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Optimization of phase unbalance in a distribution grid with demand response

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Abstract: We consider the use of demand response to address phase unbalance in a distribution grid. At present this problem is addressed by modifying the topology of the grid, but this has limited flexibility because it can only be done once or twice a year. We propose to use demand response to minimize phase unbalance on a 15-minute basis. Because phase unbalance is difficult to estimate analytically, we assess it using the OpenDSS distribution system simulator and we optimize using a black-box optimization algorithm. Our approach reduces the phase imbalance by up to 50% on the IEEE 123-bus feeder.

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1 Introduction

The integration of renewable energy sources to generate electricity in large quantities raises new challenges. One of them is the management of intermittent energy sources: the production of these technologies is based on wind or sun, and cannot be easily adjusted to demand. Another is the fact that these energy sources are often decentralised. On the other hand, the development of smart meters makes it possible to gather information in a nearly real-time basis within the distribution system.

These changes are leading to new forms of distribution grids, known as smart grids. The traditional grid with consumers on one side, who only consume, and producers on the other side, who manage generation to balance the consumption, will no longer exist. Participants can now be both consumer and producer, and the deployment of new technologies such as electric vehicles, which can be seen as batteries for the grid, leads to a new situation in which demand response that adapts consumption to generation takes on greater importance. One example is to increase consumption when renewable generation is higher. Another is to reduce demand peaks at certain hours of the day, when many people consume at the same moment, to avoid turning on costly generators. Many studies have been conducted on the topic of smart grids and demand response. In particular, [1] shows how a smart grid and demand response allow management of renewable energy sources and [2] presents the concept of virtual power plants to allow an efficient integration of renewable energy sources.

In power grids, the balance between production and consumption is not the only difficulty utilities have to face. There are other issues in distribution grids that have to be dealt with. One of them is phase unbalance, a problem that comes from the split from three-phase wire to single phase wire and the different consumption of each customer. This transformation leads to increase electric losses and electric equipment failure.

In this paper we investigate the potential of using demand response in order to minimize phase unbalance. Demand response will be used from an utility perspective to obtain the optimal consumption profile for each client in a network. The next step, which is investigating how to reach precisely these levels of consumption is not considered in this study. It is assumed that each load can modify its consumption to perfectly answer the optimal profile. As will be shown in the second part of this paper, estimating phase balance of an electrical network is difficult to do precisely. This is why we have chosen to assess it with a distribution system simulator like OpenDSS from EPRI specially developed to evaluate the state of a given network. This assures that we obtain an accurate estimation of our parameter to minimize but it also means that we will not have access to the literal formulation of our objective function and that a black-box approach will be required.

The issue with black-box optimization algorithms is that they are designed to solve problems with around 20 to 50 variables. However, our goal is to be able to apply our method on a network as close as possible to the reality, which means with a significant number of buses and time steps. Therefore our problems are of a different order of magnitude than the ones for which the black-box optimization algorithms are usually developed and an important part of this work has focused on finding a method that can deal with an important number of variables. In particular, we are able to reduce phase unbalance on a distribution network with 123 feeders and 91 loads with a sample time of 15 minutes.

This paper presents the method we have developed which is able to solve a black-box optimization problem of more than 8,500 variables and with a significant reduction of phase, up to more than 50% in certain situations.

This paper is structured as follows. In Section 2 we present the studies that have previously been done in similar fields. Section 3 focuses on the physical origin of the phase unbalancing issue and how it will be estimated. Section 4 explains the mathematical model proposed, and Section 5 describes how to solve it on a simplified version of our model. The techniques used to optimize the whole model are explained in Section 6, and Section 7 reports results based on two IEEE benchmarks. Section 8 concludes the paper.

2 State of art

In recent years there has been an important number of articles about demand response. First, many articles have demonstrated how the control of consumption can be useful in a context of smart grid. In [3] there is a modeling of the different types of loads that can be found in a residence based on their impact on comfort and how they can be controlled. These models are then used to optimize costs with some operational constraints. In [4], some tools are developed to help actors use demand response. [5] models demand response in order to reduce costs in a context of renewable energy sources on the grid.

In multiple articles considering demand response, the cost of electricity is considered variable on a small time scale in order to represent the real cost of electricity production for the utility. It is this variation of cost, or an estimation of it, that can be used in demand response programs in order to minimize the cost for the consumer. It is also possible to define a variation of price through time between the utility and the consumer as an incentive for the consumer to consume electricity when electricity is cheap for the utility to produce. Articles like [6] or [7] propose methods to define these incentive prices for it to be beneficial both for the utility and for the consumer.

On the other hand, there has been some research about how to minimize phase unbalance, which is a measure of the phase balancing issue, on a network, like in [8] where six algorithms are compared. In [9] unbalance is considered in a network with data center and distributed energy. However the method used in both of these articles to optimize unbalance is phase swapping, which means the reorganisation on the grid. The difficulty with this method is that it requires a physical modification of the grid topology, which means a human intervention or the use of particular equipment. Therefore, it is not possible to modify the network too often, and usually it is not done frequently. In this article, our goal is to optimize unbalance on a time scale of maximum one hour, which means that this kind of methods cannot be used. [10] highlights the fact that there are three issues on electric networks that demand response can help to answer and the reduction of phase unbalance is one of them. However, no method is explained on how to reduce it and there are no results about the potential benefits.

Finally some articles consider phase balancing with demand response. For example, in [11] or in [12], demand response is used for optimization and takes phase balancing into consideration but as a constraint by forcing it to remain in a certain range. However, unbalance is not considered as an objective function as it will be done in this article.

3 Estimation of phase unbalance

For our work, we consider the minimization of current phase unbalance, one of the issues that appears in electric networks. As explained in [13], three-phase unbalance is a characteristic of a grid that has different consequences such as electrical losses or deterioration of equipment. We will start by explaining the electrical origin of this issue.

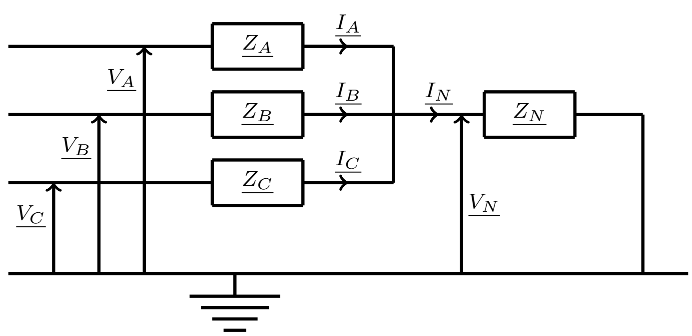


Figure 1: Graph origin unbalance

Electricity is usually transported with three-phase current to limit ohmic losses. However, most of the electrical devices work with single phase current. Therefore, it is necessary to transform the three-phase current to a single phase one. On the Figure 1, the three voltages $\underline{V_A}, \underline{V_B}, \underline{V_C}$ are the result of a transformation from a three-phase current to three mono-phase currents. We here specify that all the variables underlined are complex variables. The real part of these correspond to their physical values. On this figure, the objective is to provide electricity to the three loads A, B and C that can be considered as individual houses or as a group of residences. These corresponds to end of the system, the N line is here to make the connection to the ground. Any current that goes through the N line is therefore lost and creates unbalance: reducing phase unbalance means reducing the current going through this line.

Let us here focus on the case where the system is balanced, which means that the phase between each of the lines is exactly $2\pi/3$. This corresponds to the best case. These three voltages can be defined as:

$$\underline{V_A} = \underline{V} \tag{1}$$

$$\underline{V_B} = \underline{V}e^{2i\pi/3} \tag{2}$$

$$\underline{V_C} = \underline{V}e^{4i\pi/3} \tag{3}$$

It is important to note that in this situation, we have: $\underline{V_A} + \underline{V_B} + \underline{V_C} = 0$.

The impedance of each of the three lines are noted $\underline{Z_A}, \underline{Z_B}$ and $\underline{Z_C}$ and can be seen as the consumption on the line. The impedance is a complex value such that:

$$\forall i \in \{A, B, C\}, \underline{V_i} - \underline{V_N} = \underline{Z_i I_i} \tag{4}$$

There is also a voltage $\underline{V_N}$ and a current $\underline{I_N}$ in the N line with an impedance $\underline{Z_N}$ such that: $\underline{V_N} = \underline{Z_N I_N}$

By applying the Kirchhoff's circuit laws, one can find:

$$\underline{V_N} = (\frac{\underline{V_A}}{\underline{Z_A}} + \frac{\underline{V_B}}{\underline{Z_B}} + \frac{\underline{V_C}}{\underline{Z_C}})(\frac{1}{\underline{Z_A}} + \frac{1}{\underline{Z_B}} + \frac{1}{\underline{Z_C}} + \frac{1}{\underline{Z_N}})^{-1} \tag{5}$$

Therefore, if $\underline{Z_A} = \underline{Z_B} = \underline{Z_C}$, $\underline{V_N} = 0$, which means that $\underline{I_N} = 0$ and there are no losses linked to the transformation from three-phase to single phase current. However, if the impedances are different, $\underline{V_N} \neq 0$ which means that there are some losses. This is the origin of unbalance of an electric network. In this example it seems quite easy to understand how it occurs, its logic and how to measure it. However when there are several loads and lines, which each have their own characteristics, it becomes much more difficult to evaluate. Furthermore, this is an example in which only the loads can be unbalanced. In reality, it is possible to have a unbalanced sources as well, which makes the estimation more difficult to realize. This is why we choose the solution of using a simulator to assess it and the black-box approach.

The method of symmetrical components, presented in [14] permits to transform the three real currents I_A, I_B, I_C, that define the state of the system in our case, in three values, I_0 the zero sequence current, I_1 the positive sequence current and I_2 the negative sequence current. We have, in particular the relation $3I_0 = I_A + I_B + I_C = I_N$, which shows us that by minimizing the zero sequence current, we will reduce the current in the N line. Each of these values can be determined for each of the time periods, I_0^t, I_1^t, I_2^t. It is also possible to show that higher is the negative sequence current, higher is the current in the N line. Therefore, in our study, we will define the phase unbalance such as:

$$Unbal = \sum_{t \in T} \frac{I_0^t + I_2^t}{I_1^t} \tag{6}$$

The simulator OpenDSS from EPRI has been used to evaluate these different values. To do so, the grid considered has to be described as precisely as possible, with the type of loads, the type of lines, their distances, their resistances, etc. Once the network is characterised, the consumption of each load through time is defined. Therefore, the simulator gives us the three values of current necessary to compute the unbalance on the whole time period.

4 Optimization model

The objective of our study is to minimize the phase unbalance on a network with the use of demand response. The major idea of demand response is to reorganize the consumption of a customer through the day and to limit its impact on its comfort. For our model, we will suppose that each load is a customer, or a group of customers, which can answer exactly to any modification instruction.

Let L be the set of loads of our network that will be modified and T be the set of time periods considered. For each load, a base consumption is considered over each time period which corresponds to the consumption without demand response. This consumption is noted $(B_i^t)_{i\in L,t\in T}$. For each time period and each load, demand response will modify this consumption. The variables $(x_i^t)_{i\in L,t\in T}$ considered are the part of the base consumption actually consumed after applying demand response. At the end, the consumption with demand response will be $(x_i^t B_i^t)_{i\in L,t\in T}$.

In order to limit the impact on the comfort of the customers, at any moment the consumption after demand response cannot vary too much compared to the base consumption. To control this, two types of parameters will be defined for each load and each time period: a lower bound $(\alpha_i^t)_{i\in L,t\in T}$ and an upper bound $(\beta_i^t)_{i\in L,t\in T}$ on the modification of the consumption. Therefore, a constraint will be added for each load and each time period:

$$\alpha_i^t \le x_i^t \le \beta_i^t \qquad \forall (i,t) \in L \times T \tag{7}$$

The idea of demand response is to reorganize the consumption through a certain time period. This means that the consumption after demand response must remain close to the one without. For each customer, a constraint is added to limit the global consumption within some bounds. We define a lower bound $(\lambda_i)_{i\in L}$ and an upper bound $(\mu_i)_{i\in L}$ on the modification of the global consumption:

$$\lambda_i \sum_{t\in T} B_i^t \le \sum_{t\in T} x_i^t B_i^t \le \mu_i \sum_{t\in T} B_i^t \qquad \forall i \in L \tag{8}$$

Finally, with $Unbal((x_i^t)_{i\in L,t\in T})$ the function that gives the unbalance calculated by our simulator for a given consumption of each load and at each time period, the problem that we will try to solve is:

$$\begin{aligned} &\min Unbal((x_i^t)_{i\in L,t\in T}) \\ &\text{s.t. } (7), (8) \end{aligned} \tag{9}$$

5 Optimizing without time linking constraint

Our objective is to solve problems with up to 100 customers and with a time step of 15 minutes on a day, so 96 time periods. This means solving a problem with close to 10,000 variables. As mentioned in the introduction, black-box optimization methods are designed to work with around 20 variables and these cannot work directly with more than around 100 variables. The evaluation of a point of the black-box is usually supposed to be quite time consuming, which is why the algorithms that solve black-box optimization problems are developed to minimize the number of its evaluations. The simulator used in our study computes quickly compared to usual black-boxes, which means that it is possible to design an algorithm that makes more evaluations. However, it remains impossible just to use an existing algorithm with around 10,000 variables. The algorithm that we have developed divides our problem in a set of sub-problems with only a few variables. The sub-problems created are small enough to be solved with a black-box optimization algorithm.

The algorithm that we have chosen to solve these black-box optimization problems is the Nelder-Mead algorithm (NM) [15]. It is a well known heuristic which has been successful on numerous problems. However, this method is not designed to optimize problems with constraints, so we apply a penalty approach.

First, we focus on solving our problem without considering the constraint (8) that assures that the consumption remains close to the consumption without demand response. We just explain here how the base method works. The approach implemented to deal with this constraint will be described in Section 6.

As we have seen, unbalance is the result of a difference of consumption between some lines of our network. In particular, there is no correlation between the different time periods. Therefore, in order to minimize our objective function, it is interesting to decompose the problem by time period. For each time period, we use the Nelder-Mead algorithm to optimize the corresponding $|L|$ variables. The other variables are fixed to the current value. This way, the black-box optimization algorithm is launched $|T|$ times, only with $|L|$ variables instead of $|L| * |T|$.

Let us note $(x_i^{t*})_{i \in L, t \in T}$ the best solution found. The method that has been developed based on this idea is initialised with the situation without demand response: $x_i^{t*} = 1,\ \forall (i,t) \in L \times T$. Then, the first time period is considered: all the variables corresponding to other time periods remain fixed to their value and are not considered: $x_i^t = x_i^{t*}, \forall (i,t) \in L \times T$. Only the variables $(x_i^1)_{i \in L}$ are considered and the Nelder-Mead algorithm is launched on them to minimize the unbalance of the whole day. This method is used for each of the time periods of our problem which are considered in chronological order. It is presented in Algorithm 1.

Algorithm 1 $\min Unbal((x_i^t)_{i \in L, t \in T})$

Initialisation: $x_i^{t*} = 1,\ \forall (i,t) \in L \times T$
for $\tau \in T$ **do**
  $\forall (i,t) \in L \times T, t \neq \tau \implies x_i^t = x_i^{t*}$
  $(x_i^\tau)_{i \in L} = \underset{(x_i^\tau)_{i \in L}}{\arg\min}\, Unbal((x_i^t)_{i \in L, t \in T})$ *Using NM*
  $(x_i^{t*})_{i \in L, t \in T} = (x_i^t)_{i \in L, t \in T}$
end for

5.1 Nelder-Mead algorithm

The Nelder-Mead algorithm is a well known approach to solve an unconstrained black-box optimization problem. This algorithm is based on a simplex: a set of $(n+1)$ points (with n being the number of variables of our problem) which convex hull represents “a bounded convex polytope with nonempty interior [...] vertices” [16]. The main idea of the Nelder-Mead algorithm is to determine new points to evaluate depending on the value of the vertices of the simplex. Figure 2 represents an example of the application of the algorithm with 2 variables. The points that compose the simplex are ordered with respect to their value. In our example, x_0 is the best point and x_2 the worst. We take the mean of the n best points, x_c, and the new points are determined on the line between this point and the worst vertex of the simplex. The first one, x_r is a reflection of the worst point by x_c: $x_r = x_c + (x_c - x_n)$. The three other points are built, if needed, depending on three parameters $\kappa_e, \kappa_o, \kappa_i$ with the same structure: $x_k = x_c + \kappa_k(x_c - x_n), k \in \{e, o, i\}$. In order to be certain to explore well the space, these parameters have to respect that: $\kappa_e > 1, 0 < \kappa_o < 1, -1 < \kappa_i < 0$. In the case that none of these 4 new points are better than the worse point that compose the simplex, x_n, than a shrink in performed. This means that only the best solution, x_0, is kept and all the other points are replaced according to a parameter κ_s, such that $0 < \kappa_s < 1$: $\forall k \in [\![1, n]\!], x_{ks} = x_0 + \kappa_s(x_0 - x_k)$.

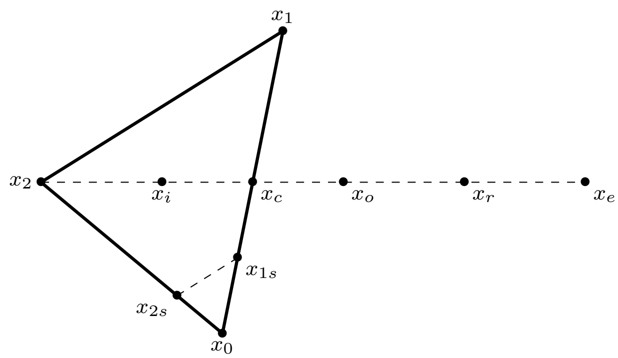


Figure 2: Figure of Nelder-Mead iteration, inspired by figure 5.1 in [16]

As explained in the manual [16], it is possible to define a normalised volume for a set of points and we use this value as a convergence criterion for our algorithm. In the beginning of the Nelder-Mead algorithm, we calculate $voln_{init}$, which is the normalised volume of our set of initial points. Then at the end of each Nelder-Mead iteration, we evaluate the new normalised volume $voln$. If $voln < \xi voln_{init}$, with ξ a parameter such that $0 < \xi < 1$, we stop the algorithm. Otherwise, we perform another iteration.

The Nelder-Mead algorithm does not specify how to choose the $n+1$ initial points of the algorithm. The method we have chosen is to define these points from the current optimal point, as developed in Algorithm 2. We only modify one of the n dimensions of this point: we try the two extreme values possible for the point to still be feasible and we keep the best one. This produces n different points to which we add the current optimal point unmodified. This method assures us to define a simplex with a non-zero volume and so a set of points to which can be applied the Nelder-Mead algorithm. As the current best point is one of the initial points, we also have the assurance that the point given by our optimization will be, in the worse case, the same point as before the optimization.

Algorithm 2 Define initial set of points S_τ for Nelder-Mead algorithm for time period τ

Initialisation: $S_\tau = \{(x_i^{t*})_{i \in L, t \in T}\}$, *The first point is the current optimal point*
for $j \in L$ **do**

$$y_{i,Low}^t = \begin{cases} x_i^{t*} & \text{if } i \neq j \text{ or } t \neq \tau \\ \alpha_i^t & \text{if } i = j \text{ and } t = \tau \end{cases}$$

$U^{Low} = Unbal((y_{i,Low}^t)_{i \in L, t \in T})$

$$y_{i,High}^t = \begin{cases} x_i^{t*} & \text{if } i \neq j \text{ or } t \neq \tau \\ \beta_i^t & \text{if } i = j \text{ and } t = \tau \end{cases}$$

$U^{High} = Unbal((y_{i,High}^t)_{i \in L, t \in T})$ *The best of the two extreme values for load j is kept*
if $U^{Low} \leq U^{High}$ **then**
$S_\tau \leftarrow S_\tau \cup (y_{i,Low}^t)_{i \in L, t \in T}$
else
$S_\tau \leftarrow S_\tau \cup (y_{i,High}^t)_{i \in L, t \in T}$
end if
end for

5.2 Applying Nelder-Mead with box constraints

As mentioned previously, the Nelder-Mead algorithm does not allow to consider constraints. Even if the conservation constraints are not considered here, the box constraints (constraint 7) have to be verified. This kind of constraint can be easily dealt with by doing a projection in the set of feasible points. However, as our convergence criterion for the Nelder-Mead algorithm is on the normalised volume of the simplex, the use of a projection sometimes leads to a null volume. Therefore, it sometimes stops the algorithm while there are still interesting points to explore. This is why we have chosen to use penalization to take constraints into account. This means that we modify our objective function in order to assure that the points given by the algorithm satisfy the constraints: if the constraint is not satisfied, then a term is added to the objective function proportionally to how much the constraint is exceeded. Therefore, the optimisation problem considered is, with η a penalty factor:

$$\begin{aligned} \min \quad & Unbal((x_i^t)_{i \in L, t \in T}) \\ & +\eta \sum_{i \in L} \sum_{t \in T} \max\{0, \alpha_i^t - x_i^t, x_i^t - \beta_i^t\} \quad (10) \\ \text{s.c} \quad & x_i^t \in \mathbb{R}, (i,t) \in L \times T \quad (11) \end{aligned}$$

This method to deal with these constraints allows for consideration, in the construction of the simplex, of points that are not feasible, but that are useful to better explore the feasible space. A penalty factor important enough assures that the best point given at the end of the algorithm is feasible. Finally this method has shown to give better results on our test cases than a projection method.

6 Solving the full model

As explained, the solution previously presented has been developed without considering the conservation constraint (8). The difficulty with this constraint is that it concerns the different time periods for each of the loads of our network, which makes the different time periods no longer independent. Therefore, there are two points of view about how to divide our problem into sub-problems.

The first one is to continue dividing it with one sub-problem per time period. This way, the reorganization of consumption between loads within a time period remains, which leads to an important reduction of the objective function. However, as the sub-problems are solved independently, there is no control on the conservation constraint.

The second point of view is to define one sub-problem for each load, with the $|T|$ variables of the day. This method would allow to deal very well with the conservation constraint, as all the variables of a constraint are in the same sub-problem. However, it would be difficult to improve the objective function as, for a given time period, it is only possible to modify one consumption.

Finally, the method we have chosen is to use the approach by time period, presented in the Algorithm 1 and to adapt it to take into account the constraint. In addition, the approach by load is also used to give more flexibility when optimizing by time period.

First, we adapt the algorithm previously presented that works with a single time period. A simple idea could be to penalize as with the box constraint. For the same reason as previously, this solution would ensure finding feasible points. However, this method does not link the different sub-problems. When optimizing a time period, the only bound that appears is the minimum or the maximum of consumption through the day, whereas this bound has to be shared between the different time periods. Therefore, solving the first time period could give us, for example, a solution which leads to little reduction of phase unbalance but that reaches the upper bound of the constraint for a customer. This would mean that for the second time period, the only possibility is to reduce the consumption of this customer to find better points, as the bound is already reached, even if the second time period could lead to better improvement of the phase unbalance. It is necessary to find a method that links the different sub-problems and which assures that when a improvement on the objective function is realized it does not have to much impact on the constraint.

We solve this issue by only accepting points that have a good balance between the gain in the objective function and the impact on the constraint. To do so, we have decided to penalise points that are feasible but which are closer to the limit of the constraint. This way we avoid considering solutions that lead to a low reduction of phase unbalance but which are close to the limit of our constraints.

Therefore, three penalty terms have been added to the one already explained. The first one follows the same logic as the penalty on the box constraints, which means penalising points that do not respect the conservation constraint in order to assure that our final solution satisfies it. The term added is, with the penalty parameter ν:

$$\nu \sum_{i\in L} \max\left\{0, \sum_{t\in T} B_i^t(x_i^t - \lambda_i), \sum_{t\in T} B_i^t(\mu_i - x_i^t)\right\}.$$

The second one has the objective of globally favouring points that do not get a constraint closer to its limit. The objective function is deteriorated for points that have a different consumption than the one without demand response. The term added is a parabola centered on this base consumption, with γ the penalty parameter for the term

$$\sum_{i\in L} \gamma \left(\sum_{t\in t} x_i^t B_i^t - \sum_{t\in t} B_i^t\right)^2.$$

The third penalty term aims to force the algorithm to find points that do not have too much impact on the constraint, but this time from the actual point. This time the term added depends on the current best solution found and not on the consumption without demand response. Therefore, before running the Nelder-Mead algorithm on the variables of a given time period, the consumption of each load is calculated given the best point found: $C_i = \sum\limits_{t\in T} x_i^{t*} B_i^t$, with $\{x_i^{t*}, (i,t) \in L \times T\}$ being the current optimal found. Once this is done, the term added to the objective function is, with δ the penalty factor:

$$\sum_{i\in L} \delta \left(\sum_{t\in t} x_i^t B_i^t - C_i\right)^2.$$

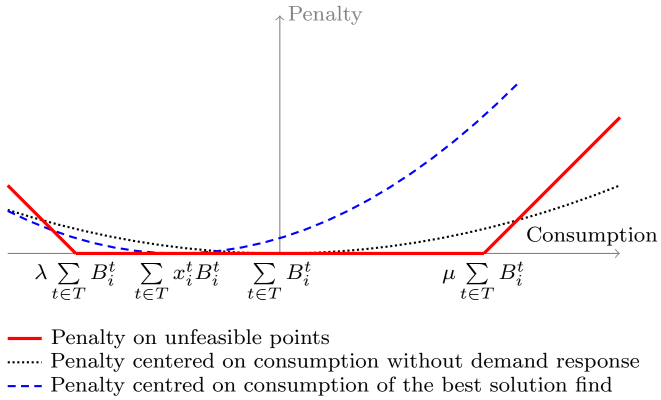
The Figure 3 shows the different penalty terms that are applied in this method. The graph of the Figure 3a represents the different terms separately and the Figure 3b represents the global penalty function applied, which is the sum of the different functions presented. Let us note:

$$\begin{aligned}
Unbal_{pena}((x_i^t)_{i\in L, t\in T}) = {} & Unbal((x_i^t)_{i\in L, t\in T}) \\
& + \eta \sum_{i\in L}\sum_{t\in T} \max\{0, \alpha_i^t - x_i^t, x_i^t - \beta_i^t\} \\
& + \nu \sum_{i\in L} \max\{0, \sum_{t\in T} B_i^t(x_i^t - \lambda_i), \sum_{t\in T} B_i^t(\mu_i - x_i^t)\\
& + \gamma \sum_{i\in L} (\sum_{t\in t} x_i^t B_i^t - \sum_{t\in t} B_i^t)^2 \\
& + \delta \sum_{i\in L} (\sum_{t\in t} x_i^t B_i^t - C_i)^2 \qquad (12)
\end{aligned}$$

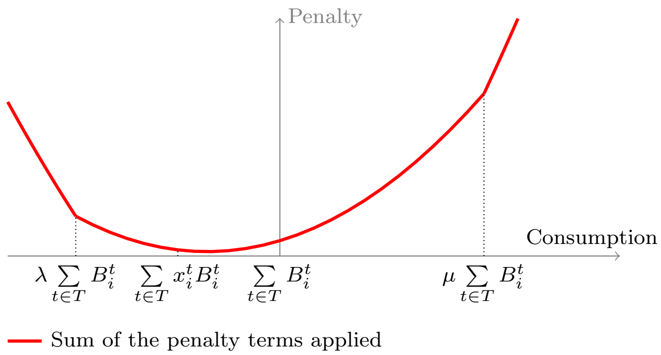
Finally, the objective function solved by the Nelder-Mead algorithm is:

$$\min \quad Unbal_{pena}((x_i^t)_{i\in L, t\in T}) \qquad (13)$$
$$s.c \quad x_i^t \in \mathbb{R}\ \forall (i,t) \in L \times T \qquad (14)$$

An important impact of this consumption conservation constraint is that the different time periods are not independent anymore. In particular, in Algorithm 1 it was possible to solve each time period independently to find an optimum. With this constraint, this is no longer possible. Therefore, the idea is to modify the objective function as just explained, and also to go through the different time periods multiple times until a certain convergence is reached. The method developed to optimize by time period in this new context is explained in Algorithm 3.



(a) Penalty terms applied



(b) Sum of the penalty terms

Figure 3: Penalty terms on the consumption conservation constraint

This penalty method is useful in order to limit, at the beginning of the optimization, the saturation of the consumption conservation constraint for the first time periods. However, this penalty is only needed inside the algorithm to find interesting points. But in the end there is no issue in saturating the constraints. Therefore, the penalty is only useful at the beginning of the algorithm, and as it goes on is less and less important. The idea we have set up is to divide the parameters γ and δ by 5 after optimizing each time period once. This value of 5 has not been optimized but seems to be the one giving the best results in our test cases.

As mentioned previously, as there is one constraint per load, it is also interesting to simply modify Algorithm 3 but to consider for each load the $|T|$ variables that represent its consumption through the day and to optimize our objective function with these. This way it is possible to reorganize the consumption through the day and to have different time periods that compensate each other directly. Therefore, we have chosen to add this method to our algorithm: after optimizing each time period once with the method previously presented, each load is optimized in order to reorganize the consumption through the day. Once it is done, each time period is once again optimized and so on. The algorithm used to optimize by load is presented in Algorithm 4. After applying this algorithm to our test case, it appears that it gives even better results when the optimizations by load are done before and after each optimization by time period. The final method implemented is presented in Algorithm 5.

Algorithm 3 $\min Unbal((x_i^t)_{i \in L, t \in T})$ by time periods

for $\tau \in T$ **do**
  $\forall (i,t) \in L \times T, t \neq \tau \implies x_i^t = x_i^{t*}$
  $(x_i^\tau)_{i \in L} = \underset{(x_i^\tau)_{i \in L}}{\arg\min}\, Unbal_{pena}((x_i^t)_{i \in L, t \in T})$ *Using NM*
  $(x_i^{t*})_{i \in L, t \in T} = (x_i^t)_{i \in L, t \in T}$
end for

Algorithm 4 $\min Unbal((x_i^t)_{i\in L,t\in T})$ by loads

for $j \in L$ **do**
  $\forall(i,t) \in L \times T, i \neq j \implies x_i^t = x_i^{t*}$
  $(x_j^t)_{t\in T} = \underset{(x_j^t)_{t\in T}}{\arg\min}\, Unbal_{pena}((x_i^t)_{i\in L,t\in T})$ *Using NM*
  $(x_i^{t*})_{i\in L,t\in T} = (x_i^t)_{i\in L,t\in T}$
end for

Algorithm 5 $\min Unbal((x_i^t)_{i\in L,t\in T})$

Initialisation: $x_i^t = 1,\ \forall(i,t) \in L \times T$
while Convergence criterion **do**
  Run Algorithm (4), optimizing by loads
  Run Algorithm (3), optimizing by time periods
  Run Algorithm (4), optimizing by loads a second time
  $\gamma \leftarrow \gamma/5$
  $\delta \leftarrow \delta/5$
end while

In this new algorithm, it has also been important to modify how the initial points are defined for the Nelder-Mead algorithm. The general idea developed in Algorithm 2 has been kept, however a few modifications have been made. The consumption conservation constraint (8), implies that the two extreme points defined in this algorithm could be infeasible. However, when working with variables of a single time period it is possible to use this constraint to calculate the upper and lower bounds for each variable according to this constraint and to the box constraint (7). Therefore we replace the bounds of constraint (7) by the bounds that consider both constraints. When working with the $|T|$ variables of a single load, it is no longer possible to define such bounds so we have decided to keep the same process to define our points even if it may produce unfeasible points.

Finally, the algorithm has to go through each sub-problem multiple times. Therefore, it is necessary to define a convergence criterion to stop the process. As explained previously, the objective of this research is to understand the potential of demand response to answer the phase unbalance issue. The question of the application of our method in real conditions is not part of this study. This is why we have decided to let the algorithm run as long as necessary. Each time a sub-problem is solved, the optimal unbalance found is saved. As long as between two optimizations of the same sub-problem, a benefit is made, the algorithm goes on. It stops when, between two optimizations of the same sub-problem, no better point has been found. For the optimization by load, which is done twice as highlighted in Algorithm 5, the optimization of a load made before and after the optimization by time period are considered different. Our convergence criterion is that if there are $2|L| + |T|$ sub-problems solved without finding a better solution, then the algorithm stops. Indeed, the method will continue to test the same points so there is no need to continue.

7 Results

In order to evaluate the effectiveness of our model, we have tested our method on two different electric grids designed by IEEE: the 13 bus feeder and the 123 bus feeder [17]. The first one has 15 different loads while the second one has 91. Our method has mostly been developed based on the 13 bus feeder with a temporal interval of one hour which means optimizing a problem of 360 variables. Different simulations have then been made with the 123 bus feeder and with a temporal interval of a quarter of an hour, which means 8,736 variables, in order to compare the behaviour on a different scale.

The two networks designed by IEEE that we have used for our simulation define each load with a nominal power in addition to the global grid (the type of line, the distances, the different devices situated on the grid, etc...). For each simulation we therefore have to specify which will be the base consumption of each customer through the day, represented by the $(B_i^t)_{i\in L,t\in T}$ parameters in the model. To do so, we have designed three different profiles: a residential, an industrial and a commercial profile. We have also considered one profile for each season of the year. These profiles have been defined based on Hydro-Quebec data for the 2009 year and give us the power consumed each quarter of an hour. These values have been brought between 0 and 1 with respect to the maximum consumption in the year. Finally we have for each type of customer (residential, commercial and industrial) and for each season an evolution between 0 and 1 of the consumption.

The values taken for the different penalization parameters are defined according to the value of the initial phase unbalance $Unbal_{init}$. Different simulations have been made to determine the best values to use. These values, with the values of the other parameters of our optimization problem, are summarized in the Table 1, in which the function E corresponds to the integer part function. The values taken for the Nelder-Mead algorithm are also presented in this table.

Table 1: Table of the values of the different parameters

$\alpha_i^t, \forall(i,t) \in L \times T$	0.9
$\beta_i^t, \forall(i,t) \in L \times T$	1.1
$\lambda_i, \forall i \in L$	0.95
$\mu_i, \forall i \in L$	1.05
η	$100E(\frac{Unbal_{init}}{3})$
ν	$100E(\frac{Unbal_{init}}{3})$
γ initial value	$5E(\frac{Unbal_{init}}{3})$
δ initial value	$5E(\frac{Unbal_{init}}{3})$
κ_e	2
κ_o	0.3
κ_i	-0.7
κ_s	0.7

7.1 13 bus feeder

In the case of the 13 bus feeder, which has 15 loads, we have decided to consider 2 industrial loads and 4 commercial, the 9 last ones being considered as residential customers. The position of these different profiles is determined randomly for each different simulation. For these simulations, we have considered a time scale of an hour which leads to an optimization problem with 360 variables. For each day considered, around 10 simulations have been realised for two different Nelder-Mead convergence criterion values ξ, as explained in Section 5 of this article. The results are presented in Table 2. The first thing that can be noticed is that a tighter convergence test does not assure better results. As each time period and each load is optimized multiple times, it is not necessary to set a high precision. The second interesting result is that with a range of 5% of flexibility on the consumption in one day, it is possible to reduce the phase unbalance, on average, of 17 to 23.3%.

Table 2: Percentage of reduction of phase unbalance for the 13 bus feeder with an hour time scale

	$\xi = 0.01$			$\xi = 10^{-5}$		
	Min	Max	Average	Min	Max	Average
Winter	14.7	24.6	20.6	16.2	23.5	19.5
Spring	9.2	28.6	17.0	11.8	19.0	15.6
Summer	10.4	77.9	23.3	5.9	43.9	16.7
Autumn	8.3	28.7	17.5	11.2	44.8	18.6

7.2 123 bus feeder

In the case of the 123 bus feeder, we have chosen to consider 5 industrial, 20 commercial and 66 residential customers for a total of 91 loads. As with the 13 bus feeder, these profiles have been positioned randomly for each simulation on the grid. The time scale considered here is 15 minutes, which means

96 time periods and leads to an optimization problem with 8,736 variables. The computing time being significantly more important in this case, and considering the results presented in Section 7.1, all our instances have been executed with a convergence criterion for the Nelder-Mead algorithm $\xi = 0.01$. The initial values of phase unbalance are between 15 and 35. For each day, 8 simulations were performed and the results are presented in Table 3. The best result we have found is for an instance in summer for which we achieve to reduce phase unbalance from 18, without demand response, to 7.9: a 55.9% reduction. In this case, without demand response, we have a mean zero sequence current of 35.8A, a mean positive sequence current of 324A and a mean negative sequence current of 44.5A. Once our method applied, we have a mean zero sequence current of 9.82A, a mean positive sequence current of 131A and a mean negative sequence current of 16.4A.

Table 3: Percentage of reduction of phase unbalance for the 123 bus feeder with a 15 minutes time scale

	$\xi = 0.01$		
	Min	Max	Average
Winter	42.5	48.0	45.5
Spring	17.1	38.6	24.8
Summer	31.7	55.9	38.3
Autumn	17.7	58.0	38.4

8 Conclusion

We have proposed a method to reduce phase unbalance on a distribution network by the use of demand response through black-box optimization. This method allows a significant reduction of the unbalance, with a minor modification of the consumption, which leads to the conclusion that demand response could be a solution to answer the phase unbalance problem. The fact that we used a simulator to compute the phase unbalance provides a stronger validation of our results.

It it is important to point out that we make the strong assumption in this study that each load can consume the exact amount of electricity given by our optimization. In reality, it may not be possible to have such control, so it is necessary in future work to account for uncertainty in the consumption. Another issue is the fact that the test cases developed here remain fairly small compared to real life networks that can have thousands of nodes. This means that the number of nodes and variables would increase by a factor of 10, which would require the development of new computational methods to deal with the increase in size. A solution to tackle this problem could be to group customers and to assume they have the exact same consumption. Therefore, all these customers could be represented by the same variable in each time period.

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