**Les Cahiers du GERAD** ISSN: 0711–2440

# Reaching Paris agreement goal through CDR/DAC development: A compact OR model

F. Babonneau, A. Haurie, M. Vielle

G–2021–71

December 2021



La publication de ces rapports de recherche est rendue possible grâce au soutien de HEC Montréal, Polytechnique Montréal, Université McGill, Université du Québec à Montréal, ainsi que du Fonds de recherche du Québec – Nature et technologies.

Dépôt légal – Bibliothèque et Archives nationales du Québec, 2021
– Bibliothèque et Archives Canada, 2021

The publication of these research reports is made possible thanks to the support of HEC Montréal, Polytechnique Montréal, McGill University, Université du Québec à Montréal, as well as the Fonds de recherche du Québec – Nature et technologies.

Legal deposit – Bibliothèque et Archives nationales du Québec, 2021
– Library and Archives Canada, 2021

**GERAD** HEC Montréal
3000, chemin de la Côte-Sainte-Catherine
Montréal (Québec) Canada H3T 2A7

**Tél. : 514 340-6053**
Téléc. : 514 340-5665
info@gerad.ca
www.gerad.ca

# Reaching Paris agreement goal through CDR/DAC development: A compact OR model


**Frédéric Babonneau** [a]

**Alain Haurie** [b, c]

**Marc Vielle** [d]

[a] *ORDECSYS, Switzerland & KEDGE Business School, France*

[b] *GERAD, Canada*

[c] *ORDECSYS & University of Geneva, Switzerland*

[d] *EPFL & LEURE, Switzerland*

frederic.babonneau@kedgebs.com
ahaurie@gmail.com
marc.vielle@epfl.ch



**December 2021**
**Les Cahiers du GERAD**
**G–2021–71**

**Abstract :** A compact operations research (OR) model is proposed to analyse the prospects of meeting the Paris Agreement targets when direct air capture technologies can be used or not. The main features of the model are (i) the representation of the economy and energy use with a nested constant elasticity of substitution production function ; (ii) the representation of climate policy through the use of a safety emissions budget concept ; and (iii) the representation of an international emissions trading scheme for the implementation of climate policy. Using dynamic optimisation, several contrasting scenarios are analysed and the potential use of the model in future developments of climate/economy modelling is discussed.

**Keywords :** Climate policy, optimal economic growth, dynamic optimisation model, market equilibrium constraints and $CO_2$ direct reduction

# 1 Introduction

It is now well established that, in order to achieve the objectives of the Paris Agreement, a regime of zero net emissions must be reached by 2050 or 2070 at the latest. Direct $CO_2$ emission reduction technologies, in particular BECCS [1] and DAC/CDR [2] technologies, will then play an important role by generating negative emissions to offset the GHGs emitted by fossil fuel technologies [3]. The objective of this article is to propose a compact OR model that can provide an economic assessment of the possible contribution of CDR/DAC technologies to the achievement of the Paris Climate Agreement objectives.

The model is based on a Ramsey-style optimal economic growth paradigm, with the economy, energy and negative emissions production described by CES functions [18]. The originality of the approach lies in the representation of useful and secondary energy production as well as the production of negative emissions from CDR/DAC, through the use of capital (plants), labour and primary fossil energy, which is the source of GHG emissions. This OR model provides a framework to explore the macroeconomic costs of climate policies to achieve the Paris Agreement targets when CDR/DAC technology can and cannot be used. By providing an overview of the more global optimal economic growth, this approach complements the work done in [4], concerning the possible role of CDR/DAC development in the climate policy of oil and gas producing countries and [5], where an oligopoly game of CDR technology development in a steady-state zero net emissions climate regime is proposed. CDR/DAC technologies have been evaluated in IAMs [4] that include an optimal economic growth paradigm "a la Ramsey" [9] and [14], but these models use a different description of the economic good and the energy production processes. The compact OR model presented here can also be linked to the stochastic control model [8] and differential game model [6, 7] already proposed to analyse global climate policy. As in these previous works, we represent climate policy through the sharing of a remaining safety emissions budget (SEB), as suggested in [3, 15]. In addition, a representation of an international emissions trading system is included in this OR model. The global supply of emission rights (permits) will determine the price of the permit and emission levels in each region will be such that the marginal abatement cost is equal to the price. Finally, to calibrate the DAC production function, we use the techno-economic analyses of [13] and [16].

In the rest of the paper, we describe in Section 2 the main features of the model and we present in Section 3 some scenarios produced using the model with different optimisation criteria and climate policy constraints. In Section 4, we discuss the results already obtained and conclude.

# 2 The model

## 2.1 Economic structure

We regroup the world countries in three "coalitions" $j$ called BRIC, [5] OECD and ROW, [6] respectively. They represent groups of nations in similar states of development. In each group, we represent an economy where a general economic good is produced with three inputs : labour $L_0$, capital $K_0$ and energy $E_0$. The energy input $E_0$, or useful energy, can be obtained from two kinds of inputs, one fossil, noted $enf_0$, and the other renewable, $enr_0$. The fossil energy input is obtained from two factors, respectively the fossil fuel power plants $K_1$ and the fossil primary energy $enp_1$. The renewable input is produced by zero-emission plants, represented by the capital $K_2$. GHG emissions are associated with the fossil energy primary source $enp_1$. Negative emissions $v$ can be produced by CDR/DAC technologies using three production factors, labor $L_3$, capital $K_3$ and useful energy $E_3$. The energy mix,

1. Biomass Energy with Carbon Capture and Sequestration.
2. Direct Air Capture and Carbon Dioxide Removal.
3. See, for example, The Economist's briefing or Shell Corp's Sky scenario where BECCS is massively used to produce negative emissions.
4. Integrated assessment models.
5. Brazil, Russia, India, China.
6. Rest of the world.

fossil vs. renewable, is supposed to be common to energy input for the general productive economy, $E_0$ and the DAC/CDR sector $E_3$.

This production structure, schematised in Figure A1 in Appendix will be mathematically described in nested CES function that are calibrated from GTAP 10 database for the reference year 2014 [1]. All economic variables are expressed in US$_{2014}$ using market exchange rates. Energy consumptions, from fossil or renewable sources are expressed in physical terms (peta-joule, PJ) ; for calibration purpose they are obtained from the energy balances published online by the International Energy Agency (IEA). [7] The $CO_2$ emissions from fuel combustion are also obtained from the IEA [11]. The DAC technology is calibrated using [16]. We use the cost estimates for a DAC system using the sodium/calcium hydroxide option (see Table 2.5 in [16]). We retain the total cost of US$ 430 per ton of $CO_2$ captured.

Population levels from 2014 to 2100, expressed in million of people in Table A1, are based on the World Population Prospects 2019 done by the United Division [17]. We use the medium variant scenario. For the whole world, it varies from 7'295 $\times 10^6$ (7.295 billion) people in 2014 up to 10'875 $\times 10^6$ (10.875 billion) people in 2100. After 2100 we assume a steady state for population in different regions.

## 2.2 Dynamics

We consider a time set $t \in \{0, 1, \ldots, T\}$, where each period corresponds to a number of years $Ny$. In this application, we take decadal periods ($Ny = 10$). The dynamic model has five state variables, which are the capital stocks $K_i(t, j); i = 0, 1, 2, 3$, and the remaining emission budget $b(t, j)$, for coalition $j$ at period $t$, and five control variables, which are the investment levels $I_i(t, j); i = 0, 1, 2, 3$, and the supply $\omega(t, j)$ of emission permits by coalition $j$ at period $t$. The state equations for capital are given in (1)–(2). In (3) the parameter $IB_3(t, j)$ is an upper bound for investment in DAC technology that limits the availability of this technology over time.

$$K_i(t, j) = K_i(t-1, j)(1-\mu_j) + Ny \cdot I_i(t-1, j), \tag{1}$$
$$K_i(j, 0) = K_i^0(j), \qquad i = 0, 1, 2, 3, \quad \forall t, \forall j, \tag{2}$$
$$I_3(t, j) \leq IB_3(t, j), \tag{3}$$

The cumulative emissions budget, since the beginning of the industrial revolution, compatible with a 60% probability of limiting the temperature increase below 1.5 °C has been evaluated at 1 trillion tons of carbon [2]. From this figure we evaluate the remaining Safety Emissions Budget (SEB) at $B = 1'170$ Gt $CO_2$. [8] The remaining SEB $b(t, j)$ for each coalition of countries will decrease by the amount of emissions permits $Ny \cdot \omega_i(t-1, j)$ supplied by the coalition, if there is a carbon market or, more directly by the emissions level $Ny \cdot em(t-1, j)$ of the coalition. The remaining SEB will be replenished by the amount of negative emissions $Ny \cdot v(t-1, j)$. The parameter $\theta_j \in [0, 1]$ is the share of the SEB given to the coalition $j$ ; one must have $\sum_j \theta_j = 1$. The parameters $\theta_j$, $j = BRIC, OECD, ROW$, summarise in this model the climate negotiations. In summary the SEB dynamics is, for all coalitions $j$

$$b(t, j) = b(t-1, j) - Ny \cdot \omega_i(t-1, j) + Ny \cdot v(t, j) \qquad t = 1 \ldots T, \tag{4}$$
$$b(0, j) = \theta_j B, \tag{5}$$

if a carbon market market exists, or

$$b(t, j) = b(t-1, j) - Ny \cdot em(t-1, j) + Ny \cdot v(t, j), \qquad t = 1 \ldots T, \tag{6}$$
$$b(0, j) = \theta_j B, \tag{7}$$
$$\sum_j b(t, j) \geq 0, \qquad t = 1 \ldots T, \tag{8}$$

7. https ://www.iea.org/reports/world-energy-balances-overview
8. Recall that 3.66 t $CO_2$ correspond to 1tC.

if there is no market. By imposing a global remaining SEB that remains nonnegative (Equation 8), we impose a climate constraint with no overshooting. We prohibit overshooting for each coalition if we impose $b(t, j) \geq 0$, $\forall t$.

## 2.3 Criteria

The periodic discount factor is given by $\beta(t) = 1/(1 + r)^{Ny \cdot t}$, with $r = 3\%$. It is used, in the performance criterion $\Phi = \sum_j \phi(j)$, which is maximised under the constraints of the dynamic model to obtain the desired scenarios. For each coalition $j$ the expression $\phi(j)$ represents the discounted sum of utility derived from consumption for its population.

$$\phi(j) = \sum_{t=0}^{T-1} \beta(t) PV \cdot L(t, j) \log(C(t, j)/L(t, j)), \quad j = \text{BRIC}, \text{OECD}, \text{ROW}, \tag{9}$$

where $PV = \sum_{s=1}^{Ny} (1 + r)^{(1-s)}$ is the present value factor at each time $t$. In (9) $\log(C(t, j)/L(t, j))$ represents the utility derived from per-capita consumption ; $C(t, j)$ is the consumption level by coalition $j$ at period $t$, given by

$$C(t, j) = Y(t, j) - \sum_{i=0,1,2,3} I_i(t, j) - \pi(t, j) enp_1(t, j), \tag{10}$$

where $\pi(t, j)$ is the price of primary fossil energy.

To compare different scenarios we shall use another welfare criterion $W(j)$ for each coalition $j$. It corresponds to the discounted sum of per-capita consumption, net of the revenue from permit trading, over the whole horizon 2020-2160. For coalition j, we have

$$W(j) = \sum_{t=0}^{T-1} \beta(t) PV \, \frac{C(t, j) + p(t)(\omega(t, j) - emf(t, j))}{L(t, j)}, \tag{11}$$

where $\omega(t, j)$ is the supply of permits by coalition $j$ and $p(t)$ is the permit price on carbon market, at period $t$.

## 2.4 Production functions

The CES production functions are introduced in the following constraints (for coalition $j$ and period $t$) :

#### General economic good production

$$Y(t, j) - A_0(j) tg(t, j) \left[ \alpha_{0K} K_0(t, j)^{\frac{s_0(j)-1}{s_0(j)}} + \alpha_{0L} L_0(t, j)^{\frac{s_0(j)-1}{s_0(j)}} + \alpha_{0E} E_0(t, j)^{\frac{s_0(j)-1}{s_0(j)}} \right]^{\frac{s_0(j)}{s_0(j)-1}} \leq 0. \tag{12}$$

#### Negative emissions production

$$v(t, j) - A_3(j) tgv(t, j) \left[ \alpha_{3K}(j) K_3(t, j)^{\frac{s_3(j)-1}{s_3(j)}} + \alpha_{3L}(j) L_3(t, j)^{\frac{s_3(j)-1}{s_3(j)}} + \alpha_{3E}(j) E_3(t, j)^{\frac{s_3(j)-1}{s_3(j)}} \right]^{\frac{s_3(j)}{s_3(j)-1}} \leq 0. \tag{13}$$

#### Labour use

$$L(t, j) \geq L_0(t, j) + L_3(t, j). \tag{14}$$

**Bounds on sequestration**

The negative emissions $v(t,j)$ must be stored. The sequestration potential $BCCS(j)$ can be limited in the different regions of the world.

$$v(t,j) \leq BCCS(j). \tag{15}$$

**Useful energy production**

$$E_0(t,j) + E_3(t,j) - A_e(j)\left[\alpha_{Ef}(j)enf(t,j)^{\frac{s_e(j)-1}{s_e(j)}} + \alpha_{Er}(j)enr(t,j)^{\frac{s_e(j)-1}{s_e(j)}}\right]^{\frac{s_e(j)}{s_e(j)-1}} \leq 0. \tag{16}$$

**Fossil secondary energy production**

$$enf(t,j) - A_1(j)\left[\alpha_{1K}(j)(tgenf(t,j)K_1(t,j))^{\frac{s_1(j)-1}{s_1(j)}} + \alpha_{1em}(j)enf_1(t,j)^{\frac{s_1(j)-1}{s_1(j)}}\right]^{\frac{s_1(j)}{s_1(j)-1}} \leq 0. \tag{17}$$

**Renewable secondary energy production**

$$enr(t,j) - A_2(j)(tgenr(t,j)K_2(t,j))^{s_2(j)} \leq 0. \tag{18}$$

The elasticity $(s.)$ and share parameters $(\alpha.)$, obtained from calibration are shown in Table A2. The parameters $tg(t,j)$, $tgv(t,j)$, $tgenf(t,j)$, $tgenr(t,j)$ are exogenously defined productivity growth factors.

## 2.5 Carbon market equilibrium

The constraints describing the international carbon market are given below. The strategic variable, for each coalition $j$, is the quantity of emission rights $\omega(t,j)$ they supply to the market at period $t$. On the carbon market the total supply of permits must be greater or equal to total emissions. The firms, in each coalition, will set their emission at a level where carbon price equals the marginal productivity of emissions (or marginal abatement cost). These two sets of conditions determine the market equilibrium :

**Emissions from primary fossil energy (for coalition $j$ at period $t$)**

$$em(t,j) = Coeff(j) \times enf_1(t,j), \tag{19}$$

where the emission rate is evaluated at $Coeff(j) = 0.004$ $GtCO_2$ per PJ of fossil energy source.

**Total supply of permits is greater or equal to total emissions (at period $t$)**

$$\sum_j \omega(t,j) - \sum_j em(t,j) \geq 0. \tag{20}$$

**Efficiency (at period $t$)**

$$p(t) = \frac{\partial Y(t,j)}{\partial em(t,j)} \tag{21}$$

$$= \frac{\partial Y(t,j)}{\partial E_0(t,j)} \frac{\partial E_0(t,j)}{\partial enp_1(t,j)} \frac{\partial enp_1(t,j)}{\partial em(t,j)}. \tag{22}$$

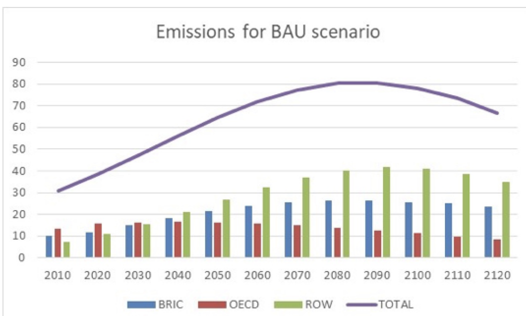
The expression of the derivatives are given in Appendix.

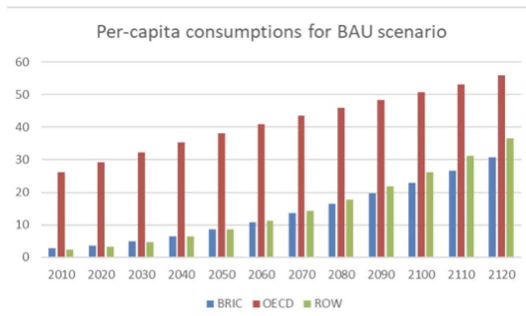
# 3 Scenarios

## 3.1 BAU optimal growth

In this simulation the sum for the three regions of the discounted utilities derived from consumption is optimized over a 150 year horizon. To eliminate the end-of-horizon effects we look only at the variable values between 2014 and 2120. In the BAU [9] scenario, there is no possibility to use CDR and one assumes a very large SEB, i.e. $B = 100'000$Gt. This eliminates the emissions constraints for all practical purpose. The resulting global and regional emission profiles that are associated with economic growth are shown in Table 1. Yearly emissions top at 80 Gt $CO_2$ in 2090 and decline afterwards. This is due to the increase in efficiency of renewable energy technologies. Cumulative emissions on the next 150 years reach more than 9'170 Gt $CO_2$. This is 8 times the nominal SEB of 1'170 Gt $CO_2$. The temperature rise and the resulting damages would be considerable. Per-capita consumption more than double for OECD countries, from 2020 to 2130, but it is multiplied by a factor 11 for BRIC and a factor 16 for ROW.

**Table 1 – BAU scenario : Emission and consumption profiles**




| | BRIC | OECD | ROW | TOTAL |
|---|---|---|---|---|
| 2014 | 11.72 | 10.76 | 6.27 | 28.75 |
| 2090 | 29.60 | 11.02 | 39.96 | 80.58 |
| 2120 | 25.70 | 7.49 | 33.72 | 66.91 |

| | BRIC | OECD | ROW |
|---|---|---|---|
| 2014 | 2.72 | 26.26 | 2.26 |
| 2090 | 19.66 | 48.44 | 21.79 |
| 2120 | 30.70 | 56.02 | 36.41 |

The considerable increase in fossil energy and renewable energy capital stocks is summarised in Table 2.

**Table 2 – BAU scenario : $K_1/L$ versus $K2/L$**

| | $K_1/L$ | | |
|---|---|---|---|
| Year | BRIC | OECD | ROW |
| 2014 | 1.56 | 3.82 | 0.89 |
| 2120 | 21.52 | 28.94 | 22.00 |

| | $K_2/L$ | | |
|---|---|---|---|
| Year | BRIC | OECD | ROW |
| 2014 | 2.17 | 7.90 | 2.32 |
| 2120 | 36.98 | 52.76 | 65.83 |

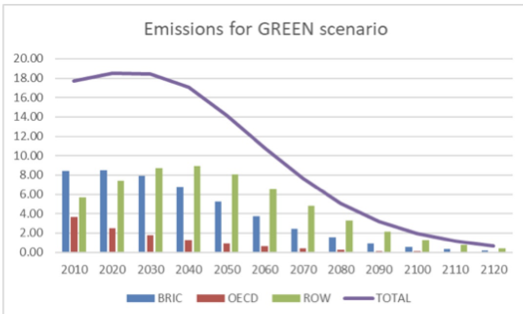
## 3.2 GREEN : Convergence to ZNE without CDR/DAC

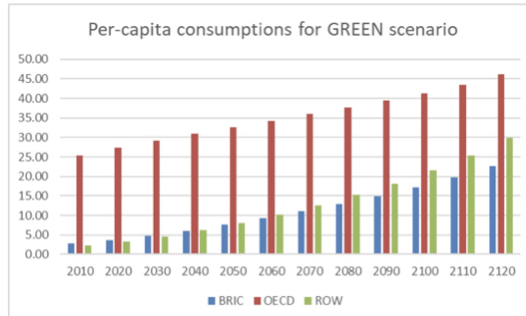
The global SEB [10] of 1'170 Gt$CO_2$ is used, cooperatively, in a program where the sum for the three regions of the discounted utilities derived from consumption is optimized over a 150 year horizon. By setting to 0 the upper limit on carbon sequestration we eliminate the possibility to use CDR/DAC technologies. The only option to reduce emissions is to switch to renewable energy source. The resulting global and regional emission profiles that are associated with economic growth are shown in Table 3.

9. Business as usual.
10. For a justification of this SEB we refer to the recent IPCC report [12]

From 2020 to 2130, per-capita consumption is multiplied by a factor 1.8 for OECD countries, but it is multiplied by a factor 8 for BRIC and a factor 13 for ROW.

**Table 3 – GREEN scenario : Emission and consumption profiles**




| Year | BRIC | OECD | ROW | TOTAL |
|---|---|---|---|---|
| 2014 | 8.44 | 4.94 | 5.60 | 18.98 |
| 2120 | 0.20 | 0.06 | 0.34 | 0.60 |

| Year | BRIC | OECD | ROW |
|---|---|---|---|
| 2014 | 2.77 | 25.29 | 2.31 |
| 2120 | 22.72 | 46.12 | 29.85 |

The fossil energy capital stock collapses and renewable energy capital stock reaches much higher value as shown in Table 4.

**Table 4 – GREEN scenario : $K_1/L$ versus $K2/L$**

| $K_1/L$ | | | |
|---|---|---|---|
| Year | BRIC | OECD | ROW |
| 2014 | 1.56 | 3.82 | 0.89 |
| 2120 | 0.39 | 0.55 | 0.51 |

| $K_2/L$ | | | |
|---|---|---|---|
| Year | BRIC | OECD | ROW |
| 2014 | 2.17 | 7.90 | 2.32 |
| 2120 | 50.10 | 80.25 | 76.28 |

## 3.3 MARKET : Optimal use of shares of SEB with carbon market

In this scenario we give the coalitions a share of the SEB and a possibility to capture $CO_2$ as indicated on Table 5.

**Table 5 – SEB shares and sequestration bounds**

| Budget shares $\theta(\cdot)$ | | | DAC-CCS Bounds (Gt $CO_2$/Y) | | |
|---|---|---|---|---|---|
| BRIC | OECD | ROW | BRIC | OECD | ROW |
| 0.4 | 0.1 | 0.5 | 8 | 5 | 10 |

A zero-net regime is reached by year 2070, as shown on Figure 1. CDR/DAC activity begins in 2060 with a rapid increase until 2090 when it reaches a steady-state at 23 Gt $CO_2$ captured each year, as shown on Figure 2. The resulting global and regional emission profiles that are associated with economic growth are shown in Table 6.

From 2020 to 2130, per-capita consumption net of revenue from permit trading is multiplied by a factor 2 for OECD countries, but it is multiplied by a factor 9.6 for BRIC and a factor 14.4 for ROW. The emissions decline until 17 Gt/Y in 2070 and reach a steady state at 21.4 Gt/Y thereafter.

Table 7 gives the per-capita consumption corrected by the revenue or spending associated with emissions trading.

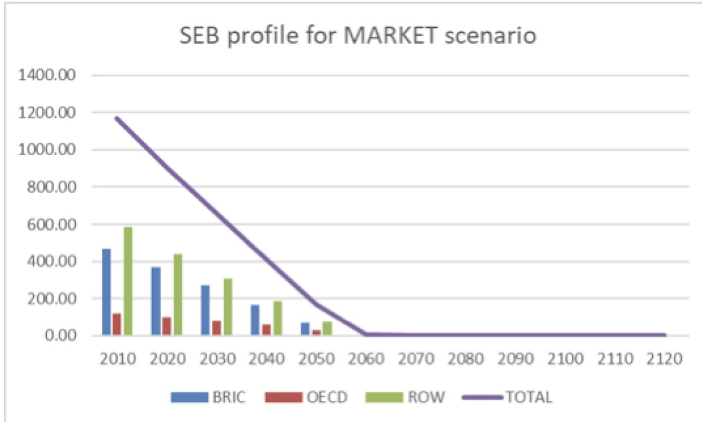



**Figure 1 – Budget Profiles**

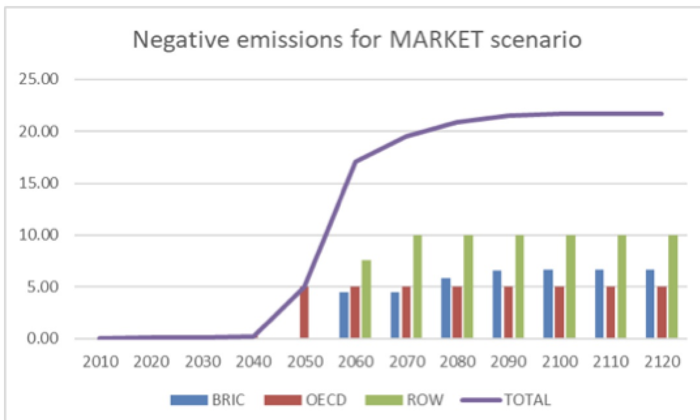
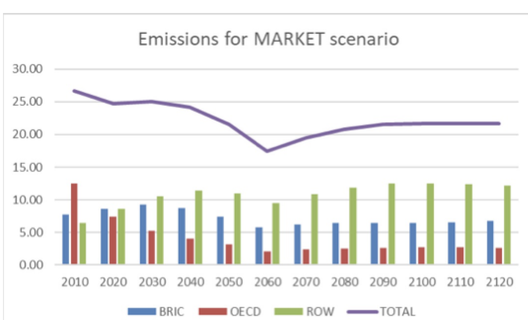
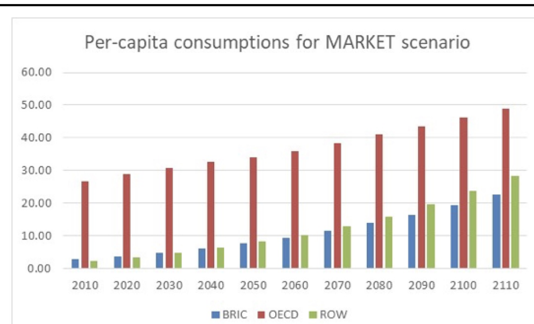



**Figure 2 – CDR/DAC activity**

**Table 6 – DAC/MARKET scenario : Emission and consumption profiles**




| | BRIC | OECD | ROW | TOTAL |
|---|---|---|---|---|
| 2014 | 7.75 | 12.47 | 6.40 | 26.63 |
| 2120 | 6.69 | 2.67 | 12.08 | 21.44 |




| | BRIC | OECD | ROW |
|---|---|---|---|
| 2014 | 2.94 | 24.54 | 2.97 |
| 2120 | 26.53 | 53.54 | 33.26 |

**Table 7 – Optimal consumption net of revenue from permit trading**

| | BRIC | OECD | ROW |
|---|---|---|---|
| 2014 | 2.77 | 25.74 | 2.30 |
| 2120 | 26.60 | 53.89 | 33.20 |

Carbon price starts at \$ 241 in 2020 and reaches \$ 828 in 2130 (See Table 8). BRIC and ROW are permit sellers and OECD is a permit buyer, as indicated on Table 8. Table 9 shows the evolution of fossil and renewable capital stocks, expressed in per-capita values.

**Table 8 – MARKET scenario : Carbon price and permits trading**

| Price (\$) | | Trading (per-capita) | | | Negative emissions (Gt $CO_2$) | | |
|---|---|---|---|---|---|---|---|
| Year | BRIC | BRIC | OECD | ROW | BRIC | OECD | ROW |
| 2014 | 241.30 | 0.20 | -2.03 | 0.66 | 0.00 | 0.00 | 0.00 |
| 2070 | 761.23 | -0.40 | 1.44 | -0.12 | 4.52 | 5.00 | 10.00 |
| 2120 | 828.36 | -0.07 | 1.43 | -0.26 | 6.44 | 5.00 | 10.00 |

**Table 9 – MARKET scenario : $K_1/L$ versus $K2/L$**

| $K_1/L$ | | | | $K_4/L$ | | | |
|---|---|---|---|---|---|---|---|
| Year | BRIC | OECD | ROW | Year | BRIC | OECD | ROW |
| 2014 | 1.56 | 3.82 | 0.89 | 2014 | 2.17 | 7.90 | 2.32 |
| 2120 | 8.11 | 10.87 | 9.82 | 2120 | 39.81 | 60.88 | 67.18 |

### 3.4 Comparing welfare

We use the discounted sum of per-capita consumption $W(j)$ defined in (11) as a welfare criterion. Table 10 shows the welfare losses with respect to the BAU scenario. From these figures it appears that introducing DAC technologies mitigates the welfare losses w.r.t. BAU scenario. We indicate also the welfare losses for a scenario OPT where the SEB is used optimally, without using a carbon market, in order to optimise the total discounted sum of the utilities derived from consumption (we do not give the details of the run due to the lack of space). It is clear that the MARKET scenario is sub-optimal but very close to the optimal one. The variations in the welfare losses seem to indicate that giving 10 % of the SEB to OECD is too generous and 50 % of the SEB to ROW is not sufficient.

In this analysis we have uniquely considered the economic variables that are affecting welfare. Indeed, BAU scenario would create considerable damages that could not be easily translated into economic losses (e.g. bio-diversity loss and species extinction). Also the stranded asset risk, which exists in a GREEN scenario where fossil energy disappears almost completely should be considered and included in a policy analysis.

**Table 10 – Welfare criteria**

| Welfare | MARKET | OPT | BAU | GREEN |
|---|---|---|---|---|
| BRIC | 231.25 | 231.60 | 254.40 | 220.161 |
| OECD | 1094.70 | 1087.24 | 1156.62 | 1029.48 |
| ROW | 248.06 | 248.70 | 262.59 | 238.551 |
| Welfare Loss | MARKET | OPT | BAU | GREEN |
| BRIC | 9% | 9% | 0% | 13% |
| OECD | 5% | 6% | 0% | 11% |
| ROW | 6% | 5% | 0% | 9% |

## 4 Discussion and conclusion

The objective of this short paper was to present a compact dynamic optimisation model that can give a first insight into the importance of developing DAC technologies to cope with anthropogenic climate change. The model is implemented in AMPL. [11] The first scenarios, BAU or GREEN, obtained with this compact OR model compare well with the scenarios constructed with much larger and more encompassing AIMs [10]. The MARKET scenario is relatively original as it provides strong support for the consideration of DAC activities as a promising way to achieve the objectives of the Paris Agreement in the long term.

The interest in having developed a compact model lies in the possibility of introducing stochastic control and/or dynamic game techniques into the modelling to address issues of uncertainty and strategic behaviour. Because it is small and based on a consistent use of nested CES production functions, this model clarifies the challenges and opportunities of developing climate policy in a framework of economic development and growth.

# Appendix

## A.1 Figure and tables

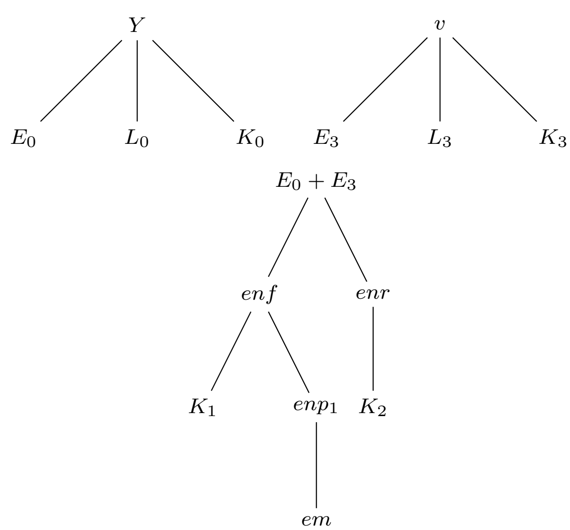



**Figure A1 – Nested structure for general economy and CDR activity**

**Table A1 – Population levels**

| Year | 2014 | 2020 | 2030 | 2040 | 2050 | 2060 | 2070 | 2080 | 2090 | 2100 |
|---|---|---|---|---|---|---|---|---|---|---|
| BRIC | 3042 | 3251 | 3375 | 3417 | 3387 | 3303 | 3182 | 3043 | 2904 | 2822 |
| OECD | 1273 | 1331 | 1365 | 1385 | 1388 | 1382 | 1372 | 1363 | 1354 | 1350 |
| ROW | 2980 | 3527 | 4081 | 4626 | 5141 | 5602 | 6001 | 6331 | 6587 | 6703 |
| TOTAL | 7295 | 8109 | 8821 | 9428 | 9916 | 10287 | 10555 | 10737 | 10845 | 10875 |

11. The source files can be obtained on request.

**Table A2 – CES functions parameters**

| $Y$ | OECD | BRIC | ROW |
|---|---|---|---|
| $A_0(\cdot)$ | 6.853 | 2.329 | 2.193 |
| $\alpha_{0K}(\cdot)$ | 0.398 | 0.298 | 0.284 |
| $\alpha_{0L}(\cdot)$ | 0.518 | 0.454 | 0.510 |
| $\alpha_{0E}(\cdot)$ | 0.085 | 0.248 | 0.207 |
| $s_0(\cdot)$ | 0.9 | 0.9 | 0.9 |

| $v$ | OECD | BRIC | ROW |
|---|---|---|---|
| $A_3(\cdot)$ | 0.006 | 0.002 | 0.002 |
| $\alpha_{3K}(\cdot)$ | 0.588 | 0.556 | 0.545 |
| $\alpha_{3L}(\cdot)$ | 0.243 | 0.286 | 0.286 |
| $\alpha_{3E}(\cdot)$ | 0.169 | 0.158 | 0.158 |
| $s_3(\cdot)$ | 0.9 | 0.9 | 0.9 |

| $E_0 + E_3$ | OECD | BRIC | ROW |
|---|---|---|---|
| $A_e(\cdot)$ | 6.853 | 2.329 | 2.193 |
| $\alpha_{Ef}(\cdot)$ | 0.706 | 0.702 | 0.616 |
| $\alpha_{Er}(\cdot)$ | 0.294 | 0.298 | 0.384 |
| $s_e(\cdot)$ | 1.5 | 1.5 | 1.5 |

| $enf$ | OECD | BRIC | ROW |
|---|---|---|---|
| $A_1(\cdot)$ | 6.853 | 2.329 | 2.193 |
| $\alpha_{1K}(\cdot)$ | 0.517 | 0.434 | 0.693 |
| $\alpha_{1em}(\cdot)$ | 0.483 | 0.566 | 0.307 |
| $s_1(\cdot)$ | 1.5 | 1.5 | 1.5 |

| $enr$ | OECD | BRIC | ROW |
|---|---|---|---|
| $A_2(\cdot)$ | 0.095 | 0.087 | 0.114 |
| $s_2(\cdot)$ | 1 | 1 | 1 |

## A.2 Derivatives of the production function

$$\frac{\partial Y(t,j)}{\partial E_0(t,j)} = A_0(t,j)tg(t.j)\left[\alpha_{0K}K_0(t,j)^{\frac{s_0(j)-1}{s_0(j)}} + \alpha_{0L}L_0(t,j)^{\frac{s_0(j)-1}{s_0(j)}} + \alpha_{0E}E_0(t,j)^{\frac{s_0(j)-1}{s_0(j)}}\right]^{\frac{s_0(j)}{s_0(j)-1}-1} \alpha_{0E}E_0(t,j)^{\frac{s_0(j)-1}{s_0(j)}-1}, \quad \text{(A1)}$$

$$\frac{\partial E_0(t,j)}{\partial enp_1(t,j)} = \frac{\partial E_0(t,j)}{\partial enf(t,j)}\frac{\partial enf(t,j)}{\partial enp_1(t,j)} = \frac{A_e(j)}{Coeff(j)}\left[\alpha_{Ef}(j)enf(t,j)^{\frac{s_e(j)-1}{s_e(j)}} + \alpha_{Er}(j)enr(t,j)^{\frac{s_e(j)-1}{s_e(j)}}\right]^{\frac{s_e(j)}{s_e(j)-1}-1} \alpha_{Ef}(j)enf(t,j)^{\frac{s_e(j)-1}{s_e(j)}-1} A_2\left[\alpha_{1K}(j)(enf(t,j)K_1(t,j))^{\frac{s_1(j)-1}{s_1(j)}} + \alpha_{1em}(j)(j)enf_1(t,j)^{\frac{s_1(j)-1}{s_1(j)}}\right]^{\frac{s_1(j)}{s_1(j)-1}-1} \alpha_{1em}(j)(j)enf_1(t,j)^{\frac{s_1(j)-1}{s_1(j)}-1} \quad \text{(A2)}$$

$$\frac{\partial enp_1(t,j)}{\partial em(t,j)} = \frac{1}{Coeff(j)}.$$

# Références


[1] A. Aguiar, M. Chepeliev, E. Corong, R. McDougall, and D. van der Mensbrugghe. The GTAP data base : Version 10. Journal of Global Economic Analysis, 4(1) :1–27, 2019.

[2] M.R. Allen, D.J. Frame, C. Huntingford, C.D. Jones, J.A. Lowe, M. Meinshausen, and Meinshausen. N. Warming caused by cumulative carbon emissions towards the trillionth tonne. Nature, 458 :1163–1166, 2009.

[3] Myles Allen. Climate 2020, chapter The scientific case for a cumulative carbon budget, pages 118–120. Witan Media, London, 2015.

[4] F. Babonneau, A. Badran, M. Benlahrech, A. Haurie, M. Schenckery, and M. Vielle. Economic assessment of the development of CO2 direct reduction technologies in long-term climate strategies of the gulf countries. Climatic Change, Published online 25 April 2021.

[5] F. Babonneau, O. Bahn, A. Haurie, and M. Vielle. An oligopoly game of CDR strategy deployment in a steady-state net-zero emission climate regime. Environmental Modelling and Assessment, Online first article, October 2020.

[6] O. Bahn and A. Haurie. A class of games with coupled constraints to model international ghg emission agreements. International Game Theory Review, Vol. 10 :337–362, 2008.

[7] O. Bahn and A. Haurie. A cost-effectiveness differential game model for climate agreements. Dynamic Games and Applications, 6(1) :1–19, 2016.

[8] O. Bahn, A. Haurie, and R. Malhamé. A stochastic control model for optimal timing of climate policies. Automatica, 44 :1545–1558., 2008.

[9] C. Chen and M. Tavoni. Direct air capture of $CO_2$ and climate stabilization : a model based assessment. Climatic Change, 118 :59–72, 2013.

[10] Kenneth Gillingham, William Nordhaus, David Anthoff, Valentina Bosetti, Haewon McJeon, Geoffrey Blanford, Peter Christensen, John Reilly, and Paul Sztorc. Modeling uncertainty in climate change : A multi-model comparison. Technical Report 13, Nota di Lavoro, 2016.

[11] International Energy Agency. $CO_2$ Emissions from Fuel Combustion, 2020.

[12] IPCC. Summary for Policymakers. In : Climate Change 2021 : The Physical Science Basis. Contribution of Working Group I to the Sixth Assessment Report of the Intergovernmental Panel on Climate Change [M. in[MassonDelmotte, V., P. Zhai, A. Pirani, S.L. Connors, C. Péan, S. Berger, N. Caud, Y. Chen, L. Goldfarb, M.I. Gomis, M. Huang, K. Leitzell, E. Lonnoy, J.B.R. Matthews, T.K. Maycock, T. Waterfield, O. Yelekçi, R. Yu, and B. Zhou (eds.)]. Cambridge University Press. In Press, 2021.

[13] D. W. Keith, G. Holmes, D. St. Angelo, and K. Heidel. A process for capturing $CO_2$ from the atmosphere. Joule, 2 :1573–1594, August 2018.

[14] A. Marcucci, V. Panos, and S. Kypreos. The road to achieving the long-term Paris targets : energy transition and the role of direct air capture. Climatic Change, 2017.

[15] Markus Ohndorf, Julia Blasch, and Renate Schubert. Emission budget approaches for burden sharing : some thoughts from an environmental economics point of view. Climatic Change, 133(3) :385–395, 2015.

[16] Robert Socolow, Michael Desmond, Roger Aines, Jason Blackstock, Olav Bolland, Tina Kaarsberg, Nathan Lewis, Marco Mazzotti, Allen Pfeffer, Karma Sawyer, Jeffrey Siirola, Berend Smit, and Jennifer Wilcox. Direct Air Capture of $CO_2$ with Chemicals : A Technology Assessment for the APS Panel on Public Affairs. 01 2011.

[17] United Nations. World population prospects : The 2019 revision. Population Division, Department of Economic and Social Affairs, 2019.

[18] H Uzawa. Production functions with constant elasticities of substitution. Review of Economic Studies, 29(4) :291–299, 1962.