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non-ferrous metal industries – Application
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Value-based production planning in non-ferrous metal industries – Application in the copper industry

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Abstract: Production planners in the non-ferrous metal industry face an inherent combinatorial complexity of the metal production process within a fast changing market environment. Herein, we study the benefit of an integrated optimization based planning approach. We present the first value-based optimization approach for operational planning in the non-ferrous metal industry that yields high economic and technical benefits. We present a mixed integer linear program for non-ferrous metal operational production planning that covers the complexity of material flows and the entire production process and is amenable for real-time application. We give insights into the practical implementation and evaluation of our modeling approach at a plant of Aurubis, a large European non-ferrous metal producer. Our results show that an optimization and value-based production planning approach provides significant benefits, including a 38% better planning solution in practice. Besides economic benefits, we highlight the technical advantages that result from a detailed techno-economic representation of the entire production process.

Keywords: Process industries, process optimization, applied optimization

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1 Introduction

Generating a metal value of more than € 600bn in 2016, the non-ferrous metal industry is one of the cornerstones of overall industrial growth (Ober 2017). Nevertheless, the non-ferrous metal industry faces the same challenges as other industries: fluctuating demands, increasing customer expectations, and high cost pressure. This forces major players to review existing planning processes and to increase overall production efficiency.

Metal producers usually face a very high complexity in their planning processes due to external market related and internal process related complexity drivers. With regard to external drivers, a very high (and further increasing) variety of input materials with a multitude of precious metals and impurities must be handled. In addition, internal processes of non-ferrous metal refinement show a high complexity with many interdependencies, e.g., bottleneck resources, cycle materials, and a multitude of technical limitations. Costs vary for different processing steps, and costs as well as processing times depend on the material content. Additionally, a variety of environmental, regulatory, and process restrictions must be considered. This complexity further increases with changes in regulations, new sources of data, and improving technologies. Despite this complexity, the efficient operation of smelting and processing capacities is inevitable to maximize profits and to meet all internal and external process restrictions.

In practice, human planners carry out the planning process manually, i.e., based on spreadsheet calculations and human intuition or 'gut feeling'. However, human planners are usually not able to take the entire production process with all interdependencies and constraints into account. Thus, planners often focus on specific production steps to reduce the overall planning complexity. This may lead to significant shortcomings in the resulting planning solution and to unforeseen production downtimes. Additionally, production planners tend to apply a more throughput-oriented perspective, i.e., focus on the maximization of the total throughput under selected processing constraints and material compositions instead of applying a value-oriented approach (cf. Sakallı and Birgoren 2009).

In order to overcome these shortcomings, mathematical optimization models are able to capture the overall complexity of the underlying planning tasks. Moreover, if following a value-based perspective and integrating physical and financial planning aspects, optimization models can be enhanced to become valuable decision support tools for companies. However, implementing such a complex optimization model is a non-trivial task in practice due to complex real-world constraints and requirements.

Against this background, we develop and implement a comprehensive optimization model for production planning in the non-ferrous metal industry and discuss challenges, procedures, and benefits of a real-world application. To set this study apart from recent work, we briefly discuss related literature, before we detail the aims and the scope of our study.

1.1 Related literature

In general, our work relates to two different research streams: *i*) value-based supply chain and production planning and *ii*) non-ferrous metal production planning. Both have been extensively discussed for various applications within different disciplines such as industrial engineering, operations management, and operations research. In the following, we give a concise overview about both streams in order to place our research in between these fields and to highlight its novelty.

Value-based planning approaches: Value-based planning approaches have been discussed for various applications and perspectives, ranging from a strategic supply-chain-planning perspective to an operational production-planning perspective. The common denominator between these heterogeneous approaches is the definition of their objective: a value-oriented objective integrates a shareholder-oriented approach that considers implications for all planning levels of a company, i.e., strategic,

tactical and operational. Often, value-driver trees are used to translate high-level indicators to tangible measures, objectives, and decision variables for lower (mid- and short-term) planning levels (see, e.g., Hahn and Kuhn 2011). To this end, a value-based perspective aligns all planning and resource allocation processes with their value creation to integrate physical and financial planning.

Among the large body of existing literature, Hahn and Kuhn (2012b) presented a comprehensive review of quantitative decision support models for value-based supply chain management and developed a modeling approach for value-based performance and risk optimization. In this line of research, Guillen et al. (2007) introduced an integrated supply chain planning and scheduling model for the chemical industry, considering financial aspects (e.g., change in equity) by integrating budgetary information. Sousa et al. (2008) developed a multi-stage planning framework for the agrochemical industry, where the first stage redesigns the supply chain network, and the second stage optimizes production and distribution decisions. Kannegiesser et al. (2009) presented a two-phase robust optimization approach for profit optimization, which bases on coordinating sales quantity, price, and supply decisions throughout the value chain. Hahn and Kuhn (2011) provided a deterministic value-based decision framework for mid-term sales and operations planning focusing on the economic value added as objective function. Hahn and Kuhn (2012a) extended this framework towards a robust framework for integrated performance and risk management that is applicable for chemical commodity production. Brandenburg (2013) introduced a conceptual framework for value-based supply chain management and derived two quantitative models to assess the impact of different value drivers of supply chain management. Motivated through applications in the semi-conductor industry, Bayram et al. (2019) focused on integrated capacity, inventory, and demand allocation decisions.

Other approaches focused on optimal control strategies, often capturing uncertainty, e.g., through a Markov decision process. Amongst others, Gavirneni (2004) analyzed an inventory system with fluctuating purchasing costs, which are modeled through a time-homogeneous Markov chain. Wu and Chen (2010) analyzed the price-inventory relationship for commodities using a rational expectations equilibrium model, herein determining optimal procurement, production, and sales policies. Liu and Yang (2015) focused on the periodical control of both raw material purchasing activities and sales prices. In this setting, raw material costs evolve in a Markovian fashion and the demand is a random variable that depends on the sales price. While most of the control models focused on a specific part of the value chain, Karabağ and Tan (2019) analyzed a discrete material flow and inventory system that allows to analyze the impact in between purchasing, production, and sales policies through a state space continuous Markov process.

Another line of research focuses on simulation models to derive estimates of expected operations performance in complex and highly dynamic or stochastic environments. For instance, Brandenburg et al. (2014) presented a conceptual framework for value-based supply chain management linked to a discrete-event simulation model. Recent work of Lin et al. (2019) combined simulation approaches with analytical methods for performance evaluation.

Non-ferrous metal production planning approaches: So far, research in non-ferrous metal production planning focused either on blending models or on integrated production planning models.

Blending problems, i.e., mixing input materials into blends to satisfy output constraints and to optimize a specific objective, were already discussed in the 1960's (cf. Dantzig 1963). Depending on the structure of the problem, blending models may be linear or nonlinear [mixed integer programs \(MIPs\)](#) (Kallrath 2000, 2005, Misener and Floudas 2009). A multitude of specific blending models exist, e.g., for the oil industry (Singh et al. 2000), the chemical industry (Kallrath 2005), or the food industry (Jank and Wäscher 1999). Further studies for the metal mining sector exist (Epstein et al. 2012, Ramazan 2007, Alonso-Ayuso et al. 2014), but applications to the non-ferrous metal production industry are still scarce. Nikolić et al. (2009) presented a first outranking approach to determine an order of copper concentrates, but were neither able to optimize a final blend nor to regard the multitude of internal and external requirements. Other approaches extended existing blending models

developing specific linear programs for the copper industry (Jovanović and Stanimirović 2012) or for brass production (Sakallı and Birgoren 2009) with extensions for uncertain parameters (Baykoç and Sakallı 2009, Sakallı and Baykoç 2011). These models do not cover the complexity of a real-world planning problem in the non-ferrous metal industry, because they simplify the problem to a few concentrates and elements. Additionally, these models omit the multitude of technical, logistical, environmental, and inventory restrictions as they neglect the interdependencies between blending and production processes. Moreover, decision relevant costs of production processes or metal losses during processing are not considered. Even advanced models that use input-output functions (Fröhling and Rentz 2010) show the same deficits. Concluding, existing blending models do not provide the level of detail that is necessary to consider the multitude of external and internal restrictions of non-ferrous metal production.

In contrast, integrated production planning models explicitly regard the technical aggregates with shared resources and cycle materials. Herein, models with different levels of detail exist. Strategic planning models aim at investment decisions anticipating the future operation of the plant with its technical and environmental restrictions (Caldentey and Mondschein 2003). However, these strategic planning models are naturally limited in scope and granularity. More detailed approaches exist for operational planning. Rentz et al. (2006) developed a material flow based approach with a multi-period planning horizon and a holistic process representation. This approach considers lower and upper concentration limits as well as minimum and maximum material mass flow amounts and specifications of product qualities. However, this approach does not consider technical and logistical restrictions in sufficient detail. Also, logistics and material handling costs are neglected.

Methodological placement: The above reviewed research streams provide important insights for value-based production planning in non-ferrous metal industries. First, value-based planning approaches indicate that it is necessary to align production planning with the overall targets of a company, i.e., to translate high level performance indicators to lower level objectives and decision variables. Second, existing production planning approaches indicate that the non-ferrous metal industry requires detailed operational planning models due to an inherent process complexity and high product quality standards.

Often value-based planning approaches are developed within a general manufacturing and assembly context and do not consider the specific requirements of the non-ferrous metal industry. Even approaches that focus on the process industry are not able to account for the variety of restrictions and the high level of detail in material flow specifications that have to be considered in non-ferrous metal production. Concluding, a detailed approach for integrated blending and production planning that considers technical, logistical, environmental, and inventory restrictions in sufficient detail, that captures the production process's complexity with respect to materials and components, and that considers the underlying business model of the non-ferrous metal industry is still missing. This work aims at providing such an integrated approach.

1.2 Aims and scope

We present the first generalized value-based optimization model for non-ferrous metal production planning that overcomes these limitations as it

- i) considers the technical characteristics of the production system at an adequate level, i.e., models individual material flows and considers internal (technical, logistical, and inventory) and external (environmental, regulatory) constraints;
- ii) considers the core business model of this industry, i.e., captures decision relevant costs and revenues from a value-based perspective;
- iii) can be applied and used in practice, i.e., shows acceptable computational times and a sufficient production schedule granularity.

Figure 1 further details these requirements. In general, such planning problems remain inherently complex and computationally intractable for standard MIPs as soon as a real-world case is addressed. Due to the problem size these real-world cases usually call for a tailored meta-heuristic or even more problem specific exact algorithms. Contrary to these approaches, we develop a sophisticated linear MIP for the outlined problem. We avoid bilinearities such that our approach remains computationally tractable for real-time application in a real-world non-ferrous metal production plant.

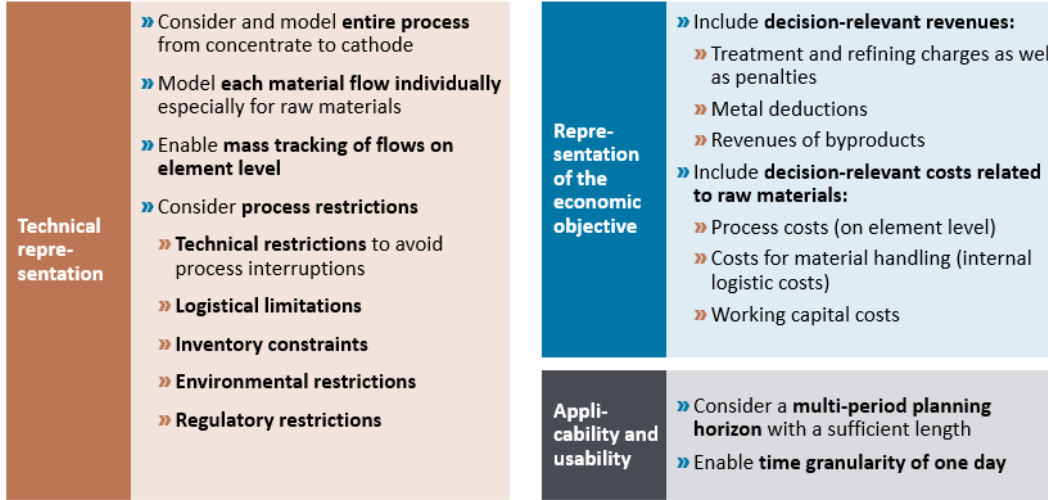


Figure 1: Requirements for an operational production planning approach.

Herein, the contribution of our work is fourfold: first, we provide a generic and comprehensive modeling approach that can easily be transferred to other non-ferrous metal processes with minimal tailoring efforts which would be necessary when transferring tailored metaheuristics or exact algorithms. Second, we implement and validate our modeling approach as a **decision support system (DSS)** at the copper production plant of Aurubis, the largest European copper producer. We discuss this application case and detail its implementation process in practice. Herein, we highlight key success factors and best practices that helped to successfully integrate the developed **DSS** in practice. Third, we evaluate the economic benefit of our modeling approach compared to the previous state-of-the-art planning in practice and show that an optimization based solution clearly outperforms previously used methods. Fourth, based on these results, we derive general managerial insights for non-ferrous metal producers.

The remainder of this paper is structured as follows. First, we detail the real-world characteristics of our planning problem and discuss necessary planning requirements. Based on these, we introduce a generic, comprehensive MIP for non-ferrous metal production planning. Then, we describe the real-world planning task, the experimental design and data, and the implementation of the **DSS** at the copper production plant of Aurubis in practice. Finally, we detail operational results from our real-world application case and highlight the benefits of our solution approach in practice, deduce general managerial insights for non-ferrous metal producers, and conclude this paper with a short summary of its main findings.

2 The non-ferrous metal production planning problem

Our main planning task is to determine a production plan that is feasible with respect to the variety of environmental, regulatory, technical, and inventory restrictions at the operational level. Furthermore, this production plan should maximize profits based on the underlying business model of the company. To create such a production plan, we take decisions on *i) the optimal blending of input materials, ii)*

the optimal amounts of material flows in processing steps, and iii) inventories. To detail this planning problem, we first focus on the production process and its material flows and afterwards on the business model of the non-ferrous metal industry. Then, we precisely define all constraints and requirements of the planning problem at hand.

2.1 Material flows and production processes

To analyze the material flows and production processes in detail, we consider external (market related) and internal (process related) drivers of complexity.

External drivers include the high number of potential input materials. More than 650 copper mines currently offer raw materials on the market (International Copper Study Group 2013). Additionally, the number of mines is increasing, while the mining projects are getting smaller due to risk minimization strategies of the mining companies. The raw material of each mine has a unique composition of elements, i.e., a unique composition of precious metals and of impurity elements that harm the refinement process. In the past decade, the proportion of impurity elements increased in relation to the metal share as a result of high tonnage operations and depletion of high grade deposits. Thus, the chemical complexity of raw materials steadily increases (Thomson Reuters 2016). As a consequence, raw materials differ strongly in their technical processability and in their value contribution. Regarding the business model, materials with a higher share of impurities usually generate higher revenues, but also higher costs due to lower processability.

Figure 2 illustrates the production process structure. As can be seen, the technical processes within non-ferrous metal production plants show a high complexity, too. The process consists of several pyro- and hydro-metallurgical steps aiming at refining the metal by removing all other elements during the process. In the following, we describe the technical process of copper production used for the case study exemplary and refer to Biswas and Davenport (2013) for a more detailed description. The process itself consists of a blending part and a refining part. During blending, raw materials are mixed according to specific characteristics (blends) that are determined by the operational production planning. Herein, a first mixing step is already carried out at the harbor (preblending) before the material is shipped. The main mixing step takes place at the production plant (blending). The refining starts at the flash smelter, which receives a continuous stream of dry raw material blends. Temperatures of over 1200 °C melt the blends and produce copper matte. The electric furnace further processes the remaining slag to recover a share of its metal content. The copper matte is transported to the converter, where amounts of sulfur and iron are removed by an injection of oxygen-enriched air. The result of this refining step is the so-called blister copper that has a copper content of 98%. In the anode furnace, further refinement increases the blister's minimum copper content to at least 99% and often even 99.99%.

Afterwards, the copper melt is casted into copper anodes, which are cooled down and inserted into the electrolysis. During the electrolysis process, copper and metals with a lower electronegativity dissolve at the anode. The metals with a higher electronegativity such as gold, silver, platinum and palladium settle down at the bottom of the electrolysis cell as undissolved anode slime which is further processed. Herein, the soluble impurities are removed from the electrolyte bleed in the chemical plant.

Copper cathodes and several by-products result from this production process. For environmental and economic reasons, many of the by-product streams of the refining process are reclaimed or further processed (e.g., in the chemical plant). The precise amount and quality of the copper cathodes as well as of the by-products depends on the raw material input, on the corresponding element content, and on the processing parameters. This results in a very complex techno-economic production planning problem with a multitude of restrictions and parameters. A variety of technical restrictions must be met to avoid process disturbances ranging from anode passivation and the formation of swimming sludges to precipitation of impurities in the copper cathode. Additionally, a multitude of concentration limits exist with regard to impurities. Table 1 details all restrictions that arise in the production process as described above.

metal deductions, and revenues for (by-)products. Costs include process costs dependent on the composition, internal logistics costs for material handling as well as working capital costs for capital tied up in the raw materials and the process.

- covers a multi-period planning horizon that is sufficiently long and has a granularity of at least one day to be able to implement it into the daily operational planning process.
- is computationally tractable for the multitude of parameters and decision variables of a real-world application.

Table 1: Restrictions in non-ferrous metal production.

Type	Restriction
Process restrictions	• Minimum and maximum heat values to cover the operational window
	• Capacity limits per process aggregate
	• Lower and upper concentration limits for process internal flows to avoid breakdowns and process interruptions as well as specifications for (by-)products
	• Ratios between element concentrations to avoid interruptions
	• Total mass amount limits per material flow
	• Limited changes in the material mix and heat balances in the production steps
Logistics & inventory restrictions	• Maximum mass amounts to be transported within the process
	• Limitations on the number of materials that can be blended in parallel
	• Maximum capacities of inventory facilities
Environmental & regulatory restrictions	• Upper concentration limits for disposal materials
	• Maximum emission of particular elements
	• Maximum total mass of particular flows
	• Maximum mass amount of refined metal

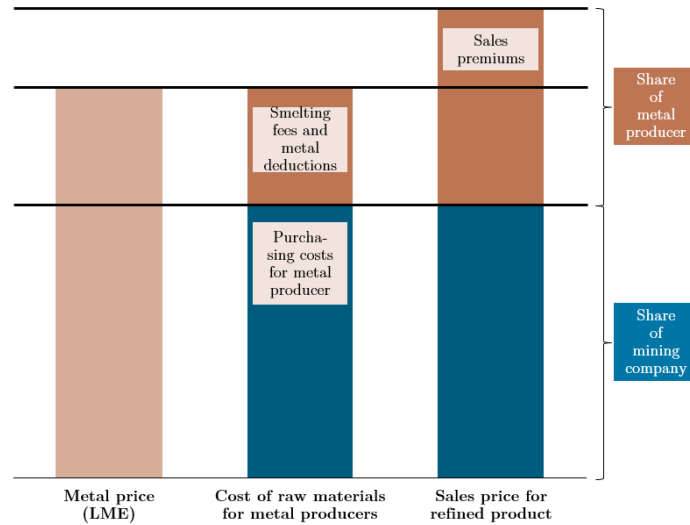


Figure 3: Added value shares of metal producers and mining companies.

Table 2 summarizes these requirements and shows which of them have already (partially) been studied in recent works. As can be seen, none of the existing approaches considers all these requirements.

Even single requirements are mostly not considered satisfactorily with respect to the high complexity in non-ferrous metal production. Therefore, we introduce the first modeling approach that covers all of these requirements in the following.

Table 2: Comparison of state-of-the-art modeling approaches for non-ferrous metal production.

	Requirement	a	b	c	d	e	f	g	h	i	j
Technical representation	Entire production process	●	●	●	●	●	●	●	●	●	●
	Each material flow on element level	●	●	●	●	●	●	●	●	●	●
	Technical restrictions	●	●	●	●	●	●	●	●	●	●
	Logistical limitations	○	○	○	○	○	○	○	○	○	●
	Inventory constraints	○	○	○	○	○	○	○	○	○	●
	Environmental/regulatory restrictions	●	●	●	●	●	●	●	●	●	●
Economic objective	Decision relevant revenues	○	●	●	○	○	○	●	●	●	●
	Decision relevant costs	○	●	●	●	●	●	●	●	●	●
Applicability and usability	Multi-period planning horizon & daily granularity	○	○	○	○	○	○	○	○	●	●
	Industry case study	●	●	●	●	●	●	●	●	●	●

Indices a–j signify publications as follows: (a) Nikolić et al. (2009); (b) Jovanović and Stanimirović (2012); (c) Jovanović et al. (2013); (d) Sakallı and Birgoren (2009); (e) Baykoç and Sakallı (2009); (f) Sakallı and Baykoç (2011); (g) Fröhling and Rentz (2010); (h) Caldentey and Mondschein (2003); (i) Rentz et al. (2006) (j) this paper.

3 Methodology

In this section we introduce a [MIP](#) for operational planning in non-ferrous metal production.

We consider a non-ferrous metal production plant and focus on all processes that are necessary to transform a wide range of raw material inputs of different composition into copper products with a very high purity level of at minimum 99%. Our objective is to maximize total profit resulting from total revenues minus total costs of processing and materials supply according to the business model of the company. In addition, we impose penalties to avoid an accumulation of impurities at the end of the limited planning horizon, and to account for process changeovers. We take decisions on the quantity and quality of input materials, the production levels of the processes, and the inventory levels. In order to account for the eminent purity level of products, we specify material flows at the chemical element level. Additionally, we model the full process chain from input of materials to the final product with all technical and legal requirements.

This modeling approach is based on the following assumptions which do not affect its applicability in practice. First, we assume that there is no demand shortage, i.e., produced products can always be sold in a dedicated planning horizon. This reflects the nature of the copper market, which is mainly based on commodity products. Second, we consider shipment arrivals for the planning horizon to be deterministic and known upfront as purchasing processes are fixed several weeks ahead of production. Analogously, material flows from other plants or entities within the company, from here on referred to as inter-company flows, are given for the entire planning horizon. Third, we assume that the metallurgical processes in each step of the production process show stationary operating points, and thus we use fixed distribution coefficients in our model. Mass conservation must be ensured for all process units.

Instead of modeling this problem straightforward, we use an enhanced concept for modeling fractional flows to avoid bilinearities and keep the [MIP](#) computationally tractable. Due to this concept and the general process complexity, the modeling approach is anything but intuitive. Hence, we derive the model stepwise and introduce some fundamentals first, before we detail the notation and the constraints, and finally discuss the objective.

3.1 Fundamentals

Modeling the non-ferrous metal production on the necessary granularity level requires the representation of fractional flows, i.e., total flows as well as element concentrations, which results in bilinearities. To avoid these bilinearities, we use a split fraction modeling approach as introduced by Quesada and Grossmann (1995). Herein, a separate flow (variable) denotes the flow between two balance areas for

a single element. Figure 4 exemplary shows how the concept of our split fraction modeling looks like for two balance areas b and b' and five element flows (k_1, \dots, k_5). Flows are modeled by arcs, such that they are directed (k_2 vs. k_5), may yield to other balance areas (k_4), or represent circular flows (k_1) that arise e.g., for slag flows.

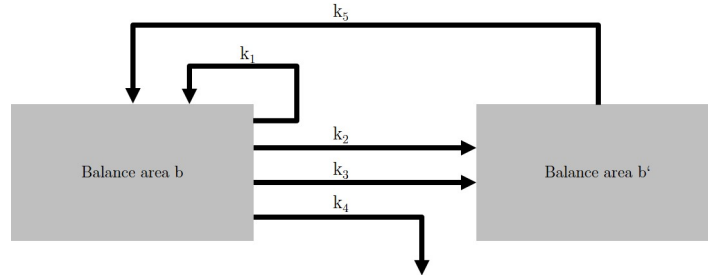


Figure 4: Exemplary illustration of the underlying multigraph structure.

We represent the complete production process by flows between balance areas. Herein, \mathcal{B} denotes the set of all balance areas $b \in \mathcal{B}$, and consists of subsets for external source balance areas \mathcal{B}^{ES} , inter-company source balance areas \mathcal{B}^{IS} (e.g., other company plants), production units \mathcal{B}^{PU} , and sink balance areas \mathcal{B}^{S} . Flows between two balance areas arise for different materials $m \in \mathcal{M}$.

3.2 Notation and constraints

Using the basic set definitions and the concept of split fractional modeling, we define our [MIP](#) on an incomplete, directed multigraph $G = (\mathcal{B}, \mathcal{K})$. Herein, \mathcal{B} denotes its balance area vertices and the arc set and \mathcal{K} denotes the balance area connecting flows. In G , a unique flow $k = (m_k, b_k, b'_k)$ is a triple of its material type m_k , its source balance area b_k , and its sink balance area b'_k , such that $\mathcal{K} \subseteq \{\mathcal{M} \times \mathcal{B} \times \mathcal{B}\}$. For the sake of conciseness, we define a cut set $\delta(\mathcal{S}) = \{k = (m_k, b_k, b'_k) \in \mathcal{K} | m_k \in \mathcal{M}, b_k \in \mathcal{S}, b'_k \in \mathcal{S}\}$ of any arbitrary subset $\mathcal{S} \subseteq \mathcal{B}$ as the set of all flows with exactly both balance areas in \mathcal{S} . Similarly $\delta^+(\mathcal{S}) = \{k = (m_k, b_k, b'_k) \in \mathcal{K} | m_k \in \mathcal{M}, b_k \in \mathcal{S}, b'_k \notin \mathcal{S}\}$ defines all outgoing flows of \mathcal{S} and $\delta^-(\mathcal{S}) = \{k = (m_k, b_k, b'_k) \in \mathcal{K} | m_k \in \mathcal{M}, b_k \notin \mathcal{S}, b'_k \in \mathcal{S}\}$ defines all ingoing flows of \mathcal{S} respectively. In the following, we apply these definitions also for singleton sets. Since our planning approach is time dependent, $\mathcal{T} = \{1, \dots, n\}$ denotes the set of time periods t . Note that for some parameters initial values for $t = 0$ are given.

To model the production process with sufficient detail, a material m consists of different elements $e \in \mathcal{E}$. Each element e has a specific heat coefficient η_e . To relate e to m , γ_{ekt} is the distribution coefficient that denotes the share of element e for material flow k in period t . Furthermore, α_{ekt} is the material concentration of element e in raw material flow k in period t . Let f_{ekt} be the amount of element e in flow k in period t , and f_{kt}^{Tot} be the total amount of flow k in time period t . In addition, f_{kt}^{IC} denotes the amount of inter-company flow for material k in period t . Besides these decision variables and parameters, two more binary variables help to model our planning task: y_{kt+1} denotes if the amount of flow k changes between period t and period $t + 1$; z_{kt} indicates if a flow k is active in period t . Shipments for material k in period t are given by σ_{kt} and i_{kt} denotes the inventory of material k at the end of period t .

Besides these basic quantities, some additional quantities are necessary to represent constraints and relations in the production process, which are to some extent interrelated between several flows or balance areas. To consider these dependencies, \mathcal{L} denotes a set of flows $\ell \in \mathcal{L}$ that have a shared limit. Analogously, \mathcal{C} denotes a set of balance areas $c \in \mathcal{C}$ with a shared limit. Additionally, \mathcal{G} depicts inventory groups $g \in \mathcal{G}$ that share inventory space. Interdependent concentration ratios have to be secured for specific flows k and elements e . Let $r \in \mathcal{R}$ be the set that denotes these interrelations. Analogously,

$q \in \mathcal{Q}$ consists of sets of flows and elements for which interdependent sum constraints hold. Herein, lower and upper bounds are given for element interdependent ratio constraints ($\zeta_{rek}^{\text{LHS}}, \zeta_{rek}^{\text{RHS}}$), as well as for element interdependent sum constraints ($\xi_{qek}^{\text{LHS}}, \xi_{qk}^{\text{RHS}}$). We refer to all other non-interdependent lower and upper bounds by a lower or upper bar on the respective quantity (see Table 3). Note that we refer to the respective subsets of $\mathcal{B}, \mathcal{K}, \mathcal{M}$ with a subset of the respective shared set, e.g., \mathcal{K}^ℓ . Table 4 summarizes this notation, which is sufficient to define a computationally tractable **MIP** for our planning task.

In the following, we derive the constraints of our model, focusing stepwise on *i) material flow constraints*, *ii) balance area constraints*, and *iii) inventory and additional constraints*.

Table 3: Boundaries used in the mixed integer linear program (MILP).

$\underline{f}_{kt}^{\text{Tot}} / \bar{f}_{kt}^{\text{Tot}}$	Lower / upper total amount limit for material flow k in time period t
$\underline{f}_{ek} / \bar{f}_{ek}$	Lower / upper amount limit of element e for material flow k
$\underline{f}_\ell / \bar{f}_\ell$	Lower / upper total amount limit for shared flow limit ℓ
$\underline{\alpha}_{ekt} / \bar{\alpha}_{ekt}$	Minimum / maximum concentration of element e in material flow k in time period t
$\underline{v}_{eb'}^{\text{Ratio}} / \bar{v}_{eb'}^{\text{Ratio}}$	Minimum / maximum ratio of element e in balance area b' coming from matte flow k^{Matte}
$\underline{\eta}_{b'} / \bar{\eta}_{b'}$	Lower / upper heat value bound for balance area b'
$\underline{q}_{b't} / \bar{q}_{b't}$	Minimum / maximum throughput of balance area b'
$\underline{q}_{eb't} / \bar{q}_{eb't}$	Minimum / maximum throughput per element e of balance area b'
$\underline{q}_c / \bar{q}_c$	Minimum / maximum boundary for shared capacity c
$\underline{v}^{\text{Ratio}} / \bar{v}^{\text{Ratio}}$	Lower / upper bound for ratio of sand to iron in converter
$\underline{i}_k / \bar{i}_k$	Minimum / maximum inventory for material flow k
$\underline{i}_{gt} / \bar{i}_{gt}$	Minimum / maximum inventory for inventory group g in time period t
$\bar{z}_{b'}$	Maximum number of incoming material flows for balance area b'

Table 4: Notation used in the MILP.

Sets	
\mathcal{M}	Set of materials
\mathcal{B}	Set of balance area vertices ($\mathcal{B} = \mathcal{B}^{\text{ES}} \cup \mathcal{B}^{\text{IS}} \cup \mathcal{B}^{\text{PU}} \cup \mathcal{B}^{\text{S}}$)
\mathcal{B}^{ES}	Set of external source balance areas
\mathcal{B}^{IS}	Set of inter-company source balance areas
\mathcal{B}^{PU}	Set of production units
\mathcal{B}^{S}	Set of sink balance areas
\mathcal{T}	Set of time periods
\mathcal{E}	Set of chemical elements and molecules
\mathcal{L}	Set of material flows
\mathcal{R}	Set of concentration interdependency ratio constraints
$(q \in) \mathcal{Q}$	Set of concentration interdependency sum constraints
\mathcal{C}	Set of balance areas with shared limits
\mathcal{G}	Set of material inventory groups with shared limits
Decision variables	
f_{ekt}	Amount of element e in flow k in time period t
f_{kt}^{Tot}	Total amount of flow k in time period t
i_{kt}	Inventory of material flow k in time period t
y_{kt+1}	Binary variable that indicates, if the flow amount changes from time period t to $t+1$
z_{kt}	Binary variable that indicates, if a flow k is active in time period t (amount > 0)
Parameters	
m_k	material of flow k
γ_{ekt}	Distribution coefficient related to element e for material flow k in time period t
α_{ekt}	Material concentration per element e in raw material flow k in time period t
f_{kt}^{IC}	Amount of inter-company material flow k from another plant in time period t
σ_{kt}	Planned incoming shipment for material flow k in time period t
ζ_{rek}^{LHS}	Left-hand side of element interdependency ratio r in material flow k
ζ_{rek}^{RHS}	Right-hand side of element interdependency ratio r in material flow k
ξ_{qek}^{LHS}	Left-hand side of element interdependency sum q in material flow k
ξ_{qk}^{RHS}	Right-hand side of element interdependency sum q in material flow k
η_e	Heat coefficient of element e

Material flow constraints

Constraints (1) enforces mass conservation between production units, while constraints (2) defines the total mass amount of each flow. Constraints (3) determine the element distribution of all chemical elements $e \in \mathcal{E}$ for balance area $b' \in \mathcal{B}$, depending on its distribution coefficient γ_{ekt} . Constraints (4) calculate the fixed overall mass amounts of inter-company flows. These flows originate from other plants or entities and are exogenously given since we consider a single plant problem. Constraints (5) split all incoming material flows (i.e., inter-company and external raw material flows) to element based flows.

Different constraints on lower and upper flow limits are given in (6)–(12). Constraints (6) obtain the lower and upper mass amount limits for each material flow. Analogously, constraints (7) obtain these limits per element. Constraints (8) secure lower and upper mass amount limits for flows with shared capacities. In addition to amount limits, constraints (9) ensure that element concentration limits are met. Interdependent concentration limits must be secured for interrelated element ratio constraints (10) and for interrelated element sum constraints (11). For the main metal flow from the smelter k' , a certain ratio of metal must be secondary material, such that the ratio of metal from primary materials is restricted (12).

$$\sum_{k \in \delta^-(b)} f_{ekt} = \sum_{k \in \delta^+(b)} f_{ekt} \quad b \in \mathcal{B}^{\text{PU}}, e \in \mathcal{E}, t \in \mathcal{T} \quad (1)$$

$$f_{kt}^{\text{Tot}} = \sum_{e \in \mathcal{E}} f_{ekt} \quad k \in \mathcal{K}, t \in \mathcal{T} \quad (2)$$

$$f_{ek't} = \gamma_{ek't} \sum_{k \in \delta^-(b_{k'})} f_{ekt} \quad e \in \mathcal{E}, k' \in \mathcal{K}, t \in \mathcal{T} \quad (3)$$

$$f_{kt}^{\text{Tot}} = f_{kt}^{\text{IC}} \quad k \in \delta^+(\mathcal{B}^{\text{IS}}), t \in \mathcal{T} \quad (4)$$

$$f_{ekt} = \alpha_{ekt} f_{kt}^{\text{Tot}} \quad e \in \mathcal{E}, k \in \delta^+(\mathcal{B}^{\text{ES}} \cup \mathcal{B}^{\text{IS}}), t \in \mathcal{T} \quad (5)$$

$$\underline{f}_{kt}^{\text{Tot}} \leq f_{kt}^{\text{Tot}} \leq \bar{f}_{kt}^{\text{Tot}} \quad k \in \mathcal{K}, t \in \mathcal{T} \quad (6)$$

$$\underline{f}_{ek} \leq f_{ekt} \leq \bar{f}_{ek} \quad e \in \mathcal{E}, k \in \mathcal{K}, t \in \mathcal{T} \quad (7)$$

$$\underline{f}_{\ell} \leq \sum_{k \in \mathcal{K}^{\ell}, e \in \mathcal{E}} f_{ekt} \leq \bar{f}_{\ell} \quad \ell \in \mathcal{L}, t \in \mathcal{T} \quad (8)$$

$$\underline{\alpha}_{ekt} f_{kt}^{\text{Tot}} \leq f_{ekt} \leq \bar{\alpha}_{ekt} f_{kt}^{\text{Tot}} \quad e \in \mathcal{E}, k \in \mathcal{K}, t \in \mathcal{T} \quad (9)$$

$$\zeta_{rek}^{\text{LHS}} f_{ekt} \leq \zeta_{rek}^{\text{RHS}} f_{ekt} \quad r \in \mathcal{R}, e \in \mathcal{E}^r, k \in \mathcal{K}^r, t \in \mathcal{T} \quad (10)$$

$$\sum_{e \in \mathcal{E}^q} \xi_{qek}^{\text{LHS}} f_{ekt} \leq \xi_{qk}^{\text{RHS}} f_{kt}^{\text{Tot}} \quad q \in \mathcal{Q}, k \in \mathcal{K}^q, t \in \mathcal{T} \quad (11)$$

$$\underline{v}_{eb'_k}^{\text{Ratio}} \sum_{k' \in \delta^-(b'_k)} f_{ek't} \leq f_{ekt} \leq \bar{v}_{eb'_k}^{\text{Ratio}} \sum_{k' \in \delta^-(b'_k)} f_{ek't} \quad e \in \mathcal{E}, t \in \mathcal{T} \quad (12)$$

Balance area constraints

The metal refining processes consist of exothermic process steps. To avoid equipment damage and downtimes, a heat balance, dependent on the heat input per element η_e must be considered. Constraints (13) secure lower (13a) and upper (13b) limits for these balances. Constraints (14) define the corridor for the total throughput of a balance area. Analogously, constraints (15) hold for the throughput of single elements. Balance areas with shared throughput limits are defined by constraints (16). Constraints (17), secure a minimum and maximum silicon (Si) to iron (Fe) ratio to ensure the slag building in the pyrometallurgical process steps. Since the maximum number of active flows is limited, constraints (18) secure this upper bound.

$$\eta_{b'} \sum_{k \in \delta^-(b')} f_{kt}^{\text{Tot}} \leq \sum_{k \in \delta^-(b'), e \in \mathcal{E}} \eta_e f_{ekt} \quad b' \in \mathcal{B}^{\text{PU}}, t \in \mathcal{T} \quad (13a)$$

$$\sum_{k \in \delta^-(b')} \sum_e \eta_e f_{ekt} \leq \bar{\eta}_{b'} \sum_{k \in \delta^-(b')} f_{kt}^{\text{Tot}} \quad b' \in \mathcal{B}^{\text{PU}}, t \in \mathcal{T} \quad (13b)$$

$$\underline{Q}_{b't} \leq \sum_{k \in \delta^-(b'), e \in \mathcal{E}} f_{ekt} \leq \bar{Q}_{b't} \quad b' \in \mathcal{B}^{\text{PU}}, t \in \mathcal{T} \quad (14)$$

$$\underline{Q}_{eb't} \leq \sum_{k \in \delta^-(b')} f_{ekt} \leq \bar{Q}_{eb't} \quad e \in \mathcal{E}, b' \in \mathcal{B}^{\text{PU}}, t \in \mathcal{T} \quad (15)$$

$$\underline{Q}_c \leq \sum_{k \in \mathcal{K}^c, e \in \mathcal{E}^c} \Lambda_{cb_k} f_{ekt} \leq \bar{Q}_c \quad c \in \mathcal{C}, t \in \mathcal{T} \quad (16)$$

$$\underline{\vartheta}^{\text{Ratio}} \sum_{k \in \delta^-(b')} f_{Fekt} \leq \sum_{k \in \delta^-(b')} f_{Sikt} \leq \bar{\vartheta}^{\text{Ratio}} \sum_{k \in \delta^-(b')} f_{Fekt} \quad t \in \mathcal{T} \quad (17)$$

$$\sum_{k \in \delta^-(b')} z_{kt} \leq \bar{z}_{b'} \quad b' \in \mathcal{B}, t \in \mathcal{T} \quad (18)$$

Inventory and additional constraints

Constraints (19) obtain the inventory balance considering raw material process inputs, inventory levels, and incoming material shipments. Additionally, lower and upper inventory limits hold for single materials (20) and for material groups (21). Note that in this formulation out-of-stock events are not allowed. However, this model can easily be extended for out-of-stock events by introducing slack variables in the inventory constraints and corresponding penalty terms in the objective function.

Non-negativity and binary constraints are stated in (22) and (23). Constraints (24) secure that $z_{kt} = 1$ holds if $f_{kt}^{\text{Tot}} > 0$. Constraints (25) ensure that $y_{kt+1} = 1$ holds if the corresponding material flow changes from time period t to $t + 1$. Certainly, we reformulate (25) by state-of-the-art techniques to avoid non-differentiability.

$$i_{kt} = i_{kt-1} + \sigma_{kt} - f_{kt}^{\text{Tot}} \quad k \in \mathcal{K}, t \in \mathcal{T} \quad (19)$$

$$\underline{i}_k \leq i_{kt} \leq \bar{i}_k \quad k \in \mathcal{K}, t \in \mathcal{T} \quad (20)$$

$$\underline{i}_{gt} \leq \sum_{k \in \mathcal{K}^g} i_{kt} \leq \bar{i}_{gt} \quad g \in \mathcal{G}, t \in \mathcal{T} \quad (21)$$

$$f_{ekt}, f_{kt}^{\text{Tot}}, i_{kt} \geq 0 \quad e \in \mathcal{E}, k \in \mathcal{K}, t \in \mathcal{T} \quad (22)$$

$$z_{kt}, y_{kt} \in \{0, 1\} \quad k \in \mathcal{K}, t \in \mathcal{T} \quad (23)$$

$$f_{kt}^{\text{Tot}} \leq M z_{kt} \quad k \in \mathcal{K}, t \in \mathcal{T} \quad (24)$$

$$|f_{kt+1}^{\text{Tot}} - f_{kt}^{\text{Tot}}| \leq M \times y_{kt+1} \quad k \in \mathcal{K}, t \in \mathcal{T} \setminus \{n\} \quad (25)$$

3.3 Objective function

Our objective function (26) considers three different quantities: revenues (*Rev*), costs (*Cost*), and penalty terms (*Pen*). Table 5 summarizes the additional parameters used to define these quantities.

$$\max z = \text{Rev} - \text{Cost} - \text{Pen} \quad (26)$$

The revenues (27) include smelting fees, metal deductions from raw materials, anticipated sales for by-products, and premiums for the production of products with a particularly high metal purity. Herein, smelting fees consist of **TCs** paid per total ton of material, **RCs** paid per ton of precious metal, and penalties paid per ton of impurity. In practice, the smelting fees are accounted at the moment the materials are inserted into the smelter. Although these fees are already fixed during contract

negotiations, they are accounted at this point to maximize the impurity utilization. This utilization is essential to reach high contribution margins.

$$Rev = \sum_{t \in \mathcal{T}, k \in \mathcal{K}} (\beta_{kt}^{\text{TC}} f_{kt}^{\text{Tot}} + \sum_{e \in \mathcal{E}} (\beta_{ekt}^{\text{RC}} + \beta_{ekt}^{\text{Pen}} + \beta_{ekt}^{\text{Ded}} \mu_{et}) f_{ekt}) + \sum_{t \in \mathcal{T}, k \in \delta^-(\mathcal{B}^S)} (\beta_k^{\text{Prod}} + \beta_{kt}^{\text{Prem}}) f_{kt}^{\text{Tot}} \quad (27)$$

The considered costs (28) comprise costs for material handling, processing costs, technical losses for unpaid metals in product streams, and imputed working capital costs for metals tied up in the process or in the inventory.

$$Cost = \sum_{t \in \mathcal{T}, k \in \mathcal{K}, e \in \mathcal{E}} (\kappa_k^{\text{Mat}} f_{kt}^{\text{Tot}} + \kappa_{ek}^{\text{Pro}} f_{ekt} + \iota_{ekt} \mu_{et} f_{ekt} + \mu_{et} \frac{wacc}{365} ((\tau_{eb'_k}^{\text{Process}} - \tau_{ek}^{\text{Payment}}) f_{ekt} + \alpha_{ekt} i_{kt})) \quad (28)$$

Additionally, we consider penalty terms (29) to account for additional soft objectives that stabilize the process or exceed our planning horizon. These include *i) penalty costs for impurities in inventories at the end of the planning horizon* and *ii) total changeover costs*. Penalty costs for impurities in inventories maximize the impurity throughput and enable a better treatability of incoming materials in later time periods that exceed the planning horizon. We differentiate changeover costs into heat value changeover costs and material changeover costs. These are caused by deviations between the time steps of either the heat values or the raw material amounts. The smaller these deviations, the less disturbances occur in the process. The values for the penalty cost factors κ_e^{Imp} , κ^{Heat} and κ^{CO} can be either determined in expert interviews or be exogenously given.

Table 5: Additional parameters for the objective function.

Revenue related parameters	
β_{kt}^{TC}	Treatment charge for material flow k : $b_k \in \mathcal{B}^{\text{ES}}$ in time period t
β_{ekt}^{RC}	Refining charge per element e for material flow k : $b_k \in \mathcal{B}^{\text{ES}}$ in time period t
β_{ekt}^{Pen}	Penalty per element e for material flow k : $b_k \in \mathcal{B}^{\text{ES}}$ in time period t
β_{ekt}^{Ded}	Percentage of metal deduction of element e for material flow k : $b_k \in \mathcal{B}^{\text{ES}}$ in time period t
μ_{et}	Metal price of metal e in time period t
β_k^{Prod}	Revenues or costs per ton of output for material product flow k : $b'_k \notin \mathcal{B}^{\text{PU}}$
β_{kt}^{Prem}	Premium per ton for products with an outstanding high purity
Cost related parameters	
κ_k^{Mat}	Material handling costs per ton for material flow k
κ_{ek}^{Pro}	Processing costs per ton of element e for material flow k
ι_{ekt}	Percentage of metal loss of element e in material flow k : $b'_k \notin \mathcal{B}^{\text{PU}}$ in time period t
$wacc$	Weighted average cost of capital
$\tau_{eb'_k}^{\text{Process}}$	Time needed for processing of element e if material is inserted in balance area b'
$\tau_{ek}^{\text{Payment}}$	Payment target of element e in raw material flow k : $b'_k \notin \mathcal{B}^{\text{PU}}$ defined as a number of days
Penalty related parameters	
κ_e^{Imp}	Imputed cost factor per ton in inventory for element e
κ^{Heat}	Changeover costs related to the change of the heat value between two time periods
κ^{CO}	Changeover costs for the change of one material flow between two time periods

$$Pen = \sum_{k \in \mathcal{K}, e \in \mathcal{E}} (\kappa_e^{\text{Imp}} \alpha_{ekn} \times i_{kn} + \sum_{t \in \mathcal{T} \setminus \{n\}} (\kappa^{\text{Heat}} \eta_e(f_{ekt+1} - f_{ekt}) + \kappa^{\text{CO}} y_{kt+1})) \quad (29)$$

4 Application in the copper industry

To validate our planning approach and show its applicability in industry by delivering tangible results for daily planning, we implemented our MIP as a DSS in the daily processes of Aurubis, the largest European copper company. This section provides details on the experiences that we gained during the design of the DSS and during its implementation. Herein, we first discuss the status quo at the plant before we implemented our DSS. Then, we detail the design of our DSS. Finally, we elaborate our work with the company and detail the integration, validation, and application of the DSS in practice. Herein, we highlight several key success factors.

4.1 Status quo

We implemented the [DSS](#) into the daily operations at the primary copper production plant of Aurubis. First, we analyzed the production process in detail. Herein, we identified about 1,500 material flows comprising 1,350 raw material flows, 100 intermediate product flows, and 50 final product flows. Besides the sheer quantity of flows, the high requirements that exist for conversion of input materials to products (cf. Tables 6 and 7) indicate the high complexity of the planning problem. Table 6 shows the range of the element concentrations in the raw materials, while Table 7 presents upper concentration limits for the copper cathodes, the main product of the company. As can be seen, the concentration of key elements varies significantly within the raw materials, while the upper concentration limits for grade A copper cathodes are very tight. Thus, reaching the high purity standards of copper products is all but intuitive and the overall complexity cannot be captured by mere human planning.

When we started our work at Aurubis, the production planners still conducted a manual planning approach based on spreadsheet calculations. Herein, production planners tried to overcome the complexity of the problem at hand by considering certain constraints but omitting others. Also, planners focused on selected production processes while other aggregates were neglected. As a result, the planners were often not able to find a fully feasible solution. Therefore, they approximated a feasible solution, consciously violating some constraints or omitting certain restrictions. The resulting shortcomings in the overall planning then had to be fixed just-in-time during the plant operations, which leads to enormous additional efforts or even products that did not fulfill market requirements. Even if the planning solution was feasible, planners often targeted on the total material throughput and did not aim at the value-contribution of the material mix. These manual planning results were the status quo when we joined the company. Thus, they serve as a baseline for the comparison with our optimized planning results at a later stage.

Table 6: Minimum and maximum concentration analysis per element in raw materials.

Element	Cu	Ag	As	Au	Bi	Cd	Fe	Ni	Pb	Pd	Pt	S	Sb	Se	Sn	Te
Min [%]	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Max [%]	100	97	32	34	2	2	51	29	27	11	3	43	12	12	18	24

Table 7: Exemplary upper concentration limits for grade A cathodes.

Element	Ag	As	Bi	Fe	Pb	S	Sb	Se	Te
Limit [%]	0.0025	0.0005	0.0002	0.001	0.0005	0.0015	0.0004	0.0002	0.0002

4.2 Design of the decision support system

To identify a suitable [DSS](#) design, we conducted workshops with different stakeholders at the operational as well as at the management side at Aurubis. We started with an initial design that worked well in previous projects. Hence, we were able to identify a suitable [DSS](#) design in only a few workshops. Herein, two key factors were essential for the design decision: first, the [DSS](#) had to be intuitive and easy to use even for users without a background in Operations Research, and second, the [DSS](#) had to provide interfaces to the existing monitoring and planning system in order to receive the required data and to pass back planning results. Focusing on these requirements, we designed a [DSS](#) with an Excel-based user interface that uses VBA extensions and operates a commercial [MILP](#) solver (in our case Gurobi 6.5.2) in the background. This [DSS](#) is run as a real-time application at the plant using a workstation with 64 GB RAM and an Intel Xeon e5-2650 processor.

Figure 5 shows the structure and the interfaces of this [DSS](#). As can be seen, we extract the required input data via interfaces from different systems such as the plant's [Enterprise Resource Planning \(ERP\)](#) system or its [Material Execution System \(MES\)](#). The interfaces have two different types of processing

macros. Green colored macros process master data that requires only monthly updates. Contrary, red colored macros process data for which we recommend a real-time update before running the optimization. With this system, the user faces only the excel front-end to fill in and check the relevant data. Data sources with low update frequencies can still be provided by corresponding functions in spreadsheets to add flexibility for modifying the input data.

After providing all data, the optimization starts via an Excel add-in. Herein, the [MIP](#) solver runs in the background, completely hidden from the user. While one may expect that the user asks for more detailed information on the model and its solution status, we experienced the opposite in practice. During the initial discussions with operational planners at Aurubis and during the implementation process, we received comprehensive feedback from the practitioners, which showed that the overall usability and acceptance of the [DSS](#) increased when the model and the corresponding solution process was hidden from the user. This holds especially if the user is missing a background in Operations Research. After the [MIP](#) is solved to optimality, the VBA code displays the results in Excel and provides additional reports.

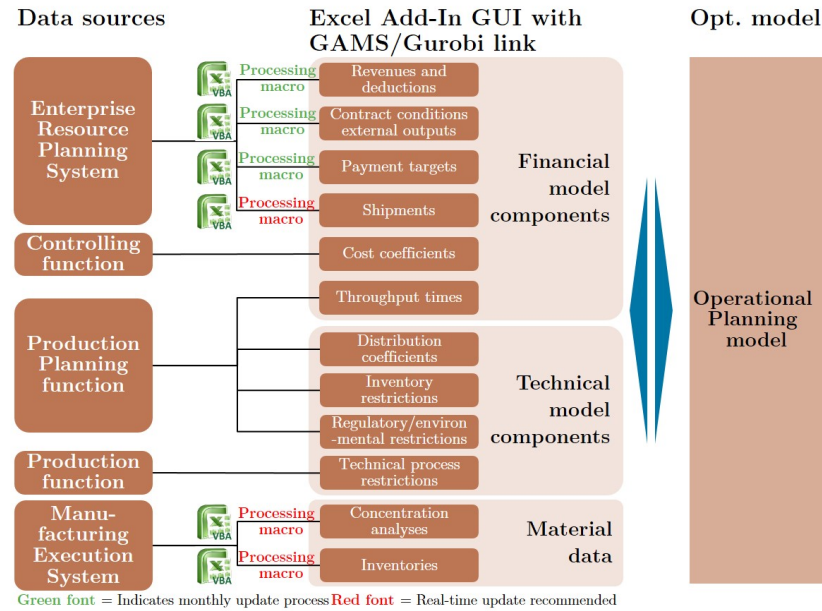


Figure 5: Design of the decision support system.

4.3 Implementation and validation

In total, we worked for 12 months with Aurubis to analyze the production process, create the modeling approach, and realize its implementation as a [DSS](#). This time span can be divided into three different periods.

In Period I (months one to five), we mainly analyzed the production process of the Aurubis plant and defined the corresponding [MIP](#) and requirements for the [DSS](#). During this offline discussion and development, we also considered alternative objective functions and tested a throughput-based optimization approach. This approach performed slightly worse than the initial status quo solution that was used by Aurubis. Already at this early stage, we paid attention to a profound stakeholder management and frequently carried out workshops to identify and discuss requirements as well as technical details. At this early stage, staying close to the operational level and considering the requirements of the planners helped to create a [DSS](#) with high usability and functionality that fostered trust in the new system. Besides hiding the detailed information on the optimization algorithm, we found that a profound and detailed visualization of all process related results and constraints helped to gain

the planners' trust into a new **DSS**. Thus, we implemented several pivot charts displaying e.g., the inventory development and all EBT related financial figures. Also, visual representations are provided displaying the tightness of process related constraints.

In Period II (months six and seven), we implemented the **DSS** and ran it in parallel to the manual planning solution. This phase was used as an alpha-test to validate the **MIP** and to fix technical errors in the **DSS**. In this period, we significantly broadened the audience of our stakeholder management. Still, a close contact at operational level was necessary to spot the last technical errors and to improve the usability of the **DSS** even further. Furthermore, reporting first results at (strategic) management level of Aurubis helped to gain the trust in and support for our project within the company.

Period III started in parallel to Period II and covered the last six months. Herein, the beta-test phase of the **DSS** was carried out. During this phase, we offered extensive on-the-job trainings for the production planners of Aurubis to guarantee increasing competences in operating the **DSS**. This helped to increase the trust of the planners in the system and secured a long-term usability without much need for support. Still, we continued the profound stakeholder management at all levels, reporting the benefits of the new system compared to the manual planning solution.

During this implementation and validation process, we identified three key success factors that helped significantly to complete this challenging project in this short period of time, and to guarantee a long-term system adoption in practice:

A high usability and easy-to-use interfaces increase the users openness and willingness to adopt the new system: The development of professional, easy-to-use interfaces for the **DSS** helped to reduce the complexity for the user and also empowered planners without a background in Operations Research to use the system. Visualizations and additional reports on operational details facilitated the understanding and analysis of the results significantly. Furthermore, the direct interface to other operational systems such as ERP or MES with an automated data preprocessing reduced the manual effort needed compared to the stats quo. This focus on ensuring a high tool usability was a crucial step to increase the willingness of the users to accept the **DSS** after years of applying a manual approach.

A transparent and extensive implementation and maintenance concept helps to gain trust in the new **DSS:** The integration of our value-based planning tool into current business processes was essential to ensure the long-term usability of the tool. Figure 6 details the structure of the business process and indicates the responsibilities of different departments within this process. To preserve the established workflow between different departments, we paid attention to implementing our **DSS** in a way that does not require changes in the organization of the business process. As can be seen, our **DSS** is closely interlinked with the main process steps, but affects only the determination of the production plan. Still, we allow for a manual evaluation check of the results of our **DSS** and permit for adjustments if discrepancies arise. Keeping changes to the organization of the business process at a minimum, allowing different departments to interact as before and permitting sanity checks on the **DSS**'s results contributed substantially to gaining the stakeholders's trust and acceptance into the new system.

We developed a governance structure, including tool management and maintenance processes. We also defined a new set of key performance indicators, tailored to the **DSS**, and integrated these into the existing rewarding system. In addition, we defined a business-to-be process to ensure that the input data is provided in time and with sufficient quality, and that results are used in an optimal way. This step turned out to be key to enable a successful implementation into the daily planning processes.

Continuously high effort in demonstrations of the benefits and in change management during the design and implementation phase helps to get the buy-in of key stakeholders: To stick to our timeline, it was essential to convince the upper management to support the implementation of the **DSS** and to provide additional resources when necessary. To get these credits, we

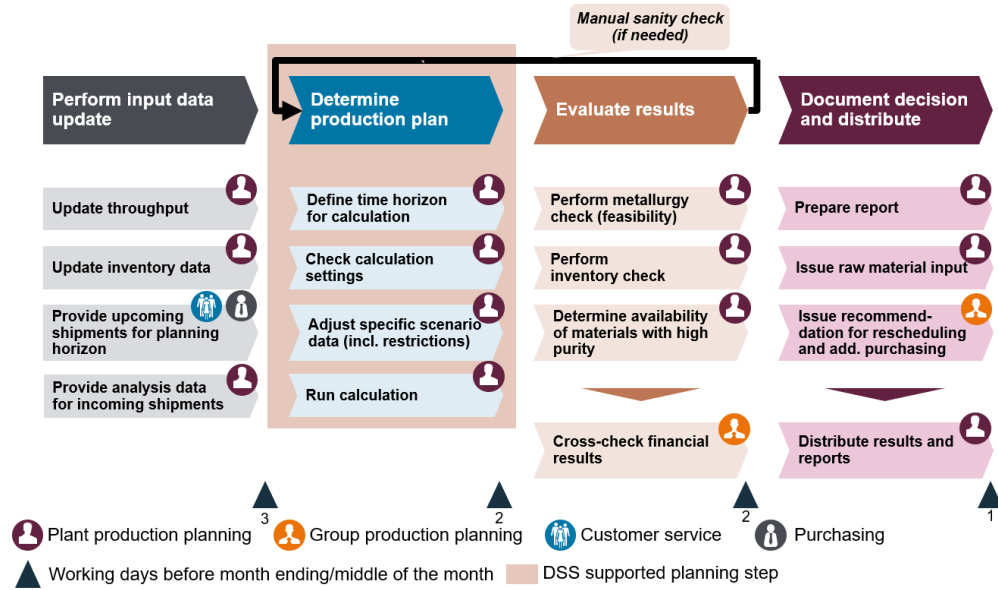


Figure 6: Business process overview, including the DSS and organizational responsibilities.

frequently emphasized the value gained from the DSS and highlighted financial as well as process benefits. In addition, we conducted a comprehensive stakeholder management to keep all involved parties and stakeholders informed.

5 Results

In this section, we discuss the results analyzed during Period III of the project. In this phase, we already ran the DSS in real-time in its final version at Aurubis, but still in parallel to the manual planning solution in order to validate results and to evaluate the performance of the DSS. During this period, the DSS was executed 18 times to create a production plan for the next 40 days, which was then partly executed in a rolling horizon mode.

In the following, we discuss this evaluation. Herein, we first focus on the realized contribution margin as the targeted objective, and secondly we analyze the quality of the solution with respect to the impact on the plant's operations. Finally, we identify different improvement levers that set the optimized solution apart from the manual planning solution. To allow for a comparison between the manual solution and the optimized solution while at the same time preserving confidentiality agreements, we average and normalize results in monetary units (MU) to the average total contribution margin of the manual planning results. However, the resulting values still reflect the improvement potential without loss of scientific relevance, and improvement ratios quantify the realized improvements accurately.

Contribution margin

To assess the economic benefit of the planning approach, we compare the average contribution margin of the manual planning as baseline with the average contribution margin attained by the DSS. Figures 7a and 7b show the composition of the average total contribution margin for the manual and for the optimized planning solution. As can be seen, the smelting fees, the process costs, and the metal result (i.e., the difference between deducted metal value and metal loss) are the main drivers of the average total contribution margin. Table 8 further details and compares these results. As can be seen, the average contribution margin of the optimal solution is 38% higher than the average contribution margin of the manual planning solution.

In practice, increasing contribution margins in production planning are often realized by reducing production costs (especially when applying optimization approaches). Analyzing Table 8 this is not the case for our study. Process costs as the main cost driver even increase after applying the DSS. Lockup costs resulting from materials in stock slightly decrease, but are not sufficient to compensate for the increasing process costs. Instead, the increased process costs are more than compensated by significantly increasing revenues, especially smelting fees and metal result revenues. This to some extent unexpected effect arises because the DSS decides not only on the production plan but also on the production program, i.e., which type and amount of product to produce.

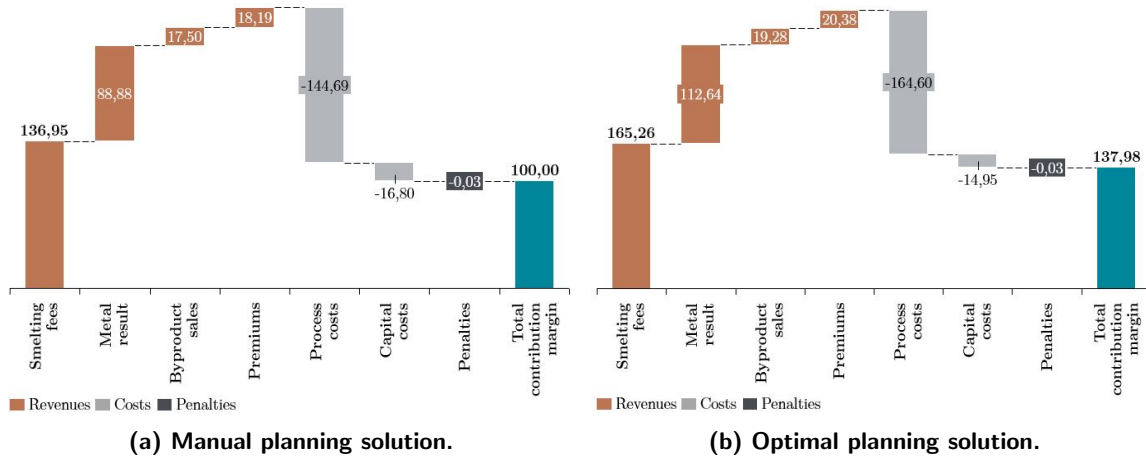


Figure 7: Contribution margin of the optimal and manual planning solution

Table 8: Economic planning results.

Component	Manual planning	Optimized planning	Deviation
Smelting fees	136.95	165.26	20.7%
Metal result	88.88	112.64	26.7%
By-product sales	17.50	19.28	10.2%
Premiums	18.19	20.38	12.0%
Process costs	-144.69	-164.60	13.8%
Capital costs	-16.80	-14.95	-11.0%
Penalties	-0.03	-0.03	0.0%
Contribution margin	100.00	137.98	38.0%

Solution quality

The optimized planning solution does not only improve the objective value, but also the technical process of the metal production and the quality of the production plan with respect to violated constraints. Figure 8 shows a box-whisker plot for the input materials mix of the manual (orange) and the optimal (blue) planning solution. Only materials that have a maximum share of more than 10% in one solution are shown. Raw materials are labeled consecutively. 'Opt' indicates the optimal share, whereas 'Man' indicates the share that results from manual planning. As can be seen, the material mix of the optimal and the manual planning solution differ significantly. The material compositions of the optimal planning show a significantly higher variation compared to the material compositions of the manual planning approach. These higher variations indicate the complexity that is captured when deriving an optimal planning solution. Contrary, the lower variations in the manual planning solution indicate that the full complexity, i.e., all degrees of freedom, cannot be captured by human planners. The manual planner derives only solutions with low material variability, since he is artificially restricting the solution space in order to decrease the planning complexity.

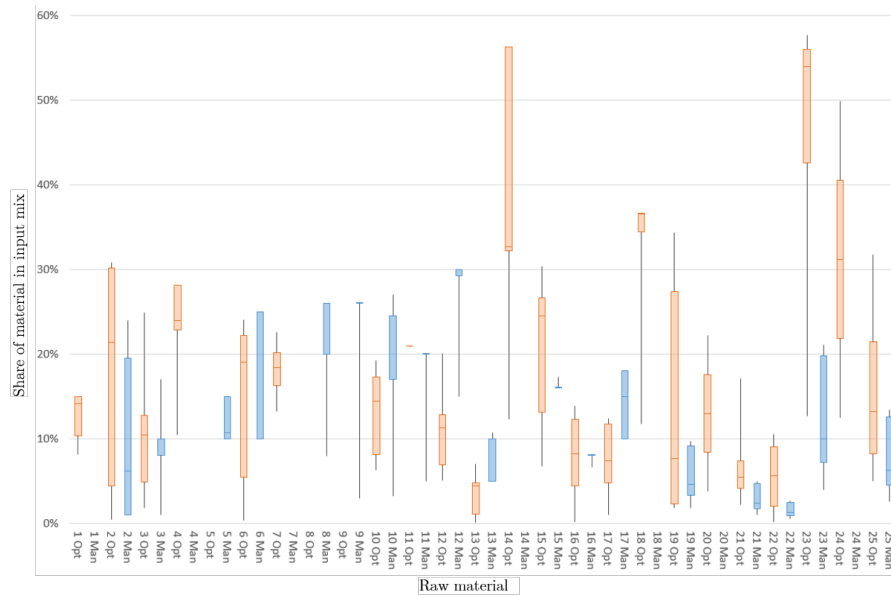


Figure 8: Comparison of material shares in the input mix between optimal and manual planning.

The higher variation in the input mix allows for improved blending to achieve a higher utilization of existing aggregate and impurity capacities. Figure 9 shows the impurity capacity utilization for both the manual and the optimized planning. As can be seen, the impurity capacity utilization of the optimal planning solution is significantly higher than the utilization of the manual planning solution. However, unused potentials with respect to impurity capacity utilizations remain even in case of optimal production planning. To leverage these unused capacities, materials with high impurities but also high contribution margin would have to be purchased, and an implementation of an integrated supply and production planning approach seems to be promising to tackle this issue in the future.

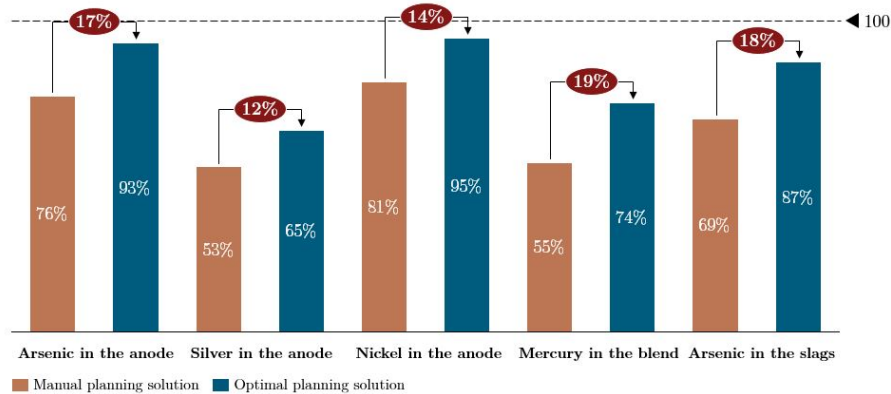


Figure 9: Average upper concentration limit utilizations.

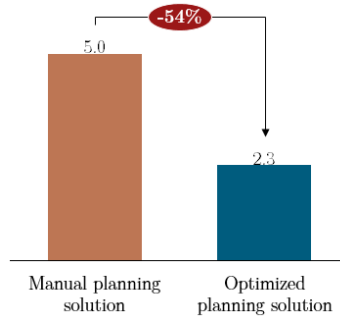
To assess the operational quality of both the manual and our optimized planning approach, we focus on the number of operational constraints that are violated by a production plan. Table 9 details violations for the different types of constraints, stating the absolute number as well as the relative share of violations. As can be seen, all optimal solutions are 100% feasible. Contrary, manual planning solutions violate approximately 3.5% of the planning constraints, since the manual planner neglects certain constraints and omits a number of technical aggregates in order to overcome planning complexity. In practice, this is partially addressed by higher safety factors for the considered aggregates.

Table 9: Number of constraint violations.

Classification	Type of constraint	Manual solution		Optimized solution	
		Absolute	Relative	Absolute	Relative
Flow constraints	Total amount limits	70	4.9%	0	0%
	Element amount limits	29	4.0%	0	0%
	Concentration limits	1000	2.8%	0	0%
	Interdependency limits	239	3.8%	0	0%
Balance area constraints	Total throughput limits	0	0.0%	0	0%
	Element throughput limits	208	8.3%	0	0%

If not addressed, this is one of the sources for unforeseen breakdowns. Thus, besides increasing economic viability, the DSS helps to reduce the risk of an unforeseen breakdown significantly.

To compare the efficiency of both planning approaches, we evaluate the time effort that is needed to carry out the different planning approaches (cf. Figure 10). In practice, we observed a reduction of the total planning effort of 54% by using the DSS. Our observations show that employees shift their effort to more strategic activities. However, a certain amount of time is still needed for data preparation and selection of scenario specific data, e.g., selecting a reasonable planning horizon or current throughput figures, although most of the data is provided from the company's information systems.

**Figure 10: Work effort in hours.**

Improvement levers

The advantages of the optimal solution are mainly due to three

improvement levers that lead to a key planning policy. In a nutshell, this policy is '*In-time use of the right material mix in the right input aggregate*'. The corresponding improvement levers hold as follows:

1. The first and most critical improvement lever is the selection of the right input material mix. Herein, materials must be selected and mixed that jointly generate a higher total contribution margin. As our analyses show, a portfolio effect exists, i.e., materials with a negative individual contribution margin act as diluter, and thus enable the usage of materials with high impurity and a high contribution margin. The identification of this portfolio effect is enabled by our value-based optimization approach that maximizes the total contribution margin.
2. The second improvement lever comprises the improved selection of the right aggregates. This results in a lower metal loss and a better metal result, because more metal ends up in product streams in which the metal content is paid. This positive effect even overcompensates the potentially increased process costs.
3. The third improvement lever realizes the early input of materials with a high material (metal) value. Thus, capital lockup costs decrease as high-value materials have a shorter cycle time. In

the manual planning solution, high-value materials stay longer in stock, which results in higher capital lockup costs because of the tied-up material value.

6 Managerial insights

In this section, we derive general managerial insights for practitioners by consolidating our findings from the results and the implementation sections. Herein, we derive four key insights:

To achieve feasible, high-quality production plans, the inherent complexity of the production process must be captured: Herein, all material and element flows and the multitude of technical, environmental, and regulatory restrictions must be considered. Material amounts and material content may vary significantly with respect to the analyzed production processes. Therefore, it is not sufficient to determine a production plan based on an excerpt of production steps and aggregates. Instead, the entire production chain must be considered end-to-end. Furthermore, a planning approach as comprehensive as the [DSS](#) helps to prevent unforeseen production downtimes caused by planning shortcomings, and additionally increases the transparency on bottlenecks and operational metrics.

A value-based planning perspective is inevitable to increase economic viability: Currently, most production processes in the non-ferrous metal industry are planned from a throughput-based perspective. Changing this planning perspective to a value-based approach helps to significantly increase economic viability. Herein, the business model of the non-ferrous industry, i.e., all decision relevant revenues and costs, must be considered throughout the planning process.

Optimization based planning helps to tackle the planning complexity: The information used in our optimization based planning approach equals the information used for the status quo planning. The differences in solution quality and operational feasibility show that the mere availability of information is not sufficient to guarantee a feasible, high-quality production plan. Independent of the data basis, a human planner cannot handle the inherent complexity of the planning task. Herein, an optimization based [DSS](#) with a user friendly interface and high usability helps to tackle the planning complexity and finds the optimal production plan.

A strong focus on the implementation is needed to ensure stakeholder acceptance and a sustainable long-term use of a [DSS](#): In order to ensure stakeholder acceptance it turned out to be key to emphasize the benefits of the [DSS](#) and to apply change management techniques. By so doing, we convinced the key user to adapt to the new system. We also gained the support of the upper management where needed. Furthermore, the integration of the [DSS](#) into the current business processes and the definition of to-be process for the tool use ensures a long-term usability and application of the tool.

The optimization approach and the [DSS](#) that we developed in this paper comprise all of these features. Although we implemented our [DSS](#) for the copper production process of Aurubis, it can easily be transferred to other copper producing companies as well as to other (non-) ferrous metal refining processes or even beyond. As we derived the underlying [MIP](#) generically, additional requirements can easily be implemented without major changes. Additional effort may only be needed to gather the required data and input parameters when transferring the [DSS](#) to other fields of application. However, this depends only on the information basis given at other companies. The interface layout of the [DSS](#) can easily be used at this point since it provides standard protocols, e.g., a connection to an ERP system.

7 Conclusion and outlook

In this paper we addressed the multi-period integrated blending and production planning problem of the non-ferrous metal industry. We developed a generalized [MIP](#) that covers the entire production

chain, i.e., the blending process and the entire production process with all aggregates in sufficient detail. Herein, we considered all process related limitations, internal technical, logistical, and inventory restrictions as well as external environmental and legal requirements. Furthermore, we applied a value-based objective that covers the business model of the non-ferrous metal industry. We embedded this optimization approach into a [DSS](#) that was implemented and is operated in practice at the largest European copper company Aurubis.

The successful integration and operation of our [DSS](#) validates our modeling approach and proves its applicability in practice. The comparison between the manual and the optimal planning results highlights the economic advantages of our modeling approach. Focusing on the contribution margin, we reach an improvement of 38% compared to the status quo. Besides realizing economic benefits, our [DSS](#) simplifies and streamlines the process of operational planning and accelerates the decision-making support while increasing the planning quality. In addition, it increases the transparency on bottlenecks and other operational metrics.

Overall, our results show the practical relevance and underline the improvement potential of optimization based decision support in the non-ferrous metal industry. Although we applied our approach to one specific copper producer, it can easily be transferred to other companies and even to other non-ferrous refining processes or industries.

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