Les Cahiers du GERAD ISSN: 0711–2440

Optimal planning of preventive maintenance tasks on power transmission systems

M. Rocha, M. F. Anjos, M. Gendreau

G–2022–06

March 2022

Citation suggérée : M. Rocha, M. F. Anjos, M. Gendreau (Mars 2022). Optimal planning of preventive maintenance tasks on power transmission systems, Rapport technique, Les Cahiers du GERAD G– 2022–06, GERAD, HEC Montréal, Canada.

Avant de citer ce rapport technique, veuillez visiter notre site Web (https://www.gerad.ca/fr/papers/G-2022-06) afin de mettre à jour vos données de référence, s'il a été publié dans une revue scientifique.

Suggested citation: M. Rocha, M. F. Anjos, M. Gendreau (March 2022). Optimal planning of preventive maintenance tasks on power transmission systems, Technical report, Les Cahiers du GERAD G–2022–06, GERAD, HEC Montréal, Canada.

Before citing this technical report, please visit our website (https://www.gerad.ca/en/papers/G-2022-06) to update your reference data, if it has been published in a scientific journal.

La publication de ces rapports de recherche est rendue possible grâce au soutien de HEC Montréal, Polytechnique Montréal, Université McGill, Université du Québec à Montréal, ainsi que du Fonds de recherche du Québec – Nature et technologies.

The publication of these research reports is made possible thanks to the support of HEC Montréal, Polytechnique Montréal, McGill University, Université du Québec à Montréal, as well as the Fonds de recherche du Québec – Nature et technologies.

Dépôt légal – Bibliothèque et Archives nationales du Québec, 2022
– Bibliothèque et Archives Canada, 2022

Legal deposit – Bibliothèque et Archives nationales du Québec, 2022
– Library and Archives Canada, 2022

GERAD HEC Montréal
3000, chemin de la Côte-Sainte-Catherine
Montréal (Québec) Canada H3T 2A7

Tél. : 514 340-6053
Téléc. : 514 340-5665
info@gerad.ca
www.gerad.ca

Optimal planning of preventive maintenance tasks on power transmission systems

Mariana Rocha [a, b, c]

Miguel F. Anjos [a, d]

Michel Gendreau [b, c]

[a] *GERAD, Montréal (Qc), Canada, H3T 1J4*

[b] *CIRRELT, Montréal (Qc), Canada, H3T 1J4*

[c] *Department of Mathematics and Industrial Engineering, Polytechnique Montréal, Montréal (Qc), Canada, H3T 1J4*

[d] *School of Mathematics, University of Edinburgh, Edinburgh, United Kingdom*

mariana.rocha@polymtl.ca
Miguel.F.Anjos@ed.ac.uk
michel.gendreau@polymtl.ca

March 2022
Les Cahiers du GERAD
G–2022–06

Abstract : Every component of an electric power system is susceptible to failure. The power transmission system connects generating units to local distribution systems, and its central operational role means that the scheduling of preventive maintenance of transmission lines must be carefully planned. This planning aims to ensure uninterrupted power supply by reducing equipment failures and accidents, increasing the quality of the energy supplied, and generally maintaining network reliability. The transmission maintenance scheduling problem is concerned with selecting the optimal periods to remove specified transmission lines from operation to carry out preventive maintenance. We propose a mixed-integer linear optimization formulation of this problem for a planning period of one year. This formulation schedules preventive maintenance while ensuring that the transmission system stays connected even in case of an unexpected line failure. The resulting large-scale optimization problem is solved using a new decomposition algorithm that divides the large problem into two smaller optimization problems. One of these problems is solved with CPLEX through Benders decomposition, and the second validates the solution found. We report computational results with the IEEE 24-bus system that demonstrate that our algorithm achieves the required accuracy and solves the problem more efficiently than solving the complete formulation without decomposition.

Keywords: Power transmission maintenance, maintenance scheduling, preventive maintenance, decomposition methods, mixed-integer linear optimization, network reliability

Acknowledgements: This research was supported by the NSERC-Hydro-Quebec-Schneider Electric Industrial Research Chair on Optimization for Smart Grids.

1 Introduction

Every component of an electric power system is susceptible to failure. Preventive maintenance aims to correct non-urgent disorders and ensure uninterrupted power supply by reducing equipment failures and accidents, increasing the quality of the energy supplied, and maintaining network reliability. The power transmission system connects the generating units to the local distribution systems, and its central operational role means that the scheduling of preventive maintenance of transmission lines must be carefully planned.

The problem of selecting which generators and transmission lines (and any other grid component) will be taken offline for maintenance and when, is known in the literature as Generation Maintenance Scheduling (GMS) and Transmission Maintenance Scheduling (TMS). The GMS, TMS or an Integrated Maintenance Scheduling (IMS) problems are optimization problems about deciding the best period for the removal of equipment for maintenance while meeting constraints of maintenance requirements and of power system operation. A significant amount of technical literature focuses on GMS while there are fewer studies on TMS [4, 6, 11, 12]. In general the proposed models are large-scale optimization problems with integer variables. The article [17] presents extensive analysis on GMS, while [3] describes maintenance scheduling problems emphasizing deregulated power systems, and [4] presents a thorough investigation on all types of maintenance scheduling problems.

Among articles on TMS, the model in [10] minimizes loss of revenue using a mixed-integer linear optimization problem (MILP) solved with Benders decomposition. The article [16] minimizes cost and a reliability feature applying a genetic algorithm, and [7] uses uncertainty in a minimization of costs and loss of profits with Benders decomposition. We note that in practice, scheduling decisions are subject to the quantity of energy produced and transported at any time, and this energy availability can be altered unexpectedly by the need to carry out urgent maintenance work.

We further note that the models in the literature often do not contain connectivity constraints that avoid islanding or grid division, contingency constraints that mitigate against the unexpected loss of a line, or account for a given target schedule for preventive maintenance. We briefly discuss each of these practical requirements in turn.

The idea of avoiding isolating buses or splitting the grid is taken into account by [8] where penalties are added when the solution contains these situations. The authors solve the TMS for the South-Wales network with coordination between a genetic algorithm and a heuristic. The idea is also considered by [6], who use a deep search algorithm to find critical lines and prevent them from being maintained, keeping the grid always connected. This TMS employs a filtering algorithm to deal with network constraints and solves the problem with a local search.

A major concern in transmission system operations is the unpredictable loss of a transmission line during the preventive maintenance of one or more other lines. This is because the network must maintain a high reliability and meet customer demand. In [9], random failures of the transmission equipment are considered through the use of the forced outage rate of each component and the existence of a demand that may not be fully met. In [15], the N-1 contingency approach handles the possibility of losing lines or generators. The N-1 security requirement means that, in the event of one emergency (loss of one equipment), the grid changes from its original state (N) to a less secure state (N-1) but still operates safely and meets customer demand to the largest possible extent.

The consideration of a target maintenance schedule is important in practice because ideally preventive maintenance is performed periodically as per the specifications of equipment manufacturers, international standards or other criteria [4, 12]. In [15], the maintenance preference of the owners of the generation and transmission facilities is an input parameter for the model, and the agreement with it is maximized for the presented IMS. The TMS in [11] uses penalties for performing maintenance outside the target period in a minimization of costs and risk of failure.

The objective of this work is to design a TMS model that computes an optimal trade-off between the maximization of energy supplied to customers and the execution of maintenance tasks as per their target times, with constraints that prevent isolating buses from the grid and take into account the unexpected loss of any transmission line.

Our main contribution is to present a TMS that models in a more comprehensive manner the constraints of urgent corrective maintenance events, so the network is fault resistant while in preventive maintenance, with the use of N-1 security inspired by the unit commitment problem, and the obligation to keep the power grid connected, with a verification of its connectivity through a flow problem. The objective is to meet the intended maintenance target time and, in the event of a line failure, to maximize energy flows to customers. The proposed TMS model is a large-scale MILP that is challenging to solve using off-the-shelf solvers. For this reason, we also propose a decomposition algorithm to solve the TMS problem in reduced computational time.

This paper is organized as follows. Section 2 describes the proposed MILP formulation for TMS. Section 3 presents the decomposition algorithm, and Section 4 presents the computational tests and results on the IEEE 24-bus Reliability Test System [14]. Section 5 concludes the paper.

2 Mathematical formulation

We represent the topology of the transmission system using a graph, where the electric buses are represented by nodes, and its transmission lines and line equipment by edges. A total horizon of one year divided into 52 weeks (time periods) is used, but the model is flexible and can be applied with other choices of time period.

The proposed model is formulated as follows:

$$\max \sum_{t\in\Lambda}(\sum_{k\in\Gamma}\sum_{c\in\Delta}\hat{d}^t_{ck} + \sum_{l\in\Delta} A^t_l y^t_l) \tag{1}$$

$$\text{s.t.}\quad y^t_l = 0 \qquad \forall\, l \in \Delta,\ t \in \Lambda^I \tag{2}$$

$$\sum_{t\in\Lambda} y^t_l = 0 \qquad \forall\, l \in \Delta^{NM} \tag{3}$$

$$\sum_{l\in\Delta^M} y^t_l \le L^t \qquad \forall\, t \in \Lambda \tag{4}$$

$$\sum_{t\in\Lambda} y^t_l = 1 \qquad \forall\, l \in \Delta^{M1} \tag{5}$$

$$\sum_{t\in\Lambda} y^t_l = 2 \qquad \forall\, l \in \Delta^{M2} \tag{6}$$

$$y^{t=2}_l \ge y^{t=1}_l \qquad \forall\, l \in \Delta^{M2} \tag{7}$$

$$y^t_l \ge y^{t-1}_l - y^{t-2}_l \qquad \forall l \in \Delta^{M2}, t \in 3..T \tag{8}$$

$$\sum_{t\in\Lambda} y^t_l = 3 \qquad \forall\, l \in \Delta^{M3} \tag{9}$$

$$y^{t=2}_l \ge y^{t=1}_l \qquad \forall\, l \in \Delta^{M3} \tag{10}$$

$$y^{t=3}_l \ge y^{t=2}_l \qquad \forall\, l \in \Delta^{M3} \tag{11}$$

$$y^t_l \ge y^{t-1}_l - \frac{1}{2}(y^{t-2}_l + y^{t-3}_l) \qquad \forall l \in \Delta^{M3}, t \in 4..T \tag{12}$$

$$y^t_l - y^t_u = 0 \qquad \forall t \in \Lambda, (l,u) \in \Xi, l \ne u \tag{13}$$

$$gen^t_k - D^t_k = \sum_{l\in\Delta} S_{kl}\ f^t_l \qquad \forall k \in \Gamma, t \in \Lambda \tag{14}$$

$$f^t_l = B_l \sum_{k\in\Gamma} S^T_{lk}\ \theta^t_k \qquad \forall l \in \Delta^{NM}, t \in \Lambda \tag{15}$$

$$\frac{f_l^t}{B_l} - \sum_{k\in\Gamma} S_{lk}^T\ \theta_k^t \geq -F_l^{Lim} y_l^t \qquad \forall l \in \Delta^M, t \in \Lambda \tag{16}$$

$$\frac{f_l^t}{B_l} - \sum_{k\in\Gamma} S_{lk}^T\ \theta_k^t \leq F_l^{Lim} y_l^t \qquad \forall l \in \Delta^M, t \in \Lambda \tag{17}$$

$$f_l^t \geq -F_l^{Lim} + F_l^{Lim} y_l^t \qquad \forall l \in \Delta, t \in \Lambda \tag{18}$$

$$f_l^t \leq F_l^{Lim} - F_l^{Lim} y_l^t \qquad \forall l \in \Delta, t \in \Lambda \tag{19}$$

$$\theta_k^t \geq -\pi \qquad \forall k \in \Gamma^{NR}, t \in \Lambda \tag{20}$$

$$\theta_k^t \leq \pi \qquad \forall k \in \Gamma^{NR}, t \in \Lambda \tag{21}$$

$$\theta_k^t = 0 \qquad \forall k \in \Gamma^{R}, t \in \Lambda \tag{22}$$

$$gen_k^t = 0 \qquad \forall\ k \in \Gamma^{NG}, t \in \Lambda \tag{23}$$

$$gen_k^t = \sum_{i\in\Upsilon^U} g_{ik}^t \qquad \forall k \in \Gamma^{G}, t \in \Lambda \tag{24}$$

$$g_{ik}^t \leq G_{ki}^+ \qquad \forall k \in \Gamma^G, i \in \Upsilon^U, t \in \Lambda \tag{25}$$

$$g_{ik}^t \geq G_{ki}^- \qquad \forall k \in \Gamma^G, i \in \Upsilon^U, t \in \Lambda \tag{26}$$

$$g\hat{e}n_{ck}^t - \hat{d}_{ck}^t = \sum_{\substack{l\in\Delta \\ l\neq c}} S_{kl}\ \hat{f}_{cl}^t \qquad \forall k \in \Gamma, t \in \Lambda, c \in \Delta \tag{27}$$

$$\hat{f}_{cl}^t = B_l \sum_{k\in\Gamma} S_{lk}^T\ \hat{\theta}_{ck}^t \qquad \forall l \in \Delta^{NM}, t \in \Lambda, c \in \Delta,\ c \neq l \tag{28}$$

$$\frac{\hat{f}_{cl}^t}{B_l} - \sum_{k\in\Gamma} S_{lk}^T \hat{\theta}_{ck}^t \geq -F_l^{Lim} y_l^t \qquad \forall l \in \Delta^M, t \in \Lambda, c \in \Delta, c \neq l \tag{29}$$

$$\frac{\hat{f}_{cl}^t}{B_l} - \sum_{k\in\Gamma} S_{lk}^T\ \hat{\theta}_{ck}^t \leq F_l^{Lim} y_l^t \qquad \forall l \in \Delta^M, t \in \Lambda, c \in \Delta, c \neq l \tag{30}$$

$$\hat{f}_{cl}^t \geq -F_l^{Lim} + F_l^{Lim} y_l^t \qquad \forall l \in \Delta, t \in \Lambda, c \in \Delta, l \neq c \tag{31}$$

$$\hat{f}_{cl}^t \leq F_l^{Lim} - F_l^{Lim} y_l^t \qquad \forall l \in \Delta, t \in \Lambda, c \in \Delta, l \neq c \tag{32}$$

$$\hat{\theta}_{ck}^t \geq -\pi \qquad \forall k \in \Gamma^{NR}, t \in \Lambda, c \in \Delta \tag{33}$$

$$\hat{\theta}_{ck}^t \leq \pi \qquad \forall k \in \Gamma^{NR}, t \in \Lambda, c \in \Delta \tag{34}$$

$$\hat{\theta}_{ck}^t = 0 \qquad \forall k \in \Gamma^{R}, t \in \Lambda, c \in \Delta \tag{35}$$

$$g\hat{e}n_{ck}^t = 0 \qquad \forall k \in \Gamma^{NG}, t \in \Lambda, c \in \Delta \tag{36}$$

$$g\hat{e}n_{ck}^t = \sum_{i\in\Upsilon^U} \hat{g}_{ik}^{ct} \qquad \forall k \in \Gamma^{G}, t \in \Lambda, c \in \Delta \tag{37}$$

$$\hat{g}_{ik}^{ct} \leq G_{ki}^+\ n_{ck}^t \qquad \forall k \in \Gamma^G, i \in \Upsilon^U, t \in \Lambda, c \in \Delta \tag{38}$$

$$\hat{g}_{ik}^{ct} \geq G_{ki}^-\ n_{ck}^t \qquad \forall k \in \Gamma^G, i \in \Upsilon^U, t \in \Lambda, c \in \Delta \tag{39}$$

$$\hat{d}_{ck}^t \leq D_k^t \qquad \forall k \in \Gamma, t \in \Lambda, c \in \Delta \tag{40}$$

$$n_{ck}^t = 1 \qquad \forall (c,k) \in \Omega^{Ok}, t \in \Lambda \tag{41}$$

$$n_{ck}^t = 1 - y_{(M_{ck})}^t \qquad \forall (c,k) \in \Omega^{N1}, t \in \Lambda \tag{42}$$

$$\hat{y}_{(O_l,R_l)}^t = y_l^t \qquad \forall l \in \Delta, t \in \Lambda^C \tag{43}$$

$$\hat{y}_{(R_l,O_l)}^t = y_l^t \qquad \forall l \in \Delta, t \in \Lambda^C \tag{44}$$

$$(1 - \hat{y}_{ij}^t) U_{ij} \geq w_{ij}^t \qquad \forall i \in \Gamma, j \in \Gamma, t \in \Lambda^C, j \neq i \tag{45}$$

$$w_{ij}^t \geq h_{ij}^{kt} \qquad \forall i, j \in \Gamma, k \in \Gamma^{N0}, t \in \Lambda^C, j \neq i \tag{46}$$

$$\sum_{\substack{j\in\Gamma\\ j\neq i}} h_{ij}^{kt} = 1 \qquad \forall i \in \Gamma^0, k \in \Gamma, t \in \Lambda^C, k \neq i \tag{47}$$

$$\sum_{\substack{i\in\Gamma\\ i\neq j}} h_{ij}^{kt} = 0 \qquad \forall j \in \Gamma^0, k \in \Gamma, t \in \Lambda^C, k \neq j \tag{48}$$

$$\sum_{\substack{i\in\Gamma\\ i\neq j}} h_{ij}^{kt} = \sum_{\substack{i\in\Gamma\\ i\neq j}} h_{ji}^{kt} \qquad \forall j, k \in \Gamma^{N0}, t \in \Lambda^C, j \neq k \tag{49}$$

$$\sum_{\substack{i\in\Gamma\\ i\neq j}} h_{ij}^{kt} \leq 1 \qquad \forall j, k \in \Gamma^{N0}, t \in \Lambda^C, j \neq k \tag{50}$$

$$\sum_{\substack{i\in\Gamma\\ i\neq j}} h_{ji}^{kt} \leq 1 \qquad \forall j, k \in \Gamma^{N0}, t \in \Lambda^C, j \neq k \tag{51}$$

$$\sum_{\substack{i\in\Gamma\\ i\neq j}} h_{ij}^{kt} = 1 \qquad \forall j \in \Gamma, k \in \Gamma^{N0}, t \in \Lambda^C, j \neq k \tag{52}$$

$$\sum_{\substack{j\in\Gamma\\ j\neq i}} h_{ij}^{kt} = 0 \qquad \forall i \in \Gamma, k \in \Gamma^{N0}, t \in \Lambda^C, i \neq k \tag{53}$$

$$h_{ij}^{kt} + h_{ji}^{kt} \leq 1 \qquad \forall i, j \in \Gamma, k \in \Gamma^{N0}, t \in \Lambda^C, j \neq i \tag{54}$$

$$y_l^t \in \{0,1\} \qquad \forall l \in \Delta, t \in \Lambda \tag{55}$$

$$g_{ik}^t \in \mathbb{R}^+ \qquad \forall k \in \Gamma^G, i \in \Upsilon^U, t \in \Lambda \tag{56}$$

$$gen_k^t \in \mathbb{R}^+ \qquad \forall k \in \Gamma, t \in \Lambda \tag{57}$$

$$\hat{g}_{ik}^{ct} \in \mathbb{R}^+ \qquad \forall c \in \Delta, k \in \Gamma^G, i \in \Upsilon^U, t \in \Lambda \tag{58}$$

$$g\hat{e}n_{ck}^t \in \mathbb{R}^+ \qquad \forall c \in \Delta, k \in \Gamma, t \in \Lambda \tag{59}$$

$$\hat{d}_{ck}^t \in \mathbb{R}^+ \qquad \forall c \in \Delta, k \in \Gamma, t \in \Lambda \tag{60}$$

$$n_{ck}^t \in \mathbb{R}^+ \qquad \forall c \in \Delta, k \in \Gamma^G, t \in \Lambda \tag{61}$$

$$\hat{y}_{ij}^t \in \mathbb{R}^+ \qquad \forall i, j \in \Gamma, t \in \Lambda^C, j \neq i \tag{62}$$

$$w_{ij}^t \in \mathbb{R}^+ \qquad \forall i, j \in \Gamma, t \in \Lambda^C \tag{63}$$

$$h_{ij}^{kt} \in \{0,1\} \qquad \forall i, j, k \in \Gamma, t \in \Lambda^C \tag{64}$$

The objective function (1) has two parts. The first part maximizes the amount of energy supplied by the grid to customers accounting for the original grid, lines in maintenance, and a potential line loss ('-1'). The second part maximizes the selection of the target time period for each maintenance task.

The first group of constraints (2)–(13) represents the general characteristics of maintenance tasks such as respecting periods inconvenient for maintenance (2) (for example, due to harsh weather), respecting the equipment that will not undergo maintenance (3), and observing the maximum number of tasks allowed per period of time (4). Each maintenance task can take one (5), two (6)–(8) or three (9)–(12) weeks to complete. Longer task durations can be implemented following the logic of constraints (5)–(12). Constraint (13) enforces the simultaneous maintenance of pairs of lines as required.

The second group of constraints (14)–(26) models the power flow. We use direct current power flow (DCPF), and the constraints guarantee the conservation of energy at each bus (14), avoid line overload (18)–(19), enforce voltage angle limits (20)–(22) and enforce the generation limits (23)–(26).

We point out that in the DCPF formulation, the energy flow in a transmission line is given by:

$$f_l^t = B_l(\Theta_{k_1}^t - \Theta_{k_2}^t) \tag{65}$$

where buses k_1 and k_2 are the origin and destination buses of line l. Equation (65) corresponds to constraint (15) and represents the energy flow in the lines that do not undergo maintenance. For our purposes, we need to turn off lines when they are under maintenance so (65) becomes:

$$f_l^t = B_l(\Theta_{k_1}^t - \Theta_{k_2}^t)(1 - y_l^t) \tag{66}$$

Clearly constraint (66) is not linear but we can linearize it using the so-called big-M method, as shown in [12] and [5]. We set the big-M equal to the transmission line capacity limit F_l^{Lim}, and thus obtain constraints (16) and (17).

The third group of constraints (27)–(42) express the N-1 security requirement that represents the possibility of an unexpected loss of a line at any time. Constraint (27) is equivalent to energy conservation (14) but in the case of an unexpected loss of one line (excluding lines already under preventive maintenance). Similarly, constraints (28)–(30) compute the load flow and (31)–(32) the limit line overload, as in (15)–(19) respectively, but now in the case of unforeseen loss of lines. The voltage angle limits (33)–(35), generation limits (36)–(39) and demand bound (40) also consider line losses. These N-1 constraints are inspired by the security-constrained unit commitment as in [1].

Lastly, if there is a generator bus connected by only two lines, constraint (42) guarantees the generation is off when one of the lines is under preventive maintenance and the other fails. Otherwise, constraint (41) keeps the generation within operation limits.

The fourth group of constraints (43)–(54) prevents the creation of islands or isolation of buses when scheduling maintenance tasks. The grid connectivity test is done through a flow problem with a single unit of artificial test flow to verify network connectivity. Constraints (43)–(44) define variable $\hat{y}$ as equivalent to y. When a line operates, (45) states it can be used in either direction by the test flow. The line capacity limit at (46) is one. In (47)–(53) a unit of test flow is sent from a super-source to each bus on the directed graph. This set of flow conservation constraints at all vertices ensure an availability of one unit at the super-source and a demand of one unit at every other bus. At most one direction linking two buses can be used for the test flow at (54). These variables and constraints are a separate construction and have no impact on the objective, nor on the rest of the solution; they only confirm the connectivity of the graph when one or more lines are under maintenance. As a consequence, it is possible that some lines can never be scheduled for maintenance; this will happen if their removal always results in a disconnected graph.

The last group of constraints (55)–(64) state the nature and the domains of the optimization variables.

The large number of constraints and the mix of binary, real and non-negative real variables characterizes this as a large problem. As the number of maintenance tasks to be scheduled increases, the computational complexity grows. Even for small systems, this optimization problem is very large and computationally expensive to solve. For this reason, we propose in the next Section a decomposition method to solve it.

3 New specialized algorithm

Considering the separability of the sets of constraints of TMS problems, we reduce the computational effort by breaking down the optimization problem (1)–(64) into two stages.

The first stage (1)–(42) and (55)–(61), contains binary and continuous variables and can be solved by Benders decomposition algorithm. The second stage contains the remaining (43)–(54)

and (62)–(64), but since these variables and constraints do not directly impact the solution of the first stage, this second stage is actually a stage of verification that no bus or set of buses is isolated from the network due to lines put in maintenance. We thus have two separate optimization problems as follows:

$$\min \ (1) \tag{67}$$

$$\text{s.t. } (2) - (42) \tag{68}$$

$$(55) - (61) \tag{69}$$

$$\max \ 0 \tag{70}$$

$$\text{s.t. } (43) - (54) \tag{71}$$

$$(62) - (64) \tag{72}$$

The first stage (67)–(69) can be solved using Benders algorithm [13], which is available via the MILP solver that we use. The solution is then verified using the second stage (70)–(72), which has a constant objective function and thus simply checks that a feasible solution exists for its constraints.

In other words, if the optimal solution of (67)–(69) is feasible for (70)–(72), then it is optimal for the complete problem (1)–(64). If the first stage solution is infeasible for the second stage, a cut (73) is generated and added to (67)–(69) and this new problem is solved using Benders decomposition.

$$\sum_{\substack{l\in\Delta,\ t\in\Lambda:\\ \hat{Y}_l^{tc}=0}} y_l^t + \sum_{\substack{l\in\Delta,\ t\in\Lambda:\\ \hat{Y}_l^{tc}=1}} (1 - y_l^t) >= 1, \quad \forall c \in 1..count \tag{73}$$

This process is repeated until either i) a solution of (67)–(69), possibly with cuts added, is feasible for (70)–(72), indicating that we have found an optimal solution for the complete problem, or ii) (67)–(69), possibly with cuts added, has no solution, indicating the complete problem has no solution.

The entire process is described in the algorithm 1.

Algorithm 1: New specialized algorithm

1 $count \leftarrow 0$;
2 Solve (67)–(69) with use of built-in Benders algorithm;
3 **if** *optimal solution found* **then**
4 $\quad Y_l^t \leftarrow y_l^t$, binary solution y_l^t found in (67)–(69) is sent as parameter Y_l^t to (70)–(72);
5 **else**
6 $\quad$ Break $\longrightarrow$ problem (1)–(64) has no solution;
7 **end**
8 **if** Y_l^t *is feasible for problem* (70)–(72) **then**
9 $\quad$ Break $\longrightarrow$ optimal solution found for (1)–(64);
10 **else**
11 $\quad count \leftarrow count + 1$;
12 $\quad \hat{Y}_l^{t\ count} \leftarrow Y_l^t$;
13 $\quad$ Generate cut[$count$] and add to problem (67)–(69);
14 **end**
15 Solve (67)–(69)+cuts with built-in Benders algorithm;
16 Go to step 3.

Equation (73) is the set of cuts generated by the second stage (70)–(72) and added to the first stage (67)–(69). The purpose of each cut is that the optimal solution previously found is eliminated from the set of possible solutions once the cut is added. These cuts are inspired by the so-called combinatorial Benders cuts well known in the literature, see e.g. [2].

In Section 4, we report the results obtained by applying the proposed model and solution algorithm to the benchmark problem from [14].

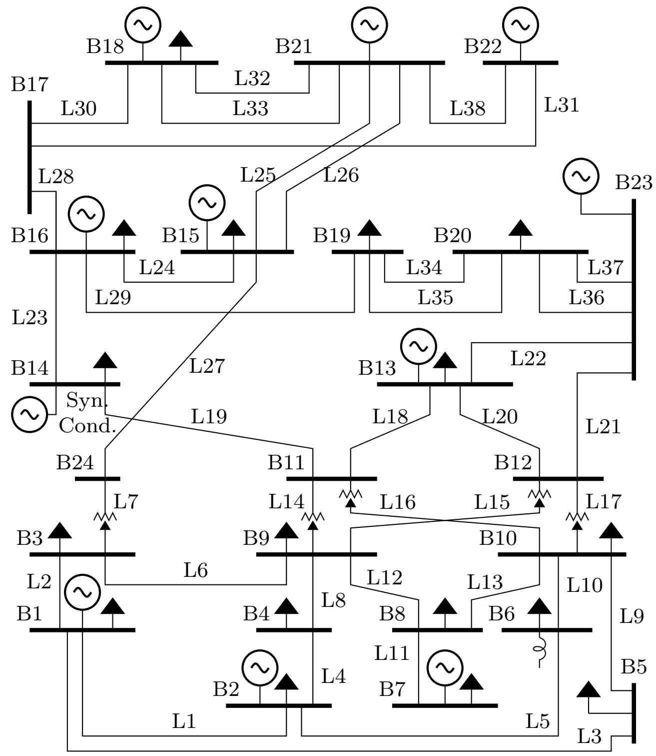


Figure 1: Transmission grid IEEE-RTS-24, detailed in [14]

4 Computational experiments

The optimization problems were solved using CPLEX 12.9 via AMPL on a Linux server with 16GB of RAM and 8 CPUs (Intel(R) Core(TM) i7-2600 CPU @ 3.40GHz).

4.1 Case study system and maintenance data

Our computational tests are based on the IEEE-24-Reliability Test System (RTS) from [14]. It has 24 buses, 38 transmission lines, 10 generation plants containing a maximum of 6 units each and 17 buses with demand customers, see Figure 1.

We introduce maintenance requirements based on the reality of maintenance planning for Hydro-Québec, a major utility in Canada. To avoid the harsh winter periods, maintenance tasks can only be carried out between May and October, specifically between weeks 15 and 47 of the year. These two weeks are indicated by vertical red lines on Figure 2.

Figure 2 also shows for each line (row) the preference level associated with each week of the year in terms of carrying out maintenance on that line. The darker the colour for a week, the stronger the preference, and these preference levels are the values of A_l^t in the objective function (1).

Table 1 gives the minimum and maximum power output of each unit of a generation plant, where each plant is identified according to the bus at which it is located in the system. These limits are the same for each period t of the year. The weekly load for the whole system, and the contribution of each bus to this total demand, are defined in the first and tenth Table of [14].

Table 2 gives the necessary information for each transmission line. The maximum power flow F_l^{Lim} values have been set to 80% of the values from [14]. Table 2 also gives, for each line, the number of weeks in a row required for its maintenance, and when it applies, the mandatory simultaneous removal

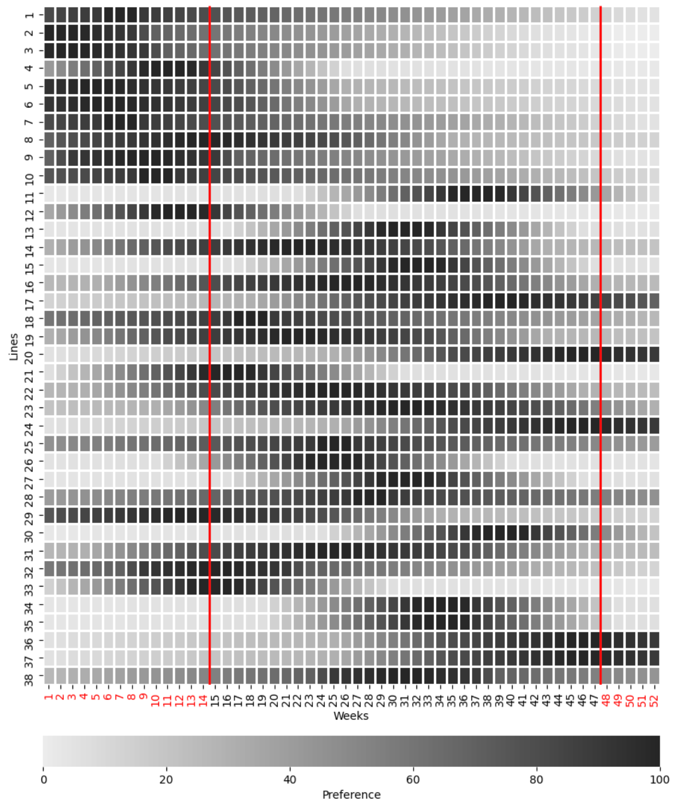


Figure 2: Preferred target time per line

of parallel lines, i.e., lines with the same origin and destination bus, to ensure worker safety as indicated by [9]. Note that in our test data, all lines require one yearly maintenance task.

Considering grid topology, the parameter M_{ck} is set to 31 when $(c, k) = (38, 22)$ and to 38 when $(c, k) = (31, 22)$. In addition, at most two tasks can be carried out simultaneously, i.e., $L_t = 2$ for all t.

Table 1: Generation range for the units of a power plant, G^{-}_{ki} - G^{+}_{ki}, in MW

	Bus									
Unit	1	2	7	13	15	16	18	21	22	23
1	16-20	16-20	25-100	69-197	2.4-12	54.3-155	100-400	100-400	10-50	54.3-155
2	16-20	16-20	25-100	69-197	2.4-12				10-50	54.3-155
3	15.2-76	15.2-76	25-100	69-197	2.4-12				10-50	140-350
4	15.2-76	15.2-76			2.4-12				10-50	
5					2.4-12				10-50	
6					54.3-155				10-50	

4.2 Case study results

To assess the impact of the different groups of constraints specified in Section 2, we started by solving a basic version of the model and then added the various groups of constraints up to the complete model. In this way we consider 5 different models, and the results are reported in Table 3 with the models listed in order of increasing complexity.

Table 3 is set up as follows. The first row indicates the lines for which maintenance is carried out. This is the full set of lines for all models except the complete model. This is because line 11 is the only

Table 2: Line data

Line l	1	2	3	4	5	6	7	8	9	10	11	12	13
F_l^{Lim} in MW	140	140	140	140	140	140	320	140	140	140	140	140	140
Mainte. duration (weeks)	1	2	1	1	2	1	1	1	1	1	1	1	1
Line l	14	15	16	17	18	19	20	21	22	23	24	25	26
F_l^{Lim} in MW	320	320	320	320	400	400	400	400	400	400	400	400	400
Mainte. duration (weeks)	1	1	1	1	1	1	1	3	2	1	1	1	1
Simulta. outage with line l												26	25
Line l	27	28	29	30	31	32	33	34	35	36	37	38	
F_l^{Lim} in MW	400	400	400	400	400	400	400	400	400	400	400	400	
Mainte. duration (weeks)	1	1	1	1	3	1	1	1	1	1	1	1	
Simulta. outage with line l						33	32	35	34	37	36		

link from bus 7 to the grid so if it is not operating, it separates the network into two parts, and this is not allowed by the connectivity constraints in the complete model. The second row of Table 3 reports the optimal objective function value for each model, and the next two rows give the contribution to the optimal value of each of the two parts of the objective function. Specifically, the third row is the amount of energy supplied by the grid to customers, and the fourth row is the preference maximization in scheduling each maintenance task. Finally, the fifth row reports the total CPU times, and the last two rows indicate the Figure showing the optimal maintenance schedule and the optimal demand for each model.

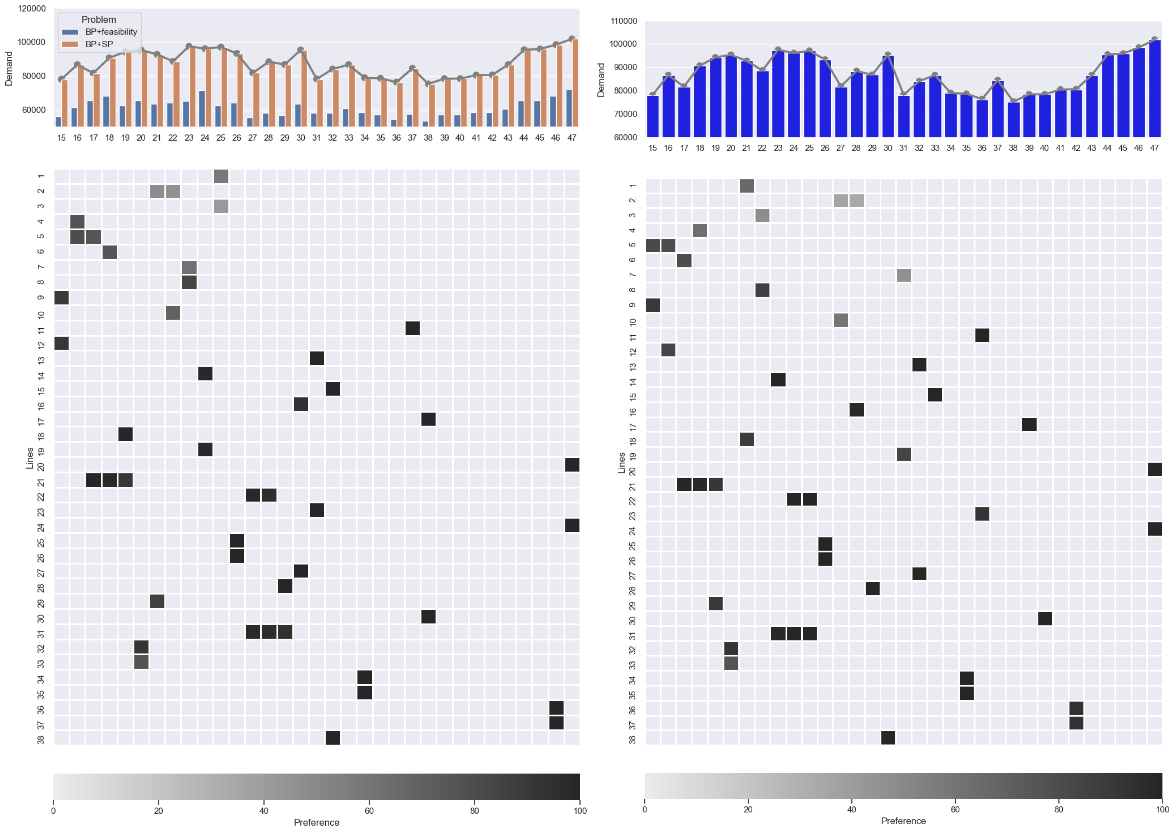
The basic model maximizes only the maintenance preferences, and has constraints (2)–(26) and (55)–(57). We solve this MILP using Benders algorithm. In this case, customer demand is fully met and an optimal scheduling is identified, as depicted in Figure 3a. However, unexpected line losses and system connectivity are not considered.

We next consider the N-1 security constraints (27)–(42). We add them to the basic model in two ways. First, we use them only to verify whether the optimal solution of the basic model satisfies the N-1 requirement. The results in Table 3 show that this is the case. Nevertheless, when in emergency maintenance due to an additional line loss, customer demand as shown in Figure 3a is overlooked as only the maintenance preferences are considered.

Second, the N-1 security constraints are integrated as a sub-problem (SP) of the basic model. This SP maximizes demand fulfillment when in emergency maintenance, which is the first part of the objective function (1). The results in Table 3 reveal that the schedule is the same but the objective value is significantly improved compared to the previous step of checking N-1 security post-optimization. Figure 3a also shows the demand supplied for both the post-optimization security check (BP+feasibility) and the security integrated as a sub-problem (BP+SP). It is clear that the demand supply is significantly different as well between the two cases.

Table 3: Results for experiments with parts of the full mathematical model

	Basic Model	Basic Model + feasibility of N-1	Basic Model + SP of N-1	Partial Model	Complete Model
Lines removed	1-38 (all)	1-38 (all)	1-38 (all)	1-38 (all)	1-10,12-38
Objective	4014	4014	4613358.74	4613613.6	4613513.6
$\sum_{t\in\Lambda}\sum_{k\in\Gamma}\sum_{c\in\Delta}\hat{d}^t_{ck}$	-	3212911.41	4609344.74	4609662.6	4609662.6
$\sum_{t\in\Lambda}\sum_{l\in\Delta} A^t_l y^t_l$	4014	4014	4014	3951	3851
CPU time (seconds)	0.6	13.7	10.9	250.7	243.5
Scheduling	see Figure 3a	see Figure 3a	see Figure 3a	see Figure 3b	see Figure 4
Demand $\hat{d}$	-	see Figure 3a	see Figure 3a	see Figure 3b	see Figure 4



(a) **(b)**

Figure 3: Schedules for (a) Basic model, and (b) Partial model

The next model we consider is a partial model consisting of objective (1) and constraints (2)–(42) and (55)–(61). This includes all the constraints except the connectivity requirement, and we solve it using the Benders algorithm. The results in Table 3 show that the objective value is further improved but that the preference part of it has deteriorated. This is corroborated by Figure 3b where we see that the schedule is noticeably different from that of the basic model, and in particular less preferred periods are assigned for the maintenance of lines 7 and 10. In general, because the solution satisfies the N-1 requirement, one line loss does not prevent system operation but demand may not be fully met. For example, we see in Figure 3b that lines 5 and 9 are in maintenance at week 15. If it happens that line 3 fails during week 15, then the demand of bus 5 (only connected by lines 3 and 9) cannot be met. In essence, in the weeks when a line is scheduled for maintenance, the N-1 requirement in our model becomes effectively an N-2 requirement, and when two lines are in maintenance it becomes an N-3 requirement.

The final step is to add the fourth group of constraints (43)–(54) to prevent islanding of sub-systems or isolated buses. This is the complete model. One consequence of integrating these system connectivity constraints is that the optimization problem is infeasible when line 11 is removed. This is because line 11 is the only link of bus 7 to the grid so any outage of that line isolates bus 7. For this reason, we consider the removal of every other line except line 11.

We solve the complete model using the new algorithm described in Section 3, and the results are shown in Table 3 and Figure 4. The optimal objective value is slightly worse that that of the partial problem. This is in large part due to line 11 not being maintained so that its preference value in the objective is zero. However, the maintenance schedule is the same, except for line 11 (that is not scheduled) and line 17 (that is maintained in a different week with the same preference level). This leads to the same demand being met if failures occur, as can be seen from Figures 3b and 4. In conclusion, the complete model successfully schedules preventive maintenance outages of lines while enforcing strict security requirements and ensuring that no buses are isolated or sub-systems islanded.

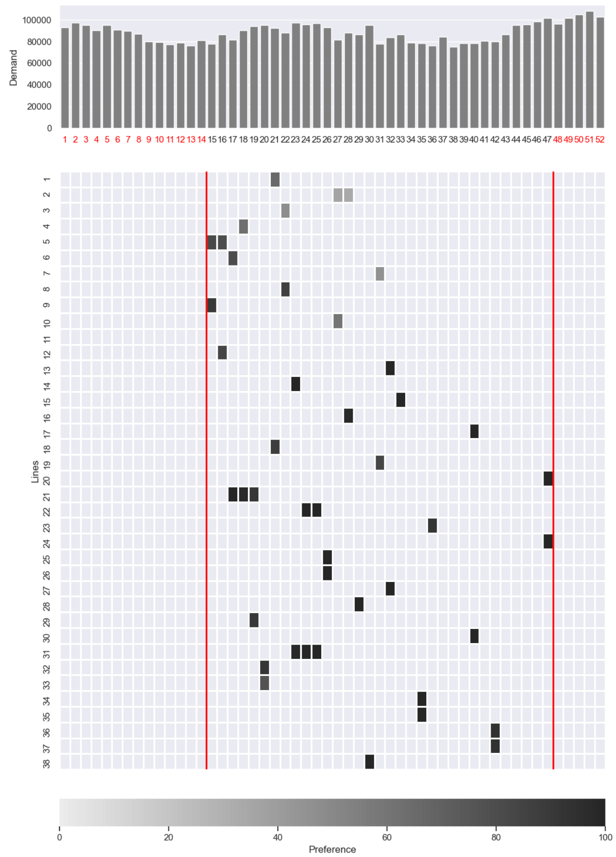


Figure 4: Schedule for the complete model

4.3 Computational costs

Lastly, let us look at the computational cost for solving our proposed model and the impact of the new algorithm. First, it is not surprising that Table 3 shows that the CPU times increase as the model's complexity increases. For completeness, we also solved the partial model with the outage of every line except line 11 and its solution (using Benders) took 241,7 seconds, which is slightly less than 243,5 seconds for the complete model.

We carried out a separate computational study using only the complete model and different sets of lines to be maintained. The results are reported in Figure 5 where the times in brown are those of the standard Benders algorithm and the times in blue are those using our new algorithm. We consider maintenance sets with single lines and with multiple lines, up to the set AL that consists of all lines except line 11 (which causes infeasibility). Generally we see that when removing few lines, the CPU times are lower and are comparable for both solution methods. When removing many lines, however, the CPU times for our new algorithm grow much more slowly with respect to the number of lines in maintenance. Moreover, the CPU times of our algorithm remain acceptable even when removing all (but one) lines, as it takes less than 5 minutes to schedule those 37 line maintenances. We thus conclude that our proposed new algorithm is more efficient than off-the-shelf CPLEX, even though the latter has a built-in Benders algorithm available.

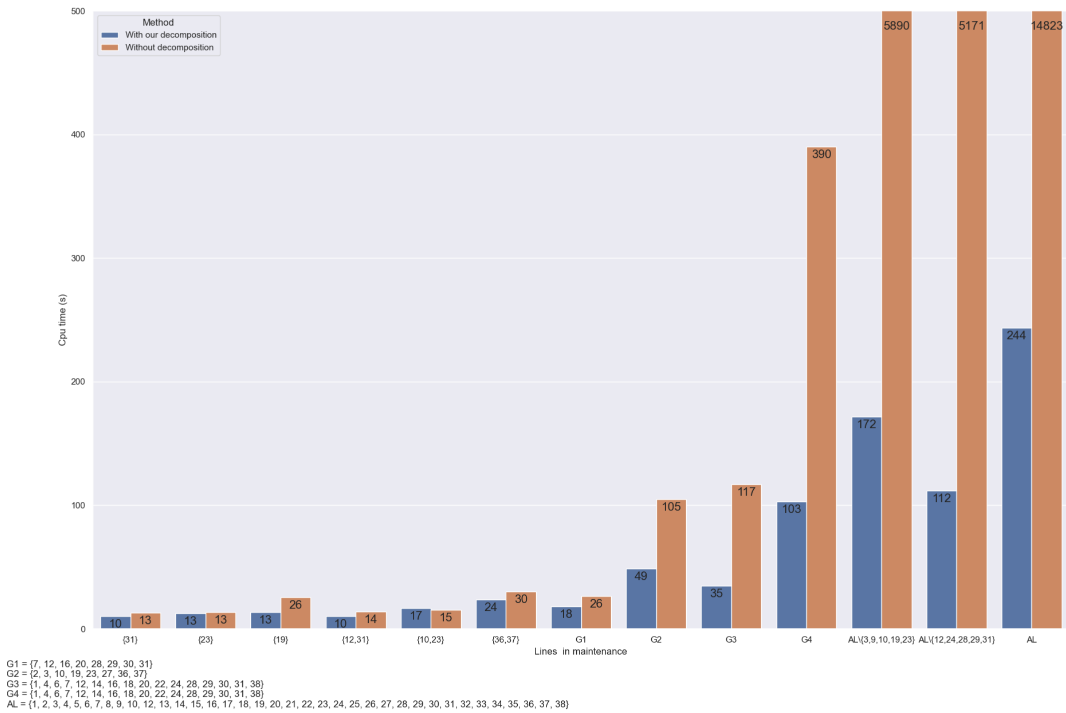


Figure 5: CPU times for different sets of lines scheduled for maintenance

5 Conclusion

This paper addresses the scheduling of preventive line maintenance tasks in electric power transmission systems. We propose a new optimization formulation that includes the main constraints from the literature on this problem plus an N-1 security requirement and the prevention of sub-system islanding or isolated buses. Because this new formulation is large-scale and challenging for state-of-the-art optimization solvers, we also propose a specialized decomposition algorithm to solve the mathematical model to optimality more efficiently. We carried out computational experiments using the IEEE-24-RTS system and the results show that the proposed model achieves the intended purpose for optimal maintenance scheduling. In particular, the model is able to identify critical lines automatically. We also confirmed that using our proposed new algorithm, the model can be solved in reasonable time to global optimality.

References

[1] M. F. Anjos and A. J. Conejo. Unit commitment in electric energy systems. Foundations and Trends® in Electric Energy Systems, 1(4):220–310, Dec. 2017.

[2] G. Codato and M. Fischetti. Combinatorial benders' cuts for mixed-integer linear programming. Operations Research, 54(4):756–766, 2006.

[3] K. P. Dahal. A review of maintenance scheduling approaches in deregulated power systems. In Proc. of the Int. Conf. on Power Systems, Kathmandu, Nepal, 2004.

[4] A. Froger, M. Gendreau, J. E. Mendoza, E. Pinson, and L-M. Rousseau. Maintenance scheduling in the electricity industry: A literature review. Europ. Journal of Operat. Research, 251(3):695–706, 2016.

[5] L. P. Garcés, A. J. Conejo, R. García-Bertrand, and R. Romero. A bilevel approach to transmission expansion planning within a market environment. IEEE Trans. Po. Sys., 24(3):1513–1522, 2009.

[6] N. Gomes, R. Pinheiro, Z. Vale, and C. Ramos. Scheduling maintenance activities of electric power transmission networks using an hybrid constraint method. Internat. Journal of Eng. Intelligent Syst. for Electrical Eng. and Communicat.-new series-, 15(3):127, 2007.

[7] R. Kulkarni. Maintenance planning of transmission assets under uncertainty for long-term horizon. Master's thesis, Delft Univ. of Technol., 2017.

[8] W. B. Langdon and PC Treleaven. Scheduling maintenance of electrical power transmission networks using genetic programming. IEEE Power Series, pages 220–237, 1997.

[9] C. Lv, J. Wang, S. You, and Z. Zhang. Short-term transmission maintenance scheduling based on the benders decomposition. Int. Trans. on Electr. Energy Systems, 25(4):697–712, 2015.

[10] M.K.C. Marwali and S.M. Shahidehpour. Short-term transmission line maintenance scheduling in a deregulated system. In Proc. of 21st IEEE Int. Conf. on Power Ind. Comp. Applic., pages 31–37, Santa Clara, CA, 1999.

[11] J. E. A. Neto and C. A. Castro. Optimal maintenance scheduling of transmission assets in the brazilian electric system. Journal of Control, Automation and Electr. Systems, 32(2):482–491, 2021.

[12] H. Pandzic, A. J. Conejo, I. Kuzle, and E. Caro. Yearly maintenance scheduling of transmission lines within a market environment. IEEE Trans. on Power Systems, 27(1):407–415, Feb. 2012.

[13] R. Rahmaniani, T. G. Crainic, M. Gendreau, and W. Rei. The benders decomposition algorithm: A literature review. Europ. Journal of Operat. Research, 259(3):801–817, 2017.

[14] Probability Methods Subcommittee. IEEE reliability test system. IEEE Trans. on power apparat. and syst., PAS-98(6):2047–2054, Dec. 1979.

[15] Y. Wang, H. Zhong, Q. Xia, D. S. Kirschen, and C. Kang. An approach for integrated generation and transmission maintenance scheduling considering n-1 contingencies. IEEE Trans. on Power Systems, 31(3):2225–2233, 2016.

[16] S. Xianyong and S. Quanyu. Multi-objective optimization of transmission line annual maintenance scheduling based on flower pollination algorithm. DEStech Trans. on Comp. Science and Eng., (csae), 2017.

[17] Z. A. Yamayee. Maintenance scheduling: description, literature survey, and interface with overall operations scheduling. IEEE Trans. on Power Apparatus and Systems, PAS-101(8):2770–2779, Aug. 1982.