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A linear mixed-integer formulation of the short-term hydropower problem

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Abstract: This paper presents a linear mixed-integer formulation to solve the short-term unit commitment problem. It determines the pair of maximum efficiency points of water discharge and the power produced at the maximal storage for all possible combinations of turbines. The goal is to maximize total energy for all periods. The objective function is calculated using the correction between the power produced at the current volume and the maximal storage and penalizes unit start-ups. Constraints on the maximal number of turbine changes are imposed to find a viable solution in practice. Numerical results are conducted on thirty cases for two powerhouses with five turbines each located in the Saguenay Lac-St-Jean region in the province of Quebec.

Keywords: Hydro unit commitment, hydropower optimization, short-term, mixed integer linear programming

1 Introduction

Hydropower is the third source of renewable energy in the world according to the International Energy Agency (IEA) in 2017 [1]. Electricity producers seek to manage their production either by maximizing the energy or minimizing the operational cost. Managing hydroelectric systems is complex and requires different optimization processes. Long-term optimization models are used to determine the future production potential for a few years of planning horizon and take into account the uncertainty of the inflows [8, 13]. Medium-term models are used to define the quantity of water available in the reservoir for hydropower production for a weekly scheduling horizon [7, 11]. Short-term models aims at defining the optimal strategy of the daily production by dispatching the quantity of water between the turbines in order to maximize the energy or minimize costs [16]. This paper focuses on short-term optimization. The purpose of the short-term models is to determine for each powerhouse and for each time stage the water discharge, the volume of the reservoirs, and the state of each unit (on or off) respecting some constraints. For the short-term problem, these constraints usually involve water balance constraints, energy demand, water discharge constraints, reservoir limits and start-ups of the units [4,6]. The study of the unit commitment problem has taken place during the past few decades. Several methods and algorithms are used to solve this problem: dynamic programming [4, 15], lagrangian relaxation [18], Benders decomposition [3] and network flows [14] among others. One of the widely used formulation of the short-term model is the Mixed Integer Linear Programming (MILP). The advantage of the MILP is that the discrete nature of the problem like the start-ups of the unit can be introduced by adding integer variables or constraints [17]. In addition, MILP can solve large size scheduling problem in power energy systems [9].

In [12] a MILP was proposed to solve the unit commitment problem in order to minimize the operational costs. In this model, the decision variables were the start-up and shutdown of the units and the water releases. The formulation of the problem considers the variation of the net head water and the non-linearity of the production function. This non-linearity was accounted for with a three-dimensional interpolation technique. The model was tested on a real case in China with one reservoir and 32 heterogeneous generating units. In the case where there are more reservoirs, this modeling would quickly become difficult to apply, given the number of variables and constraints.

Another formulation [10] uses the MILP to maximize power efficiency. The problem was split into two phases. The first phase was an preprocessing based on unit commitment and a piecewise linear generation function to define the water discharge and the total powerhouse generation, considering the maximum discharge bound, the efficiency curves and restricted operating zones for each unit. In the second phase, a MILP formulation with fewer binary variables was built based on the preprocessing phase in order to maximize the final storage energy. This formulation was interesting but the problem was solved in two phases and in the operational reality it is suggested to have only one model.

In [5] a unit commitment problem with head dependent reservoir was developed to find the optimal scheduling of a multiunit pump-storage hydropower system. A mixed integer linear model was formulated to solve the problem. The continuous variables are the water discharge, the volume of the reservoir, the produced power, the water spillage and the binary variables are the start-ups and shutdowns of the units. An enhanced linearization technique was used to solve the non-linearity of the relationship between power and water flow. However, this technique was difficult to apply in practice, given the size of the problem.

Previous works used MILP to facilitate the introduction of the integer variables and the linearization of non-linear functions. In [20] a MILP was developed to maximize the profits by estimating the income of produced power and the start-up/shut-down costs. An analysis of the linearization effects of non-linear functions and related constraints on solution feasibility was conducted. It was found that the linearization may result in infeasible solutions, like the restricted operational zone constraint. To obtain feasible solutions an approximation of the error caused by MILP approximation formulation was done.

All of the cited works are interesting and allow to solve the problem, but the solutions provided by the models often do not reflect the operational reality. This research develops a mathematical formulation of the unit commitment problem that allows to implement directly the solution found by solving the optimization problem. To do so, a formulation based on the efficiency curves is proposed and fixed discretizations of the water discharge are permitted, where only the volume of the reservoir is a continuous variable.

This paper is organized as follows. Section 2 presents the characteristics of the problem. Section 3 presents the notation used throughout the paper and the mathematical model developed. This model was tested on thirty cases for two powerhouses with five turbines for each. The results are discussed in Section 4 and finally concluding remarks are presented in Section 5.

2 Short-term hydro-power scheduling problem

The unit commitment problem is used to determine the optimal production plan for the next day or so up to one week. The purpose is to maximize energy production and penalize unit start-ups. To maximize energy produced, several factors are considered. This section defines the different factors and describes the short-term problem.

2.1 Power production

The production function depends on the water discharge q in m^3/s, the efficiency of the unit η, the gravitational acceleration G in m/s^2 and the net water head h_n in m which in turn depends on the total water discharge Q in m^3/s and the volume of the reservoir v in hm^3 [19]. The power output P in KW from hydro generating unit can be given by the equation:

$$P = G * \eta * h_n(Q, v) * q \tag{1}$$

The net water head is the difference between the forbay elevation h_f, tailrace elevation h_t and the losses caused by the friction in the penstock h_p. The net water head is calculated by:

$$h_n(Q, v) = h_f(v) - h_t(Q) - h_p(Q, q) \tag{2}$$

2.2 Turbine efficiency

Turbine efficiency is the most important factor in regards to the production of the unit. It is a measure that represents the relation between the mechanical energy produced and the potential energy of the water discharge. The turbine efficiency depends on the water discharge and the net water head. Each turbine has its own efficiency curve and for the same value of water discharge and volume, the turbines produce different power.

2.3 Combinations of the turbines

The total power output depends on the total water discharge and the number of active turbines. In the operational reality, the active turbines are grouped into combinations.Table 1 shows an example of the possible combinations of a powerhouse that has 4 available turbines. The operator can use all or less than 4 turbines. In the operation of a powerhouse, a minimum number of active turbines (for example 3 turbines) are required due to the physical constraints.

As explained in Section 2.2, each turbine has its own efficiency. The combination of active turbines has also its own efficiency. Figure 1 shows the power output depending on the water discharge for all possible combinations and this for a given forbay elevation.

Table 1: Combination of 4 available turbines

3 active turbines	4 active turbines
123 - 124	1234
134 - 234	

As shown in Figure 1, the power output decreases when the maximum water discharge of the turbine combination has been reached. For example, in the case where the active turbines are 234 (curve with 'square' marker) the power output decreases once the maximum water discharge is reached ($510m^3$/s). It is useless to increase the quantity of water discharge because it will be spilled, therefore the tailrace elevation will be increased, consequently, the water head will be reduced. For this reason the power output is decreased.

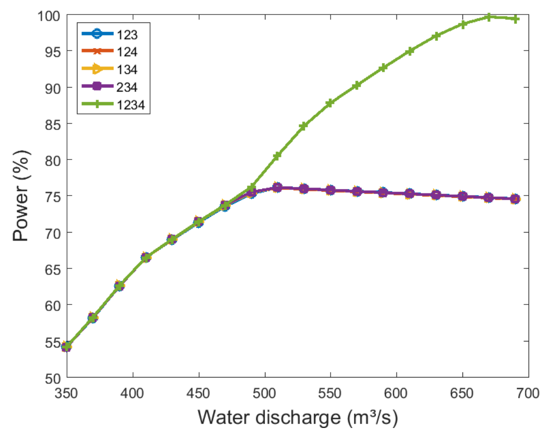

Figure 1: Power output for all combinations

2.4 Start-ups

One of the objectives of the problem is to determine for each period the best combinations of active turbines in order to maximize the energy. The model can select different combination from one period to another. However, there is an important concept that must be taken into account which is the start-ups of the turbines. Frequent start-ups cause maintenance costs and decrease the life cycle of the turbines. In addition, in the operational reality, it is recommended to have a limited number of start-ups.

2.5 Problem description

The objective is to maximize energy production and penalize unit start-ups. Hydropower production function can be modelled in many different ways. Polynomial equations, which determines an analytic expression of a polynomial of degree n passing through data points (P,H_n), where P is the produced power and H_n is the gross water head. The wrong choice of the degree influences the results. Splines allow to split the points into subsets and use a polynomial approximation for each subset of points, then connect them. These methods lead to nonlinear models and the resolution of the problem becomes more difficult.

Another way is to select a set of points of water discharge representing the powerhouse management. These points correspond to the maximum water discharge and the maximum efficiency of each available combinations group. However, selecting only these points can limit the optimization and lead to infeasible solution. In this regard, a set of adjacent points to the maximum should be defined. To do so, the convex hull of the efficiency curves for each combinations group is traced, then the adjacent points are defined. These points belong to the convex hull and are chosen with $\pm\zeta(m^3/s)$ of the maximum efficiency of water discharge where ζ are real variables. The choice of these variables depends on the management of the powerhouses.

For example, Figure 2 shows the efficiency curve of the water discharge for a powerhouse that has 4 available turbines. As explained in Section 2.3, the number of active turbines can be 3 (curve with dashed line) or 4 (curve with solid line). For each curve, we define the maximum efficiency (point with $'*'$ marker). The maximum of water discharge (point with $'o'$ marker) is defined only for the curve with the maximum number of turbines because it is the only case where spillage is an option.

The adjacent point with $'X'$ marker are defined as -3 and -6m^3/s and 2 equidistant points between the maximum water discharge and the efficiency water discharge. For the curve with a dashed line (3 active turbines) the adjacent points are defined with $\pm$ $3m^3/s$ and $\pm$ $6m^3/s$. Once these points are defined we determine the power produced associated to the points in order to obtain the pair of points (P,Q) as shown in Figure 3.

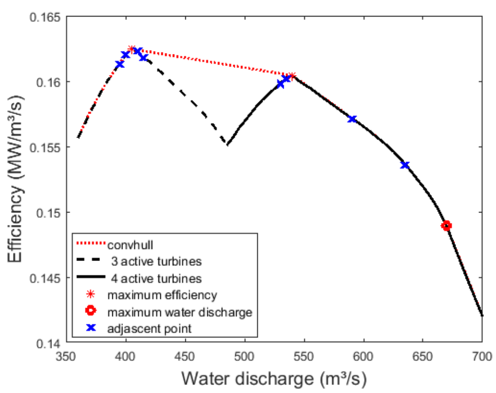

Figure 2: Efficiency curve of water discharge

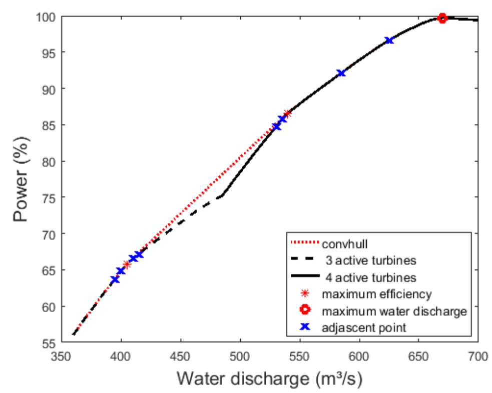

Figure 3: The set of points

The operator must select the best point of the set respecting some constraints like the water balance. Once the point is selected for the period t, the model should use another combination to reach a new point for the next period. To decrease the number of changes, a penalty factor must be added at each start-up. The set of the pair points presented in the previous curves is obtained with a given forbay elevation. The idea is to fix the value of the forbay elevation at the maximum volume of the reservoir V_{max}. Using V_{max} reduces the number of variables and facilitates the resolution of the problem. However, in the reality, the reservoir is not always full. The presented model in this paper will take into account this aspect and make a correction between the produced power at V_{max} and the power obtained at the real volume.

3 Mathematical model

This section presents the mathematical model used to solve the short-term unit commitment problem. The problem is formulated using an MILP. The following notation is used throughout the paper.

Notation	
$t \in \{1, 2, 3, .., T\}$:	index of periods.
$c \in \{1, 2, 3, .., C\}$:	index of powerhouses.
$l \in \{1, 2, 3, .., U^c\}$:	index of upstream powerhouses of each powerhouse c.
$j \in \{1, 2, 3, .., J_t\}$:	index of turbines associated to period t.
$b \in \{1, 2, 3, .., B^c\}$:	index of combinations of each powerhouse c.
$k \in \{1, 2, 3, .., K_c^b\}$:	index of points (the maximum and the adjacent points) associated to powerhouse c and combination b.
$k \in \{1, 2, 3, .., K_l^b\}$:	index of points (the maximum and the adjacent points) associated to upstream powerhouse l and combination b.
$P_{k,t}^c$:	power output of powerhouse c at period t and point k (MW)
$q_{k,t}^c$:	water discharge at powerhouse c at period t and point k (m^3/s).
$q_{k,t}^l$:	water discharge at powerhouse l at period t and point h (m^3/s).
δ_t^c:	inflow of powerhouse c at period t (m^3/s).
β:	conversion factor from (m^3/s) to (hm^3/h).
θ^c:	estimated energy losses from maximal storage (MW) at powerhouse c.
ϵ^c:	start-up penalty of turbine (MW) at powerhouse c.
N_{max}:	maximal number of the start-ups.
V_{max}^c:	maximal volume of the reservoir c.
v_{ini}^c:	initial volume of the reservoir c.
v_{target}^c:	target volume of the reservoir c.
Δt:	the time interval between periods (h).
$A_{t,k,j}^c =$	$\begin{cases} 1 & \text{if the turbine } j \text{ of powerhouse } c \text{ at the point } k \text{ is activated at period } t \\ 0 & \text{otherwise} \end{cases}$
The decision variables are :	$y_{k,t}^c = \begin{cases} 1 & \text{if the point } k \text{ is chosen at period } t \text{ for powerhouse } c \\ 0 & \text{otherwise} \end{cases}$ $y_{k,t}^l = \begin{cases} 1 & \text{if the point } k \text{ is chosen at period } t \text{ for powerhouse } l \\ 0 & \text{otherwise} \end{cases}$ $z_{j,t}^c = \begin{cases} 1 & \text{if the turbine } j \text{ of powerhouse } c \text{ is started at period } t \\ 0 & \text{otherwise} \end{cases}$
v_t^c:	volume of the reservoir of powerhouse c at period t (hm^3).
d_t^c:	water spillage at powerhouse c at period t (m^3/s).
d_t^l:	water spillage at powerhouse l at period t (m^3/s).

The first computes the power output $P_{k,t}^c$ at each point for each combination at each period. This power is determined at a maximum volume of the reservoir V_{max}. Since the power output is calculated at V_{max}, the second component allows to make a correction between the power produced at the current volume and V_{max}. In this regard, the second component of the objective function computes the difference between the volumes is multiplied by an estimate energy losses θ^c. The last component reduces the number of changes by penalizing unit start-ups. Finally the three components are multiplied by Δ_t to obtain the energy. The optimization problem is:

$$max(\sum_{c\in C}\sum_{t\in T}\sum_{b\in B}\sum_{k\in K_b^c} P_{k,t}^c * \ y_{k,t}^c - \sum_{c\in C}\sum_{t\in T}(V_{max}^c - v_t^c) * \theta^c - \sum_{c\in C}\sum_{t\in T}\sum_{j\in J} z_{j,t}^c * \varepsilon^c) * \Delta_t \tag{3}$$

Subject to:

$$\begin{aligned} v_{t+1}^c =& v_t^c - \sum_{b\in B}\sum_{k\in K_b^c}(q_{t,k}^c * \ y_{k,t}^c * \beta) + (\delta_t^c * \beta) - d_t^c \\ &+ \sum_{l\in U^c}\sum_{b\in B}\sum_{k\in k_b^l}(q_{t,k}^l * \ y_{k,t}^l * \beta) + d_t^l, \qquad \forall c \in C, \forall t \in T, \end{aligned} \tag{4}$$

$$\sum_{b\in B}\sum_{k\in K_b^c} y_{k,t}^c = 1, \qquad \forall c \in C, \forall t \in T, \tag{5}$$

$$\begin{aligned} &\sum_{b\in B}\sum_{k\in K_b^c} y_{k,t+1}^c * A_{t+1,k,j}^c \\ &-\sum_{b\in B}\sum_{k\in K_b^c} y_{k,t}^c * A_{t,k,j}^c \le z_{j,t}^c \qquad \forall c \in C, \forall t \in T, \forall j \in J, \end{aligned} \tag{6}$$

$$\sum_{t\in T}\sum_{j\in J} z_{j,t}^c \le N_{max}, \qquad \forall c \in C, \tag{7}$$

$$v_{min}^c \le v_t^c \le V_{max}^c, \qquad \forall c \in C, \forall t \in T, \tag{8}$$

$$v_1^c = v_{ini}^c, \qquad \forall c \in C, \tag{9}$$
$$v_T^c \geq v_{target}^c, \qquad \forall c \in C, \tag{10}$$
$$d_t^c \geq 0, \qquad \forall c \in C, \forall t \in T, \tag{11}$$
$$q_{k,t}^c, P_{k,t}^c \geq 0, \qquad \forall c \in C, \forall t \in T, \forall b \in B, \forall k \in K_b^c, \tag{12}$$
$$q_{k,t}^l \geq 0, \qquad \forall c \in C, \forall t \in T, \forall b \in B, \forall k \in K_b^c, \forall k \in K_b^c, \tag{13}$$
$$y_{k,t}^c, y_{k,t}^l, z_{j,t}^c, A_{t,j}^c \in \mathcal{B}, \qquad \forall c \in C, \forall t \in T, \forall b \in B, \forall k \in K_b^c, \tag{14}$$
$$d_t^c, d_t^l, v_t^c, q_{k,t}^c, q_{k,t}^l, P_{k,t}^c \in \mathcal{R}, \qquad \forall c \in C, \forall t \in T, \forall b \in B, \forall k \in K_b^c. \tag{15}$$

The constraints (4) ensure that the water balance of the powerhouses are met. In the case where the powerhouses are in series (U^c not empty), we add the water discharge $q_{k,t}^l$ and the water spillage d_t^l of the previous powerhouses to the volume v_{t+1}^c. Otherwise, there will not be taken into account. By this way, we ensure that the trade-off between the powerhouses is met. Constraints (5) force the model to choose only one point at each period for each powerhouse. Constraints (6) are the link between start-up variables and combination choice considering the set of points. To limit the number of turbine change during the planning horizon a maximum number of start-ups N_{max} is imposed with constraints (7). Constraints (8) limit the volume of the reservoirs and (9)–(10) specify initial and final volumes. Non-negativity of the variables are taken into account by constraints (11)–(13). Finally, (14) impose binary variables and (15) impose real variables.

4 Case study

The formulated MILP model presented in the previous section is tested on the Saguenay-Lac-St-Jean hydroelectric system owned by Rio Tinto. Rio Tinto is a company that produces aluminum in the Saguenay region in the province of Quebec. The production of aluminum requires a lot of energy. In this regard, Rio Tinto has a hydropower system that allows it to produce energy to fill its needs. Rio Tinto has five powerhouses with 44 turbines. Five reservoirs are available with different capacities.

The model presented in this paper is tested on two powerhouses in the system: Chute-Du-Diable and Chute-Savane. These two powerhouses are in series and each has five turbines. To schedule hydropower production, Rio Tinto aims to use the maximum efficiency of their turbines in order to maximize the energy by defining the best combination at each period. To do that, they use a dynamic programming algorithm and rely on their staff's experience. The problem is that the selection of the quantity of water discharge and the combination of active turbines are done manually. Moreover, actually there is no correlation between the powerhouses. The purpose of this paper is to define an automatic procedure which allows to determine the best schedule by defining the best combination of active turbines taking into account the power loss caused by the start-ups.

4.1 Numerical tests

For both powerhouses of five turbines, operational restrictions require a minimum of three active turbines. In this case, the number of possible combinations will be 16 for each powerhouse as shown in Table 2: 10 combinations of three turbines, 5 combinations of four turbines and 1 combination of five turbines. For each combination, we define the pair of points of water discharge and power produced (P,Q) as explained in Section 2.1. The discretization of the water discharge is done in $5m^3/s$ in practice. In this regard, the adjacent points are defined with $\pm 5m^3/s$ and $\pm 10m^3/s$ of water discharge and 2 equidistant point between the maximum water discharge and the efficiency water discharge where the number of active turbines is equal to the number of available turbines.

Table 2: Combination of 5 available turbines

3 active turbines	4 active turbines	5 active turbines
123 - 124	1234	12345
125 - 134	1235	
135 - 145	1245	
234 - 235	1345	
245 - 345	2345	

The model is tested on 30 test cases. The duration of each is four days partitioned into 96 hours. The quantity of natural inflows and the availability of the turbines are known and provided from the historical database. The model is deterministic, therefore there is no uncertainty in the inflows. Initial final volumes of each reservoir are also provided.

The Xpress [2] solver with Python programming language are used to solve this problem. A linear mixed-integer program with 4032 binary variables, 386 real variables and 1920 constraints has been created. To solve this unit commitment problem, the computational time is fluctuating between 5 and 10 minutes for each case.

4.2 Interpretation of the results

The results from the proposed model are compared with historical results. The historical periods with turbines unavailable due to maintenance or repair, the periods with a contingency energy request, and the periods with high water spillage were not selected for comparison in order to obtain accurate results.

Figure 4 is a histogram comparing the average improvements between optimized and historical solutions. The figure shows that all the values of the average improvement are positive which indicates that the optimizer solution is better than the historical, moreover, 50% of the cases allows to have an improvement of more than 2%.

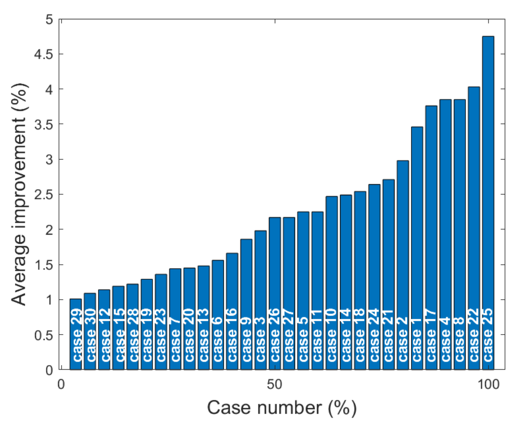

Figure 4: Average improvement

Let us analyze case 1 where the average improvement is high. Figure 5 shows the difference between the number of active turbines for powerhouses Chute-Du-Diable (CD) and Chute-Savane (CS). At the powerhouse CD, unlike the optimizer solution (solid line) that requires the activation of four turbines during the 3 first periods followed by the activation of five turbines, the historical solution (dash line) activates three turbines for the 38 first periods followed by four turbines until period 49, then three turbines for the 2 next periods and finally four turbines until the last period. The difference between the number of active turbines is clear at powerhouse CS. In the optimizer case, the number of active turbines is unchanged (five turbines) during the whole planning horizon, but in the operational case, the number fluctuates between three, four and five. The optimizer the solution requires the activation of five turbines practically for all periods. For this reason, the produced energy provided from optimizer solution is more than the historical solution. In addition, the number of the start-ups for the optimizer case is less than the reality case. At the powerhouse CD, there is one start-up in the optimizer solution versus 2 in the historical. At powerhouse CS, there is no start-ups in the optimizer solution versus 3 start-ups in the historical.

Operating with a maximum number of active turbines for all periods does not necessarily mean that the produced energy will increase. Let's analyze the case 6. Figure 6 shows that the optimized solution and the historical solution require respectively four turbines and five turbines for both powerhouses (CD and CS) during all periods. However, the optimized solution produces more energy than the historical solution, whereas the number of active turbines is lower. This difference is due to the good choice of the quantity of water discharge. Figures 7 and 8 show the results of the quantity of water discharge and the volume of the reservoir at each period for each powerhouse. The optimized solution is presented by a solid line and the historical solution by a dashed line. The Figures 7 and 8 shows that in the historical solution the quantity of the water discharge used with 5 turbines is ineffective that's why the power produced is low.

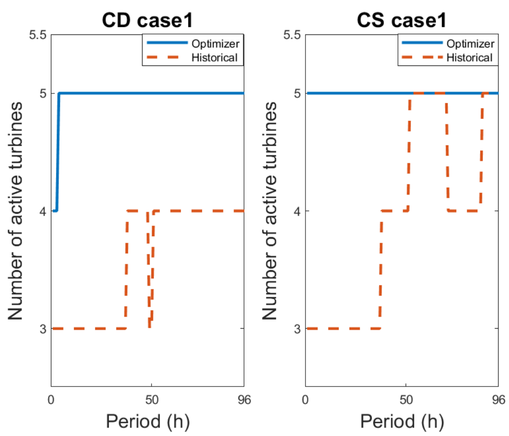

Figure 5: Number of active turbines for case 1

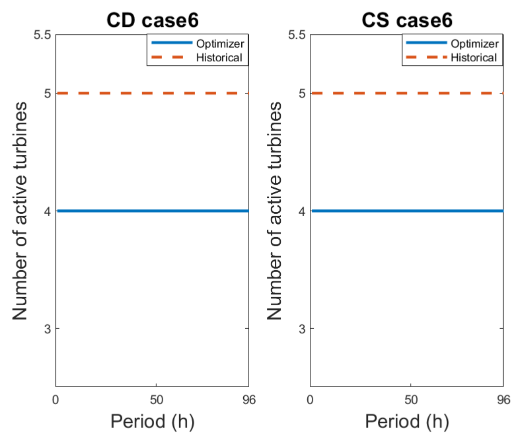

Figure 6: Number of active turbines for case 6

The lowest average improvement is reached for case 29. Figures 9 and 10 shows that the curve with a dash line of water discharge follows the same pace as that of the solid curve for both powerhouses which indicate that the quantity of water discharge of the historical solution passes practically through the efficiency points. In this regard, the produced power obtained from optimized solutions and historical solutions are closer, therefore the average improvement is weak.

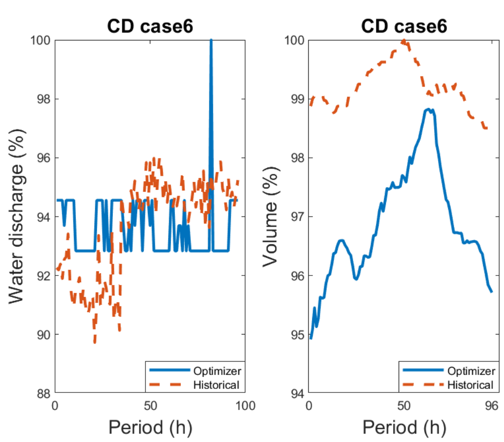

Figure 7: Water discharge and volume at powerhouse CD for case 6

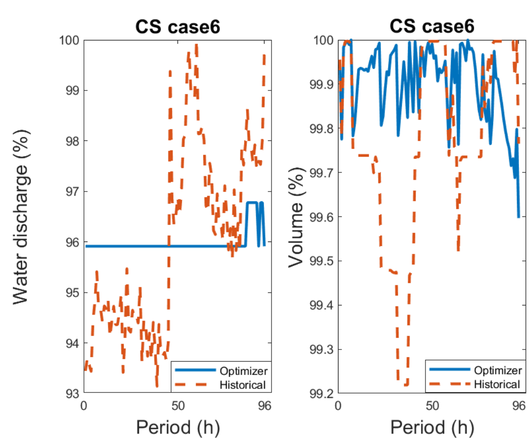

Figure 8: Water discharge and volume at powerhouse CS for case 6

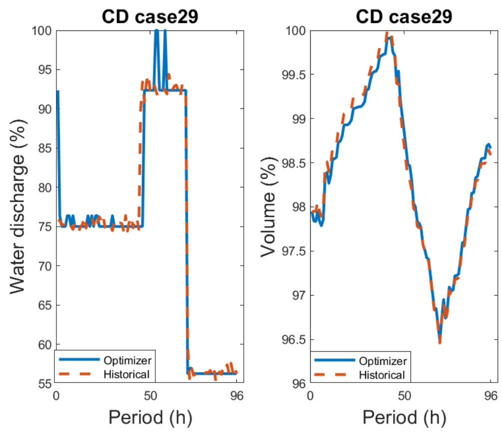

Figure 9: Water discharge and volume at powerhouse CD for case 29

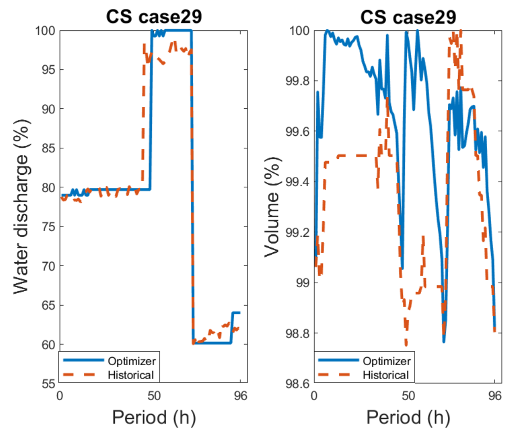

Figure 10: Water discharge and volume at powerhouse CS for case 29

5 Conclusion

This paper introduces an innovative method to optimize the unit commitment of hydropower generating units. This method allows the producer to implement directly the optimized solution since it reflects the operational reality. The innovation is to determine the pair of maximum efficiency points of water discharge and power produced at the maximal storage then operate the turbines using these points. The points are determined for all possible combinations of turbines. Since the reservoir is not always full, a correction of produced power will be done. To avoid the start-ups, maximal number of turbine changes are imposed to find a viable solution in practice. The problem is formulated using an MILP to find the exact combination of turbines and the best point that maximizes total energy but also penalizes the start-up. For this paper, the model is tested on two powerhouses and the optimal operation schedules have been successfully obtained. The study has shown that the optimized solution is better than reality.

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