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É. Edom, M.F. Anjos,
C. D'Ambrosio, W. van Ackooij,
P. Côté, S. Séguin

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GERAD HEC Montréal
3000, chemin de la Côte-Sainte-Catherine
Montréal (Québec) Canada H3T 2A7

Tél. : 514 340-6053
Téléc. : 514 340-5665
info@gerad.ca
www.gerad.ca

On the impact of the power production function approximation on hydropower maintenance scheduling

Éloïse Edom [a]

Miguel F. Anjos [b]

Claudia D'Ambrosio [c]

Wim van Ackooij [d]

Pascal Côté [e]

Sara Séguin [f]

[a] *GERAD & Department of Mathematics and Industrial Engineering, Polytechnique Montréal Montréal (Québec) Canada*

[b] *GERAD & School of Mathematics, University of Edinburgh, Edinburgh, UK*

[c] *LIX, CNRS & École Polytechnique, Palaiseau, France*

[d] *EDF R&D. OSIRIS, Palaiseau, France*

[e] *GERAD & Énergie Électrique, Rio Tinto Aluminium, (Québec), Canada*

[f] *GERAD & Department of Computer Science and Mathematics, Université du Québec à Chicoutimi, Chicoutimi, (Québec), Canada*

eloise.edom@polymtl.ca
anjos@stanfordalumni.org
sdambrosio@lix.polytechnique.fr
wim.van-ackooij@edf.fr
pascal.cote@riotinto.com
sara.seguin@uqac.ca

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Abstract: Maintenance planning for hydropower plants is a crucial problem. In this paper, we evaluate the impact of the Hydropower Production Function (HPF) formulation on the maintenance scheduling. Based on an existing model for Generator Maintenance Scheduling that uses a convex hull approximation for representing the HPF, we developed two additional approximations, one that is piecewise linear approximation and another that uses a polynomial function. Then, we compare these three approximations: first a convex hull approximation, then a piecewise linear approximation and third, a nonlinear approach using a polynomial function fitted on real data. We experiment with two test cases based on real-world hydroelectric systems. The results show that for a one-month planning horizon, depending on the approximation used, maintenance tasks can be shifted by up to 5 days, and the difference in energy production can reach 8,300 MWh.

Keywords: Hydroelectricity, maintenance scheduling, hydropower production function, mixed-integer linear optimization, nonlinear optimization

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1 Introduction

In hydropower systems, maintenance is a key element to ensure reliability of the system. The main decisions involved in maintenance management are when to stop or start a generator and which resources to allocate. There are three types of maintenance: corrective maintenance occurs after a generator breakdown; condition-based maintenance is carried out depending on the condition of the generator; and preventive maintenance takes place at regular intervals to reduce the risk of failure. In this paper, we focus on preventive maintenance. In the literature, preventive maintenance scheduling in the power industry is known as the Generator Maintenance Scheduling Problem (GMSP). We are concerned with how significantly preventive maintenance decisions planned using the GMSP may be influenced by the modelling of the Hydropower Production Function (HPF).

Adequately representing the HPF is difficult in itself. Usually it is nonconcave due to the turbine efficiency and the net water head [12]. For each turbine, the general form of the HPF is:

$$P = \rho\; g\; \gamma\; u\; h\; \eta(u,h) \tag{1}$$

where P is the power output $[MW]$, ρ the water density $[kg/m^3]$, g the gravitational acceleration $[m/s^2]$, γ a conversion factor $[10^{-6}]$, u the water discharge $[m^3/s]$, h the net water head $[m]$ and $\eta(u,h)$ the turbine efficiency. In addition, the quantity of water available depends on the weather which causes uncertainty. Moreover, the power stations are in cascade which means they are placed one after the other as in Figure 1. This configuration implies a spatial and temporal interdependency.

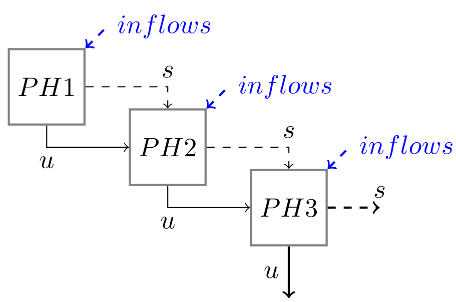


Figure 1: Hydroelectric complex with 3 powerhouses in cascade ($PH1, PH2, PH3$**)**

We assume that the hydroelectric complexes of interest do not have analytic representations for the production functions of their powerhouses, as in generally the case. The two hydroelectric power complexes in this study currently plan maintenance separately from generation scheduling. These are two different problems that are solved iteratively. Previous research considered the nonlinearity of the HPF but usually these works do not address maintenance planning. Catalão et al. [2] used a nonlinear function of the water discharge and water storage to approximate the power generation. Diniz and Piñeiro Maceira [4] developed an approximation of the HPF based on linear inequalities. Subsequently Diniz et al. [5] adapted this formulation to apply it on aggregated cascade systems for long-term planning. Séguin et al. [12] presented a two-stage optimization approach using smoothing splines to represent the HPF for short-term scheduling. Piecewise linear (PWL) approximations have also been proposed. Borghetti et al. [1] refined a PWL representation to consider many of the hydroelectric system characteristics. More recently, Hjelmeland et al. [9] worked on a PWL approximation and they evaluated the impact of modelling details on the HPF. van Ackooij et al. [13] also used a PWL approximation in their study of unit commitment under wind uncertainty to properly capture the nonlinear behavior of the HPF.

Additional literature focused on the GMSP. Foong et al. [7] proposed an Ant Colony Optimization heuristic. Kuzle et al. [10] used a Benders decomposition with a mixed integer linear optimization formulation. The uncertainty of the power output was taken into account by Feng et al. [6] using fuzzy variables. These variables represent the uncertainty of inflows and generators failure. Guedes et al. [8] worked with a nonlinear formulation of the GMSP where only storage variables are explicit; due to the complexity of this formulation, some simplifying assumptions were made about the HPF. Sometimes the power output is considered constant, e.g. in [7], while in other studies, the nonlinearity is not considered [10, 6]. All these papers use valid approximations of the HPF, but none of them evaluate the impact of the HPF on the solution obtained.

In this paper, we evaluate the impact of the HPF formulation on the maintenance scheduling. We compare three approximations: first a convex hull approximation based on the model developed by Rodriguez et al. [11]; second a piecewise linear approximation; third a nonlinear approach using a polynomial function fitting real data. These three models are applied to two existing hydroelectric systems, one in the southeast of Brazil and the other in the region of Saguenay-Lac-St-Jean in Canada. We compare the results in terms of power generation and maintenance schedules.

The paper is organized as follows. Firstly, the mathematical formulation of the model used is presented in Section 2. Section 3 describes the two additional approximations of the HPF that we considered. In Section 4, we present and discuss our computational results on the two hydroelectric systems considered. Concluding remarks are given in Section 5.

2 Mathematical formulation

Throughout this paper, parameters are represented by uppercase and variables by lowercase.

2.1 Objective function

In this study, we use the model developed by [11]. This model was chosen because the convex hull approximation of the HPF that it uses has good accuracy for representing the power production function. The GMSP is formulated as miximizing the profit defined by the value of the electricity production plus the value of the stored water minus the maintenance costs. We write this objective function as:

$$\max_{\substack{w^+,w^-\\u,v,s,\\r,p,y,z}} \sum_{t\in\mathcal{T}}(B_t^+w_t^+ + B_t^-w_t^-) + \sum_{i\in\mathcal{I}}(A_i s_{iT}) - \sum_{m\in\mathcal{M},t\in\mathcal{T}(m)} C_{mt}y_{mt} \tag{2}$$

The first part of the function represents the value of the electricity corresponding to the net benefit of electricity trade: $(B_t^+w_t^+ + B_t^-w_t^-)$, where $(B_t^+w_t^+)$ is the revenue from electricity sales, and $(B_t^-w_t^-)$ is the cost of purchasing electricity. The value of stored water at the end of the planning horizon is given by $A_i s_{iT}$, and the cost of each maintenance m by $C_{mt}y_{mt}$.

2.2 Hydro constraints

The mass balance equation (3a) means that the difference between the water volume at the beginning of the time period and the end of the time period is equal to external inflows plus inflows coming from the upstream powerhouses minus total outflows. The energy balance equation (3b) defines that total energy production plus purchased energy equals total load plus energy sales.

$$s_{it} - s_{i(t-1)} = Q(F_{it} + \sum_{g\in\mathcal{U}(i)}[u_{gt} + v_{gt}] - u_{it} - v_{it}), \qquad \forall\, i\in\mathcal{I}, t\in\mathcal{T}, \tag{3a}$$

$$\sum_{i\in\mathcal{I}} p_{it} + w_t^- = d_t + w_t^+, \qquad \forall\, t\in\mathcal{T}. \tag{3b}$$

Bounds are imposed on water discharges, water spills, water volumes, and electricity trade variables by (4a)–(4e).

$$0 \le u_{it}\bar{U}_i, \qquad \forall\, i\in\mathcal{I},\ t\in\mathcal{T}, \tag{4a}$$

$$0 \leq v_{it} \leq \bar{V}_i, \qquad \forall\, i \in \mathcal{I},\ t \in \mathcal{T}, \tag{4b}$$
$$\underline{S}_i \leq s_{it} \leq \bar{S}_i, \qquad \forall\, i \in \mathcal{I},\ t \in \mathcal{T}, \tag{4c}$$
$$0 \leq w_t^+ \leq \bar{W}^+, \qquad \forall\, t \in \mathcal{T}, \tag{4d}$$
$$0 \leq w_t^- \leq \bar{W}^-, \qquad \forall\, t \in \mathcal{T}. \tag{4e}$$

The constraints for decision variables for production and maintenance are defined by (5a)–(5e).

$$\sum_{k \in \mathcal{K}(i,t)} z_{itk} = 1, \qquad \forall\, i \in \mathcal{I},\ t \in \mathcal{T}, \tag{5a}$$
$$\sum_{t \in \mathcal{T}(m)} y_{mt} = 1, \qquad \forall\, m \in \mathcal{M}, \tag{5b}$$
$$\sum_{\substack{m \in \mathcal{M}(i) \\ t' \in \{\mathcal{T}(m) | \\ (t - D_m + 1) \leq t' \leq t\}}} y_{mt'} = r_{it}, \qquad \forall\, i \in \mathcal{I}, t \in \mathcal{T}, \tag{5c}$$
$$0 \leq r_{it} \leq O_{it}, \qquad \forall\, i \in \mathcal{I}, t \in \mathcal{T}, \tag{5d}$$
$$r_{it} + \sum_{k \in \mathcal{K}(i,t)} k z_{itk} = \bar{G}_{it}, \qquad \forall\, i \in \mathcal{I},\ t \in \mathcal{T}. \tag{5e}$$

2.3 Estimation of the power production

The next group of constraints concerns the power generation. This part has been modified from the reference model [11] to test the three different HPF representations. For the convex hull approximation with hyperplanes used in [11], the constraints are:

$$p_{itk} \leq \beta_h^0 + \beta_h^u u_{it} + \beta_h^s s_{it}, \qquad \forall\, i \in \mathcal{I}, t \in \mathcal{T}, k \in \mathcal{K}(i,t), h \in \mathcal{H}(i,k), \tag{6a}$$
$$p_{itk} \leq z_{ik} \bar{P}_{ik}, \qquad \forall\, i \in \mathcal{I},\ t \in \mathcal{T},\ k \in \mathcal{K}(i,t), \tag{6b}$$
$$\sum_{k \in \mathcal{K}(i,t)} p_{itk} = p_{it}, \qquad \forall\, i \in \mathcal{I},\ t \in \mathcal{T}. \tag{6c}$$

3 Alternative power production representations

We propose two alternative approaches to represent the HPF. The first one is based on a two-variable piecewise linear approximation method, called rectangle method, presented by [3]. We adapt it to the hydroelectricity production context. Consequently, the power generation constraints become:

$$\sum_{f \in \mathcal{F}-1} h_{ikft} = 1, \tag{7a}$$
$$\alpha_{ikft} \leq h_{ik,f-1,t} + h_{ikft}, \tag{7b}$$
$$\sum_{f \in \mathcal{F}} \alpha_{ikft} = 1, \tag{7c}$$
$$u_{it} = \sum_{f \in \mathcal{F}} \alpha_{ikft} B_{ikf}^u, \tag{7d}$$
$$\sum_{e \in \mathcal{E}-1} \beta_{iket} = 1, \tag{7e}$$
$$s_{it} = \sum_{e \in \mathcal{E}-1} \beta_{iket} B_{ike}^s + \gamma_{iket}(B_{ik,e+1}^s - B_{ike}^s), \tag{7f}$$
$$\gamma_{iket} \leq \beta_{iket}, \tag{7g}$$
$$\phi_{iket} = \sum_{f \in \mathcal{F}} \alpha_{ikft} B_{ikef}^p, \tag{7h}$$

$$p_{itk} \leq \phi_{iket} + \gamma_{iket} K_{ikef} + M(2 - \beta_{iket} - h_{ikft}), \tag{7i}$$

$$p_{itk} \leq z_{ik} \bar{P}_{ik}, \tag{7j}$$

$$\sum_{k \in \mathcal{K}(i,t)} p_{itk} = p_{it}, \tag{7k}$$

where i is the index of powerhouses, k is the index of active generators configuration, e and f are the indexes of sampling points for the water discharge and volume, and t is the index of time period. In addition, h_{ikft} and β_{iket} are binary variables that select the right interval of the approximation of the HPF; B^u_{ikf}, B^s_{ike} and B^p_{ikef} are the breakpoints for respectively the water discharge, the water volume and the power output; α_{ikft} and γ_{iket} are continuous variables; ϕ_{iket} is the estimation of the power as a function of the water discharge; K_{ikef} is a correction parameter; and M is a suitable large value.

The second alternative is a mixed integer nonlinear formulation using a polynomial function fitted to real data. This function has the form

$$\begin{aligned} f_{itk} = {} & c_1 + sc_2 + uc_3 + s^2c_4 + usc_5 + u^2c_6 + s^2uc_7 \\ & + su^2c_8 + u^3c_9 + s^2u^2c_{10} + su^3c_{11} + u^4c_{12} \\ & + s^2u^3c_{13} + u^4sc_{14} + u^5c_{15} \end{aligned}$$

where $c_1, ..., c_{15}$ are the coefficients of the polynomial; u is the water discharge; and s the stored volume. Thus the power generation constraints are expressed as

$$p_{itk} \leq f_{itk}, \qquad \forall\, i \in \mathcal{I},\, t \in \mathcal{T},\, k \in \mathcal{K}(i,t), \tag{8a}$$

$$p_{itk} \leq z_{ik} \bar{P}_{ik}, \qquad \forall\, i \in \mathcal{I},\, t \in \mathcal{T},\, k \in \mathcal{K}(i,t), \tag{8b}$$

$$\sum_{k \in \mathcal{K}(i,t)} p_{itk} = p_{it}, \qquad \forall\, i \in \mathcal{I},\, t \in \mathcal{T}. \tag{8c}$$

4 Computational experiments

4.1 Case study

The three models are tested on two hydroelectric systems. The first one is in the southeast region of Brazil. It is composed of two hydroelectric plants: Itumbiara and Cachoeira Dourada. The second case studied consists of three hydroelectric plants on the lower part of Rio Tinto Alcan's system in Saguenay, Canada. The basic characteristics of the plants are presented in Table 1.

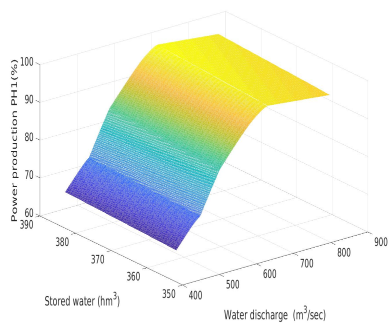
Table 1: Characteristics of the Brazilian and Canadian hydroelectric complexes

Powerhouse	Capacity (MW)	Number of generators	System type
Itumbiara	2082	6	Reservoir
Cach. Dourada	658	10	Run of river
Chute-du-Diable	205	5	Reservoir
Chute-à-la-Savane	210	5	Run of river
Isle-Maligne	402	12	Reservoir

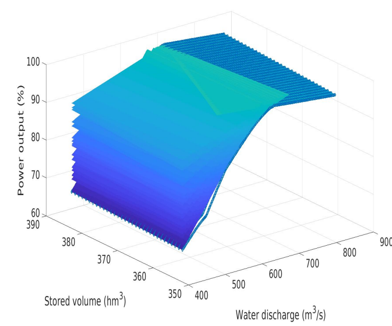
The one-month planning horizon is partitioned into 30 time periods with each period representing one day. The models are deterministic but for the second case studied, 3 inflows scenarios are tested.

As mentioned earlier, the systems studied do not have an analytical representation of their power production function. Currently, Rio Tinto Alcan uses a dynamic programming algorithm to compute specific points of the HPF [12]. The overall HPF can be approximated using a tight grid of points in the plane defined by water discharge and stored volume, with respectively steps of $0.5m^3/s$ and $1hm^3$, and applying the dynamic programming algorithm at each point. A similar procedure was used for the Brazilian hydroelectric complex. This grid approach serves as baseline to evaluate the convex hull, piecewise linear and fitted polynomial approximations.

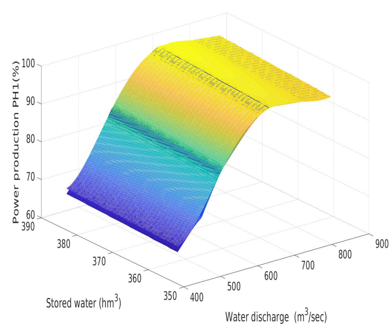
Figure 2(a) shows the real data plotted using the tight grid for the Saguenay hydroelectric complex. Figure 2(b) and 2(c) show the hyperplanes from the convex hull approximation and the polynomial function. The piecewise linear approximation is computed directly in the optimization model. We considered five breakpoints. The load and the purchased electricity are fixed at zero. In addition, to allow the comparison between the models, the same target water volume is fixed at the end of the time horizon in order to ensure that the total amount of turbined water is the same from a model to another. The solution time is consistently of the order of a few minutes regardless of the HPF approximation used. Consequently CPU time is not among our criteria for comparison.



(a) Real data for PH1



(b) 20 Hyperplanes for PH1



(c) Polynomial of degree 5 for PH1

Figure 2: Case Saguenay: Power production (MW) for 4 active generators according to water discharge (m^3/s) and stored volume (hm^3)

To be able to compare the results, we first use the optimization models to compute water discharge, stored volume, and maintenance schedules. These values are then used as inputs for a program using the baseline data to compute the power output.

4.2 Results and discussion

Table 2 shows maintenance parameters of the 6 tasks of PH1 from the Brazilian case. The last two columns represent the boundaries between which the tasks can begin. The duration of each task is given as a number of time periods. Maintenance schedules and power output are presented in Figure 3, 4, 5 and 6. For the Brazilian case, the results shown correspond to a high inflows scenario based on historical data of December (high rainfall season) and 13 maintenance tasks. For the Saguenay case, the results are for a low inflows scenario and 20 maintenance tasks. The maintenance schedules are presented using Gantt charts where the black rectangles are the maintenance tasks and each vertical separation equals one time period.

The last lines of Table 3 and Table 4 present the gap, in percentage, between the objective value computed by the optimization models and the baseline data (with inputs of water volume and water

Table 2: Exemple of parameters for Maintenance Tasks (MT)

Powerhouse	Maintenance task	Duration	Time window Earliest	Latest
PH1	MT1	5	1	6
	MT2	3	5	9
	MT3	3	9	14
	MT4	3	13	18
	MT5	4	17	20
	MT6	2	21	26

discharge from the optimization models). For the Brazilian system the results are shown for 13 maintenance tasks, while for the Saguenay system the results are for three inflow scenarios and two sets of maintenance parameters, the first set having 10 maintenance tasks and the second one having 20. In addition, the first two lines of Table 3 and Table 5 show the energy generated by the optimization models and by the real data for the 13 MT and 20 MT cases respectively. Table 6 shows the profit difference when the maintenance schedule of an approximation is imposed as maintenance schedule for the two other approximations.

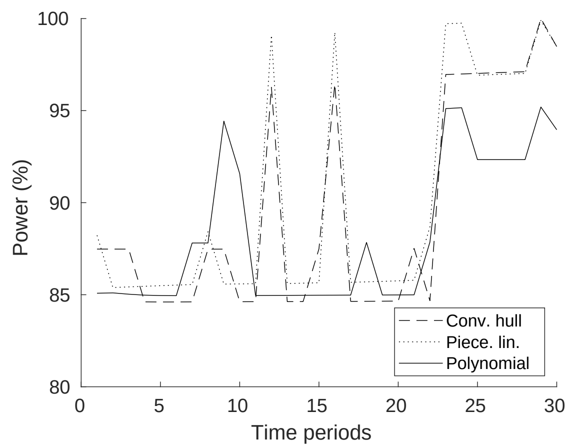
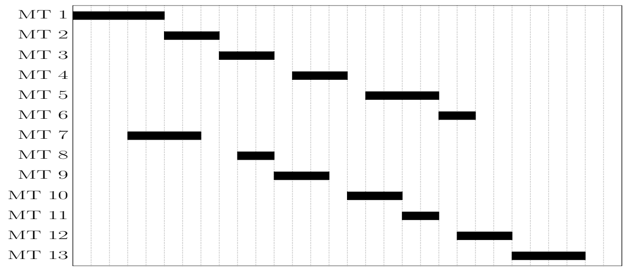


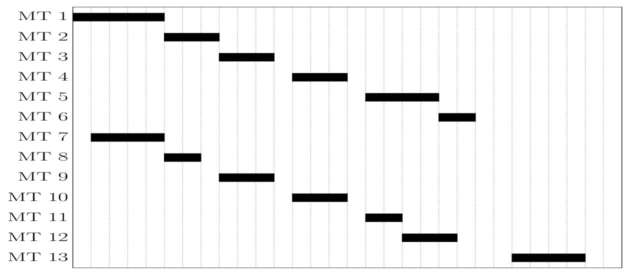
Figure 3: Case Southeast Brazil: Total power production (%)

In Tables 3 and 4, one can observe that the gaps between the objective values computed from the optimization models and from the real data are significant. A negative gap means that production is underestimated by the approximation of the HPF while a positive gap means that production is overestimated. These differences are of up to 1.5%, and at the average price of energy in North American market can represent more than \$400,000 for one month. This is equivalent to 8,300 MWh which is approximately the annual production of a wind turbine of $4MW$ with an average capacity factor of 24%.

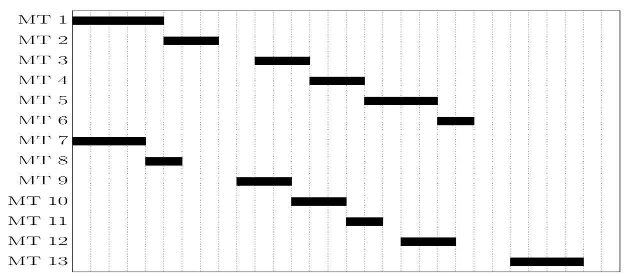
For the Brazilian case, the largest gap (1.397%) is given by the polynomial approximation, and the smallest (0.232%) by the piecewise approximation. For the Canadian case, the largest gaps (up to 1.474%) are observed for the piecewise linear approximation, and the smallest (0.093%) for the convex hull approximation. Table 4 shows that when there are 20 MT, the gaps are lower. This makes sense because the more tasks there are to schedule, the less flexibility there is. Thus, there is less possibility that the maintenance decisions will differ depending on the approximation used. Occasionally, we can still observe important differences, e.g., Figure 6 shows a big difference between the power output of the polynomial approximation and the two other ones at the beginning of the time periods. This can be understood from Figure 5 where we see that two maintenance tasks start later for the polynomial approximation so more power can be generated during this period. As shown in Figure 4 and 5, between two approximations, the start of a maintenance task can be shifted by up to 5 time periods for the Brazilian case and 3 time periods for the Saguenay case.



(a) Convex hull approximation



(b) Piecewise linear approximation



(c) Fitted polynomial approximation

Figure 4: Case Southeast Brazil: Maintenance planning for 30 time periods, 13 maintenance tasks, 6 for PH1, 7 for PH2

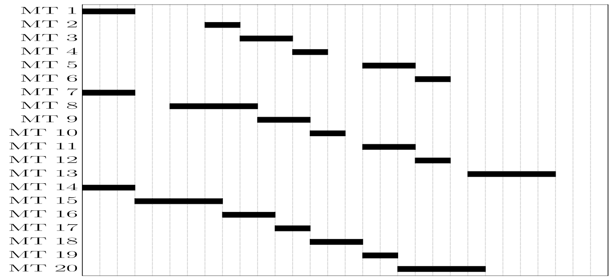
By imposing the maintenance schedule obtained from the convex hull model to the other two models for the Saguenay system, we can see in Table 6 that there are non-negligible gaps. It is also the case when imposing the maintenance schedule obtained from the piecewise linear model or from the polynomial one. Thus, for the same maintenance schedule, the quantities of energy produced computed by the three approximations of the HPF are different from each other.

Table 3: Case Southeast Brazil: Energy produced (MWh) for 13 maintenance tasks

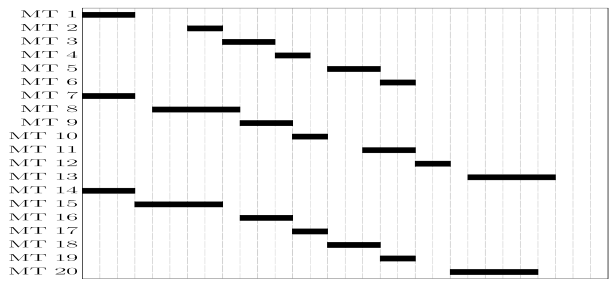
	Conv. Hull	Piecewise Lin.	Polynomial
From optimization	1907849.4	1881410.2	1892179.3
From real data	1883598.1	1855485.5	1896586.7
Gap (%)	1.287	-0.232	1.397

Table 4: Case Saguenay: Profit difference (%) for 10 and 20 maintenance tasks

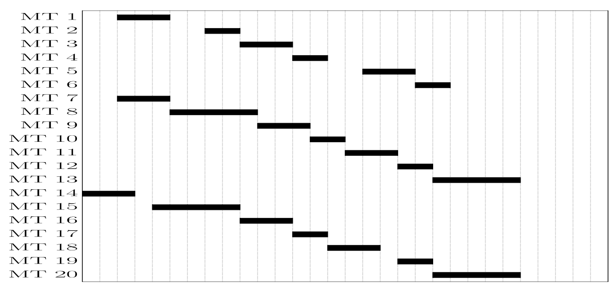
	Conv. Hull			Piecewise Lin.			Polynomial		
Inflows scenario	High	Med.	Low	High	Med.	Low	High	Med.	Low
Obj. value, 10MT	0.346	0.500	0.187	-0.881	-1.057	-1.474	-0.722	-0.238	-1.088
Obj. value, 20MT	0.215	0.259	0.093	0.114	-0.017	-1.462	0.565	0.529	-0.766



(a) Convex hull approximation



(b) Piecewise linear approximation



(c) Fitted polynomial approximation

Figure 5: Case Saguenay: Maintenance planning for low inflows, 30 time periods, 20 maintenance tasks, 6 for PH1, 7 for PH2 and 7 for PH3

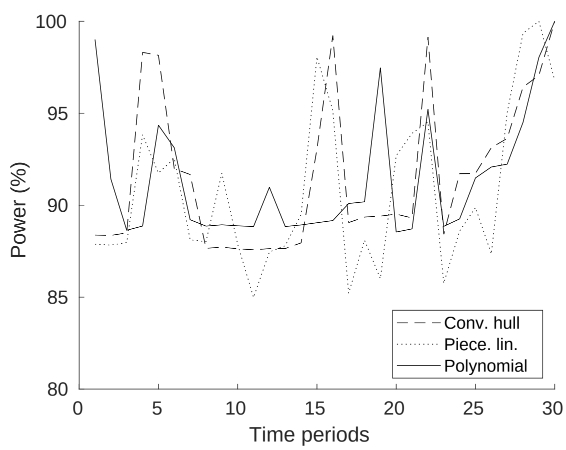


Figure 6: Case Saguenay: Total power production (%) for low inflows scenario, 20 maintenance task

Table 5: Case Saguenay: Energy produced (MWh) for 20 maintenance tasks

	Conv. Hull			Piecewise Lin.			Polynomial		
Inflows scenario	High	Med.	Low	High	Med.	Low	High	Med.	Low
From optimization	589782.2	589684.8	573708.6	580835.4	577354.7	559229.1	586026.0	585884.9	566662.0
From real data	588515.6	588160.8	573173.5	580176.1	577451.6	567528.9	582733.3	582800.0	571035.1

Table 6: Case Saguenay: Profit gap (%) when maintenance schedule is exchanged, for low inflows scenario and 20 maintenance tasks

Maint. sched. from	Conv. Hull	Piecewise Lin.	Polynomial
Conv. Hull	0	-0.200	0.436
Piecewise Lin.	0.315	0	0.818
Polynomial	-0.429	0.638	0

4.3 Operational considerations

We performed an additional experiment for the Saguenay case study. In the previous section, the results shown are based directly on the real data produced by the dynamic programming algorithm. But in practice, as mentioned by [14], due to mechanical aspects such as vibration or loss of efficiency, the operators avoid certain restricted zones.

Thus, in some hydroelectric complexes, the basic data used is actually an upper envelope of the surface of the real data. This does not change the nonlinear behaviour of the HPF. Figure 7 represents this operational data for the same powerhouse and the same turbine configuration as in Figure 2(a). The same experiments as in Section 4.2 have been conducted. The results are shown in Table 7.

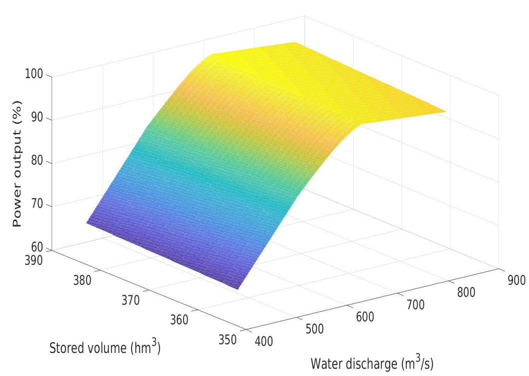
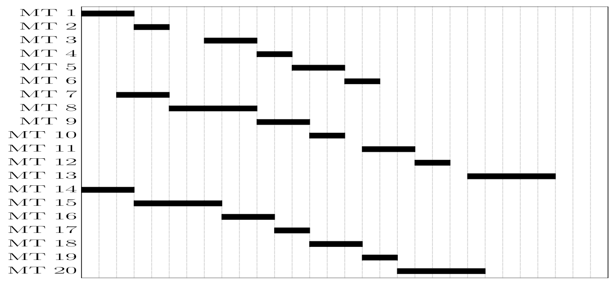


Figure 7: Case Saguenay, operational data: Power for PH1, with 4 active turbines out of 5

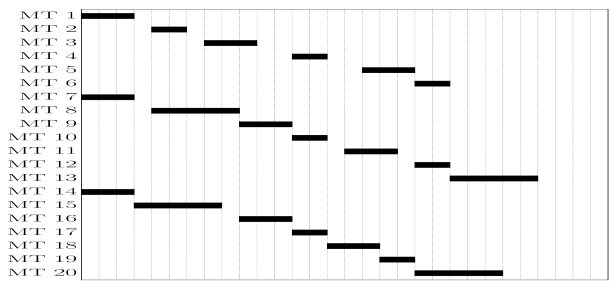
Table 7: Case Saguenay, operational data: Profit difference (%) for 10 and 20 maintenance tasks from operational data

	Conv. Hull			Piecewise Lin.			Polynomial		
Inflows scenario	High	Med.	Low	High	Med.	Low	High	Med.	Low
Obj. value, 10MT	0.460	0.489	0.204	-0.311	-0.326	-0.289	0.117	0.010	0.080
Obj. value, 20MT	0.541	0.777	1.371	-0.221	-0.291	-0.481	-0.140	0.617	-0.721

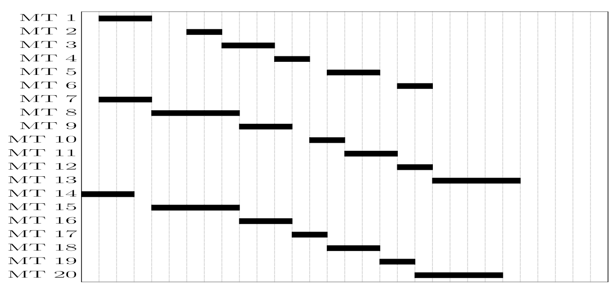
When comparing Table 4 with Table 7, one can see that the orders of magnitude of the gaps are similar. In that case, the polynomial is the approximation that gives the most accurate estimation of the power with a minimum gap of 0.010% for the medium inflows scenario, while the convex hull approximation gives the largest gaps. Looking at the beginning of the time horizon, Figure 8 shows that the model using the convex hull approximation schedules two maintenance tasks at the same time, while there are three tasks for the piecewise linear approximation and only one for the model using the polynomial. This behaviour is clearly observable in Figure 9, showing the total power generated.



(a) Convex hull approximation



(b) Piecewise linear approximation



(c) Fitted polynomial approximation

Figure 8: Case Saguenay, operational data: Maintenance planning for 30 time periods, 20 maintenance tasks, 6 for PH1, 7 for PH2 and 7 for PH3

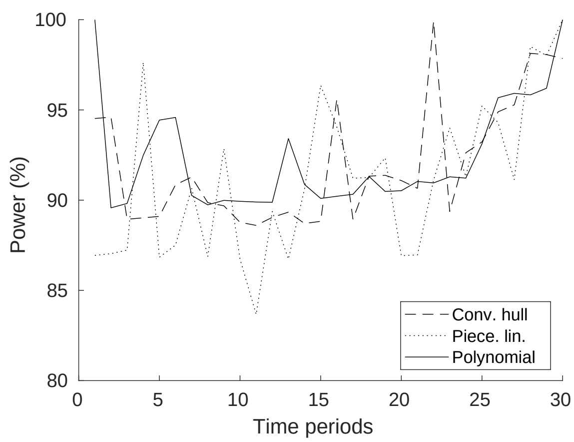


Figure 9: Case Saguenay, operational data: Total power production (%) for low inflows scenario

From these experiments based on the real data and the operational data, one observes that none of the approximations always estimates the power generated better than the others. The choice will depend on the needs. The first two approximations are linear which is well supported by current solvers, but, as shown in Table 8, the number of variables for the piecewise linear approximation is more than 5 times the one using the convex hull approximation. For very large hydroelectric complexes with many powerhouses, this aspect can become very restrictive. However, to be implemented, the piecewise linear approximation needs less initial information about the HPF and less pretreatment than the convex hull and polynomial approximations. The non-linearity of the third approximation can make it difficult to reach global optimality. Small size cases are easily solved but it is harder for larger ones.

Table 8: Size of the models

	Model	Variables	Constraints
Case Brazil	Conv. Hull	584	2748
	Piecewise Lin.	2984	4921
	Polynomial	584	1273
Case Saguenay	Conv. Hull	848	3908
	Piecewise Lin.	4508	7841
	Polynomial	848	2210

5 Conclusion

The aim of the study is to explore the impact on maintenance scheduling from the choice of approximation of the power production function in hydroelectric generation. Based on an existing model for the Generator Maintenance Scheduling Problem, we considered two alternative models. The model used as starting point approximates the Hydropower Production Function using the convex hull whereas the first alternative model uses piecewise linear approximation and the second one uses a polynomial function. We compare the impact of the choice of approximation on the maintenance schedules for two real cases: a hydroelectric system from southeast Brazil and one from Saguenay in Canada. In the Saguenay case, we experimented with two sets of data, one produced directly by a dynamic programming algorithm that evaluates the power generated, and the second produced by taking a concave envelope of the first set (and thus better represents practical operational conditions). The results show for a one-month planning horizon, some maintenance tasks can be shifted by up to 5 days depending on which approximation is used. The impact is most significant when the number of maintenance tasks to be scheduled is small. In terms of energy produced, the difference between the baseline data and the optimization models can reach 8,300 MWh for the month. Each of the three approaches has advantages, and the model based on the convex hull approximation offers the best compromise between the size and complexity of the optimization problem to be solved and the deviation from the reference data. Future work will look into characterizing additional variables and parameters that influence the outcomes of maintenance scheduling for hydroelectric generators. These characterizations will help select the most suitable option for the hydropower production function depending on the operational needs.

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