro and thermal plants to meet variable energy demands (Pereira and Pinto 1991, Shapiro et al. 2013). The HGSP involves a multi-stage decision process. Each decision stage in the HGSP (usually a month) involves determining the water outflow for power generation and spillage. Other applications include portfolio optimization in finance (Valladão et al. 2019, Bhattacharya et al. 2023) and lot sizing in manufacturing management (Quezada et al. 2023, Thevenin et al. 2022). It has been shown that SDDP can well solve the HGSP or other multi-stage stochastic optimization problems with "simple" inter-stage linkages. In particular, in the HGSP, stages are connected such that the storage level at the end of each stage provides an initial storage level for the next stage.

Compared with the HGSP and other problems typically solved by SDDP, the SFPTMP has two salient features that make it more difficult to solve. First, the decisions in a stage of the SFPTMP may be affected by a series of decisions made in multiple previous stages, including the freight services procured in the first stage, the shipping volumes decided in previous stages (shipments may take multiple periods), and the last-period inventory and backlog levels in the preceding stage. Second, each decision stage in the SFPTMP is still a multi-period decision problem, involving a set of interrelated decisions in shipping management and inventory and backlog control. These challenges limit the effectiveness of traditional SDDP methods (refer to Section 6). We thus devised tailored methods for enhancing the performance of the basic SDDP approach (refer to Sections 5.2.2 and 5.4). To the best of our knowledge, we are the first to develop a highly scalable framework with convergence guarantees for such a challenging problem in transportation-inventory management.

3 Problem description and notation

In the SFPTMP, a single commodity is shipped from a set of supply sites (suppliers) $\mathcal{I}^S$ to a set of demand sites (customers) $\mathcal{I}^D$ over a discrete and finite time horizon which consists of a set $\mathcal{T} = \{1, 2, .., \bar{t}\}$ of periods. Let $\mathcal{I} = \mathcal{I}^S \cup \mathcal{I}^D$ be the set of all sites. The commodity's production or demand at site $i \in \mathcal{I}$ in period $t \in \mathcal{T}$ is given by $d_{i,t}$, where $d_{i,t} \geq 0$ if $i \in \mathcal{I}^S$ and $d_{i,t} \leq 0$ if $i \in \mathcal{I}^D$. Each site $i \in \mathcal{I}$ has a maximum inventory limit $\overline{q}_i$ and starts with an initial inventory q_i^0 at the beginning of the planning horizon. Excess supply or unmet demand can be backlogged, incurring higher costs for backlogged supplies due to additional warehousing and for delayed production due to backlogged demands. Unit inventory holding costs are h_i, and unit backlogging penalty costs are e_i with $e_i > h_i$ for all $i \in \mathcal{I}$. Let $\mathcal{L} = \{(i,j) | i \in \mathcal{I}^S, j \in \mathcal{I}^D\}$ be the set of (directed) lanes between the supply and the demand sites. It takes $o_{i,j} \in \mathbb{Z}^+$ periods to complete one shipment on lane $(i,j) \in \mathcal{L}$.

For freight procurement, the shipper makes inquiries to carriers regarding their services on the lanes in $\mathcal{L}$ and the carriers respond by providing a group of bids on each lane. Capacity contracts (e.g., COAs in sea transportation) for freight services are negotiated based on these bids. Let $\mathcal{B}_{i,j}$ be the set of bids for lane $(i,j) \in \mathcal{L}$ and $\mathcal{B} = \bigcup_{(i,j)\in\mathcal{L}} \mathcal{B}_{i,j}$. Let $i(b) \in \mathcal{I}^S$ and $j(b) \in \mathcal{I}^D$ represent the supply site and the demand site associated with the bid $b \in \mathcal{B}$. Each bid $b \in \mathcal{B}$ contains a set $\mathcal{R}_b$ of shipments. Let $\mathcal{R} = \bigcup_{b\in B} \mathcal{R}_b$. Let $t_1(r) \in \mathcal{T}$ and $t_2(r) \in \mathcal{T}$ denote the periods in which the shipment starts and ends in shipment $r \in \mathcal{R}$.

All shipments in a bid $b \in \mathcal{B}$ have the same capacity within the range $[\underline{m}_b, \overline{m}_b] \subseteq \mathbb{R}^+$. A fixed cost F_b is incurred once a capacity contract is settled for a bid b and an additional cost g_b is paid per unit shipped under that bid. In addition to capacity contracts, the shipper can also transport the commodity through non-volume-based freight rates (referred to as non-contractual freight rates) without capacity limits. We denote by $c_{i,j}$ the non-contractual freight rate on lane $(i,j) \in \mathcal{L}$. Typically, at a given point in time, the non-contractual rate on a lane is higher than the contractual rates for the same lane (HandyBulk LLC 2023). Furthermore, for a regular shipper, non-contractual rates are typically hedged through the adoption of freight futures contracts in practice (Beullens et al. 2023). Since the non-contractual rates are available at the time of capacity contract decisions, they are deterministic parameters in the problem.

In practice, the supplies or demands $(d_{i,t})$ are uncertain at the time of negotiating capacity contracts and are typically revealed gradually during the planning horizon. To characterize this uncertainty, we define vectors $\boldsymbol{d}_t = (d_{i,t} | i \in \mathcal{I})$ for each period t and $\boldsymbol{d} = (\boldsymbol{d}_t | t \in \mathcal{T})$ for the entire horizon. We assume that $\boldsymbol{d}$ evolves as a discrete-time stochastic process with finite support. The process contains a set $\mathcal{P} = \{1, ..., \overline{p}\}$ of *stages*. Each stage $p \in \mathcal{P}$ spans a set $\mathcal{T}_p = \{\underline{t}_p, ..., \bar{t}_p\} \subseteq \mathcal{T}$ of periods. We have $\bigcup_{p\in\mathcal{P}} \mathcal{T}_p = \mathcal{T}$ and $\mathcal{T}_p \cap \mathcal{T}_{p'} = \varnothing, \forall p, p' \in \mathcal{P}, p \neq p'$.

At the beginning of each stage $p \in \mathcal{P}$, the shipper observes the realizations of $(\boldsymbol{d}_t)_{t=1}^{\bar{t}_p}$. The possible realizations of $(\boldsymbol{d}_t)_{t=\underline{t}_p}^{\bar{t}_p}$ in stage $p \in \mathcal{P}$ is captured through a set of scenarios Ω_p. For a scenario $\omega \in \Omega_p$, $d_{i,t}^{\omega}$ represents the realized supply or demand at site i for period $t \in \mathcal{T}_p$. The scenarios across different stages are independent, with ϱ_ω denoting the probability of scenario ω in Ω_p. Note that while the stage-wise independence assumption requires the distribution of scenarios to be memoryless, any trend or seasonality of $\boldsymbol{d}_t$ can still be represented using the scenarios (Fattahi and Govindan 2018).

The full set of scenarios for all stages in the problem can be represented by a scenario tree. Figure 1 shows an example scenario tree with three stages (i.e., $|\mathcal{P}| = 3$). Each stage contains three periods and two scenarios (i.e., $|\mathcal{T}_p| = 3$ and $|\Omega_p| = 2$). The path from a node in stage 1 to a node in stage $\overline{p}$, denoted by $\{\omega_1, ..., \omega_{\overline{p}}\}$, where $\omega_p \in \Omega_p$, $\forall p \in \mathcal{P}$, corresponds to a scenario ξ for a realization of $\boldsymbol{d}$. Let Ξ be the set of all such scenarios. For each scenario $\xi \in \Xi$, let $d_{i,t}^{\xi}$ denote the supply or demand at site $i \in \mathcal{I}$ in period $t \in \mathcal{T}$ under this scenario. The probability of scenario $\xi \in \Xi$ is denoted by ρ_ξ. For ease of presentation, in this paper, we will refer to scenarios in Ξ simply as *scenarios* and refer to scenarios ω in any Ω_p as *stage scenarios*. For each scenario $\xi \in \Xi$, we let

$\omega_p(\xi) \in \Omega_p$ denote the index of the stage scenario in stage $p \in \mathcal{P}$ associated with this scenario. To impose the non-anticipativity constraints in the SFPTMP, for each $p \in \mathcal{P}$, we introduce the set $\Lambda_p = \{(\xi_1, \xi_2) \in \Xi \times \Xi | \omega_{p'}(\xi_1) = \omega_{p'}(\xi_2), \forall p' = 1, ..., p\}$ which contains all pairs of scenarios $(\xi_1, \xi_2) \in \Xi \times \Xi$ that are indistinguishable in stage p. In the SFPTMP, the shipper makes decisions

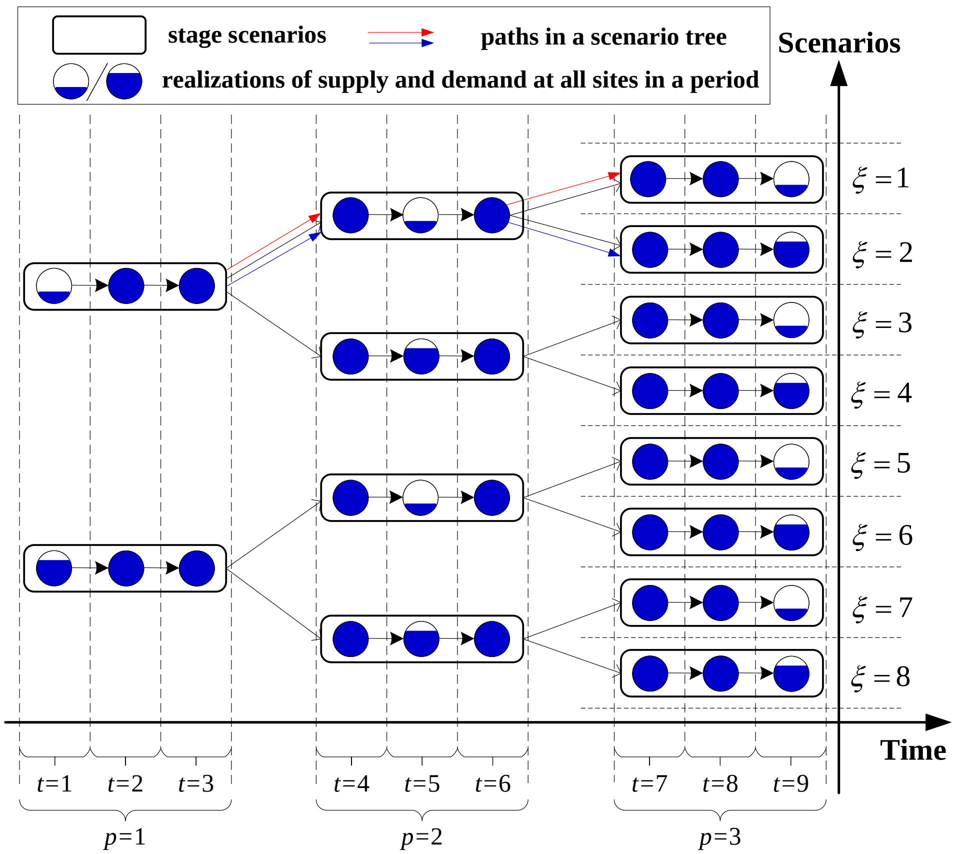


Figure 1: An illustration of a Scenario Tree.

in $1 + \overline{p}$ stages. We refer to the first decision stage as stage 0 and let $\mathcal{P}^+ = \{0\} \cup \mathcal{P}$ denote the full set of decision stages. In stage 0, without knowing any accurate supply or demand information, the shipper selects bids and determines the shipment capacities by signing capacity contracts. In stage $p \in \mathcal{P}$, the specific amount of supply and demand, $(\boldsymbol{d}_t)_{t=1}^{\overline{t}_p}$, has been revealed up to this stage. Given the decisions made in the previous stages and the observed supplies and demands in the current stage, the shipper determines how to: (i) allocate the loading quantity for each shipment in the capacity contracts that starts in this stage, (ii) determine the quantity to be transported through the non-contractual freight rate on each lane in each period within this stage, (iii) control the inventory levels at each site in each period within this stage, and (iv) determine the backlogged supply or demand at each site in each period within this stage. The objective is to formulate a joint freight service procurement and transportation-inventory plan with the minimum expected total cost, which includes expenses incurred by freight procurement (both contractual and non-contractual), inventory storage, and backlog management. The complexity of the SFPTMP is given in the following theorem:

Theorem 1. *The SFPTMP is NP-hard.*

4 Model formulation

To formulate the SFPTMP, we first construct a space-time network.

4.1 Space-time network

The space-time network is given by $\mathcal{G} = (\mathcal{N}, \mathcal{A})$, where $\mathcal{N}$ and $\mathcal{A}$ represent the set of nodes and arcs, respectively. Set $\mathcal{N}$ consists of $|\mathcal{T}|$ copies of each site $i \in \mathcal{I}$, i.e., $\mathcal{N} = \{(i,t)|i \in \mathcal{I}, t \in T\}$. Let $\mathcal{N}^S = \{(i,t) \in \mathcal{N}|i \in \mathcal{I}^S\}$ and $\mathcal{N}^D = \{(i,t) \in \mathcal{N}|i \in \mathcal{I}^D\}$ be the sets of nodes associated with supply and demand sites, respectively. The arc set is defined as $\mathcal{A} = \mathcal{A}^1 \cup \mathcal{A}^2$. Set $\mathcal{A}^1$ contains arcs that represent shipments in capacity contracts and is defined as $\mathcal{A}^1 = \bigcup_{b\in\mathcal{B}} \mathcal{A}^1_b$, where for each bid $b \in \mathcal{B}$, $\mathcal{A}^1_b = \{((i,t_1),(j,t_2))|(i,t_1),(j,t_2) \in \mathcal{N}, i = i(b), j = j(b), t_1 = t_1(r), t_2 = t_2(r), r \in \mathcal{R}_b\}$. Set $\mathcal{A}^2$ represents shipments through the non-contractual freight rates and is defined as $\mathcal{A}^2 = \{((i,t_1),(j,t_2))|t_2 = t_1 + o_{i,j}, (i,t_1) \in \mathcal{N}^S, (j,t_2) \in \mathcal{N}^D\}$. Let $\mathcal{N}_p = \{(i,t) \in \mathcal{N}|t \in \mathcal{T}_p\}$ and $\mathcal{A}_p = \{((i,t_1),(j,t_2)) \in \mathcal{A}|(i,t_1) \in \mathcal{N}_p\}$ represent the sets of nodes and arcs associated with stage $p \in \mathcal{P}$, respectively.

For notational simplicity, we use n and (i,t) interchangeably to represent a node, and use a and $((i,t_1),(j,t_2))$ interchangeably to represent an arc. Given any node $n \in \mathcal{N}$, the sets of its outgoing and incoming arcs are written as $A^+(n) = \{((i,t_1),(j,t_2)) \in \mathcal{A}|(i,t_1) = n\}$ and $A^-(n) = \{((i,t_1),(j,t_2)) \in \mathcal{A}|(j,t_2) = n\}$, respectively.

With slight abuse of notation, we redefine some parameters to cast them into the network structure. First, for each node $n = (i,1) \in \mathcal{N}$, let $q^0_n = q^0_i$ be the initial inventory and for each node $n = (i,t) \in \mathcal{N}$, let $\overline{q}_n = \overline{q}_i$ be the upper bound of the inventory. Second, we use $d_{\omega,n} = d^{\omega}_{i,t}$ to represent the supply or demand at node $n = (i,t) \in \mathcal{N}_p$ under stage scenario $\omega \in \Omega_p$ in any stage $p \in \mathcal{P}$. We also use $d_{\xi,n} = d^{\xi}_{i,t}$ to represent the supply or demand at node $n = (i,t) \in \mathcal{N}$ under scenario $\xi \in \Xi$. Third, for each node $n = (i,t) \in \mathcal{N}$, let $h_n = h_i$ denote its unit inventory holding cost and let $e_n = e_i$ denote the unit penalty cost associated with the backlogged supply or demand. Finally, for each arc $a \in \mathcal{A}$, we use c_a to represent the unit (variable) transportation cost on this arc. For each $a \in \mathcal{A}$, c_a is set as:

$$c_a = \begin{cases} g_b, & \text{if } \ a \in \mathcal{A}^1_b, b \in \mathcal{B}, \\ c_{i,j}, & \text{if } \ a = ((i,t_1),(j,t_2)) \in \mathcal{A}^2. \end{cases}$$

4.2 The compact model

We formulate the problem as a multi-stage stochastic optimization model in a compact form. Table 1 lists the decision variables used in the model.

Table 1: Decision Variables in the SFPTMP.

Decision variables in stage 0:	
x_b	binary variable taking value 1 if and only if the shipper accepts bid $b \in \mathcal{B}$.
y_b	continuous variable representing the capacity purchased for each shipment associated with bid $b \in \mathcal{B}$.
Decision variables in stages $p \in \mathcal{P}$:	
$z_{\xi,a}$	continuous variable representing the volume of the commodity allocated on arc $a \in \mathcal{A}$ under scenario $\xi \in \Xi$.
$u_{\xi,n}$	continuous variable representing the inventory level at node $n \in \mathcal{N}$ under scenario $\xi \in \Xi$.
$v_{\xi,n}$	continuous variable representing the volume of the supply or demand backlogged at node $n \in \mathcal{N}$ under scenario $\xi \in \Xi$.

The SFPTMP can be formulated as an MILP model denoted by $\mathbf{P}$ as follows:

$$\mathbf{P} = \min \sum_{b\in\mathcal{B}} F_b y_b + \sum_{\xi\in\Xi} \rho_\xi \left(\sum_{n\in\mathcal{N}} (h_n u_{\xi,n} + e_n v_{\xi,n}) + \sum_{a\in\mathcal{A}} c_a z_{\xi,a} \right) \tag{1}$$

$$\begin{aligned}
\text{s.t.} \quad & y_b \geq \underline{m}_b x_b && \forall b \in \mathcal{B} && (2)\\
& y_b \leq \overline{m}_b x_b && \forall b \in \mathcal{B} && (3)\\
& z_{\xi,a} \leq y_b && \forall a \in \mathcal{A}_b^1, \forall b \in \mathcal{B}, \forall \xi \in \Xi && (4)\\
& u_{\xi,n_1} + v_{\xi,n_1} = d_{\xi,n_1} + u_{\xi,n_2} + v_{\xi,n_2} - \sum_{a\in A^+(n_1)} z_{\xi,a} && \forall n_1 = (i,t), n_2 = (i,t-1) \in \mathcal{N}^S, \forall \xi \in \Xi && (5)\\
& u_{\xi,n} + v_{\xi,n} = d_{\xi,n} + q_n^0 - \sum_{a\in A^+(n)} z_{\xi,a} && \forall n = (i,1) \in \mathcal{N}^S, \forall \xi \in \Xi && (6)\\
& u_{\xi,n_1} - v_{\xi,n_1} = d_{\xi,n_1} + u_{\xi,n_2} - v_{\xi,n_2} + \sum_{a\in A^-(n_1)} z_{\xi,a} && \forall n_1 = (i,t), n_2 = (i,t-1) \in \mathcal{N}^D, \forall \xi \in \Xi && (7)\\
& u_{\xi,n} - v_{\xi,n} = d_{\xi,n} + q_n^0 + \sum_{a\in A^-(n)} z_{\xi,a} && \forall n = (i,1) \in \mathcal{N}^D, \forall \xi \in \Xi && (8)\\
& u_{\xi,n} \leq \overline{q}_n && \forall n \in \mathcal{N}, \forall \xi \in \Xi && (9)\\
& z_{\xi_1,a} = z_{\xi_2,a} && \forall a \in \mathcal{A}_p, \forall (\xi_1,\xi_2) \in \Lambda_p, \forall p \in \mathcal{P} && (10)\\
& x_b \in \{0,1\} && \forall b \in \mathcal{B} && (11)\\
& y_b \geq 0 && \forall b \in \mathcal{B} && (12)\\
& u_{\xi,n}, v_{\xi,n}, z_{\xi,a} \geq 0 && \forall n \in \mathcal{N}, \forall a \in \mathcal{A}, \forall \xi \in \Xi. && (13)
\end{aligned}$$

The objective function (1) aims to minimize the total cost, including the cost of purchasing freight services using capacity contracts and the expected total cost incurred by holding inventories and backlogging supplies or demands and sending flows on the arcs. Constraints (2) and (3) set the lower bound and upper bound for the capacity of each shipment associated with each bid, respectively. Constraints (4) ensure that under any scenario, the actual transport volumes on the arcs associated with shipments in capacity contracts do not exceed their contractual capacities. Constraints (5)–(8) define the relationship among inventory levels and backlogged quantities at the nodes and the flow volumes allocated on the relevant arcs. Constraints (9) require the inventory level at a node to not exceed its upper bound. Constraints (10) are non-anticipativity constraints. Finally, constraints (11)–(13) define the domains for the decision variables. In the model, non-anticipativity requirements are imposed only on flow variables $z_{\xi,a}$, through constraints (10). Proposition 1 shows that these constraints are sufficient to impose non-anticipativity requirements for the SFPTMP.

Proposition 1. *Problem* $\mathbf{P}$ *satisfies the non-anticipativity requirements in the SFPTMP.*

Given a scenario tree, the above formulation forms a deterministic optimization problem. However, the scale of the problem grows exponentially with the number of stages ($|\mathcal{P}|$) and the number of stage scenarios ($|\Omega_p|$, $p \in \mathcal{P}$). As a result, only very small instances of the model can be directly solved using a general-purpose MILP solver. In next section, we propose an SDDP approach for solving instances of very large scale.

5 A stochastic dual dynamic programming approach

The SDDP approach follows an iterative procedure and each iteration in the approach consists of a *sampling* step, a *forward* step, and a *backward* step. The framework of the SDDP approach for solving the SFPTMP is presented in Section EC.1 of the electronic companion.

5.1 The sampling step

In iteration l, a subset of scenarios (denoted by Ξ^l) is sampled from the original set Ξ. The scenarios in Ξ^l are sampled randomly from the original scenario set Ξ based on the distribution $\{\rho_\xi : \xi \in \Xi\}$.

5.2 The forward step

In the forward step of iteration l, we solve problem $\mathbf{P}$ under each scenario $\xi \in \Xi^l$ by decomposing the problem into $|\mathcal{P}^+|$ subproblems, each corresponding to a decision stage. We let $\mathbf{P}_0$ denote the first-stage problem, and let $\mathbf{P}_{\xi,p}$ denote the problems in stages $p \in \mathcal{P}$ under each scenario $\xi \in \Xi^l$.

As a problem in a stage is partially defined by the decisions made in the previous stages, the communication between problems in different stages must be carefully established. Also, because of the stage-wise solution procedure, such inter-stage communication only exists between problems in adjacent stages. In particular, given a scenario $\xi \in \Xi^l$, the problem $\mathbf{P}_{\xi,p}$ in any stage $p \in \mathcal{P}$, is characterized by a set of *state variables* derived from the solution of its *parent problem* i.e., problem $\mathbf{P}_{\xi,p-1}$ or $\mathbf{P}_0$. We next explain how to derive these state variables.

To begin with, given a scenario $\xi \in \Xi^l$, the capacities of shipments in the capacity contracts parameterized in problem $\mathbf{P}_{\xi,p}$ in any stage $p \in \mathcal{P}$ should remain consistent with those determined in the parent problem in stage $p-1$. We let $\bar{\boldsymbol{y}}_{\xi,p-1} = (\bar{y}_{\xi,p-1,b} | b \in \mathcal{B})$ be the vector of such capacities obtained by solving the parent problem.

In addition, suppose $p \geq 2$, then the problem $\mathbf{P}_{\xi,p}$ (for any $\xi \in \Xi^l$) is also subject to the inventory levels, backlogged supplies or demands of certain nodes, and the flows of certain arcs that are determined in its parent problem. In order to characterize this information, we define the following sets of nodes and arcs that are critical for transferring information between stages. First, for the nodes, we let $\widetilde{\mathcal{N}}_p = \{n | n = (i, \bar{t}_p) \in \mathcal{N}_p\}$ denote the set of nodes whose inventory levels and backlogged supplies or demands are passed on to the next stage, $\forall p \in \mathcal{P} \backslash \{\bar{p}\}$. Especially, for stage 0, we define $\widetilde{\mathcal{N}}_0 = \{(i, 0) | i \in \mathcal{I}\}$. Second, as for the arcs, we use $\overrightarrow{\mathcal{A}}_{(p_1,p_2)} = \{a | a = (n_1, n_2) \in \mathcal{A}, n_1 \in \mathcal{N}_{p_1}, n_2 \in \mathcal{N}_{p_2}\}$ to represent the set of arcs directed from nodes in stage p_1 to nodes in stage p_2, where $p_1, p_2 \in \mathcal{P}$ and $p_2 > p_1$. Further, $\forall p \in \mathcal{P}$, let $\widetilde{\mathcal{A}}_p = \bigcup_{p_1=1}^{p} \bigcup_{p_2=p+1}^{\bar{p}} \overrightarrow{\mathcal{A}}_{(p_1,p_2)}$ be set of arcs that link nodes $n \in \bigcup_{p_1=1}^{p} \mathcal{N}_{p_1}$ with nodes $n \in \bigcup_{p_2=p+1}^{\bar{p}} \mathcal{N}_{p_2}$. Also, we especially have $\widetilde{\mathcal{A}}_0 = \varnothing$ and $\widetilde{\mathcal{A}}_{\bar{p}} = \varnothing$.

Based on these sets of nodes and arcs, for problem $\mathbf{P}_{\xi,p}$ (where $p \in \mathcal{P}$ and $\xi \in \Xi^l$), we define the following state variables that are determined in its parent problem (i.e., $\mathbf{P}_{\xi,p-1}$ or $\mathbf{P}_0$) and affect this problem. First, for each node $n \in \widetilde{\mathcal{N}}_{p-1}$, we let $\bar{u}_{\xi,p-1,n}$ and $\bar{v}_{\xi,p-1,n}$ denote the inventory level and the backlogged quantity at node n that are determined by solving the parent problem in stage $p-1$. Especially, for $p-1=0$, we define $\bar{u}_{\xi,0,n} = q_i^0$ and $\bar{v}_{\xi,0,n} = 0$, $\forall n = (i, 0) \in \widetilde{\mathcal{N}}_0$. Besides, for each arc $a \in \widetilde{\mathcal{A}}_{p-1}$, we use $\bar{z}_{\xi,p-1,a}$ to represent the flow on this arc determined by solving the parent problem in stage $p-1$. Let $\bar{\boldsymbol{u}}_{\xi,p} = (\bar{u}_{\xi,p,n} | n \in \widetilde{\mathcal{N}}_p)$, $\bar{\boldsymbol{v}}_{\xi,p} = (\bar{v}_{\xi,p,n} | n \in \widetilde{\mathcal{N}}_p)$, and $\bar{\boldsymbol{z}}_{\xi,p} = (\bar{z}_{\xi,p,a} | a \in \widetilde{\mathcal{A}}_p)$. To further simplify the notation, we define $\bar{\boldsymbol{\chi}}_{\xi,p} = ((\bar{\boldsymbol{y}}_{\xi,p})^\top, (\bar{\boldsymbol{u}}_{\xi,p})^\top, (\bar{\boldsymbol{v}}_{\xi,p})^\top, (\bar{\boldsymbol{z}}_{\xi,p})^\top)^\top$.

5.2.1 The Problem in the bidding stage

Decisions made in the bidding stage of the problem (i.e., $\mathbf{P}_0$) include the selection of bids and the purchase of capacities for the shipments associated with the bids. The associated decision variables are given in the vectors $\boldsymbol{x} = (x_b | b \in \mathcal{B})$ and $\boldsymbol{y} = (y_b | b \in \mathcal{B})$. The problem is also formulated based on a cost-to-go function, denoted by $\Psi_0(\boldsymbol{y})$, which approximates the expected total cost incurred in the subsequent stages and is defined as follows:

$$
\begin{aligned}
\Psi_0(\boldsymbol{y}) = \quad & \min\{\eta_0 : \\
& \qquad \eta_0 \geq 0, && (14)\\
& \qquad \eta_0 \geq (\mu_0^k + (\boldsymbol{\nu}_0^k)^\top \boldsymbol{y}), \ \ \forall k \in \mathcal{K}_0\}, && (15)
\end{aligned}
$$

where $\mu_0^k \in \mathbb{R}$ and $\boldsymbol{\nu}_0^k \in \mathbb{R}^{|\mathcal{B}|}$ are parameters, and $\mathcal{K}_0$ denotes the initial set of valid inequalities for the cost-to-go functions Ψ_0.

Problem $\mathbf{P}_0$ can be formulated as the following MILP model:

$$\mathbf{P}_0(\Psi_0) = \min_{\boldsymbol{x},\boldsymbol{y},\eta_0} \sum_{b\in\mathcal{B}} F_b y_b + \eta_0 \quad (16)$$
$$\text{s.t. } (2), (3), (11), (12), (14), (15).$$

Objective function (16) minimizes the sum of the cost of capacity purchase and the value of the cost-to-go function.

Let $\boldsymbol{y}^*$ be the vector of optimal solution values of the y variables obtained by solving the above model. We then obtain $\bar{\boldsymbol{\chi}}_{\xi,0}$ for characterizing the problems in stage 1 by letting $\bar{y}_{\xi,0,b} = y_b^*$, $\forall b \in \mathcal{B}, \forall \xi \in \Xi^l$.

5.2.2 Problems in the shipping stages

In stage $p \in \mathcal{P}$ of the shipping stages, we solve a problem denoted by $\mathbf{P}_{\xi,p}$ under scenario $\xi \in \Xi^l$ in the forward step of iteration l. Decision variables used for formulating the problem can be partitioned into two groups.

The first group consists of variables in the following vectors: $\boldsymbol{u}_{\xi,p} = (u_{\xi,n}|n \in \mathcal{N}_p)$, $\boldsymbol{v}_{\xi,p} = (v_{\xi,n}|n \in \mathcal{N}_p)$, and $\boldsymbol{z}_{\xi,p} = (z_{\xi,a}|a \in \mathcal{A}_p)$. These variables control inventory levels, determine backlogged quantities, and allocate flows for the nodes and arcs in $\mathcal{N}_p$ and $\mathcal{A}_p$, respectively.

Variables in the second group are auxiliary variables that make local copies of the variables determined in the parent problem of $\mathbf{P}_{\xi,p}$. In particular, we use the set of variables in vector $\boldsymbol{y}'_{\xi,p} = (y'_{\xi,p,b}|b \in \mathcal{B})$ to represent the "copied" capacity of each shipment in the bids. Besides, for the nodes $n \in \widetilde{\mathcal{N}}_{p-1}$, we use variables in $\boldsymbol{u}'_{\xi,p} = (u'_{\xi,p,n}|n \in \widetilde{\mathcal{N}}_{p-1})$ and $\boldsymbol{v}'_{\xi,p} = (v'_{\xi,p,n}|n \in \widetilde{\mathcal{N}}_{p-1})$, respectively, as the local copies of the inventory and backlogging decisions determined in the parent problem of $\mathbf{P}_{\xi,p}$. For each arc $a \in \widetilde{\mathcal{A}}_{p-1}$, we introduce the variable $z'_{\xi,p,a}$ to copy the flow allocated on it and let $\boldsymbol{z}'_{\xi,p} = (z'_{\xi,p,a}|a \in \widetilde{\mathcal{A}}_{p-1})$. Finally, for notational simplicity, we use $\boldsymbol{X}_{\xi,p}$ to represent the vector that includes all the decision variables from these two groups in problem $\mathbf{P}_{\xi,p}$.

Problem $\mathbf{P}_{\xi,p}$ is also characterized by a cost-to-go function denoted by $\Psi_p(\boldsymbol{X}_{\xi,p})$ which approximates the expected total cost incurred in the subsequent stages and is defined as follows:

$$\Psi_p(\boldsymbol{X}_{\xi,p}) = \min\{\eta_{\xi,p} :$$
$$\eta_{\xi,p} \geq 0, \quad (17)$$
$$\eta_{\xi,p} \geq \mu_p^k + (\boldsymbol{\nu}_p^k)^\top \boldsymbol{X}_{\xi,p}, \forall k \in \mathcal{K}_p\}, \quad (18)$$

where $\mu_p^k \in \mathbb{R}$ and $\boldsymbol{\nu}_p^k \in \mathbb{R}^N$ are parameters with $N = |\mathcal{B}| + 2|\mathcal{N}_p| + |\mathcal{A}_p| + 2|\widetilde{\mathcal{N}}_{p-1}| + |\widetilde{\mathcal{A}}_{p-1}| + |\mathcal{N}_p \cap \mathcal{N}^D|$. Especially, we have $\mathcal{K}_p = \varnothing$, if $p = \overline{p}$.

Deriving feasibility inequalities. A notable difficulty in formulating problems in shipping stages is that these problems can be infeasible. In particular, given any problem $\mathbf{P}_{\xi,p}$, consider any two stages p_1 and p_2 such that $p_1, p_2 \in \mathcal{P}$, $p_1 < p$, and $p_2 = p$, the flows on the arcs in the set $\overrightarrow{\mathcal{A}}_{(p_1,p_2)}$ are determined in the problem in stage p_1 without explicitly considering the inventory restrictions for the corresponding head nodes in $\mathcal{N}_p$ in stage p. This is insufficient to ensure feasibility of problem $\mathbf{P}_{\xi,p}$ as such flows can lead to inventories at certain nodes in $\mathcal{N}_p$ exceeding their upper bounds.

To avoid infeasibilities caused by arc flows between stages, a common method in SDDP is to use large costs (big-M) to penalize the amounts of overflowed inventories (i.e., inventories beyond $\overline{q}_n$). However, the use of big-M terms leads to poor solutions generated in the forward step and weak cuts for cost-to-go functions obtained in the backward step. To resolve this issue, we propose using valid inequalities to impose the feasibility of the stage-wise problems. The feasibility inequalities are derived as follows.

Consider any $\mathbf{P}_{\xi,p_1}$, with $\xi \in \Xi^l$ and $p_1 \in \mathcal{P} \setminus \{\bar{p}\}$. After solving $\mathbf{P}_{\xi,p_1}$, we obtain $\bar{\boldsymbol{\chi}}_{\xi,p_1}$, which defines $\mathbf{P}_{\xi,p_2}$ where $p_2 = p_1 + 1$. Additionally, $\bar{\boldsymbol{\chi}}_{\xi,p_1}$ may influence the formulations of any problem $\mathbf{P}_{\xi,p_2}$ where $p_2 > p_1 + 1$. To ensure the feasibility of these subsequent problems, the inventory level at each node (j,t), $j \in \mathcal{I}^D$, $t \in \mathcal{T}_{p_2} : p_2 \geq p_1$ must be able to remain within the limit $\bar{q}_j$. To this end, it is sufficient to maintain this constraint for any node (j,t) under the scenario with the minimum cumulative demand from $\underline{t}_{p_1+1}$ to t, where $t \in \mathcal{T}_{p_2}$. In view of this, let $\overline{d}_{j,p_1,t}$ be the minimum amount of demand at node (j,t) accumulated from the first period in stage p_1 (i.e., $\underline{t}_{p_1}$) to any period $t \in \mathcal{T}_{p_2}$ where $p_1, p_2 \in \mathcal{P}$ and $p_2 \geq p_1$ under all scenarios $\xi \in \Xi$. $\overline{d}_{j,p_1,t}$ can be calculated by:

$$\overline{d}_{j,p_1,t} = \sum_{p=p_1}^{p_2-1} \max_{\omega \in \Omega_p} \sum_{t'=\underline{t}_p}^{\bar{t}_p} d^{\omega}_{j,t'} + \max_{\omega \in \Omega_{p_2}} \sum_{t'=\underline{t}_{p_2}}^{t} d^{\omega}_{j,t'}. \tag{19}$$

Using these minimum accumulated demands, we have the following Lemma.

Lemma 1. *The following inequalities are valid for problem* $\mathbf{P}$*:*

$$u_{\xi,n_1} - v_{\xi,n_1} + \sum_{t=\underline{t}_{p+1}}^{t_2} \sum_{n=(i,t)\in\mathcal{N}} \sum_{p'=1}^{p} \sum_{a \in A^-(n)\cap\mathcal{A}_{p'}} z_{\xi,a} + \overline{d}_{i,p+1,t_2} \leq \overline{q}_{n_2}$$
$$\forall n_1 = (i,\bar{t}_p), n_2 = (i,t_2) \in \mathcal{N}^D, t_2 \geq \underline{t}_{p+1}, \forall p \in \mathcal{P} \setminus \{\overline{p}\}, \forall \xi \in \Xi. \tag{20}$$

Based on Lemma 1, we derive the feasibility inequalities (21) that are valid for problems $\mathbf{P}_{\xi,p}$, where $p \in \mathcal{P} \setminus \{\overline{p}\}$ and $\xi \in \Xi$:

$$u_{\xi,n_1} - v_{\xi,n_1} + \sum_{t=\underline{t}_{p+1}}^{t_2} \sum_{n=(j,t)\in\mathcal{N}} \sum_{a\in A^-(n)\cap\mathcal{A}_p} z_{\xi,a} + \sum_{t=\underline{t}_{p+1}}^{t_2} \sum_{n=(j,t)\in\mathcal{N}} \sum_{a\in A^-(n)\cap\widetilde{\mathcal{A}}_{p-1}} z'_{\xi,p,a} + \overline{d}_{j,p+1,t_2} \leq \overline{q}_{n_2}$$
$$\forall n_1 = (j,\bar{t}_p), n_2 = (j,t_2) \in \mathcal{N}^D, \forall((i,t_1),(j,t_2)) \in \widetilde{\mathcal{A}}_p. \tag{21}$$

The formulation. We are now ready to present the formulation for problem $\mathbf{P}_{\xi,p}$, which is an LP model written as follows:

$$\mathbf{P}_{\xi,p}(\bar{\boldsymbol{\chi}}_{\xi,p-1}, \Psi_p) = \min_{\boldsymbol{X}_{\xi,p},\eta_{\xi,p}} \sum_{n\in\mathcal{N}_p} (h_n u_{\xi,n} + e_n v_{\xi,n}) + \sum_{a\in\mathcal{A}_p} c_a z_{\xi,a} + \eta_{\xi,p} \tag{22}$$

s.t. (17), (18), (21)

$$z_{\xi,a} \leq y'_{\xi,p,b} \quad \forall a \in \mathcal{A}^1_b \cap \mathcal{A}_p, \forall b \in \mathcal{B} \tag{23}$$

$$u_{\xi,n_1} + v_{\xi,n_1} = d_{\xi,n_1} + u_{\xi,n_2} + v_{\xi,n_2} - \sum_{a\in A^+(n_1)} z_{\xi,a}$$
$$\forall n_1 = (i,t), n_2 = (i,t-1) \in \mathcal{N}_p \cap \mathcal{N}^S \tag{24}$$

$$u_{\xi,n_1} + v_{\xi,n_1} = d_{\xi,n_1} + u'_{\xi,p,n_2} + v'_{\xi,p,n_2} - \sum_{a\in A^+(n_1)} z_{\xi,a}$$
$$\forall n_1 = (i,\underline{t}_p) \in \mathcal{N}_p, \forall n_2 = (i,\bar{t}_{p-1}) \in \widetilde{\mathcal{N}}_{p-1}, \forall i \in \mathcal{I}^S \tag{25}$$

$$u_{\xi,n_1} - v_{\xi,n_1} = d_{\xi,n_1} + u_{\xi,n_2} - v_{\xi,n_2} + \sum_{a\in A^-(n_1)\cap\widetilde{\mathcal{A}}_{p-1}} z'_{\xi,p,a} + \sum_{a\in A^-(n_1)\cap\mathcal{A}_p} z_{\xi,a}$$
$$\forall n_1 = (i,t), n_2 = (i,t-1) \in \mathcal{N}_p \cap \mathcal{N}^D \tag{26}$$

$$u_{\xi,n_1} - v_{\xi,n_1} = d_{\xi,n_1} + u'_{\xi,p,n_2} - v'_{\xi,p,n_2} + \sum_{a \in A^-(n_1) \cap \widetilde{\mathcal{A}}_{p-1}} z'_{\xi,p,a} + \sum_{a \in A^-(n_1) \cap \mathcal{A}_p} z_{\xi,a}$$
$$\forall n_1 = (i, \underline{t}_p) \in \mathcal{N}_p, \forall n_2 = (i, \bar{t}_{p-1}) \in \widetilde{\mathcal{N}}_{p-1}, \forall i \in \mathcal{I}^D \quad (27)$$
$$u_{\xi,n} \leq \bar{q}_n \quad \forall n \in \mathcal{N}_p \quad (28)$$
$$y'_{\xi,p,b} = \bar{y}_{\xi,p-1,b} \quad \forall b \in \mathcal{B} \quad (29)$$
$$u'_{\xi,p,n} = \bar{u}_{\xi,p-1,n} \quad \forall n \in \widetilde{\mathcal{N}}_{p-1} \quad (30)$$
$$v'_{\xi,p,n} = \bar{v}_{\xi,p-1,n} \quad \forall n \in \widetilde{\mathcal{N}}_{p-1} \quad (31)$$
$$z'_{\xi,p,a} = \bar{z}_{\xi,p-1,a} \quad \forall a \in \widetilde{\mathcal{A}}_{p-1} \quad (32)$$
$$u_{\xi,n}, v_{\xi,n}, z_{\xi,a} \geq 0 \quad \forall n \in \mathcal{N}_p, \forall a \in \mathcal{A}_p. \quad (33)$$

The objective function (22) minimizes the sum of three terms, including the total inventory holding cost and the total backlogging cost at the nodes in $\mathcal{N}_p$, the total shipping cost for sending flows on the arcs in $\mathcal{A}_p$, and the value of the cost-to-go function. Constraints (23) set upper bounds for the flows on the arcs associated with shipments in the bids. Constraints (24) and (25) track the inventory levels and backlogged supplies at the nodes associated with the supply sites in stage p. Similarly, constraints (26) and (27) track the inventory levels and backlogged demands at the nodes associated with the demand sites in stage p. Constraints (28) require that the inventory stored at each node be maintained under the upper limit. Constraints (29)–(32) link the decision variables determined in the parent problem with their local copies in problem $\mathbf{P}_{\xi,p}$. The last set of constraints define the domains of the decision variables.

Proposition 2. *Problems $\mathbf{P}_{\xi,p}$ are always feasible, $\forall p \in \mathcal{P}$, $\forall \xi \in \Xi$.*

Finally, let $\boldsymbol{X}^*_{\xi,p}$ be the vector of the solution values of the decision variables in $\boldsymbol{X}_{\xi,p}$ obtained by solving $\mathbf{P}_{\xi,p}$. If $p < \bar{p}$, we obtain $\bar{\chi}_{\xi,p}$, which will be used for defining problem $\mathbf{P}_{\xi,p+1}$, by letting (i) $\bar{y}_{\xi,p,b} = y'^{,*}_{\xi,p,b}$, $\forall b \in \mathcal{B}$, (ii) $\bar{u}_{\xi,p,n} = u^*_{\xi,n}$, $\forall n \in \widetilde{\mathcal{N}}_p$, (iii) $\bar{v}_{\xi,p,n} = v^*_{\xi,n}$, $\forall n \in \widetilde{\mathcal{N}}_p$, (iv) $\bar{z}_{\xi,p,a} = z^*_{\xi,a}$, $\forall a \in \widetilde{\mathcal{A}}_p \cap \mathcal{A}_p$, and (v) $\bar{z}_{\xi,p,a} = z'^{,*}_{\xi,p,a}$, $a \in \widetilde{\mathcal{A}}_p \setminus \mathcal{A}_p$.

5.3 The backward step

When all the forward-step problems for each sampled scenario $\xi \in \Xi^l$ are solved in iteration l, the backward step starts from the last stage $p = \bar{p}$. It then moves backward, stage by stage, until reaching stage $p = 1$. In each stage, a set of problems are solved. The goal of the backward step is to update the cost-to-go functions for problems in the forward step.

5.3.1 Problems in backward step

In iteration l of the SDDP approach, for each sampled scenario $\xi \in \Xi^l$, we solve $|\Omega_p|$ problems in the backward step in stage $p \in \mathcal{P}$. Each of the problems corresponds to a stage scenario $\omega \in \Omega_p$ in stage p in the *original scenario tree*. Let $\mathbf{Q}_{\xi,\omega,p}$ denote the problem that is associated with scenario $\xi \in \Xi^l$ and stage scenario $\omega \in \Omega_p$ in stage $p \in \mathcal{P}$ in the backward step.

Given $\xi \in \Xi^l$, and $\omega \in \Omega_p$ in stage $p \in \mathcal{P}$, problem $\mathbf{Q}_{\xi,\omega,p}$ and problem $\mathbf{P}_{\xi,p}$ in the forward step are characterized by the same set of state variables obtained by solving the parent problem $\mathbf{P}_{\xi,p-1}$ (or $\mathbf{P}_0$) and the same cost-to-go function (i.e., $\bar{\chi}_{\xi,p-1}$ and Ψ_p).

The decision variables for $\mathbf{Q}_{\xi,\omega,p}$ include those contained in the vector $\boldsymbol{X}_{\omega,p}$ and variable $\eta_{\omega,p}$. Here, there is a one-to-one correspondence between variables in $\boldsymbol{X}_{\omega,p}$ and those in $\boldsymbol{X}_{\xi,p}$ of problem $\mathbf{P}_{\xi,p}$ in the forward step. To be more specific, for every variable defined for the scenario ξ in $\boldsymbol{X}_{\xi,p}$, there is a corresponding variable in $\boldsymbol{X}_{\omega,p}$ defined for the stage scenario ω. In addition, $\eta_{\omega,p}$ represents the value returned by the cost-to-go function Ψ_p.

By respectively replacing the variables in $\boldsymbol{X}_{\xi,p}$ and $\eta_{\xi,p}$ and the parameters in $\boldsymbol{d}_{\xi,p} = (d_{\xi,n} | n \in \mathcal{N}_p)$ with their counterparts in $\boldsymbol{X}_{\omega,p}$, $\eta_{\omega,p}$, and $\boldsymbol{d}_{\omega,p} = (d_{\omega,n} | n \in \mathcal{N}_p)$ in constraints (17), (18), (21), (23)–(28), and (33), problem $\mathbf{Q}_{\xi,\omega,p}$ can be formulated as the following LP model:

$$\mathbf{Q}_{\xi,\omega,p}(\bar{\boldsymbol{\mathcal{X}}}_{\xi,p-1}, \Psi_p) = \min_{\boldsymbol{X}_{\omega,p},\eta_{\omega,p}} \sum_{n \in \mathcal{N}_p} (h_n u_{\omega,n} + e_n v_{\omega,n}) + \sum_{a \in \mathcal{A}_p} c_a z_{\omega,a} + \eta_{\omega,p} \tag{34}$$
$$\text{s.t.} \qquad (17), (18), (21), (23) - (33).$$

5.3.2 Update of cost-to-go functions

In the SDDP, we solve the dual problem of $\mathbf{Q}_{\xi,\omega,p}$, denoted by $\mathbf{D}_{\xi,\omega,p}$, $\forall p \in \mathcal{P}, \omega \in \Omega_p, \xi \in \Xi^l$ to generate valid inequalities for updating the cost-to-go functions. In particular, by solving $\mathbf{D}_{\xi,\omega,p}$ to optimality, let $\zeta_{\xi,\omega,p}$ be the optimal objective function value of $\mathbf{D}_{\xi,\omega,p}$, and let $\phi_{\xi,\omega,p,b}$ ($\forall b \in \mathcal{B}$), $\pi_{\xi,\omega,p,n}$ ($\forall n \in \widetilde{\mathcal{N}}_{p-1}$), $\varpi_{\xi,\omega,p,n}$ ($\forall n \in \widetilde{\mathcal{N}}_{p-1}$), and $\theta_{\xi,\omega,p,a}$ ($\forall a \in \widetilde{\mathcal{A}}_{p-1}$) be the optimal solution values of the dual variables associated with constraints (29)–(32) of $\mathbf{Q}_{\xi,\omega,p}$, respectively. Given these results, we update the cost-to-go functions as follows.

First, for the cost-to-go function Ψ_0 in stage 0, the following set of inequalities are valid:

$$\eta_0 \geq \sum_{\omega \in \Omega_1} \varrho_\omega \zeta_{\xi,\omega,1} + \sum_{\omega \in \Omega_1} \varrho_\omega \sum_{b \in \mathcal{B}} \phi_{\xi,\omega,1,b}(y_b - \bar{y}_{\xi,0,b}) \;\; \forall \xi \in \Xi^l. \tag{35}$$

Let $\mathcal{K}_0^+$ denote the set of these inequalities. We update the cost-to-go function $\Psi_0(\boldsymbol{y})$ by letting $\mathcal{K}_0 = \mathcal{K}_0 \cup \mathcal{K}_0^+$.

Similarly, for any scenario $\xi' \in \Xi$, we obtain the following set of valid inequalities for the cost-to-go functions Ψ_p in stage $p \in \mathcal{P} \setminus \{\overline{p}\}$:

$$\begin{aligned}
\eta_{\xi',p} \geq & \sum_{\omega \in \Omega_{p+1}} \varrho_\omega \zeta_{\xi,\omega,p+1} + \sum_{\omega \in \Omega_{p+1}} \varrho_\omega \sum_{b \in \mathcal{B}} \phi_{\xi,\omega,p+1,b}(y'_{\xi',p,b} - \bar{y}_{\xi,p,b}) \\
& + \sum_{\omega \in \Omega_{p+1}} \varrho_\omega \sum_{n \in \widetilde{\mathcal{N}}_p} \pi_{\xi,\omega,p+1,n}(u_{\xi',n} - \bar{u}_{\xi,p,n}) + \sum_{\omega \in \Omega_{p+1}} \varrho_\omega \sum_{n \in \widetilde{\mathcal{N}}_p} \varpi_{\xi,\omega,p+1,n}(v_{\xi',n} - \bar{v}_{\xi,p,n}) \\
& + \sum_{\omega \in \Omega_{p+1}} \varrho_\omega \sum_{a \in \widetilde{\mathcal{A}}_p \cap \mathcal{A}_p} \theta_{\xi,\omega,p+1,a}(z_{\xi',a} - \bar{z}_{\xi,p,a}) \\
& + \sum_{\omega \in \Omega_{p+1}} \varrho_\omega \sum_{a \in \widetilde{\mathcal{A}}_p \setminus \mathcal{A}_p} \theta_{\xi,\omega,p+1,a}(z'_{\xi',p,a} - \bar{z}_{\xi,p,a}) \;\; \forall \xi \in \Xi^l.
\end{aligned} \tag{36}$$

Let $\mathcal{K}_p^+$ denote the set of inequalities (36) for problems in stage $p \in \mathcal{P} \setminus \{\overline{p}\}$. We update the cost-to-go functions $\Psi_p(\boldsymbol{X}_{\xi',p})$ by letting $\mathcal{K}_p = \mathcal{K}_p \cup \mathcal{K}_p^+$.

5.4 Enhancements to the SDDP approach

In this section, we describe several important enhancements to the SDDP approach.

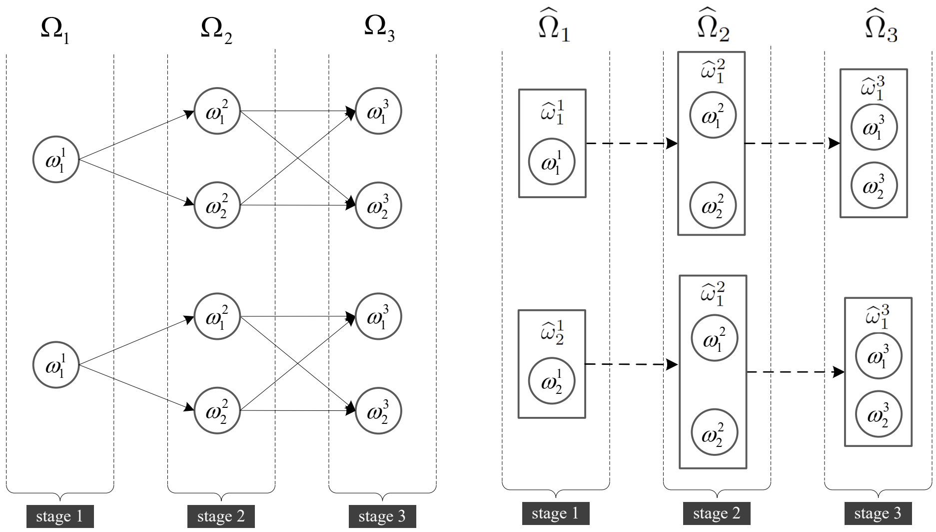
5.4.1 Optimality inequalities

In the SDDP, the quality of the lower bound depends on the quality of the cost-to-go functions which are obtained by solving problems $\mathbf{Q}_{\xi,\omega,p}$ in the backward step. Note that problems $\mathbf{Q}_{\xi,\omega,p}$ are parameterized by solutions obtained from solving problems $\mathbf{P}_0$ and $\mathbf{P}_{\xi,p}$ in the forward step. Therefore, having high-quality solutions for problems $\mathbf{P}_0$ and $\mathbf{P}_{\xi,p}$ is critical for generating high-quality cost-to-go functions. However, due to the stage-wise solution procedure, especially in the initial iterations of the SDDP approach, both the lower bound and solutions of problems in the forward step tend to have low quality.

To address these issues, we propose to improve the lower bound and the quality of solutions to problems $\mathbf{P}_0$ and $\mathbf{P}_{\xi,p}$ through the use of optimality inequalities. In particular, for problem $\mathbf{P}_0$ or $\mathbf{P}_{\xi,p}$, the associated optimality inequalities estimate the lower bound of the cost incurred in the subsequent stages. Our method of generating optimality inequalities is inspired by Theorem 1 in Chapter 10 of Birge and Louveaux (2011), which derives a valid lower bound of a multi-stage stochastic linear program based on consistent partitions of the stage scenarios. We extend this idea by constructing an *approximate scenario tree* for each stage $p \in \mathcal{P}^+ \setminus \{\overline{p}\}$ and formulating the optimality inequalities based on the approximate scenario tree.

Approximate scenario tree construction. For generating the optimality inequalities, for each stage $p \in \mathcal{P}^+ \setminus \{\overline{p}\}$ in the original scenario tree (*original tree*), we construct an approximate scenario tree (*approximate tree*), denoted by $\mathscr{T}_p$. This construction is achieved by employing a *consistent partition* of the stage scenarios from stages $p+1$ to $\overline{p}$. A consistent partition ensures that in the approximate tree, the aggregation of the original scenarios preserves the same stage-wise relationships between the scenarios in the original tree (Birge and Louveaux 2011).

Each approximate tree $\mathscr{T}_p$, $p \in \mathcal{P}^+ \setminus \{\overline{p}\}$, consists of a set $\widehat{\mathcal{P}}_p$ of stages, where $\widehat{\mathcal{P}}_p = \{p+1, .., \overline{p}\}$. There is a set $\widehat{\Omega}_k$ of stage scenarios in stage $k \in \widehat{\mathcal{P}}_p$ of the approximate tree $\mathscr{T}_p$. Each stage scenario $\widehat{\omega} \in \widehat{\Omega}_k$ maps a subset of stage scenarios in Ω_k in the original tree, which is denoted by $\Omega_k(\widehat{\omega}) \subseteq \Omega_k$. For any $k \in \widehat{\mathcal{P}}_p$ and $p \in \mathcal{P}^+ \setminus \{\overline{p}\}$, the mapping between $\widehat{\omega} \in \widehat{\Omega}_k$ and $\omega \in \Omega_k$ satisfies (i) $\bigcup_{\widehat{\omega} \in \widehat{\Omega}_k} \Omega_k(\widehat{\omega}) = \Omega_k$, and (ii) $\Omega_k(\widehat{\omega}_1) \cap \Omega_k(\widehat{\omega}_2) = \varnothing, \quad \forall \widehat{\omega}_1, \widehat{\omega}_2 \in \widehat{\Omega}_k, \widehat{\omega}_1 \neq \widehat{\omega}_2$. Figure 2 provides an example of generating an approximate tree for stage 1 (i.e., $\mathscr{T}_1$), illustrated in Figure 2(b), from the original tree shown in Figure 2(a) using a consistent partition.



(a) original tree (b) approximate tree with a consistent partition

Figure 2: Example of constructing approximate scenario trees.

Given a stage scenario $\widehat{\omega} \in \widehat{\Omega}_k$ in stage $k \in \widehat{\mathcal{P}}_p$ of an approximate tree $\mathscr{T}_p$, we set its realization probability $\widehat{\varrho}_{\widehat{\omega}}$ as $\widehat{\varrho}_{\widehat{\omega}} = \sum_{\omega \in \Omega_k(\widehat{\omega})} \varrho_\omega$. The supply or demand of each node $n \in \mathcal{N}_k$ under stage scenario $\widehat{\omega} \in \widehat{\Omega}_k$, denoted by $\widehat{d}_{\widehat{\omega},n}$, is set as $\widehat{d}_{\widehat{\omega},n} = \sum_{\omega \in \Omega_k(\widehat{\omega})} \frac{\varrho_\omega}{\widehat{\varrho}_{\widehat{\omega}}} d_{\omega,n}$.

For an approximate tree $\mathscr{T}_p$, any path in the form of $\{\widehat{\omega}_{p+1}, ..., \widehat{\omega}_{\overline{p}}\}$ in the tree, where $\widehat{\omega}_k \in \widehat{\Omega}_k$, represents an *approximate scenario*. Let $\widehat{\Xi}_p$ be the set of approximate scenarios associated with $\mathscr{T}_p$. For each $\widehat{\xi} \in \widehat{\Xi}_p$, we denote by $\widehat{\omega}_k(\widehat{\xi}) \in \widehat{\Omega}_k$ the index of the stage scenario in stage $k \in \widehat{\mathcal{P}}_p$ associated with this scenario in the approximate tree $\mathscr{T}_p$. Accordingly, the probability of each scenario $\widehat{\xi} \in \widehat{\Xi}_p$, denoted by $\widehat{\rho}_{\widehat{\xi}}$, is calculated by $\widehat{\rho}_{\widehat{\xi}} = \prod_{k\in\widehat{\mathcal{P}}_p} \widehat{\varrho}_{\widehat{\omega}_k(\widehat{\xi})}$. The supply or demand of any node $n \in \mathcal{N}_k$ for any $k \in \widehat{\mathcal{P}}_p$ in the scenario tree $\mathscr{T}_p$ under scenario $\widehat{\xi} \in \widehat{\Xi}_p$ is denoted by $\widehat{d}_{\widehat{\xi},n}$ and is set equal to $\widehat{d}_{\widehat{\omega}_k(\widehat{\xi}),n}$. Further, for each scenario tree $\mathscr{T}_p$, $p \in \mathcal{P}^+ \setminus \{\bar{p}\}$, we define the set $\widehat{\Lambda}_{p,k} = \{(\widehat{\xi}_1, \widehat{\xi}_2) \in \widehat{\Xi}_p \times \widehat{\Xi}_p | \widehat{\omega}_{k'}(\widehat{\xi}_1) = \widehat{\omega}_{k'}(\widehat{\xi}_2), \forall k' = p+1, ..., k\}$ which contains all pairs of scenarios $(\widehat{\xi}_1, \widehat{\xi}_2) \in \widehat{\Xi}_p \times \widehat{\Xi}_p$ that are indistinguishable in stage $k \in \widehat{\mathcal{P}}_p$ in the approximate tree.

One can verify that for any stage scenario $\omega \in \Omega_p$, $p \in \mathcal{P}^+ \setminus \{\bar{p}\}$, the approximate tree $\mathscr{T}_p$ forms a consistent partition (Birge and Louveaux 2011) of the stage scenarios in the subtree associated with ω in the original tree.

Deriving optimal inequalities. The optimal inequalities for problems $\mathbf{P}_0$ and $\mathbf{P}_{\xi,p}$ are then derived based on the approximate trees $\mathscr{T}_p$, where $p \in \mathcal{P}^+ \setminus \{\bar{p}\}$. Details of the derivation are explained in EC.2 of the electronic companion.

Note that given any approximate tree $\mathscr{T}_p$, where $p \in \mathcal{P}^+ \setminus \{\bar{p}\}$, if we have $|\widehat{\Omega}_k| = 1$ for any stage $k \in \widehat{P}_p$, then the optimal inequalities are defined based on a single scenario determined by the average supplies and demands across all scenarios. We have the following proposition showing the value of constructing approximate trees with multiple stage scenarios.

Proposition 3. *The optimality inequalities defined on any $\mathscr{T}_p$, are at least as strong as those defined for $\mathscr{T}_p$ with $|\widehat{\Omega}_k| = 1$ for any $k \in \widehat{P}_p$ and $p \in \mathcal{P}^+ \setminus \{\bar{p}\}$.*

5.4.2 Primal-dual lifting

In any iteration l of the SDDP approach, given a forward-step problem $\mathbf{P}_{\xi,p}$ in stage $p \in \mathcal{P} \setminus \{\bar{p}\}$ under scenario $\xi \in \Xi^l$, let $\eta^*_{\xi,p}$ be the optimal solution value of variable $\eta_{\xi,p}$ of the problem, and let $\zeta^*_{\xi,\omega,p+1}$ be the optimal objective function values of the dual backward-step problems $\mathbf{D}_{\xi,\omega,p+1}$, $\forall \omega \in \Omega_{p+1}$. The primal-dual lifting method strengthens $\mathbf{P}_{\xi,p}$ by iterating between solving problem $\mathbf{P}_{\xi,p}$ and problems $\mathbf{D}_{\xi,\omega,p+1}$ $(\omega \in \Omega_{p+1})$ to generate inequalities (36) for the cost-to-go function Ψ_p until a *local convergence* is reached such that we have:

$$\eta^*_{\xi,p} \geq (1-\epsilon) \sum_{\omega\in\Omega_{p+1}} \varrho_\omega \zeta^*_{\xi,\omega,p+1}, \tag{37}$$

where $\epsilon \in [0, 1]$ is a preset parameter. Note that convergence is guaranteed due to the limited number of extreme points of the polyhedral feasible regions for the problems $\mathbf{D}_{\xi,\omega,p+1}$ (Benders 1962).

6 Computational experiments

In this section, we first introduce the experimental settings in Section 6.1. Section 6.2 explains how the testing instances were generated. We then discuss computational results in two parts: Section 6.3 evaluates the impacts of the enhancement techniques on the performance of the SDDP approach, and Section 6.4 compares the performance of the approach with that of other commonly used solution methods. Interested readers can find our code implementation, data sets used, detailed results, and associated user instructions at `https://github.com/LingxiaoWu2021/SFPTMP`.

6.1 Computational settings

In order to provide a thorough computational assessment of our proposed SDDP approach, we have implemented the following three variants of the SDDP approach:

1. S0 solves the problem using the basic SDDP approach proposed in Sections 5.1–5.3;
2. S1 is similar to S0 but also uses the optimality inequalities of Section 5.4.1;
3. S2 is similar to S1 but also uses the primal-dual lifting strategy of Section 5.4.2.

All variants of the approach were implemented in a three-phase framework. In the first phase, the integrality constraints of $\mathbf{P}_0$ are dropped. In the second phase, integrality constraints on $\mathbf{P}_0$ are imposed. In the last phase, we solve problem $\mathbf{P}_0$ with the (final) updated cost-to-go function Ψ_0 to obtain the final lower bound and solutions to $\mathbf{P}_0$.

To ensure fair comparisons, we allocated the same computational times for three variants of the SDDP on any instance. In particular, given any instance with $\bar{p}$ stages, we set the computational times to be $360\bar{p}$ and $1200\bar{p}$ seconds for the first and second phases, respectively. The time limit for solving the MILP model of $\mathbf{P}_0$ in any iteration of any phase is set to 1,200 seconds.

We implemented our algorithms in C++, and all the experiments were conducted on the Cedar cluster of Compute Canada with 128GB of RAM in a single-threaded Linux environment. We used CPLEX 20.1.0 for solving the MILP and LP models.

6.1.1 Lower bounds and upper bounds

In order to evaluate the performance of an approach, we derive the lower bound (LB) and upper bound (UB) obtained by the approach for an instance as follows.

Let Z_0^* and δ be the (sub)optimal objective function value and the optimality gap obtained by solving problem $\mathbf{P}_0$ in the third phase of the SDDP approach. Let also $\boldsymbol{y}^*$ be the solutions of y_b variables obtained by solving this problem.

The lower bound of the instance is calculated as $LB = Z_0^*(1-\delta)$. To obtain the upper bound, a sample set $\Xi' \subseteq \Xi$ is created. If $|\Xi| \leq 10,000$, we let $\Xi' = \Xi$. Otherwise, we independently and randomly sample 10,000 scenarios from Ξ to construct Ξ'. The probability of each scenario $\xi \in \Xi'$ is set as $\rho'_\xi = \frac{\rho_\xi}{\sum_{\xi \in \Xi'} \rho_\xi}$. Then, for each scenario $\xi \in \Xi'$ we solve problems $\mathbf{P}_{\xi,p}$ in each stage $p \in \mathcal{P}$ with the given $\boldsymbol{y}^*$. Let $\gamma^*_{\xi,p}$ and $\eta^*_{\xi,p}$ be the optimal objective function value and the optimal solution value of $\eta_{\xi,p}$ obtained by solving problem $\mathbf{P}_{\xi,p}$, respectively. We let $Z^*_{\xi,p} = \gamma^*_{\xi,p} - \eta^*_{\xi,p}$, which represents the total cost associated with the decisions made in stage p under scenario ξ. Let also $\mu_\xi = \sum_{p\in\mathcal{P}} Z^*_{\xi,p}$, $\hat{\mu} = \sum_{\xi\in\Xi'} \rho'_\xi \mu_\xi$, and $\sigma^2 = \frac{1}{|\Xi'|-1} \sum_{\xi\in\Xi'} (\mu_\xi - \hat{\mu})^2$.

Finally, for the case with $|\Xi| \leq 10,000$, we set $UB = \sum_{b\in\mathcal{B}} F_b y_b^* + \hat{\mu}$, which is the "true" upper bound for the instance. For the case with $|\Xi| > 10,000$, we set $UB = \sum_{b\in\mathcal{B}} F_b y_b^* + \hat{\mu} + 1.96\frac{\sigma^2}{\sqrt{|\Xi'|}}$, which represents a 95%-confidence statistical upper bound for the instance. Given LB and UB, the optimality gap of the instance is calculated by $GAP = 100(UB - LB)/LB$.

6.2 Instance generation

To test the performance of the SDDP approaches, we generate 450 instances. These instances were generated from five *cases*, each of which represents a deterministic FPTMP instance (i.e., an SFPTMP instance with a sole scenario). The cases were adapted from the instances originally created by Papageorgiou et al. (2014) for the maritime inventory routing problem (MIRP) and have been widely used in the literature.

In each case, we let a period represent one week and each stage contains six periods (i.e., six weeks). In each instance, we let the number of stages $|\mathcal{P}| \in \{3, 6, 9, 12, 15, 18\}$. For the number of stage scenarios at each stage, we let $|\Omega_p| \in \{10, 20, 50\}$. The stage scenarios in any Ω_p ($p \in \mathcal{P}$) were generated by a Monte-Carlo simulation in which the demand of each demand site $i \in \mathcal{I}^D$ in each period $t \in \mathcal{T}$ is independent and generated through the uniform distribution $U[\bar{d}_{it}(1-\Delta), \bar{d}_{it}(1+\Delta)]$, where $\bar{d}_{it}$ is the *nominal demand* in a case and $\Delta \in \{0.1, 0.2, 0.3, 0.4, 0.5\}$ is a selected deviation ratio. Hence, there are 90 ($6 \times 3 \times 5$) different combinations of $|\mathcal{P}|$, $|\Omega_p|$, and Δ, and we accordingly generated 90 SFPTMP instances based on each case, leading to 450 instances in total.

Details of the cases and the settings of other parameters in the SFPTMP instances are explained in Section EC.4 of the electronic companion.

6.3 Impacts of enhancements

We have implemented approaches S0, S1, and S2 to solve the instances. To better present the results, we classify all instances into three categories: small instances (with 3 and 6 stages), medium instances (with 9 and 12 stages), and large instances (with 15 and 18 stages). The results of small, medium, and large instances are reported in Tables 2, 3, and 4, respectively. In these tables, the results of five instances sharing the same $|\mathcal{P}|$, $|\Omega_p|$ and Δ are reported as a group. The first three columns show the settings of $|\mathcal{P}|$, $|\Omega_p|$ and Δ of each instance group. Columns *LB*, *UB* and *GAP* report the average lower bound, average upper bound and average optimality gap generated by these approaches, respectively. For each group, we use boldface to indicate the best results obtained among the three approaches.

Table 2: Computational Results of the SDDP Algorithms on Small Instances.

$\|\mathcal{P}\|$	$\|\Omega_p\|$	Δ	LB ($\times 10^3$)			UB ($\times 10^3$)			GAP (%)		
			S0	S1	S2	S0	S1	S2	S0	S1	S2
3	10	0.1	**200.5**	200.4	200.4	201.1	**201.0**	**201.0**	**0.3**	**0.3**	**0.3**
3	10	0.2	201.5	**201.6**	**201.6**	202.9	203.0	**202.6**	0.7	0.7	**0.5**
3	10	0.3	205.0	**205.1**	205.0	**206.1**	**206.1**	206.2	0.6	**0.5**	0.6
3	10	0.4	**206.8**	**206.8**	206.7	208.4	208.4	**208.2**	**0.8**	**0.8**	**0.8**
3	10	0.5	**205.8**	**205.8**	205.7	**206.7**	207.1	207.0	**0.5**	0.7	0.6
Average			**203.9**	**203.9**	**203.9**	205.1	205.1	**205.0**	**0.6**	**0.6**	**0.6**
3	20	0.1	**200.4**	**200.4**	**200.4**	201.8	**201.0**	201.1	0.7	**0.3**	0.4
3	20	0.2	202.7	**202.9**	202.8	203.9	204.1	**203.8**	0.6	0.6	**0.5**
3	20	0.3	204.8	**204.9**	204.8	205.8	**205.7**	205.9	0.5	**0.4**	0.5
3	20	0.4	206.3	**206.4**	206.3	208.3	207.7	**207.6**	1.0	**0.7**	**0.7**
3	20	0.5	209.3	**209.4**	209.3	210.8	**210.6**	210.9	0.7	**0.6**	0.8
Average			204.7	**204.8**	204.7	206.1	**205.8**	205.9	0.7	**0.5**	0.6
3	50	0.1	201.7	**202.0**	**202.0**	203.8	**202.9**	**202.9**	1.0	**0.4**	**0.4**
3	50	0.2	200.6	**201.1**	**201.0**	202.9	**202.4**	202.6	1.2	**0.7**	0.8
3	50	0.3	205.1	**205.4**	205.3	207.1	207.4	**206.9**	1.0	1.0	**0.8**
3	50	0.4	206.8	**207.0**	206.9	**209.3**	**209.3**	209.7	1.2	**1.1**	1.4
3	50	0.5	209.3	**209.4**	209.3	211.7	**211.5**	211.8	1.2	**1.1**	1.3
Average			204.7	**205.0**	204.9	207.0	**206.7**	206.8	1.1	**0.9**	**0.9**
6	10	0.1	329.2	**329.9**	329.8	334.1	**331.7**	**331.7**	1.5	**0.6**	**0.6**
6	10	0.2	329.5	330.2	**330.4**	334.2	333.6	**332.9**	1.4	1.0	**0.8**
6	10	0.3	335.0	335.6	**335.8**	340.5	340.8	**339.8**	1.6	1.5	**1.2**
6	10	0.4	339.7	340.1	**340.3**	345.2	343.9	**343.8**	1.6	1.1	**1.0**
6	10	0.5	341.9	342.2	**342.6**	349.5	**348.5**	**348.5**	2.2	**1.8**	**1.8**
Average			335.0	335.6	**335.8**	340.7	339.7	**339.3**	1.7	1.2	**1.1**
6	20	0.1	328.3	329.8	**330.0**	335.7	332.6	**332.4**	2.3	0.9	**0.8**
6	20	0.2	328.5	329.9	**330.3**	336.7	**333.9**	334.3	2.5	**1.2**	**1.2**
6	20	0.3	334.9	335.9	**336.4**	342.6	341.7	**341.1**	2.3	1.7	**1.4**
6	20	0.4	339.4	340.2	**340.3**	**344.9**	345.0	345.0	1.7	**1.4**	**1.4**
6	20	0.5	345.9	346.5	**346.9**	354.7	352.7	**352.5**	2.6	1.8	**1.6**
Average			335.4	336.5	**336.8**	342.9	341.2	**341.1**	2.3	1.4	**1.3**
6	50	0.1	325.6	328.9	**329.1**	338.1	**331.6**	331.7	3.8	**0.8**	**0.8**
6	50	0.2	328.3	330.9	**331.0**	337.4	334.5	**334.4**	2.8	1.1	**1.0**
6	50	0.3	330.9	332.9	**333.3**	339.5	**338.0**	339.6	2.6	**1.5**	1.9
6	50	0.4	337.5	339.0	**339.2**	346.3	**345.2**	346.3	2.6	**1.8**	2.1
6	50	0.5	344.0	345.1	**345.5**	353.8	**353.0**	353.7	2.9	**2.3**	2.4
Average			333.2	335.4	**335.6**	343.0	**340.4**	341.2	2.9	**1.5**	1.7

Table 3: Computational Results of the SDDP Algorithms on Medium Instances.

$\|\mathcal{P}\|$	$\|\Omega_p\|$	Δ	LB ($\times 10^3$)			UB ($\times 10^3$)			GAP (%)		
			S0	S1	S2	S0	S1	S2	S0	S1	S2
9	10	0.1	462.6	465.4	**465.5**	473.4	**468.7**	468.9	2.4	**0.7**	**0.7**
9	10	0.2	469.1	470.9	**471.5**	480.2	**477.0**	**477.0**	2.4	1.3	**1.2**
9	10	0.3	475.8	477.8	**478.9**	487.2	**485.5**	486.2	2.4	1.6	**1.5**
9	10	0.4	480.2	481.6	**482.6**	490.9	**491.2**	492.4	2.2	**2.0**	2.1
9	10	0.5	484.8	485.9	**486.8**	494.7	**494.7**	495.1	2.1	1.8	**1.7**
Average			474.5	476.3	**477.1**	485.3	**483.4**	483.9	2.3	1.5	**1.4**
9	20	0.1	461.7	465.7	**465.9**	475.8	**468.8**	469.0	3.1	**0.7**	**0.7**
9	20	0.2	464.6	469.1	**469.5**	481.3	477.6	**476.7**	3.6	1.8	**1.5**
9	20	0.3	472.4	475.3	**476.2**	485.6	485.7	**484.5**	2.8	2.2	**1.7**
9	20	0.4	481.4	483.5	**484.7**	495.6	493.3	**491.6**	3.0	2.0	**1.4**
9	20	0.5	493.1	494.6	**496.0**	507.7	507.7	**505.5**	3.0	2.7	**1.9**
Average			474.6	477.6	**478.5**	489.2	486.6	**485.4**	3.1	1.9	**1.5**
9	50	0.1	454.3	463.1	**463.3**	480.9	**466.8**	**466.8**	6.0	**0.8**	**0.8**
9	50	0.2	460.1	467.2	**468.0**	481.9	**475.0**	475.2	4.7	1.7	**1.5**
9	50	0.3	466.2	472.5	**473.8**	488.5	**483.9**	484.2	4.8	2.5	**2.2**
9	50	0.4	476.3	479.6	**481.0**	494.9	**492.1**	492.2	4.0	2.6	**2.3**
9	50	0.5	492.8	494.9	**496.5**	508.5	510.0	**508.8**	3.2	3.0	**2.5**
Average			470.0	475.5	**476.5**	491.0	485.6	**485.4**	4.5	2.1	**1.9**
12	10	0.1	612.8	617.9	**618.0**	639.3	**623.4**	**623.4**	4.5	**0.9**	**0.9**
12	10	0.2	615.4	618.6	**620.3**	632.6	628.3	**626.7**	2.8	1.6	**1.1**
12	10	0.3	622.2	626.0	**627.8**	641.3	638.6	**636.6**	3.1	2.0	**1.4**
12	10	0.4	627.9	630.6	**633.1**	649.5	646.8	**643.8**	3.5	2.6	**1.7**
12	10	0.5	644.8	646.2	**649.2**	668.1	663.2	**662.7**	3.6	2.7	**2.1**
Average			624.6	627.9	**629.7**	646.2	640.1	**638.6**	3.5	2.0	**1.4**
12	20	0.1	609.3	615.9	**616.2**	635.7	622.2	**621.8**	4.3	1.0	**0.9**
12	20	0.2	612.7	621.0	**621.8**	646.2	**631.8**	631.9	5.5	1.8	**1.6**
12	20	0.3	618.8	625.1	**627.6**	645.9	639.6	**638.5**	4.5	2.4	**1.8**
12	20	0.4	631.0	635.7	**638.2**	661.9	654.5	**650.5**	5.2	3.1	**2.0**
12	20	0.5	647.8	650.6	**652.8**	670.5	665.9	**665.5**	3.5	2.4	**2.0**
Average			623.9	629.7	**631.3**	652.0	642.8	**641.6**	4.6	2.1	**1.7**
12	50	0.1	599.3	612.6	**612.7**	642.7	**618.4**	**618.4**	7.4	1.0	**0.9**
12	50	0.2	608.4	620.6	**621.7**	654.4	**633.4**	633.5	7.7	2.1	**2.0**
12	50	0.3	618.5	629.0	**630.9**	654.7	**644.8**	645.3	5.9	2.5	**2.3**
12	50	0.4	629.0	638.1	**640.7**	667.8	659.6	**657.3**	6.4	3.4	**2.7**
12	50	0.5	643.2	649.3	**653.3**	679.6	678.9	**672.4**	5.7	4.5	**2.9**
Average			619.7	629.9	**631.9**	659.9	647.0	**645.4**	6.6	2.7	**2.2**

By comparing the results of approaches S0 and S1 in Tables 2–4, we found that incorporating optimality inequalities improves the lower and upper bounds in addition to reducing the optimality gaps across all instances. As instance scale increases, the improvement in optimality gaps becomes more noticeable. For S1 and S2, the differences in lower bounds, upper bounds, and optimality gaps are not significant in small-scale instances. However, we can see that S2, which uses the primal-dual lifting technique, reports the best lower bounds and secures the smallest optimality gaps in all but one of the medium and large-scale instances. The technique is of greater value for instances with longer planning horizons and greater uncertainties in demand and supply.

6.4 Comparisons with alternative methods

We next compare the performance of the SDDP approach with that of three alternative solution methods, including a commonly used optimization solver (CPLEX) solving the compact MILP model and two benchmark methods that simulate common decision policies used in practice.

Table 4: Computational Results of the SDDP Algorithms on Large Instances.

$\|\mathcal{P}\|$	$\|\Omega_p\|$	Δ	LB ($\times 10^3$)			UB ($\times 10^3$)			GAP (%)		
			S0	S1	S2	S0	S1	S2	S0	S1	S2
15	10	0.1	750.8	759.2	**759.6**	793.2	766.0	**765.4**	5.8	0.9	**0.8**
15	10	0.2	757.0	765.6	**767.2**	791.9	**781.0**	780.1	4.8	2.0	**1.7**
15	10	0.3	771.3	776.3	**778.9**	800.3	793.7	**792.6**	3.8	2.2	**1.8**
15	10	0.4	784.1	789.4	**793.1**	821.7	**810.2**	811.3	4.8	2.7	**2.4**
15	10	0.5	802.2	804.9	**808.4**	829.1	825.3	**822.7**	3.4	2.5	**1.8**
Average			773.1	779.1	**781.4**	807.2	795.2	**794.4**	4.5	2.1	**1.7**
15	20	0.1	744.5	**758.8**	**758.8**	788.7	766.1	**765.9**	6.1	**1.0**	**1.0**
15	20	0.2	751.8	766.3	**767.0**	798.5	**780.1**	780.5	6.4	**1.8**	**1.8**
15	20	0.3	765.9	776.3	**778.5**	805.8	**795.7**	796.2	5.4	2.5	**2.3**
15	20	0.4	781.5	787.9	**792.4**	819.9	816.6	**808.9**	5.0	3.7	**2.1**
15	20	0.5	801.9	806.6	**811.1**	840.0	834.2	**829.0**	4.9	3.5	**2.3**
Average			769.1	779.2	**781.6**	810.6	798.5	**796.1**	5.6	2.5	**1.9**
15	50	0.1	731.9	758.7	**758.8**	805.9	**765.0**	765.5	10.2	**0.9**	**0.9**
15	50	0.2	745.3	765.4	**766.2**	813.8	780.3	**778.3**	9.5	2.0	**1.6**
15	50	0.3	757.6	779.6	**782.7**	826.5	806.6	**803.0**	9.3	3.6	**2.7**
15	50	0.4	770.5	784.3	**788.9**	825.2	819.3	**809.5**	7.2	4.6	**2.7**
15	50	0.5	803.0	812.1	**817.4**	855.5	850.1	**839.8**	6.8	4.7	**2.8**
Average			761.6	780.0	**782.8**	825.4	804.3	**799.2**	8.6	3.1	**2.1**
18	10	0.1	894.3	909.4	**910.0**	947.3	919.0	**918.6**	6.0	1.1	**1.0**
18	10	0.2	904.6	916.0	**917.3**	948.3	935.0	**933.5**	4.9	2.1	**1.8**
18	10	0.3	913.8	923.4	**927.5**	965.5	949.4	**945.6**	5.8	2.8	**2.0**
18	10	0.4	925.6	933.8	**939.1**	974.5	967.7	**961.9**	5.4	3.7	**2.5**
18	10	0.5	954.7	958.4	**965.1**	995.1	993.6	**991.3**	4.4	3.7	**2.9**
Average			918.6	928.2	**931.8**	966.1	952.9	**950.2**	5.3	2.7	**2.0**
18	20	0.1	881.5	902.6	**902.7**	941.0	**911.6**	911.8	6.9	**1.0**	**1.0**
18	20	0.2	895.6	913.0	**914.0**	952.4	**929.9**	930.0	6.5	1.9	**1.8**
18	20	0.3	907.8	926.2	**929.8**	982.0	954.0	**951.2**	8.3	3.0	**2.3**
18	20	0.4	927.4	940.8	**946.1**	992.3	976.6	**971.8**	7.2	3.8	**2.8**
18	20	0.5	957.7	964.5	**971.4**	1007.9	1008.6	**994.4**	5.4	4.6	**2.4**
Average			914.0	929.4	**932.8**	975.1	956.1	**951.9**	6.8	2.9	**2.1**
18	50	0.1	869.9	906.7	**907.0**	977.6	916.0	**915.5**	12.8	**1.0**	**1.0**
18	50	0.2	879.0	912.1	**913.1**	1000.9	932.9	**931.0**	14.5	2.3	**2.0**
18	50	0.3	894.4	921.3	**925.4**	996.6	954.2	**950.4**	11.9	3.6	**2.7**
18	50	0.4	917.5	937.8	**944.7**	997.2	978.0	**973.8**	8.8	4.2	**3.1**
18	50	0.5	950.3	963.4	**970.4**	1016.9	1003.9	**999.2**	7.1	4.2	**3.1**
Average			902.2	928.2	**932.1**	997.8	957.0	**954.0**	11.0	3.1	**2.4**

6.4.1 Comparisons with CPLEX

We applied CPLEX on the MILP model of problem **P** to solve the instances. The maximum computational time of CPLEX is set to be $1800 \times \bar{p}$ for any instance with $\bar{p}$ stages, while other computational settings remain the same as described in Section 6.1. Table 5 summarizes the results produced by CPLEX and S2. Columns *LB* and *UB* report the average lower bound and upper bound obtained by CPLEX and S2, respectively. Column *UBG* reports the average gaps (in percentage) of the upper bounds obtained by S2 against those obtained by CPLEX.

As shown in Table 5, due to the memory limit, CPLEX can only obtain feasible solutions for the smallest instances with three stages and 10 stage scenarios. For these instances, S2 can also obtain near-optimal solutions. Furthermore, as shown in Tables 2–4, for instances that are out of the capacity of CPLEX, the SDDP approach serves as a highly reliable and efficient alternative.

Table 5: Computational Results of CPLEX and S2.

$\|\mathcal{P}\|$	$\|\Omega_p\|$	Δ	LB ($\times 10^3$)		UB($\times 10^3$)		UBG (%)
			CPLEX	S2	CPLEX	S2	
3	10	0.1	200.6	200.4	200.6	201.0	0.2
3	10	0.2	201.9	201.6	201.9	202.6	0.3
3	10	0.3	205.4	205.0	205.4	206.2	0.4
3	10	0.4	207.3	206.7	207.3	208.2	0.5
3	10	0.5	206.2	205.7	206.2	207.0	0.4
Average			204.3	203.9	204.3	205.0	0.3
3	20	0.1	198.3	200.4	–	201.1	–
3	20	0.2	198.8	202.8	–	203.8	–
3	20	0.3	199.7	204.8	–	205.9	–
3	20	0.4	201.2	206.3	–	207.6	–
3	20	0.5	203.6	209.3	–	210.9	–
Average			–	204.7	–	205.9	–
6	10	0.1	–	329.8	–	331.7	–
6	10	0.2	–	330.4	–	332.9	–
6	10	0.3	–	335.8	–	339.8	–
6	10	0.4	–	340.3	–	343.8	–
6	10	0.5	–	342.6	–	348.5	–
Average			–	335.8	–	339.3	–

"–": CPLEX failed to generate feasible solutions due to the memory limit.

6.4.2 Comparisons with other benchmark methods

We have also compared the performance of the SDDP approach with that of two benchmark solution methods. The first method (BM1) simulates a decision policy ignoring capacity contracts, and the second method (BM2) is a two-stage stochastic optimization solution approach that simulates a myopic decision policy. Details of these two methods are explained in EC.5 of the electronic companion. For an instance, the upper bound obtained by BM1 or BM2 is derived using a large set of scenarios generated by the method as described in Section 6.1.1. To evaluate the savings generated by the SDDP approach, in Table 6, we report the average gaps of upper bounds produced by BM1 and BM2 against those produced by S2, respectively, in columns *UBG1* and *UBG2*.

From Table 6, we find that securing long-term capacities with the carriers (as in S2) rather than transporting all commodities using non-contractual freight rates (as in BM1) can reduce the total cost for a shipper by 17.3% to 27.9%. Additionally, in BM2, we formulate the SFPTMP as a myopic two-stage stochastic model, which overlooks the interconnection between multiple decision stages. The performance of S2 against BM2 demonstrates the value of multi-stage stochastic optimization for solving the SFPTMP.

7 Conclusions

In this study, we have introduced an SFPTMP in the supply chain management of a shipper that sources freight services from the 3PL carriers. We have formulated the problem as a multi-stage stochastic programming model and have developed an SDDP approach for solving the model. We have embedded valid feasibility inequalities in the stage-wise problems of the approach. To improve the performance of the approach, we have further derived optimality inequalities into the stage-wise problems and proposed a primal-dual lifting procedure. Using synthetic instances, we have demonstrated that the enhancement strategies can significantly improve the performance of the approach. These results also demonstrated that the approach can obtain near-optimal solutions to instances of realistic scale and that it significantly outperforms other solution methods commonly used in practice.

Table 6: Improvements Generated by SDDP Against the Benchmark Methods.

$\|\mathcal{P}\|$	$\|\Omega_p\|$	Δ	UBG1 (%)	UBG2 (%)	$\|\mathcal{P}\|$	$\|\Omega_p\|$	Δ	UBG1 (%)	UBG2 (%)	$\|\mathcal{P}\|$	$\|\Omega_p\|$	Δ	UBG1 (%)	UBG2 (%)
3	10	0.1	20.0	9.1	3	20	0.1	19.8	10.9	3	50	0.1	19.3	7.5
3	10	0.2	19.6	8.6	3	20	0.2	18.9	12.6	3	50	0.2	19.4	11.0
3	10	0.3	18.4	13.5	3	20	0.3	18.8	7.8	3	50	0.3	18.4	7.5
3	10	0.4	17.9	12.2	3	20	0.4	18.2	11.5	3	50	0.4	17.3	10.6
3	10	0.5	17.8	10.0	3	20	0.5	17.3	9.6	3	50	0.5	17.3	8.8
Average			18.7	10.7	Average			18.6	10.5	Average			18.3	9.1
6	10	0.1	24.0	31.8	6	20	0.1	23.8	24.4	6	50	0.1	23.9	26.4
6	10	0.2	23.9	29.5	6	20	0.2	23.5	25.3	6	50	0.2	23.7	28.9
6	10	0.3	22.9	24.4	6	20	0.3	22.4	23.9	6	50	0.3	22.7	23.1
6	10	0.4	22.0	26.9	6	20	0.4	22.0	30.0	6	50	0.4	21.5	27.7
6	10	0.5	20.8	31.5	6	20	0.5	20.8	23.9	6	50	0.5	20.6	27.4
Average			22.7	28.8	Average			22.5	25.5	Average			22.5	26.7
9	10	0.1	26.0	32.0	9	20	0.1	25.9	23.1	9	50	0.1	26.3	34.4
9	10	0.2	25.1	30.2	9	20	0.2	25.1	35.1	9	50	0.2	25.3	29.2
9	10	0.3	23.8	40.2	9	20	0.3	24.3	31.0	9	50	0.3	24.2	29.4
9	10	0.4	23.4	34.6	9	20	0.4	23.4	32.5	9	50	0.4	23.5	30.5
9	10	0.5	23.1	40.9	9	20	0.5	22.4	27.1	9	50	0.5	21.5	34.9
Average			24.3	35.6	Average			24.2	29.7	Average			24.2	31.7
12	10	0.1	26.1	31.6	12	20	0.1	26.3	28.8	12	50	0.1	26.7	30.2
12	10	0.2	25.9	27.9	12	20	0.2	25.5	33.2	12	50	0.2	25.2	42.0
12	10	0.3	25.0	34.0	12	20	0.3	24.9	46.7	12	50	0.3	24.2	32.7
12	10	0.4	24.4	38.0	12	20	0.4	23.8	34.9	12	50	0.4	23.3	41.2
12	10	0.5	23.2	31.7	12	20	0.5	22.6	37.3	12	50	0.5	22.2	41.4
Average			24.9	32.7	Average			24.6	36.2	Average			24.3	37.5
15	10	0.1	27.3	44.3	15	20	0.1	27.3	46.2	15	50	0.1	27.4	34.5
15	10	0.2	26.2	47.2	15	20	0.2	26.3	42.5	15	50	0.2	26.5	35.7
15	10	0.3	25.4	26.3	15	20	0.3	25.2	46.9	15	50	0.3	24.5	44.7
15	10	0.4	24.3	29.8	15	20	0.4	24.4	30.5	15	50	0.4	24.3	36.5
15	10	0.5	23.5	41.6	15	20	0.5	23.1	47.1	15	50	0.5	22.4	38.9
Average			25.3	37.8	Average			25.3	42.6	Average			25.0	38.1
18	10	0.1	27.3	33.8	18	20	0.1	27.9	32.9	18	50	0.1	27.7	32.8
18	10	0.2	26.5	35.1	18	20	0.2	26.7	33.5	18	50	0.2	26.9	37.1
18	10	0.3	25.7	45.8	18	20	0.3	25.4	46.0	18	50	0.3	25.5	37.4
18	10	0.4	25.1	45.2	18	20	0.4	24.3	47.1	18	50	0.4	24.2	34.5
18	10	0.5	23.3	35.4	18	20	0.5	23.4	40.1	18	50	0.5	23.2	40.7
Average			25.6	39.1	Average			25.5	39.9	Average			25.5	36.5

While this study assumes that the probabilistic distribution of the uncertain parameters is available, this distribution can be unknown in practice especially due to limited data. Hence, future research should investigate robust optimization methods for solving the FPTMP under uncertainty such that the distribution information of uncertain parameters is not fully available.

Appendix

A1 The SDDP framework

This section presents the pseudocode of the SDDP approach.

Algorithm A1 Stochastic Dual Dynamic Programming (SDDP).

Initialize: $LB \leftarrow -\infty$, $l \leftarrow 1$, and initial cost-to-go functions Ψ_p, $p \in \mathcal{P}^+$
while Some stopping criterion is not satisfied **do**
  /*Sampling step*/
  Sample a set of scenarios $\Xi^l \subseteq \Xi$
  /*Forward step*/
  Solve the problem $\mathbf{P}_0(\Psi_0)$
  Collect $\bar{\boldsymbol{\chi}}_{\xi,0}$, $\forall \xi \in \Xi^l$
  Set LB equal to the optimal value of $\mathbf{P}_0(\Psi_0)$
  for $\xi \in \Xi^l$ **do**
    for $p = 1, ..., \overline{p}$ **do**
      Solve the problem $\mathbf{P}_{\xi,p}(\bar{\boldsymbol{\chi}}_{\xi,p-1}, \Psi_p)$
      Collect $\bar{\boldsymbol{\chi}}_{\xi,p}$
    end for
  end for
  /*Backward step*/
  for $\xi \in \Xi^l$ **do**
    for $p = \overline{p}, ..., 1$ **do**
      for $\omega \in \Omega_p$ **do**
        Solve the dual of problem $\mathbf{Q}_{\xi,\omega,p}(\bar{\boldsymbol{\chi}}_{\xi,p-1}, \Psi_p)$
      end for
      Update Ψ_{p-1} by adding valid cuts
    end for
  end for
  $l \leftarrow l + 1$
end while

A2 Deriving optimality inequalities

The optimality inequalities for problems $\mathbf{P}_0$ and $\mathbf{P}_{\xi,p}$ in the SDDP are generated based on the approximate trees $\mathscr{T}_p$, where $p \in \mathcal{P}^+ \setminus \{\overline{p}\}$ and $\xi \in \Xi$. Given an approximate tree $\mathscr{T}_p$ and its associated approximate scenario set $\widehat{\Xi}_p$ ($p \in \mathcal{P}^+ \setminus \{\overline{p}\}$), the associated optimality inequalities make use of the following additional variables:

$\widehat{z}_{\widehat{\xi},a}$ continuous variable, which represents the volume of the commodity allocated on arc $a \in \mathcal{A}_k$, $k \in \widehat{\mathcal{P}}_p$ under scenario $\widehat{\xi} \in \widehat{\Xi}_p$;

$\widehat{u}_{\widehat{\xi},n}$ continuous variable, which represents the inventory level at node $n \in \mathcal{N}_k$, $k \in \widehat{\mathcal{P}}_p$ under scenario $\widehat{\xi} \in \widehat{\Xi}_p$;

$\widehat{v}_{\widehat{\xi},n}$ continuous variable, which represents the volume of the supply or demand backlogged at node $n \in \mathcal{N}_k$, $k \in \widehat{\mathcal{P}}_p$ under scenario $\widehat{\xi} \in \widehat{\Xi}_p$.

For problem $\mathbf{P}_0$, where $\widehat{\mathcal{P}}_0 = \mathcal{P}$, we have the following valid inequalities:

$$\eta_0 \geq \sum_{\widehat{\xi} \in \widehat{\Xi}_0} \widehat{\rho}_{\widehat{\xi}} \left(\sum_{n \in \mathcal{N}} \left(h_n \widehat{u}_{\widehat{\xi},n} + e_n \widehat{v}_{\widehat{\xi},n} \right) + \sum_{a \in \mathcal{A}} c_a \widehat{z}_{\widehat{\xi},a} \right) \tag{A1}$$

$$\widehat{z}_{\widehat{\xi},a} \leq y_b \quad \forall a \in \mathcal{A}_b^1, \forall b \in \mathcal{B}, \forall \widehat{\xi} \in \widehat{\Xi}_0 \tag{A2}$$

$$\widehat{u}_{\widehat{\xi},n_1} + \widehat{v}_{\widehat{\xi},n_1} = \widehat{d}_{\widehat{\xi},n_1} + \widehat{u}_{\widehat{\xi},n_2} + \widehat{v}_{\widehat{\xi},n_2} - \sum_{a \in A^+(n_1)} \widehat{z}_{\widehat{\xi},a}$$
$$\forall n_1 = (i,t), n_2 = (i,t-1) \in \mathcal{N}^S, \forall \widehat{\xi} \in \widehat{\Xi}_0 \tag{A3}$$

$$\widehat{u}_{\widehat{\xi},n} + \widehat{v}_{\widehat{\xi},n} = \widehat{d}_{\widehat{\xi},n} + q_n^0 - \sum_{a \in A^+(n)} \widehat{z}_{\widehat{\xi},a} \quad \forall n = (i,1) \in \mathcal{N}^S, \forall \widehat{\xi} \in \widehat{\Xi}_0 \tag{A4}$$

$$\widehat{u}_{\widehat{\xi},n_1} - \widehat{v}_{\widehat{\xi},n_1} = \widehat{d}_{\widehat{\xi},n_1} + \widehat{u}_{\widehat{\xi},n_2} - \widehat{v}_{\widehat{\xi},n_2} + \sum_{a \in A^-(n_1)} \widehat{z}_{\widehat{\xi},a}$$
$$\forall n_1 = (i,t), n_2 = (i,t-1) \in \mathcal{N}^D, \forall \widehat{\xi} \in \widehat{\Xi}_0 \tag{A5}$$

$$\widehat{u}_{\widehat{\xi},n} - \widehat{v}_{\widehat{\xi},n} = \widehat{d}_{\widehat{\xi},n} + q_n^0 + \sum_{a \in A^-(n)} \widehat{z}_{\widehat{\xi},a} \quad \forall n = (i,1) \in \mathcal{N}^D, \forall \widehat{\xi} \in \widehat{\Xi}_0 \tag{A6}$$

$$\widehat{u}_{\widehat{\xi},n} \leq \overline{q}_n \quad \forall n \in \mathcal{N}, \forall \widehat{\xi} \in \widehat{\Xi}_0 \tag{A7}$$

$$\widehat{z}_{\widehat{\xi}_1,a} = \widehat{z}_{\widehat{\xi}_2,a} \;\; \forall a \in \mathcal{A}_k, \forall(\widehat{\xi}_1,\widehat{\xi}_2) \in \widehat{\Lambda}_{0,k}, \forall k \in \widehat{\mathcal{P}}_0 \tag{A8}$$

$$\widehat{u}_{\widehat{\xi},n_1} - \widehat{v}_{\widehat{\xi},n_1} + \sum_{t=\underline{t}_{p+1}}^{t_2} \sum_{n=(j,t)\in\mathcal{N}} \sum_{a\in A^-(n)} \widehat{z}_{\widehat{\xi},a} + \overline{d}_{j,p+1,t_2} \leq \overline{q}_{n_2}$$
$$\forall n_1 = (j,\overline{t}_p), n_2 = (j,t_2) \in \mathcal{N}^D, \forall((i,t_1),(j,t_2)) \in \widetilde{\mathcal{A}}_p, \forall p \in \mathcal{P}\setminus\{\overline{p}\}, \forall \widehat{\xi} \in \widehat{\Xi}_0 \tag{A9}$$

$$\widehat{u}_{\widehat{\xi},n}, \widehat{v}_{\widehat{\xi},n}, \widehat{z}_{\widehat{\xi},a} \geq 0 \;\; \forall n \in \mathcal{N}, \forall a \in \mathcal{A}, \forall \widehat{\xi} \in \widehat{\Xi}_0. \tag{A10}$$

Proposition 4. *The optimality inequalities* (A1)–(A10) *are valid for problem* $\mathbf{P}_0$.

Proof of Proposition 4. Given any feasible solution $(\boldsymbol{x},\boldsymbol{y})$ from stage 0, consider the problems $\mathbf{R}(\boldsymbol{x},\boldsymbol{y})$ and $\widehat{\mathbf{R}}(\boldsymbol{x},\boldsymbol{y})$ which are formulated as follows:

$$\mathbf{R}(\boldsymbol{x},\boldsymbol{y}) = \min \sum_{\xi\in\Xi} \rho_\xi \left(\sum_{n\in\mathcal{N}} (h_n u_{\xi,n} + e_n v_{\xi,n}) + \sum_{a\in\mathcal{A}} c_a z_{\xi,a} \right) \tag{A11}$$
$$\text{s.t.}(4) - (10), (13)$$

$$\widehat{\mathbf{R}}(\boldsymbol{x},\boldsymbol{y}) = \min \sum_{\widehat{\xi}\in\widehat{\Xi}_0} \widehat{\rho}_{\widehat{\xi}} \left(\sum_{n\in\mathcal{N}} \left(h_n \widehat{u}_{\widehat{\xi},n} + e_n \widehat{v}_{\widehat{\xi},n} \right) + \sum_{a\in\mathcal{A}} c_a \widehat{z}_{\widehat{\xi},a} \right) \tag{A12}$$
$$\text{s.t.}(A2) - (A10).$$

One can easily verify that these problems are feasible and bounded. Let Z_1 and Z_2 denote the optimal objective function values of $\mathbf{R}$ and $\widehat{\mathbf{R}}$, respectively. Then, because of Theorem 1 in Chapter 10 of Birge and Louveaux (2011), we have $Z_2 \leq Z_1$. The validity of the optimality inequalities (A1)–(A10) follows directly from the result. □

Moreover, for problems $\mathbf{P}_{\xi,p}$ with $p \in \mathcal{P}\setminus\{\overline{p}\}$ and $\xi \in \Xi$, we have the following valid inequalities:

$$\eta_{\xi,p} \geq \sum_{\widehat{\xi}\in\widehat{\Xi}_p} \widehat{\rho}_{\widehat{\xi}} \left(\sum_{k\in\widehat{\mathcal{P}}_p} \left(\sum_{n\in\mathcal{N}_k} \left(h_n \widehat{u}_{\widehat{\xi},n} + e_n \widehat{v}_{\widehat{\xi},n} \right) + \sum_{a\in\mathcal{A}_k} c_a \widehat{z}_{\widehat{\xi},a} \right) \right) \tag{A13}$$

$$\widehat{z}_{\widehat{\xi},a} \leq y'_{\xi,p,b} \;\; \forall a \in \mathcal{A}_b^1 \cap \mathcal{A}_k, \forall k \in \widehat{\mathcal{P}}_p, \forall b \in \mathcal{B}, \forall \widehat{\xi} \in \widehat{\Xi}_p \tag{A14}$$

$$\widehat{u}_{\widehat{\xi},n_1} + \widehat{v}_{\widehat{\xi},n_1} = \widehat{d}_{\widehat{\xi},n_1} + \widehat{u}_{\widehat{\xi},n_2} + \widehat{v}_{\widehat{\xi},n_2} - \sum_{a\in A^+(n_1)} \widehat{z}_{\widehat{\xi},a}$$
$$\forall n_1 = (i,t), n_2 = (i,t-1) \in \mathcal{N}_k \cap \mathcal{N}^S, \forall k \in \widehat{\mathcal{P}}_p, \forall \widehat{\xi} \in \widehat{\Xi}_p \tag{A15}$$

$$\widehat{u}_{\widehat{\xi},n_1} + \widehat{v}_{\widehat{\xi},n_1} = \widehat{d}_{\widehat{\xi},n_1} + u_{\xi,n_2} + v_{\xi,n_2} - \sum_{a\in A^+(n_1)} \widehat{z}_{\widehat{\xi},a}$$
$$\forall n_2 = (i,\overline{t}_p), n_1 = (i,\underline{t}_{p+1}) \in \mathcal{N}^S, \forall \widehat{\xi} \in \widehat{\Xi}_p \tag{A16}$$

$$\widehat{u}_{\widehat{\xi},n_1} - \widehat{v}_{\widehat{\xi},n_1} = \widehat{d}_{\widehat{\xi},n_1} + \widehat{u}_{\widehat{\xi},n_2} - \widehat{v}_{\widehat{\xi},n_2} + \sum_{a\in A^-(n_1)\cap\widetilde{\mathcal{A}}_{p-1}} z'_{\xi,p,a} + \sum_{a\in A^-(n_1)\cap\mathcal{A}_p} z_{\xi,a}$$
$$+ \sum_{k\in\widehat{\mathcal{P}}_p} \sum_{a\in A^-(n_1)\cap\mathcal{A}_k} \widehat{z}_{\widehat{\xi},a} \;\; \forall n_1 = (i,t), n_2 = (i,t-1) \in \mathcal{N}_k \cap \mathcal{N}^D, \forall k \in \widehat{\mathcal{P}}_p, \forall \widehat{\xi} \in \widehat{\Xi}_p \tag{A17}$$

$$\widehat{u}_{\widehat{\xi},n} - \widehat{v}_{\widehat{\xi},n} = \widehat{d}_{\widehat{\xi},n_1} + u_{\xi,n_2} - v_{\xi,n_2} + \sum_{a\in A^-(n_1)\cap\widetilde{\mathcal{A}}_{p-1}} z'_{\xi,p,a} + \sum_{a\in A^-(n_1)\cap\mathcal{A}_p} z_{\xi,a}$$
$$+ \sum_{k\in\widehat{\mathcal{P}}_p} \sum_{a\in A^-(n_1)\cap\mathcal{A}_k} \widehat{z}_{\widehat{\xi},a} \;\; \forall n_2 = (i,\overline{t}_p), n_1 = (i,\underline{t}_{p+1}) \in \mathcal{N}^D, \forall \widehat{\xi} \in \widehat{\Xi}_p \tag{A18}$$

$$\widehat{u}_{\widehat{\xi},n} \leq \overline{q}_n \;\; \forall n \in \mathcal{N}_k, \forall k \in \widehat{\mathcal{P}}_p, \forall \widehat{\xi} \in \widehat{\Xi}_p \tag{A19}$$

$$\widehat{z}_{\widehat{\xi}_1,a} = \widehat{z}_{\widehat{\xi}_2,a} \;\; \forall a \in \mathcal{A}_k, \forall(\widehat{\xi}_1,\widehat{\xi}_2) \in \widehat{\Lambda}_{p,k}, \forall k \in \widehat{\mathcal{P}}_p \tag{A20}$$

$$\widehat{u}_{\widehat{\xi},n_1} - \widehat{v}_{\widehat{\xi},n_1} + \sum_{t=\underline{t}_{k+1}}^{t_2} \sum_{n=(j,t)\in\mathcal{N}} \left(\sum_{a\in A^-(n)\cap\widetilde{\mathcal{A}}_{p-1}} z'_{\xi,p,a} + \sum_{a\in A^-(n)\cap\mathcal{A}_p} z_{\xi,a} + \sum_{k'\in\widehat{\mathcal{P}}_p} \sum_{a\in A^-(n)\cap\mathcal{A}_{k'}} \widehat{z}_{\widehat{\xi},a} \right)$$
$$+\bar{d}_{j,k+1,t_2} \le \bar{q}_{n_2}$$
$$\forall n_1 = (j,\bar{t}_k), n_2 = (j,t_2) \in \mathcal{N}^D, \forall((i,t_1),(j,t_2)) \in \widetilde{\mathcal{A}}_k, \forall k \in \widehat{\mathcal{P}}_p \setminus \{\bar{p}\}, \forall \widehat{\xi} \in \widehat{\Xi}_p \quad \text{(A21)}$$

$$\widehat{u}_{\widehat{\xi},n}, \widehat{v}_{\widehat{\xi},n}, \widehat{z}_{\widehat{\xi},a} \ge 0 \;\; \forall n \in \mathcal{N}_k, \forall a \in \mathcal{A}_k, \forall k \in \widehat{\mathcal{P}}_p, \forall \widehat{\xi} \in \widehat{\Xi}_p. \quad \text{(A22)}$$

Proposition 5. *The optimality inequalities (A13)–(A22) are valid for problem $\mathbf{P}_{\xi,p}$, where $p \in \mathcal{P} \setminus \{\bar{p}\}$ and $\xi \in \Xi$.*

The proof is similar to that of Proposition 4 and is thus omitted here.

A3 Mathematical proofs

This section presents the proofs to the theorems, propositions, and lemmas introduced in the main text.

A3.1 Proof of Theorem 1

Proof. We only need to prove the case with a single scenario and a single period (i.e., $|\Xi| = 1$ and $|\mathcal{T}| = 1$), to which any case with $|\Xi| \ge 1$ and $|\mathcal{T}| \ge 1$ can be reduced. To prove its NP-hardness, we use a reduction from the following NP-complete problem (Garey and Johnson 1983).

Subset Sum Problem (SSP). Given a finite set $N = \{1, ..., n\}$, size $d_i \in \mathbb{Z}^+$, $\forall i \in N$, and a positive integer B, is there a subset $N' \subseteq N$ such that $\sum_{i\in N'} d_i = B$? We only consider the case with $\sum_{i\in N} d_i > B$, as otherwise, the problem is trivial.

For any arbitrary instance of SSP with $\sum_{i\in N} d_i > B$, consider the following polynomial reduction to an instance of the SFPTMP with $|\Xi| = 1$ and $|\mathcal{T}| = 1$. Let each element $i \in N$ indicate a supply site with supply d_i, then we have $\mathcal{I}^S = \{1, ..., n\}$. Let $\mathcal{I}^D = \{D_1, D_2\}$ be the set of demand sites with demand $d_{D_1} = -B$ and $d_{D_2} = B - \sum_{i\in\mathcal{I}^S} d_i$. We set the initial inventory levels $q_i^0 = 0$, $\forall i \in \mathcal{I}$. The upper bounds for holding inventories are set as $\bar{q}_i = 0$, $\forall i \in \mathcal{I}$.

Let $\mathcal{L} = \{(i,j) | i \in \mathcal{I}^S, j \in \mathcal{I}^D\}$ be the set of lanes, and each lane $(i,j) \in \mathcal{L}$ has a shipment time $o_{i,j} = 0$ (i.e., shipments can be completed within one period). Lane $(i,j) \in \mathcal{L}$ is associated with only one bid and let $\mathcal{B}_{(i,j)} = \{b_{(i,j)}\}$. The capacity range of any bid $b_{(i,j)}$ is set as $[d_i, d_i]$, $\forall (i,j) \in \mathcal{L}$. Further, the bid $b_{(i,j)}$ of any lane $(i,j) \in \mathcal{L}$ contains one shipment $r_{(i,j)}$ such that $t_1(r_{(i,j)}) = t_2(r_{(i,j)}) = 1$. For any lane $(i,j) \in \mathcal{L}$, we set the freight rate for purchasing capacity from the bid $b_{(i,j)}$ as $f_{b_{(i,j)}} = 1/d_i$. Besides, the variable shipping costs in the bids are set to be $g_{b_{(i,j)}} = 0$, $\forall (i,j) \in \mathcal{L}$. In addition, for any site $i \in \mathcal{I}$, we set the unit inventory holding cost and the unit backlog cost to be $h_i = 2n$ and $e_i = 3n$, respectively. Finally, the non-contractual freight rates are set as $c_{i,j} = 2n$, $\forall (i,j) \in \mathcal{L}$. Now we prove that the minimum total cost of the instance is at most n if and only if the answer to the SSP is "yes".

On the one hand, suppose there exists such a subset N' for the SSP. Let $x_b \in \{0,1\}$ denote whether a bid $b \in \mathcal{B}$ is selected in the solution of the SFPTMP instance. Then for any $i \in N'$, we select bid $b_{(i,D_1)}$ (i.e., $x_{b_{(i,D_1)}} = 1$) and set its capacity to be d_i. Meanwhile for any $i \in N \setminus N'$, we select bid $b_{(i,D_2)}$ (i.e., $x_{b_{(i,D_2)}} = 1$) and set its capacity to be d_i. Let $z^1_{(i,j)}$ and $z^2_{(i,j)}$ denote the volume of the commodity shipped on lane $(i,j) \in \mathcal{L}$ through the shipment $r_{(i,j)}$ in bid $b_{(i,j)}$ and the non-contractual freight rate, respectively. We then set $z^1_{(i,j)} = d_i$, if $x_{b_{(i,j)}} = 1$ and $z^1_{(i,j)} = 0$, otherwise, $\forall (i,j) \in \mathcal{L}$. Meanwhile, we let $z^2_{(i,j)} = 0$, $\forall (i,j) \in \mathcal{L}$. Because $\sum_{i\in N'} d_i = -d_{D_1}$, $\sum_{i\in N\setminus N'} d_i = -d_{D_2}$, and $\sum_{i\in N} d_i = -(d_{D_1} + d_{D_2})$, the demand at each demand site is exactly satisfied with zero inventories and backlogs at all sites. Therefore, this is a feasible solution. From the settings of the cost components, the total cost is n, indicating that the instance has a minimum total cost no larger than n.

On the other hand, suppose that we have an optimal solution to the SFPTMP instance with a total cost no larger than n. Consider the subset $N' = \{i \in \mathcal{I}^S | x_{b_{(i,D_1)}} = 1\}$. We next prove that $\sum_{i\in N'} d_i = B$, indicating that the answer to the SSP is "yes". To this end, we first show that for any supply site $i \in \mathcal{I}^S$, the equation $x_{b_{(i,D_1)}} + x_{b_{(i,D_2)}} = 1$ must hold in the optimal solution. We prove this by contradiction.

First, suppose that in the optimal solution, there exists a supply site $i' \in \mathcal{I}^S$ such that $x_{b_{(i',D_1)}} + x_{b_{(i',D_2)}} = 0$. In this case, let $\delta_{i'}$ denote the remaining (unshipped) supply at this site, indicating that a total volume of $(d_{i'} - \delta_{i'})$ is shipped from this site through the non-contractual rates. As a result, the shipping cost of this solution should at least be $2n(d_{i'} - \delta_{i'})$. In addition, because $q_i^0 = 0$ and $\bar{q}_i = 0$ with $i \in \mathcal{I}$, the backlog level at site i' equals $\delta_{i'}$. Further, because $\sum_{i\in\mathcal{I}^S} d_i = -(d_{D_1} + d_{D_2})$, the total backlog volume at the demand sites must also at least be $\delta_{i'}$. These indicate that the backlog cost of the solution is at least $6n\delta_{i'}$. Summing these costs, we have that the total cost of this solution should at least be $2nd_{i'} + 4n\delta_{i'} > n$, which forms a contradiction. Therefore, the equation $x_{b_{(i,D_1)}} + x_{b_{(i,D_2)}} \geq 1$ must hold for any $i \in \mathcal{I}^S$ in the optimal solution.

Second, suppose there exists a supply site $i' \in \mathcal{I}^S$ such that $x_{b_{(i',D_1)}} + x_{b_{(i',D_2)}} = 2$. For any lane $(i,j) \in \mathcal{L}$, bid $b_{(i,j)}$ is associated with a freight rate $1/d_i$ and a capacity range $[d_i, d_i]$, which indicates that the cost of selecting any bid equals 1. Therefore, the total cost of selecting bids in the solution can be calculated as $\sum_{i\in\mathcal{I}^S\setminus\{i'\}}(x_{b_{(i,D_1)}} + x_{b_{(i,D_2)}}) + x_{b_{(i',D_1)}} + x_{b_{(i',D_2)}}$. Since we have proved that $x_{b_{(i,D_1)}} + x_{b_{(i,D_2)}} \geq 1, \forall i \in \mathcal{I}^S$, if $x_{b_{(i',D_1)}} + x_{b_{(i',D_2)}} = 2$, the cost of bid selection in this solution should at least be $n+1$, which again forms a contradiction. Therefore, we have $x_{b_{(i,D_1)}} + x_{b_{(i,D_2)}} = 1$, for any $i \in \mathcal{I}^S$ in the optimal solution.

Because $x_{b_{(i,D_1)}} + x_{b_{(i,D_2)}} = 1, \forall i \in \mathcal{I}^S$, the cost for bid selection in the solution equals n. This further indicates that the inventory and backlog levels at all sites should be zero and the volume of the shipment acquired through the non-contractual freight rate at any lane should also be zero in the solution. For these conditions to hold, the solution must satisfy $\sum_{i\in N'} d_i = B$. This completes the proof. □

A3.2 Proof of Proposition 1

Let $\boldsymbol{x}^* = (x_b^* | b \in \mathcal{B})$, $\boldsymbol{y}^* = (y_b^* | b \in \mathcal{B})$, $\boldsymbol{z}^* = (z_{\xi,a}^* | a \in \mathcal{A}, \xi \in \Xi)$, $\boldsymbol{u}^* = (u_{\xi,n}^* | n \in \mathcal{N}, \xi \in \Xi)$, and $\boldsymbol{v}^* = (v_{\xi,n}^* | n \in \mathcal{N}, \xi \in \Xi)$ be the vectors for the values of variables x_b, y_b, $z_{\xi,a}$, $u_{\xi,n}$, and $v_{\xi,n}$ in an *optimal* solution (denoted by $\boldsymbol{X}^*$) of $\mathbf{P}$. We have the following lemma.

Lemma 2. *$\boldsymbol{X}^*$ satisfies the following equalities:*

$$\min\{v_{\xi,n}^*, \overline{q}_n - u_{\xi,n}^*\} = 0 \;\; \forall n \in \mathcal{N}^S, \forall \xi \in \Xi, \tag{A23}$$

$$\min\{u_{\xi,n}^*, v_{\xi,n}^*\} = 0 \;\; \forall n \in \mathcal{N}^D, \forall \xi \in \Xi. \tag{A24}$$

Proof of Lemma 2. Supposing (A23) do not hold, for some $n \in \mathcal{N}^S$, we must have $u_{\xi,n}^* < \overline{q}_n$ and $v_{\xi,n}^* > 0$. Let $\sigma = \min\{v_{\xi,n}^*, \overline{q}_n - u_{\xi,n}^*\}$. We have $\sigma > 0$.

Consider a solution (denoted by $\boldsymbol{X}'$) for problem $\mathbf{P}$ in which $u_{\xi,n} = u_{\xi,n}^* + \sigma$ and $v_{\xi,n} = v_{\xi,n}^* - \sigma$ and other variables remain the same as in $\boldsymbol{X}^*$. It is easy to check that $\boldsymbol{X}'$ is feasible. Let Z' and Z^* denote objective function values associated with $\boldsymbol{X}'$ and $\boldsymbol{X}^*$, respectively. We have $Z' - Z^* = (h_n - e_n)\sigma$. Because $h_n < e_n$, $Z' - Z^* < 0$, which is a contradiction of the optimality of $\boldsymbol{X}^*$. Therefore, (A23) must hold for any optimal solution of $\mathbf{P}$.

The process to show that (A24) must hold for any optimal solution $\mathbf{P}$ is similar, and thus we omit it here. □

Proof of Proposition 1. From the definition of Λ_p, we have $(\xi_1, \xi_2) \in \Lambda_p$ if and only if $(\xi_1, \xi_2) \in \Lambda_{p'}$, $\forall p' \in \{1, ..., p\}$, where $p \in \mathcal{P}$. Then, given any $(\xi_1, \xi_2) \in \Lambda_p$ and $p \in \mathcal{P}$, due to constraints (10), one must have

$$z^*_{\xi_1,a} = z^*_{\xi_2,a} \ \ \forall a \in \mathcal{A}_{p'}, \forall p' \in \{1, ..., p\}. \tag{A25}$$

It is therefore easy to infer that

$$\sum_{a\in A^+(n)} z^*_{\xi_1,a} = \sum_{a\in A^+(n)} z^*_{\xi_2,a} \ \ \forall n \in \mathcal{N}_{p'}, \forall p' \in \{1, ..., p\}, \tag{A26}$$

$$\sum_{a\in A^-(n)} z^*_{\xi_1,a} = \sum_{a\in A^-(n)} z^*_{\xi_2,a} \ \ \forall n \in \mathcal{N}_{p'}, \forall p' \in \{1, ..., p\}. \tag{A27}$$

Further, combining these two equations with constraints (5)–(8) gives us:

$$u^*_{\xi_1,n} + v^*_{\xi_1,n} = u^*_{\xi_2,n} + v^*_{\xi_2,n} \ \ \forall n \in \mathcal{N}_{p'} \cap \mathcal{N}^S, \forall p' \in \{1, ..., p\}, \tag{A28}$$

$$u^*_{\xi_1,n} - v^*_{\xi_1,n} = u^*_{\xi_2,n} - v^*_{\xi_2,n} \ \ \forall n \in \mathcal{N}_{p'} \cap \mathcal{N}^D, \forall p' \in \{1, ..., p\}. \tag{A29}$$

Finally, from the results in Lemma 2 and equations (A28) and (A29), we have

$$u^*_{\xi_1,n} = u^*_{\xi_2,n} \ \ \forall n \in \mathcal{N}_{p'}, \forall p' \in \{1, ..., p\}, \tag{A30}$$

$$v^*_{\xi_1,n} = v^*_{\xi_2,n} \ \ \forall n \in \mathcal{N}_{p'}, \forall p' \in \{1, ..., p\}. \tag{A31}$$

Therefore, by solving **P** to optimality, we have identical decisions under scenarios $\xi_1, \xi_2 \in \Lambda_p$ in any stage $p' \in \{1, ..., p\}$. This completes the proof. □

A3.3 Proof of Lemma 1

Proof. Given any site $i \in \mathcal{I}^D$, let $\bar{\omega}^i_p = \arg\max_{\omega\in\Omega_p} \sum_{t=\underline{t}_p}^{\bar{t}_p} d^{\omega}_{i,t}$, and let $\bar{\omega}^i_{p,t} = \arg\max_{\omega\in\Omega_p} \sum_{t'=\underline{t}_p}^{t} d^{\omega}_{i,t'}$, where $t \in \mathcal{T}_p$ and $p \in \mathcal{P}$.

Given any stages $p_1, p_2 \in \mathcal{P}$ with $p_2 > p_1$ and a period $t_2 \in \mathcal{T}_{p_2}$, for any $i \in \mathcal{I}^D$, let Ξ^0 be the set of scenarios such that $\forall \xi \in \Xi^0$, $\omega_p(\xi) = \bar{\omega}^i_p$, $\forall p \in \{p_1 + 1, ..., p_2 - 1\}$ and $\omega_{p_2}(\xi) = \bar{\omega}^i_{p_2,t_2}$. By summing constraints (7) for site i under any scenario $\xi^0 \in \Xi^0$ over all periods $t' \in \{\underline{t}_{p_1+1}, ..., t_2\}$ we have

$$u_{\xi^0,n_2} - v_{\xi^0,n_2} = u_{\xi^0,n_1} - v_{\xi^0,n_1} + \bar{d}_{i,p_1+1,t_2} + \sum_{t'=\underline{t}_{p_1+1}}^{t_2} \sum_{n=(i,t')\in\mathcal{N}} \sum_{a\in A^-(n)} z_{\xi^0,a} \ \ \forall \xi^0 \in \Xi^0, \tag{A32}$$

where $n_1 = (i, \bar{t}_{p_1})$ and $n_2 = (i, t_2)$.

Because $v_{\xi^0,n_2} \geq 0$ and $z_{\xi^0,a} \geq 0$, $\forall a \in A^-(n)$ the following inequality holds:

$$u_{\xi^0,n_2} \geq u_{\xi^0,n_1} - v_{\xi^0,n_1} + \bar{d}_{i,p_1+1,t_2} + \sum_{t'=\underline{t}_{p_1+1}}^{t_2} \sum_{n=(i,t')\in\mathcal{N}} \sum_{p'=1}^{p_1} \sum_{a\in A^-(n)\cap\mathcal{A}_{p'}} z_{\xi^0,a} \ \ \forall \xi^0 \in \Xi^0 \tag{A33}$$

Due to constraints (9), we have

$$u_{\xi^0,n_1} - v_{\xi^0,n_1} + \bar{d}_{i,p_1+1,t_2} + \sum_{t'=\underline{t}_{p_1+1}}^{t} \sum_{n=(i,t')\in\mathcal{N}} \sum_{p'=1}^{p_1} \sum_{a\in A^-(n)\cap\mathcal{A}_{p'}} z_{\xi^0,a} \leq \overline{q}_{n_2} \ \ \forall \xi^0 \in \Xi^0. \tag{A34}$$

Further, given any $\xi^0 \in \Xi^0$, let $\Xi(\xi^0) \subseteq \Xi$ be the set of scenarios such that $\Xi(\xi^0) = \{\xi \in \Xi | \xi = \xi^0 \vee (\xi, \xi^0) \in \Lambda_{p_1}\}$. From Proposition 1, we have

$$u_{\xi,n_1} - v_{\xi,n_1} + \bar{d}_{i,p_1+1,t_2} + \sum_{t'=\underline{t}_{p+1}}^{t} \sum_{n=(i,t')\in\mathcal{N}} \sum_{p'=1}^{p_1} \sum_{a\in A^-(n)\cap\mathcal{A}_{p'}} z_{\xi,a} \leq \overline{q}_{n_2} \forall \xi \in \Xi(\xi^0), \forall \xi^0 \in \Xi^0. \tag{A35}$$

In addition, the structure of the scenario tree implies that $\bigcup_{\xi^0\in\Xi^0} \Xi(\xi^0) = \Xi$, and the final result follows directly. □

A3.4 Proof of Proposition 2

Proof. We show that problem $\mathbf{P}'_{\xi,p}$, where $p \in \mathcal{P}$ and $\xi \in \Xi$, is feasible by constructing a feasible solution ($\mathcal{S}$) to the problem as follows.

First, in $\mathcal{S}$, we let $z_{\xi,a} = 0$, $\forall a \in \mathcal{A}_p$. In the sequel, for the solution to be feasible, we must have

$$u_{\xi,n_2} = \bar{u}_{\xi,p-1,n_1} + \bar{v}_{\xi,p-1,n_1} + \sum_{t'=\underline{t}_p}^{t} \sum_{n=(j,t')\in\mathcal{N}} d_{\xi,n} - v_{\xi,n_2}$$
$$\forall n_1 = (j,\bar{t}_{p-1}), n_2 = (j,t) \in \mathcal{N}, \forall j \in \mathcal{I}^S, \forall t \in \mathcal{T}_p, \tag{A36}$$
$$u_{\xi,n_2} = \bar{u}_{\xi,p-1,n_1} - \bar{v}_{\xi,p-1,n_1} + \sum_{t'=\underline{t}_p}^{t} \sum_{n=(j,t')\in\mathcal{N}} (d_{\xi,n} + \sum_{a\in A^-(n)\cap\widetilde{\mathcal{A}}_{p-1}} \bar{z}_{\xi,p-1,a}) + v_{\xi,n_2}$$
$$\forall n_1 = (i,\bar{t}_{p-1}), n_2 = (j,t) \in \mathcal{N}, \forall j \in \mathcal{I}^D, \forall t \in \mathcal{T}_p. \tag{A37}$$

To show that such a feasible $\mathcal{S}$ exists, it suffices to show that for any $n_2 = (j,t) \in \mathcal{N}_p$ there exists a $v_{\xi,n_2} \geq 0$ such that:

$$\bar{u}_{\xi,p-1,n_1} + \bar{v}_{\xi,p-1,n_1} + \sum_{t'=\underline{t}_p}^{t} \sum_{n=(j,t')\in\mathcal{N}} d_{\xi,n} - v_{\xi,n_2} \geq 0, \tag{A38}$$
$$\bar{u}_{\xi,p-1,n_1} + \bar{v}_{\xi,p-1,n_1} + \sum_{t'=\underline{t}_p}^{t} \sum_{n=(j,t')\in\mathcal{N}} d_{\xi,n} - v_{\xi,n_2} \leq \overline{q}_{n_2}, \tag{A39}$$

if $j \in \mathcal{I}^S$ and

$$\bar{u}_{\xi,p-1,n_1} - \bar{v}_{\xi,p-1,n_1} + \sum_{t'=\underline{t}_p}^{t} \sum_{n=(j,t')\in\mathcal{N}} (d_{\xi,n} + \sum_{a\in A^-(n)\cap\widetilde{\mathcal{A}}_{p-1}} \bar{z}_{\xi,p-1,a}) + v_{\xi,n_2} \geq 0, \tag{A40}$$
$$\bar{u}_{\xi,p-1,n_1} - \bar{v}_{\xi,p-1,n_1} + \sum_{t'=\underline{t}_p}^{t} \sum_{n=(j,t')\in\mathcal{N}} (d_{\xi,n} + \sum_{a\in A^-(n)\cap\widetilde{\mathcal{A}}_{p-1}} \bar{z}_{\xi,p-1,a}) + v_{\xi,n_2} \leq \overline{q}_{n_2}, \tag{A41}$$

if $j \in \mathcal{I}^D$, where $n_1 = (j,\bar{t}_{p-1}) \in \mathcal{N}$.

One can easily verify that inequalities (A38)–(A41) hold as long as we have:

$$\bar{u}_{\xi,p-1,n_1} - \bar{v}_{\xi,p-1,n_1} + \sum_{t'=\underline{t}_p}^{t} \sum_{n=(j,t')\in\mathcal{N}} (d_{\xi,n} + \sum_{a\in A^-(n)\cap\widetilde{\mathcal{A}}_{p-1}} \bar{z}_{\xi,p-1,a}) \leq \overline{q}_{n_2} \tag{A42}$$

for the case $j \in \mathcal{I}^D$.

Note that $d_{\xi,n} \leq 0$, $\forall n \in \mathcal{N}^D$ and $\overline{q}_{n_2} = \overline{q}_j$, $\forall n_2 = (j,t) \in \mathcal{N}^D$. Hence, if $p = 1$, we have $\widetilde{\mathcal{A}}_0 = \varnothing$ and (A42) holds directly as long as the original problem $\mathbf{P}$ is feasible. If $p > 1$, (A42) holds if we have:

$$\bar{u}_{\xi,p-1,n_1} - \bar{v}_{\xi,p-1,n_1} + \sum_{t'=\underline{t}_p}^{t} \sum_{n=(j,t')\in\mathcal{N}} \sum_{a\in A^-(n)\cap\widetilde{\mathcal{A}}_{p-1}} \bar{z}_{\xi,p-1,a} + \sum_{t'=\underline{t}_p}^{t} \sum_{n=(j,t')\in\mathcal{N}} d_{\xi,n} \leq \overline{q}_{n_2}$$
$$\forall n_1 = (i,\bar{t}_{p-1}), n_2 = (j,t_2) \in \mathcal{N}^D, \forall((i,t_1),(j,t_2)) \in \widetilde{\mathcal{A}}_{p-1}. \tag{A43}$$

By definition, we have $\sum_{t'=\underline{t}_p}^{t} \sum_{n=(j,t')\in\mathcal{N}} d_{\xi,n} \leq \bar{d}_{j,p,t}$. Therefore, from constraints (21), we have that (A42) is valid for $\mathbf{P}'_{\xi,p}$ with $p > 1$. This completes the proof. □

A3.5 Proof of Proposition 3

Proof. We prove correctness of the proposition only for inequalities (A1)−(A10), as the result can be easily extended to inequalities (A13)−(A22). Let $\mathscr{T}_0$ and $\widehat{\Xi}_0$ denote the approximate tree for stage 0 and the set of scenarios associated with it, respectively. Consider the problem $\widehat{\mathbf{R}}(\boldsymbol{x},\boldsymbol{y})$ which is formulated as follows:

$$\begin{aligned} \widehat{\mathbf{R}}(\boldsymbol{x},\boldsymbol{y}) \quad &= \quad \min \sum_{\widehat{\xi}\in\widehat{\Xi}_0} \widehat{\rho}_{\widehat{\xi}} \left(\sum_{n\in\mathcal{N}} \left(h_n \widehat{u}_{\widehat{\xi},n} + e_n \widehat{v}_{\widehat{\xi},n} \right) + \sum_{a\in\mathcal{A}} c_a \widehat{z}_{\widehat{\xi},a} \right) \\ &\quad \text{s.t.} \quad (\text{A2}) - (\text{A10}). \end{aligned} \tag{A44}$$

Additionally, let $\mathscr{T}_0'$ be an approximate tree for the same stage such that $|\widehat{\Omega}_k| = 1$ for any stage $k \in \widehat{P}_0$. Following Proposition 4, the inequalities (A1)−(A10) defined on $\mathscr{T}_0'$ are valid for the problem $\widehat{\mathbf{R}}(\boldsymbol{x},\boldsymbol{y})$. This completes the proof. □

A4 Details of instance generation

All instances were created from five cases which were generated based on five instances selected from the instance set provided by Papageorgiou et al. (2014) for the maritime inventory-routing problem (MIRP). In each of the five selected MIRP instances, there are one supply port and eight demand ports. Each instance covers a planning horizon of 360 days and the (deterministic) daily supply or demand generated at each port is provided.

We proceed as follows to convert an MIRP instance into an SFPTMP case. To begin with, each supply (demand) port in the MIRP instance corresponds to a supply (demand) site in a case. Second, in each case, we let a period $t \in \mathcal{T}$ contain seven consecutive days (a week) and the planning horizon consists of 54 periods (378 days). The nominal demand $\bar{d}_{it}$ at site $i \in \mathcal{I}^D$ in period $t \in \mathcal{T}$ is set equal to the sum of the daily demands of the corresponding port that are associated with period t in the MIRP instance (we set the daily demands of the days later than the 360th day equal to those of the 360th day in the MIRP instance).

Other parameters in a case were generated as follows. The initial (q_i^0) and maximum inventory levels ($\overline{q}_i$) at each site $i \in \mathcal{I}$ were set equal to those at the corresponding port in the related MIRP instance. The unit inventory cost h_i was set to 0 for all sites $i \in \mathcal{I}$, aligning with the MIRP instance settings. For sites in $\mathcal{I}^S$, the unit backlogging cost e_i was set at 0.05. For sites in $\mathcal{I}^D$, it was set at $1.1(\max_{j\in\mathcal{I}^S} c_{j,i})$, where $c_{j,i}$ represents the non-contractual freight rate on lane $(j,i) \in \mathcal{L}$.

The commodity can be shipped on the lane between any supply site and any demand site. We let the shipping time $o_{i,j} = \lceil \bar{o}_{i,j}/7 \rceil$ for all $(i,j) \in \mathcal{L}$, where $\bar{o}_{i,j}$ (in days) is the travel time between the corresponding ports in the associated MIRP instance. The non-contractual freight rate on each lane $(i,j) \in \mathcal{L}$ is set as $c_{i,j} = 0.0005 DIS_{i,j}$, where $DIS_{i,j}$ represents the distance (km) between the corresponding ports in the original MIRP instance.

The bids were created as follows. Each bid is characterized by a shipment capacity range and a shipment frequency. Let $\overline{C}$ be the maximum of the vessel capacities in the original MIRP instance, and let $\overline{Q}_j = \min\{\overline{C}, \overline{q}_j\}$, $\forall j \in \mathcal{I}$. For generating the bids on a lane $(i,j) \in \mathcal{L}$, three shipment capacity ranges were used, which are $[\lfloor 0.25\overline{Q}_j \rfloor, \lfloor 0.5\overline{Q}_j \rfloor]$, $[\lfloor 0.5\overline{Q}_j \rfloor + 1, \lfloor 0.75\overline{Q}_j \rfloor]$, and $[\lfloor 0.75\overline{Q}_j \rfloor + 1, \lfloor \overline{Q}_j \rfloor]$. We also used three shipping frequencies, where the intervals between two consecutive shipments in a bid are set to two, four, and six periods. There are thus nine combinations of capacity ranges and shipping frequencies, and for each combination, we generate a bid. Hence, we have $|\mathcal{B}_{i,j}| = 9$, $\forall (i,j) \in \mathcal{L}$. The freight rate f_b of a bid $b \in \mathcal{B}_{i,j}$ was set as follows:

$$f_b = \begin{cases} 0.8 c_{i,j}, & \text{if } [\underline{m}_b, \overline{m}_b] = [\lfloor 0.25\overline{Q}_j \rfloor, \lfloor 0.5\overline{Q}_j \rfloor], \\ 0.7 c_{i,j}, & \text{if } [\underline{m}_b, \overline{m}_b] = [\lfloor 0.5\overline{Q}_j \rfloor + 1, \lfloor 0.75\overline{Q}_j \rfloor], \\ 0.6 c_{i,j}, & \text{if } [\underline{m}_b, \overline{m}_b] = [\lfloor 0.75\overline{Q}_j \rfloor + 1, \lfloor \overline{Q}_j \rfloor]. \end{cases}$$

Further, the variable transportation cost in each bid $b \in \mathcal{B}$ was set as $g_b = 0$. As for the shipment schedules, given any bid b, the start time of its first shipment was randomly selected from the set of periods $\{1, 2, 3\}$ and the start times of subsequent shipments were set according to the shipping frequency. The number of shipments in the bid was set to the maximum number of shipments that can be completed within the planning horizon, which was determined by the start time of the first shipment, the shipping frequency, and the transportation time of a shipment in the bid.

In any SFPTMP instance, supplies are generated only in the first period ($\underline{t}_p$) in any stage $p \in \mathcal{P}$. That is, we let $d^{\omega}_{i,t} = 0$, $\forall t \in \mathcal{T}_p \setminus \{\underline{t}_p\}$, $\forall p \in \mathcal{P}$, $\forall i \in \mathcal{I}^S$. We assume that supplies and demands are balanced in each stage. In particular, in any of these instances, given a stage $p \in \mathcal{P}$ and a stage scenario $\omega \in \Omega_p$, the supply produced in the (sole) supply site in period $\underline{t}_p$ under this scenario was set equal to $-\sum_{i \in \mathcal{I}^D} \sum_{t \in \mathcal{T}_p} d^{\omega}_{i,t}$.

A5 Details of benchmark methods

We adopted two benchmark methods for solving the SFPTMP. Their implementation details are explained below.

When applying BM1 to solve an instance, we run the SDDP approach (i.e., S2) in which the x variables in problem $\mathbf{P}_0$ are set to zero. Besides, when applying BM2 to solve an instance, we first solve a two-stage stochastic optimization version of problem $\mathbf{P}$ for selecting the bids. In two-stage stochastic optimization, the shipper determines which bids to accept at the first stage while the second stage contains $|\Omega_1|$ shipment subproblems. Each shipment subproblem is associated with a stage scenario $\omega \in \Omega_1$. For the subproblem associated with $\omega \in \Omega_1$, let $d'_{\omega,n}$ denote the demand or supply at node $n \in \mathcal{N}$. We have $d'_{\omega,n} = d_{\omega,n}$, $\forall n \in \mathcal{N}_1$ and $d'_{\omega,n} = \frac{\sum_{\varpi \in \Omega_p} d_{\varpi,n}}{|\Omega_p|}$, $\forall n \in \mathcal{N}_p$, $p \in \{2, ..., \bar{p}\}$.

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