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Are retailers' private labels always detrimental to national brand manufacturers? A differential game perspective

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Abstract: In this paper, we study the competition between national brands and private labels (or store brands) by analyzing the impacts of their presence on strategies, sales, and profits in a vertical channel structure. We use a differential game, where the channel members control price and non–price marketing variables, and investigate two scenarios. The first is used as a benchmark. It considers an exclusive retailer that distributes only a national brand provided by a manufacturer, who invests in national advertising to build its brand's goodwill. In the second scenario, the retailer owns a private label that competes with the national brand. By computing the results under both scenarios, we provide answers to the following research questions: (i) What should the price and the non–price marketing strategies be, with and without the private label? (ii) How do they compare? (iii) Is the presence of a private label always profitable for the retailer and harmful to the manufacturer, as the literature suggests? One of our main results indicates that the manufacturer is not necessarily always hurt by the private label. More specifically, we find that, when the local advertising competition between both brands is lower than the price competition, the national brand's manufacturer could charge a higher transfer price to a retailer carrying a private label. This result brings into question an oft-cited explanation given in the literature, namely, that retailers choose to sell private labels as a counter-strategy to overcome double marginalization and to gain market power against the manufacturer.

Keywords: Differential games, private labels, channels, feedback Stackelberg equilibrium

Résumé : Dans cet article, nous étudions la concurrence entre marques nationales et marques privées (ou marques de distributeurs) en analysant l'effet de la présence de marques privées sur les stratégies, les ventes et les profits des membres d'un circuit de distribution. Nous utilisons un jeu différentiel dans lequel les agents contrôlent les prix ainsi que d'autres variables marketing (publicité et efforts promotionnels) et étudions deux scénarios. Le premier est utilisé comme scénario de référence. Dans ce premier scénario on considère un détaillant exclusif qui distribue uniquement une marque nationale fournie par un fabricant, qui investit dans la publicité nationale pour accroitre l'image de sa marque. Dans le second scénario, le détaillant possède une marque privée qu'il distribue en même temps que la marque nationale. En calculant les résultats dans les deux scénarios, nous apportons des réponses aux questions de recherche suivantes: (i) Quelles devraient être les stratégies de prix et de marketing, avec et sans la marque privée ? (ii) Comment se comparent-elles ? (iii) La présence d'une marque privée est-elle toujours rentable pour le détaillant et nuisible pour le fabricant, comme le suggère la littérature ? L'un de nos principaux résultats indique que la marque privée ne nuit pas forcément au fabricant. Plus précisément, nous constatons que lorsque la concurrence sur les efforts promotionnels entre les deux marques est inférieure à la concurrence sur les prix, le fabricant de la marque nationale pourrait fixer un prix de transfert plus élevé à un détaillant qui distribue une marque maison. Ce résultat remet en question une explication souvent citée donnée dans la littérature, à savoir que les détaillants choisissent de vendre leurs marques privées en guise de contre-stratégie pour surmonter la double marginalisation et gagner un pouvoir de marché contre le fabricant.

Mots clés : Jeux différentiels, marques privées, circuits de distribution, équilibre Stackelberg

1 Introduction

The Private Label Manufacturers Association (PLMA) recently announced that, in 2018, the market share of private labels (PLs) had reached almost 25% in US and over 30% in Europe. According to a Nielsen report, PLs are part of a new "retail revolution," and their market shares are increasing at a faster rate than national brands.[1] In such a context, it is important for national brands manufacturers that formerly dealt with exclusive retailers or competed only with other national brands to be prepared to face the competition driven by retailers' private labels. These could be a serious threat to national brands manufacturers for various reasons. First, because the retail price of PLs is often lower than that of national brands.[2] Second, since retailers control the shelf-space in their stores, they can decide to allocate less space to national brands in order to give their store brand more visibility. Finally, retailers may be tempted to put more effort into local promotional activities to boost their brand instead of promoting the national brand. How manufacturers and retailers adjust their prices and make other marketing decision variables in a channel where the retailer may or may not own a private label is a very important question in this context, especially because channel members have more than one "marketing" instrument to react or to influence the other's decisions and outcomes.

This paper explores this issue. It examines the price and non–price marketing decisions (e.g., national and local advertising efforts) taken by national brand manufacturers and by retailers, when the latter owns a PL or not. Our main objective is to provide answers to the following questions: (i) What should the prices and the non–price marketing efforts be? (ii) How do channel members' strategies, profits, and sales for both products compare when the retailer offers (or not) a PL? (iii) Is the presence of a PL always profitable for the retailer and harmful to the manufacturer?

To the best of our knowledge, most of the game-theoretical literature that models the interactions between national brand manufacturers and retailers selling PLs is static, and in most cases, price and non–price marketing decisions are investigated separately. More particularly, no studies consider a setting where a manufacturer fixes the transfer price and invests in national advertising and, at the same time, a retailer controls the retail price and the local promotional activities of both brands. Only two studies simultaneously examine price and non–price decisions, but both of them consider only the scenario where a national brand and a PL compete in the market, and they focus on issues related to the type of advertising between brands (i.e., competitive or informative advertising). Hence, the results obtained in these studies do not provide answers to the research questions $(i) - (iii)$ that we address in this study.

The paper proceeds as follows. In the next section, we provide a brief review of the literature that has studied marketing channels, and where the effects of private labels are analyzed. In Section 3, we introduce our model and the scenarios investigated. In Section 4, we compute the equilibria and compare the results. Section 5 concludes.

2 Literature review

The existing literature investigating the impacts of PLs on the profits and strategies of vertical channel members can be classified into two broad lines of research: a) studies that developed empirical models and where scanner data were used for validation purposes; and b) theoretical models adopting a game-theoretical framework to model this issue. These models are mainly static, but recent research has extended the study of this topic to a dynamic framework in order to account for the carryover effects of some marketing decisions and the multiple interactions taking place in the marketing channel.

[1] PLMA webpage, retrieved April 12th, 2018.

[2] According to the PLMA, audits comparing the prices of PLs vs national brands for 100 market basket items revealed that consumers saved an average of 72 cents per item—about $32 billion on grocery—by buying PLs instead of national brands.

2.1 Empirical models

Most of the papers in this line of research used scanner data from large supermarket chains, reporting sales, prices, marketing activities, and/or market shares for one or multiple product categories where a retailer had introduced its own PL.

In Putsis (1997), the author examined the impact of industry concentration on the proliferation of brands and demonstrated that when the number of branded products increases, their retail prices increase, but if the industry is very concentrated, then prices decrease. Where the new brand introduced into the market is a private label, the author found that the retail prices of the national brands decreased.

Chintagunta et al. (2002) examined the data for two product categories: oats and frozen pasta. They empirically tested the effects of private label introduction in these product categories on prices, price sensitivity, and sales. In the case of oats, only one major brand was handled by the retailer before the entry of the store brand, while the frozen pasta product category included four national brands.

The authors found that the transfer prices of the existing firms decreased in most cases for both product categories. In the oats category, an increase in the retailer's margin for the national brand coincided with the introduction of the PL into the market. The link between both observations allowed the authors to conclude that PLs shift the bargaining power from manufacturers to retailers, since the competition introduced by the PL pushes the manufacturers to reduce their transfer prices. The authors also observed a decrease in the PL's retail price for oats, which may explain the drop observed in the sales of the national brand (by more than 5%), while sales for the whole product category increased. Chintagunta et al. (2002) also examined the impacts of PL introduction on the non–price marketing variables. They reported that the retailer's promotional spending for the national brand was cut by almost 50% , while promotional spending for the PL was fixed at a higher level (w.r.t. the national brand). According to the authors, the increase in the retailer's margin and the decrease in the national brand's promotional spending confirm the change in the nature of the interactions between channel members, with the benefits going to the retailer.

In Bonfrer and Chintagunta (2004), the authors analyzed the sales and prices of brands in different product categories in order to assess the impact of introducing a new brand on the prices of the incumbent brands. Their empirical results indicated that when the new brand is a national brand, the prices in the product category go down. However, when the new brand is a PL, the effects are mixed among the different product categories they studied.

2.2 Game-theoretical models

The game-theoretic literature on PLs is mainly made up of papers adopting a static setting. Raju et al. (1995) examined a situation where a PL is launched in a market where two incumbent national brands compete. They used a model where they considered prices (i.e., wholesale and retail) as the sole decision variables and tested their results with scanner data. They demonstrated that a higher cross-price elasticity between the national brands and the store brand increases the retailer's profit. This result is attributed to the fact that the higher levels of competition lead to higher sales of the PL, but also to an increase in its retail price, which still remains lower than the prices of the national brands. They also observed that the sales of the national brands decrease with the presence of the PL. Hence, the increase in the retailer's profit for the whole product category is mainly driven by the introduction of the PL, since the authors found that the retailer's profit from selling the national brands decreases under this condition. Another interesting result in Raju et al. (1995) states that, when only two national brands are sold at the retailer's store, the retailer is only interested in introducing its PL if the competition between the existing national brands is low.

Mills (1995) argued that the introduction of a PL is a strategic instrument that a retailer can use to counterbalance the inefficiencies in decentralized channels. The PL allows the retailer to capture a portion of the channel profit that would be lost otherwise, because the competitive effect caused by the PL's presence leads to a drop in retail prices, and hence, mitigates the double-marginalization problem. According to the author, PLs not only increase the channel's total profit, but they also help the retailer to strengthen its channel power and to increase its profits, to the detriment of the manufacturer. Indeed, the manufacturer may be tempted to fix higher transfer prices if the national brand is the only product sold in the market. With the threat of PLs, the manufacturer is pressured to reduce the transfer price, since the introduction of a PL is only viable for the retailer when the national product is so expensive that the margin the retailer gets from selling it becomes too low. Under such circumstances, the retailer might be motivated to introduce another brand in order to gain a higher margin. In such a case, launching a PL rather than introducing a second national brand would allow the retailer to better reach this objective.

According to Mills (1995), retailer's increase in profit surpasses the manufacturer's profit loss, and the consumer surplus is higher when a PL is available in a product category.

Narasimhan and Wilcox (1998) provided additional evidence on the positive effect of PL introduction on the retailer's profit and negotiation power. They also found that PLs lower the manufacturer's profit and increase the consumer surplus. The authors also proved that retailers benefit from a lower transfer price when they introduce a PL, even in the situation where multiple national brands compete at the store level.

Soberman and Parker (2004) introduced the advertising effect in their model by considering that the main difference between national brands and PLs is the goodwill (i.e., brand equity or image) that national brands manufacturers build through advertising. Hence, advertising leads not only to an increase in consumers' willingness to pay, but also creates heterogeneity by distinguishing between consumers that are brand seekers *versus* those that are product seekers. The retailer takes advantage of this psychologically based differentiation by offering a PL as an alternative to the highly advertized national brand. This allows it to capture the portion of the market that has a lower willingness to pay. The authors predict an increase in the average price of a product category that includes a PL when the portion of product seekers is small and the cost of advertising is high.

Karray and Zaccour (2005) were among the first to bring up the topic of PLs in the dynamic games literature. They considered an infinite-horizon game taking place in a bilateral monopoly where the manufacturer's national advertising builds the brand's goodwill. The authors examined a similar setting to the one we are studying in this paper, but considered constant margins. In their study, the retailer controls the promotional efforts for the national and store brands. These efforts aim at increasing the sales of both brands. The results in Karray and Zaccour (2005) confirm that the presence of a PL reduces the manufacturer's profits, at the retailer's benefit. Furthermore, the authors found that the presence of a PL has no effect on the manufacturer's national advertising investment, but does affect the retailer's local advertising efforts for both brands: the retailer systematically reduces its effort to promote the national brand and can even stop promoting it whenever its marginal revenues are found to be lower than the marginal revenues driven by the PL. On the other hand, as long as the unit margin of the PL is higher than that of the national brand, the retailer will always promote its PL.

The authors demonstrated that a cooperative advertising program mitigates the negative effects of launching a PL and suggest, as an extension of their model, to introduce prices as endogenous control variables. Our paper answers this call by simultaneously considering price and non–price marketing decisions in a similar setting to the one used in Karray and Zaccour (2005).

In the literature, seldom have price and non–price marketing variables been introduced in the same model, despite the fact that they often interact with each others and have different impacts on the optimal strategies of the channel members. The main arguments given by the authors for this have been that they wanted to focus on the effect of only one of these variables or wanted to keep their model simple in order to obtain analytical results.[3]

[3]The introduction of additional variables is often a challenge for model tractability.

Amrouche et al. (2008a,b) and Karray and Martín-Herrán (2009) are the only exceptions: they investigate price and advertising decisions in a channel where a PL and a national brand are sold. Surprisingly, neither of these studies answers the questions of how prices, efforts, and individual and total channel profits compare when the retailer owns (or not) a PL. Indeed, in Amrouche et al. (2008a,b), the authors investigated the pricing and advertising strategies when the retailer offers both a PL and a national brand. While Amrouche et al. (2008a) computed Feedback Nash equilibria, the information structure in Amrouche et al. (2008b), is feedback Stackelberg. In both studies, the focus of the authors is to characterize channel members' strategies for the particular case where each member's advertising efforts hurt the goodwill stock of the other.

Karray and Martín-Herrán (2009) also examined the interactions between pricing and advertising decisions of channel members when a PL competes with a national brand. They considered two situations in which the advertising for one brand could have either a competitive or complementary (i.e., informative) impact on the other brand's sales. One of the main results in their study states that the relationship between advertising and pricing strategies depends on the nature of the advertising effect. In particular, when the retailer's advertising negatively affects the national brand's sales, the manufacturer reacts to the increase in the retailer's advertising for the store brand by lowering the advertising effort and transfer price for its national brand. The retailer's reaction to an increase in the manufacturer's advertising effort when the latter is competitive depends on the price competition level and the strength of the competitive advertising effects. When advertising contributes to an increase in both brands' sales (i.e., is informative), prices and advertising efforts are set at higher levels for the national and the store brands, and revenues increase for both channel members. Although interesting, these results are obtained only under the scenario where both the PL and the national brand compete in the market. Hence, they do not indicate how, in this scenario, prices, advertising, or profits compare to their respective values when the national brand does not face competition from a PL.

3 The model

We consider a bilateral monopoly where the manufacturer (M) is the channel leader,[4] and we investigate two scenarios. In the first scenario, the retailer (R) distributes only the manufacturer's national brand, denoted by N. In the second scenario, we add a second brand, owned by the retailer as a PL, denoted by P. We follow the literature and consider that the PL manufacturer is a non–strategic player, and denote by $c_P > 0$ and $c_N > 0$ the production cost of the PL and of the national brand, respectively.[5]

The manufacturer controls the transfer price $w(t)$ and the national advertising investment $a_N(t)$ for the national brand. National advertising investment encompasses all advertising efforts devoted to building the brand's goodwill at the national level (e.g., TV and online advertising). The national brand's goodwill, considered as the state variable, is denoted by $G(t)$. It evolves according to the Nerlove and Arrow (1962) dynamics given by

$$\dot{G}(t) = a_N(t) - \delta G(t), \quad G(0) = G_0 \geq 0, \tag{1}$$

where the parameter $\delta > 0$ is the goodwill decay rate.

In both scenarios, we consider that the retailer controls the retail prices and local advertising for the products carried at its store. We denote by $p_i(t)$ and $a_{iR}(t)$ the retail price and the local advertising (e.g. promotional activities in the store) for brand i, where $i \in \{N, P\}$. This hypothesis is plausible since retailers are often interested in commercial activities designed to increase traffic and sales for the whole product category sold in their stores. The responsibility of investing in brand's goodwill is often left to the manufacturers. Another argument justifying this hypothesis is that retailers carry various

[4] This is a standard hypothesis in the game-theoretic channel literature. Amrouche et al. (2008b) give a list of papers where this hypothesis is used.

[5] The retailer bears this cost when it offers the PL.

brands and cannot invest in national advertising for all of them, unless they benefit from incentives offered by manufacturers (e.g. Cooperative advertising programs).

The local and national advertising costs for both products are given by the following expression:

$$C(a_j) = \frac{1}{2}(a_j)^2; j \in \{N, NR, PR\}. \tag{2}$$

Under scenario I, when only the national brand is sold in the marketplace, we use the following expression for the demand function:

$$Q_N^I(t) = \alpha - \beta p_N(t) + \theta a_{NR}(t) + \rho G(t). \tag{3}$$

When the retailer introduces its private brand (i.e., scenario II), we use the following demand functions:[6]

$$Q_N^{II}(t) = \alpha - \beta p_N(t) + \gamma p_P(t) + \theta a_{NR}(t) - \tau a_{PR}(t) + \rho G(t) \tag{4}$$
$$Q_P(t) = \alpha - \beta p_P(t) + \gamma p_N(t) + \theta a_{PR}(t) - \tau a_{NR}(t) \tag{5}$$

$\alpha, \beta, \gamma, \theta, \rho$, and τ are all positive parameters: α denotes the market potential for the product category; β is the impact of a brand's retail price on its own sales; and γ captures the cross-price effect (i.e., price competition). The parameter θ represents the impact of the retailer's local advertising for each brand on the sales of that brand, while τ captures the impact of the retailer's local advertising for one brand on the sales of the other (i.e., advertising competition). A standard hypothesis in the economic literature states that a product's demand is more sensitive to its own price and advertising effort than those of its competitor. Hence, the model's parameters compare as follows:

$$\beta > \gamma, \qquad \theta > \tau. \tag{6}$$

Three remarks can be added about equations (4) and (5): First, for simplicity, we consider that the direct and cross-effect of prices and local advertising on demand are the same for both brands.[7] Second, we consider that the retailer does not invest in national advertising activities to build goodwill for its private brand. Third, we add the term $\rho G(t)$ in the national brand's demand specification in Equation (4) in order to give higher baseline sales for the national brand with respect to the PL. This is to capture the fact that consumers always prefer buying well-known national brands instead of PLs whenever the retailer decides to sell both brands at the same price and to spend the same effort on local advertising.[8]

We consider that both the manufacturer's and the retailer's objective under the two scenarios is the maximization of their discounted profit streams over an infinite horizon, assuming a common discount rate $r > 0$. In what follows, we will omit the time argument when no confusion may arise. The manufacturer's and retailer's optimization problems in the two scenarios read as follows:

[6]The superscripts (I, II) in the expressions of Q_N refer to the scenarios I and II.

[7]Note that in the literature, direct and cross-price effects among brands could be taken either similar or different. Raju et al. (1995), Sayman et al. (2002), and Amrouche et al. (2008b) are some examples of studies where these parameters are considered symmetric, as we do in our study. By using this hypothesis, we focus on a situation where the PL is positioned very close the national brand with respect to price and perceived quality. This could be the case for premium PLs. This hypothesis should not have an impact on our qualitative results or the structure of the game we study. It can be easily relaxed, but at the cost of having very long analytical expressions.

[8] Narasimhan and Wilcox (1998) attribute the fact that consumers often prefer national brands to PLs to several factors: One of them is the superior perceived quality of national brands w.r.t. PLs. Another is the investment made by branded-product manufacturers in image-building advertising.

- Scenario I:

$$\max_{w,a_N} J_M^I = \int_0^{+\infty} e^{-rt}\left\{(w-c_N)\,Q_N^I - \frac{1}{2}(a_N)^2\right\}dt,$$

$$\max_{p_N,a_{NR}} J_R^I = \int_0^{+\infty} e^{-rt}\left\{(p_N-w)\,Q_N^I - \frac{1}{2}(a_{NR})^2\right\}dt,$$

where Q_N^I is given by (3).
- Scenario II:

$$\max_{w,a_N} J_M^{II} = \int_0^{+\infty} e^{-rt}\left\{(w-c_N)\,Q_N^{II} - \frac{1}{2}(a_N)^2\right\}dt,$$

$$\max_{p_N,p_R a_{NR},a_{PR}} = \int_0^{+\infty} e^{-rt}\left\{(p_N-w)\,Q_N^{II} + (p_P-c_P)Q_P - \frac{1}{2}(a_{NR})^2 - \frac{1}{2}(a_{PR})^2\right\}dt.$$

where Q_N^{II} and Q_P are given by (4) and (5), respectively.

4 The results

In this section, we compute the equilibria under scenarios I and II and compare the results in order to answer the research questions addressed in this study. In both scenarios, we consider a feedback information structure. Hence, we look for prices and advertising strategies that depend on the state variable (i.e., the goodwill stock). We consider the retailer to be the follower; therefore in the following subsections, we first compute its best reaction function, and then the Stackelberg equilibrium strategies of both players.

4.1 Scenario I

The first order optimality conditions (FOC) of the HJB equation associated with the follower's optimization problem (see Equation (14) in Appendix A) are

$$\alpha - 2\beta p_N + \theta a_{NR} + \rho G + \beta w = 0, \tag{7}$$

$$\theta\,(p_N - w) - a_{NR} = 0. \tag{8}$$

Note that the second equation gives the following equality:

$$a_{NR} = \theta\,(p_N - w)\,.$$

This result indicates that the retailer will advertise the national brand locally[9] only if the unit margin that he gets from selling this brand is positive ($p_N - w > 0$). Otherwise, the retailer won't spend any local advertising effort to promote the national brand. By solving the system of Equations (7) and (8), we obtain the following expressions for the retailer's reaction functions:

$$\bar{p}_N\,(w,G) = \frac{\rho G + \left(\beta - \theta^2\right)w + \alpha}{2\beta - \theta^2}, \qquad \bar{a}_{NR}\,(w,G) = \frac{(\alpha - \beta w + \rho G)\,\theta}{2\beta - \theta^2}, \tag{9}$$

where $2\beta - \theta^2 > 0$.[10]

One interesting piece of information that can be obtained from the first reaction function is the type of strategic interaction linking the manufacturer's and the retailer's pricing strategies. According to Moorthy (1988), channel members' pricing strategies may be substitutes, complements, or independent, depending on the sign of the derivative $\frac{\partial p_N}{\partial w}$.

[9] I.e., $a_{NR} > 0$, thus the solution is interior.

[10] The positivity of this expressions originates from the second–order condition that guarantees that p_N and a_{NR} are maxima.

The first retailer's reaction function indicates that $\frac{\partial p_N}{\partial w} = \beta - \theta^2$. Since this result could be positive, negative, or null, we can conclude that, depending on how the price and local advertising effects on demand compare, each channel member could modify, (or not modify) its pricing strategy in the same or in the opposite directions as the price variations of the other channel's member.

The second expression clearly indicates that the retailer decreases its local advertising investment in promoting the national brand whenever the manufacturer decides to increase the national brand's transfer price ($\frac{\partial a_{NR}}{\partial w} < 0$).

The manufacturer, who acts as the channel leader, takes into account the retailer's reaction functions when computing its optimal transfer price and the national advertising for its brand. The following proposition gives strategies and profits at equilibrium under scenario I.

Proposition 1 *Assuming an interior solution, the manufacturer's and the retailer's pricing and advertising strategies at equilibrium under scenario I are given by the following expressions:*

$$\begin{aligned} w^I(G) &= \frac{\rho G + \alpha + \beta c_N}{2\beta}, \\ a_N^I(G) &= M_1 G + M_2, \\ p_N^I(G) &= \frac{\rho\left(3\beta - \theta^2\right)G + \beta\left(3\alpha + c_N\left(\beta - \theta^2\right)\right) - \alpha\theta^2}{2\beta\left(2\beta - \theta^2\right)}, \\ a_{NR}^I(G) &= \frac{\theta(\rho G + \alpha - \beta c_N)}{2\left(2\beta - \theta^2\right)}. \end{aligned}$$

Channel members' value functions are

$$V_M^I(G) = \frac{1}{2}M_1G^2 + M_2G + M_3, \quad V_R^I(G) = \frac{1}{2}R_1G^2 + R_2G + R_3, \tag{10}$$

the expressions of $M_1, M_2, M_3, R_1, R_2,$ and R_3 are given in Appendix A.

Proof. See Appendix A □

Proposition 1 indicates that both channel members' strategies are linear in the state variable and that the value functions are quadratic. These results are expected because of the structure of the game. Moreover, it is very easy to observe that the national brand's transfer price increases when the goodwill level increases. This means that manufacturers selling brands that have a high goodwill will charge a higher transfer price with respect to the price that they would charge if their brand image was low.

The expressions of $p_N^I(G)$ and $a_{NR}^I(G)$ indicate that the national brand's retail price and the retailer's local advertising effort to promote the national brand both increase with the level of the brand's goodwill.

4.2 Scenario II

In scenario II, we compute the channel members' strategies and profits in the situation where the retailer offers a PL on top of selling a national brand. The results are obtained by following the same steps as in the previous scenario. An interesting result to be reported here relates to the conditions for the retailer to allocate local advertising effort for both brands (i.e., the national brand and the PL), and the condition under which the retailer might allocate more (or less) local advertising effort to promote one brand at the expense of the other. Indeed, by computing the partial derivatives of the retailer's HJB (see Equation (15) in Appendix B) with respect to a_{NR} and a_{PR} and equating to zero, we obtain

$$\bar{a}_{NR} = \theta\left(p_N - w\right) - \tau\left(p_P - c_P\right), \tag{11}$$

$$\bar{a}_{PR} = \theta\left(p_P - c_P\right) - \tau\left(p_N - w\right). \tag{12}$$

This result indicates that the retailer will allocate effort to advertising a brand ($a_{NR} > 0$ or $a_{PR} > 0$) locally only if the marginal revenue resulting from promoting it surpasses the marginal loss from promoting the other brand. Karray and Zaccour (2005) obtained a similar condition for the case where channel members make only advertising decisions while margins are constant.

Furthermore, by computing the difference given by

$$\bar{a}_{NR} - \bar{a}_{PR} = (\theta + \tau)\left((p_N - w) - (p_P - c_P)\right), \tag{13}$$

we can conclude that the retailer will always allocate more (less) local advertising effort to promote the brand with the highest (lowest) profit margin.

The results under scenario II are given in the following proposition.

Proposition 2 *Assuming an interior solution, the manufacturer's and the retailer's strategies at the equilibrium under scenario II are given by the following expressions:*

$$w^{II}(G) = \frac{-\rho Y G + T}{2U}, \qquad a_N^{II}(G) = N_1 G + N_2,$$
$$a_{NR}^{II}(G) = \frac{C_1 G + D_1}{2SU}, \qquad a_{PR}(G) = \frac{C_2 G + D_2}{2SU},$$
$$p_P(G) = \frac{C_3 G + D_3}{2SU}, \qquad p_N^{II}(G) = \frac{C_4 G + D_4}{2SU},$$

where

$$\begin{aligned} Y &= 2\gamma\theta\tau - (U + \gamma^2(\theta^2 + \tau^2))/\beta, \\ T &= \alpha K + c_N U + 2 c_P Z, \\ U &= 2\beta^3 - \beta^2(\theta^2 + \tau^2) - 2\beta\gamma(\gamma - 2\theta\tau) - \gamma^2(\theta^2 + \tau^2) > 0, \\ S &= 4\beta^2 - 4\beta(\theta^2 + \tau^2) - 4\gamma^2 + 8\gamma\theta\tau + (\theta^2 - \tau^2)^2 > 0. \end{aligned}$$

$C_1, C_2, C_3, C_4, D_1, D_2, D_3$ and D_4 are parameters given in Appendix B.

The channel members' value functions are

$$V_M^{II}(G) = \frac{1}{2}N_1 G^2 + N_2 G + N_3, \qquad V_R^{II}(G) = \frac{1}{2}Z_1 G^2 + Z_2 G + Z_3,$$

the expressions of the parameters $N_1, N_2,$ and N_3 are given in Appendix B.[11]

Proof. See Appendix B □

The results in Proposition 2 indicate that all prices and national and local advertising strategies for both the national brand and the PL are linear in the goodwill stock. The expressions of parameters $C_1, C_2, C_3,$ and C_4 do not allow us to determine whether they are positive or negative. Hence, additional comments on how channel members' strategies compare, and how they are affected by the goodwill cannot be made about these analytical expressions.

As in scenario I, the channel members' value functions are quadratic in the goodwill stock.

[11]The coefficients Z_1, Z_2, and Z_3 have very long expressions. We do not write them explicitly, since they do not affect the optimal strategies. We will need their values in the numerical section to compute the optimal profits throughout the value functions. In any case, the *Mathematica* code for generating the results is available from the authors upon request.

4.3 Comparing results

To provide answers to this paper's research questions (ii) and (iii), it is necessary to compare the strategies and profits of both channel members under scenarios I and II. Although we obtained analytical solutions for both scenarios, comparing the results analytically was challenging because of the complicated expressions obtained under scenario II. Hence, we chose to perform some numerical simulations, where we fixed some of the initial model's parameters and examined the impacts of two key parameters on the model: the price and the advertising competition effects (i.e., parameters γ and τ, respectively).

In the next subsection, we provide additional details about these simulations and the results obtained. We are interested in the long-term behavior of prices, advertising efforts, goodwill stocks, brand demands, and profits. Therefore, the values of the variables under investigation are compared at the steady state.

4.3.1 Numerical illustration

We start by computing the results under both scenarios by using a benchmark case,[12] where the following values for the model's initial parameters are used. Some parameters (discount rate, decay rate, and production cost) are taken from Amrouche et al. (2008b). The other parameters are used to calibrate the model in order to obtain admissible and realistic values.[13]

The demand parameters are $\alpha = 4$; $\beta = 1$, $\theta = 1$, $\gamma \in [0, 1[$, $\tau \in [0, 1[$.

The cost parameters are $c_N = 0.01$, $c_P = 0$.

The dynamic parameters are $\rho = 0.01$ and $\delta = 0.1$.

The discount rate is given by $r = 0.03$.

Note that with this set of parameters, we normalize to 1 the values of β and θ, which capture the effects of own-price and own–local advertising on a product's demand, and we allow the parameters γ and τ to vary in the interval $[0, 1[$, according to (6). The choice to vary these parameters is justified by their importance in capturing the competition between the PL and the national brand.

In all our simulations, we checked the stability conditions and considered only the results guaranteeing the positiveness of demands, strategies and profits.[14] Figure (1) shows the feasible region in the space defined by γ and τ where all these constraints are fulfilled.

We generated a non–uniform grid of points (more dense close to the border of the feasible region), where the different values of the analyzed variables are computed. Moreover a mesh size of 0.1 was built to verify possible monotonicity behaviors.

The following figures are obtained by computing the difference between the results obtained under scenario II and scenario I. In each figure, the "++" ("- -") regions correspond to the feasible values of γ and τ, such that the value of the represented variable is greater (smaller) in scenario II, i.e., in the presence of the PL, w.r.t. its value in scenario I.

Figure (2) illustrates how the manufacturer adjusts its national advertising investments when the retailer owns a PL with respect to the case where the retailer sells only the national brand. Figure (3) shows the effects of the PL's presence on the national brand's goodwill.

[12] All the claims obtained under this section have been checked for robustness after varying the values of parameters $\alpha, \delta, \rho, c_P$ and the initial goodwill level G_0.

[13] E.g., since PL retail prices are generally equal to or lower than the retail prices of national brands, we chose a set of parameters that reflect this reality.

[14] We carried out the simulations using *Mathematica 11.1.1;* the code for generating the numerical results is available from the authors upon request.

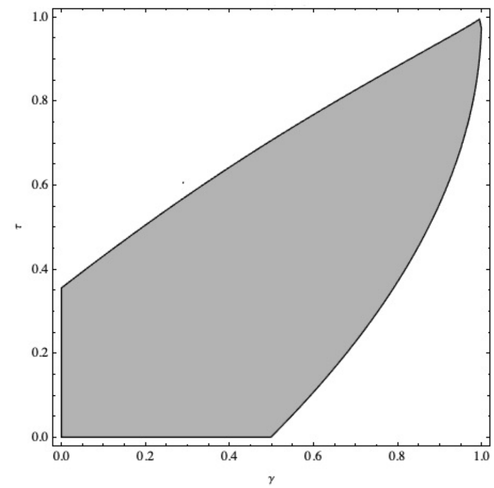

Figure 1: Feasibility region

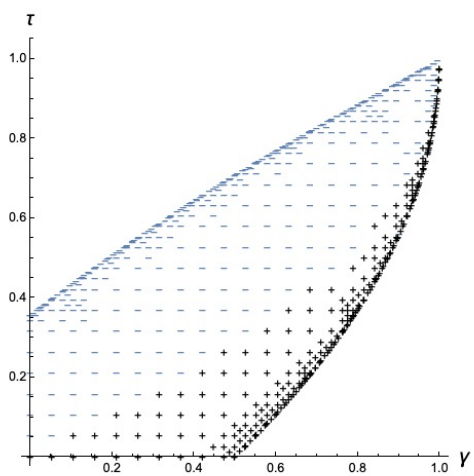

Figure 2: Variation of a_{NM}

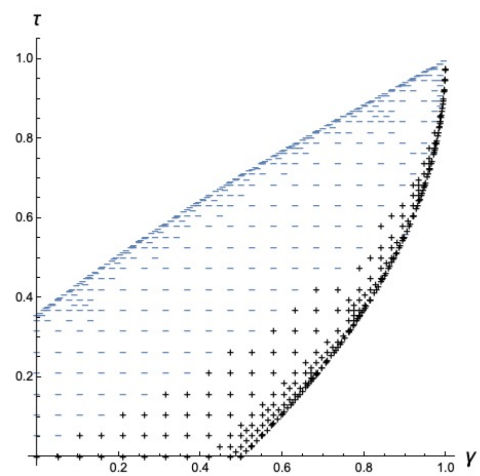

Figure 3: Variation of G

The results indicate that, depending on the combination of values for the parameters capturing the price competition (γ) and the local advertising competition (τ), the manufacturer of the national brand could react to the launch of the PL by either increasing or decreasing its investments in national advertising. As a result, the brand's goodwill could be either higher or lower than its level when the retailer has a PL. This result is not in line with the result obtained in Karray and Zaccour (2005) that the manufacturer's national advertising investment is not affected by the presence or absence of the PL. Since both our study and the study of Karray and Zaccour (2005) share multiple similarities with respect to the game structure and channel setting, we attribute this result to the endogeneity of prices in our model and their interplay with the non–price marketing decisions.

The "++" region in Figure (2) shows that there are some combinations where the manufacturer's best reaction to the introduction of a PL is to invest more in national advertising (with respect to the situation where it holds a monopolistic power at the retailer's store). More particularly, we can see that such a reaction can be observed only for parameters combinations characterized by an advertising competition effect that is lower than the price competition effect.[15] For all combinations where τ is higher than γ, the manufacturer reduces the national advertising for its brand.

Furthermore, Figure (2) indicates that for all parameters combinations, the region where the manufacturer invests less in national advertising under the presence of the PL is bigger than the region where the manufacturer invests more in national advertising. Stated differently, we can see that there are more situations where the manufacturer invests less in national advertising than where it does the opposite.

Since the brand's goodwill is built through national advertising investments, the results we observe in Figure (3) are expected: in the region where the manufacturer reduces the national advertising investment, we observe a decrease in the national brand's goodwill after the introduction of the PL, while in the region where the manufacturer reacts to the PL's introduction by increasing its national advertising investment, the goodwill stock increases.

The result in Figure (4) reveals that, depending on the same combinations of values for the parameters γ and τ identified in the previous figures, the manufacturer of the national brand could react to the presence of the PL by fixing either a higher or a lower transfer price. Interestingly, this result contradicts previous results in the channels literature, which state that the pressure resulting from introducing a PL always forces the manufacturer to concede a portion of its profits to the retailer by decreasing the transfer price. As mentioned in Section 2, Chen (2010) finds that the manufacturer should reduce its transfer price whenever the national brand faces a high level of price competition.

The "++" region in Figure (4) indicates that there are some combinations of the values of γ and τ for which the manufacturer's best reaction to the presence of the PL is to increase the transfer price. Such a reaction can be observed in the same region as where the manufacturer invests more in the national brand advertising, and where the goodwill stock is higher when the retailer holds a PL. Hence, the only situations where the manufacturer could increase the national brand's transfer price are observed when the competitive effect of local advertising is lower than the level of advertising competition. For all combinations where τ is higher than γ, that is, in the region where the manufacturer reduces its investments in national advertising and the goodwill stock is lower, the manufacturer fixes a lower transfer price when the retailer holds a PL.

Here again, we can observe that the region where the manufacturer decreases the transfer price under the presence of a PL is bigger than the region where the manufacturer increases the transfer price. Stated differently, we can see that there are more situations where the manufacturer decreases the transfer price rather than doing the opposite.

Figure (5) illustrates the difference in the national brand's retail price under both scenarios. The results indicate that the retailer may either increase or decrease the retail price, depending on the combinations of the parameters γ and τ in the feasibility region. The "- -" zone shows all the parameters combinations for which the retailer fixes a lower retail price for the national brand when selling a PL in the market. The results clearly show that the retailer fixes a lower price for the national brand only when the price competition is low ($\gamma < 0.6$) and $\tau > \gamma$. In all situations where the price competition is higher than 0.6, the retailer always fixes a higher price for the national brand. Interestingly, we can observe, by comparing the "- -" zone in Figure (5) with the "- -" zone in Figure (4), that the retailer decreases the retail price when carrying a PL only in the situations where the manufacturer reduces the national brand's transfer price. However, the retailer does not always pass-through this transfer price reduction to consumers, (i.e., the "- -" zone in Figure (4) is bigger than the "- -" zone in Figure (5). Finally, comparing the "++" zones in both figures indicates that, in all the situations where the manufacturer fixes a higher transfer price for the national brand, the retailer reacts by setting a higher retail price for the national brand.

[15] Although there are many combinations of γ and τ for which the manufacturer decreases the national advertising investment with $\gamma < \tau$.

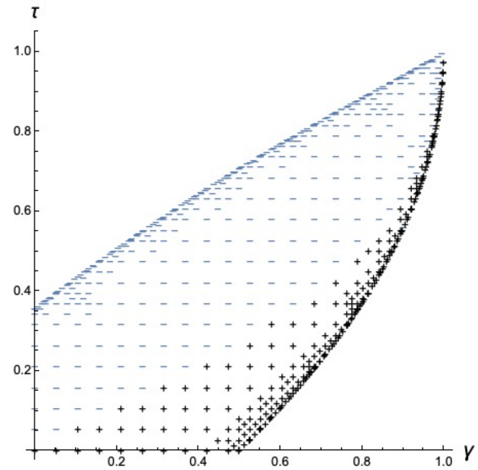

Figure 4: Variation of w

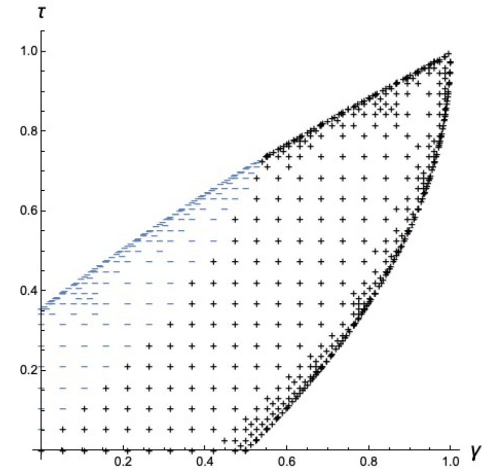

Figure 5: Variation of p_{NR}

Figure (6) depicts the difference in local advertising. It indicates that when the retailer carries a PL, it may either allocate higher or lower local advertising to the national brand (w.r.t. scenario I). In any case, our simulations indicate that a retailer carrying a PL always spends more on the local advertising effort to promote its brand. This is because the PL's unit margin is higher than the unit margin of the national brand. This condition is demonstrated in Subsection 4.2. and its fulfillment is observed in all our simulations.

Furthermore, our results reveal that the retailer always reacts to an increase in manufacturer's national advertising and transfer price by increasing the local advertising effort, but does not always reduce its local advertising for the national brand in the mirror situation. Since the "++" region is bigger in Figure (6) than in Figure (2), we can observe that there are many parameters combinations in which the manufacturer can decrease its national advertising investment and transfer price, while the retailer increases its local advertising effort for the national brand. Finally, the results indicate that an increase in the retailer's local advertising is observed only for admissible parameters values involving a τ lower than γ.

Figure (7) shows that the demand for the national brand could also be higher or lower when a PL is available on the market. Interestingly, the "- -" and "++" zones in this figure coincide with the "- -" and "++" zones in Figures (2), (3), and (4). Hence, the increase in the national brand's demand, observed mainly for situations where γ is higher than τ, can be attributed to the manufacturer's higher investment in national advertising in this parameter region and the resulting increase in the brand's goodwill. The effect of the increase in the brand's goodwill on the demand for the national brand seems to compensate for the increase in the national brand's retail price in this region.

An additional observation (not depicted in the figure) indicates that in all the numerical simulations, we find that if a retailer introduces a PL, the demand for the PL is always higher than the demand for the national brand. This result can be attributed to the fact that the retail price of the PL is always lower than that of the national brand, and the retailer always allocates more local advertising effort to the PL than to the national brand.

The analysis of the variations in the individual and total channel profits indicates that, for all the admissible parameters values, the retailer always benefits from having its own store brand. This result is consistent with the previous results reported in the literature review section (see e.g. Raju et al. (1995)).

Since the retailer's margin is higher for the PL than for the national brand, the retailer's profit stemming from the PL sales in scenario II is higher than the profit the retailer obtains from selling

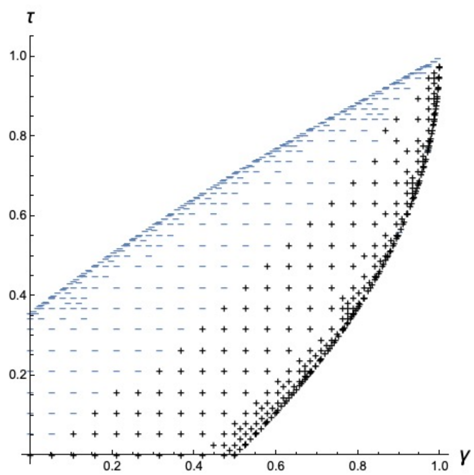

Figure 6: Variation of a_{NR}

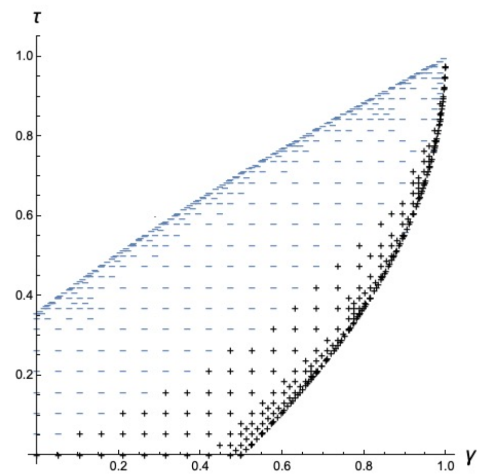

Figure 7: Variation of National brand demand

the national brand under this same scenario. Furthermore, comparing the retailer's profit stemming from sales of the national brand, under the presence or not of a PL, reveals that the national brand is more profitable for the retailer when it is sold exclusively in its store. When the retailer carries a PL, the national brand becomes less profitable than the PL.

For the manufacturer, we find mixed results, depending on the combinations of parameters γ and τ. These results are illustrated in Figure (8).

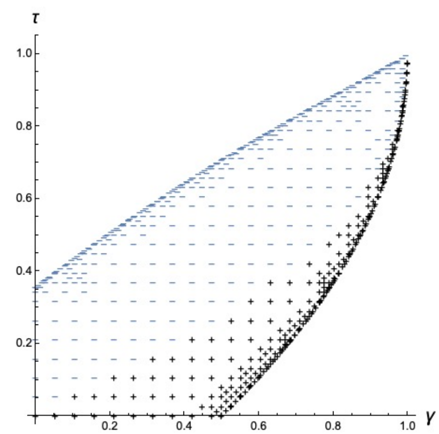

Figure 8: Variation of manufacturer's profits

This figure captures a very interesting result that diverges from the existing literature that has compared the impact of PLs on the strategies and profits of channel members. Indeed, unlike previous results, which have suggested that the manufacturer always looses profits, at the PL–owning retailer's benefit, our results indicate that there are situations where the manufacturer can also benefit from the presence of the PL. These situations correspond to some of the parameter combinations characterized by a higher value of γ than of τ.

The increase in manufacturer's profit in the "++" region can be attributed to the rise in the demand for the national brand and in the transfer price in this same region (as depicted in Figures (7) and (4)).

Finally, our numerical simulations indicate that, for all the admissible values of the parameters γ and τ, the presence of a PL increases the overall channel efficiency. This result is consistent with previous results in the literature. It indicates that, despite the fact that there are some regions in which the manufacturer's profit is hurt by the presence of a PL, the increase in the retailer's profit compensates for the decrease in the manufacturer's profit. This means that in the "- -" region in Figure (8), the decrease in the manufacturer's profit is lower than the increase in the retailer's profit resulting from the launch of the PL.

5 Conclusion

This paper investigates price and non–price marketing decisions (i.e., national and local advertising) in a bilateral monopoly where a retailer may or may not own a private label in addition to distributing a national brand. We build a dynamic model that takes into account the carryover effects of the manufacturer's national advertising, and study two scenarios where the national brand does and does not face competition from a private label. One of our key results indicates that the presence of the PL is not always detrimental to the national brand manufacturer, as has been suggested in the existing literature. More particularly, we find that, when the competitive effect of local advertising between the PL and the national brand is lower than the competitive effect of prices, the national brand's manufacturer could charge a higher transfer price to the retailer carrying a PL (with respect to the situation where the retailer sells only the national brand). This result raises questions about the main reasons advanced by Mills (1995) on why retailers launch PLs. In that, where only pricing decisions are taken into account, the author considers PLs counterstrategies used by retailers to push manufacturers to reduce transfer prices. We explain the difference in the results by the fact that our study considers that the manufacturer and the retailer have multiple control variables that allow them to find the appropriate combination of instruments when maximizing their objectives. By considering the interplay between the price and non–price marketing variables, each channel member uses both instruments to exercise its influence on the decisions and outcomes of the other channel member and the channel.

These results highlight the importance of considering both the price and the non–price marketing variables in this literature, as suggested in Karray and Zaccour (2005).

The following are a few possible extensions of this study:

- To investigate if some incentives strategies can be implemented by the manufacturer in the regions where the introduction of the LP hurts its profits: For example, Mills (1999) examined whether or not the manufacturer can employ some counterstrategies in order to appropriate a portion of the profit surplus resulting from the PLs' launch. He demonstrated that the use of a coupons program and the implementation of a two-part tariff (e.g., quantity discounts or retail price maintenance) can allow the manufacturer to reach this objective, while lump sum payments (e.g., shelf payments and slotting allowances) are not effective in counterbalancing the retailer's PL strategy. In a dynamic setting, Karray and Zaccour (2005) found that a cooperative advertising program can not only mitigate the manufacturer's profit loss, but is also Pareto-improving for both channel members. Chen (2010) investigated this question in a static setting where the channel members make price and advertising decisions, and found that channel members will benefit and implement a cooperative advertising program in a similar context only when the level of competition between the national brand and the PL is high.
- To examine the situation where the retailer also invests in national advertising to build goodwill for its brand, and, as in Amrouche et al. (2008a,b), to consider that the national advertising investments of one channel member could hurt the goodwill stock of the other channel member
- To consider, a second national brand manufacturer in the channel, in a dynamic game involving price and non–price marketing variables. Competition here will be observed in the horizontal and vertical channel structures (i.e., competition between the two national brands, and competition between the national brand and the retailer's PL).

Appendix

Appendix A

Since the retailer is the follower, we start by writing its HJB equation:

$$rV_R^I(G) = \max_{p_N, a_{NR}} \left\{ (p_N - w)\left(\alpha - \beta p_N + \theta a_{NR} + \rho G\right) - \frac{1}{2}a_{NR}^2 + \frac{dV_R^I(G)}{dG}\left(a_N - \delta G\right) \right\}. \tag{14}$$

Computing the partial derivatives of the RHS in Equation (14) with respect to p_N and a_{NR} and setting them equal to zero gives the retailer's reaction functions under scenario I given by Equation (9). As a leader, the manufacturer maximizes its profit functional subject to the retailer's reaction functions.

The Hamilton-Jacobi-Bellman (HJB) associated with the manufacturer's optimization problem is given by the following equation:

$$rV_M^I(G) = \max_{w, a_N} \left\{ (w - c_N)\left(\alpha - \beta \bar{p}_N + \theta \bar{a}_{NR} + \rho G\right) - \frac{1}{2}a_N^2 + \frac{dV_R^I(G)}{dG}\left(a_N - \delta G\right) \right\}.$$

We substitute the retailer's reaction functions in it and compute the derivatives w.r.t. the manufacturer's control variables w and a_N :

$$w(G) = \frac{\rho G + \alpha + \beta c_N}{2\beta}, \quad a_N(G) = \frac{\partial V_M^I(G)}{\partial G}.$$

The sufficient conditions for a stationary feedback Stackelberg equilibrium require us to find bounded and continuously differentiable functions, $V_R^I(G)$ and $V_M^I(G)$, for the retailer and the manufacturer, respectively, which satisfy for all $G(t) \geq 0$, the HJB equations obtained after the substitution of $w(G)$ and $a_N(G)$.

Guided by the model's linear-quadratic structure, we conjecture that the functions $V_R^I(G)$ and $V_M^I(G)$ are quadratic and given by the expressions in the proposition. The coefficients R_1, R_2, R_3, M_1, M_2 and M_3 are obtained by identification after replacing $V_R^I(G)$ and $V_M^I(G)$ as well as their first derivatives into the HJB equations. There are two triplets of coefficients associated to the manufacturer's problem: (M_{1+}, M_{2+}, M_{3+}) and (M_{1-}, M_{2-}, M_{3-}), where

$$\begin{aligned}
M_{1+} &= \frac{1}{2}\left((r+2\delta) + \sqrt{(r+2\delta)^2 - \frac{2\rho^2}{2\beta - \theta^2}} \right), \\
M_{1-} &= \frac{1}{2}\left((r+2\delta) - \sqrt{(r+2\delta)^2 - \frac{2\rho^2}{2\beta - \theta^2}} \right), \\
M_{2+} &= \frac{(\alpha - c_N\beta)\rho}{(2\beta - \theta^2)\left(r - \sqrt{(r+2\delta)^2 - \frac{2\rho^2}{2\beta-\theta^2}} \right)}, \\
M_{2-} &= \frac{(\alpha - c_N\beta)\rho}{(2\beta - \theta^2)\left(r + \sqrt{(r+2\delta)^2 - \frac{2\rho^2}{2\beta-\theta^2}} \right)}, \\
M_{3+} &= \frac{(\alpha - c_N\beta)^2\left(r^2 + 2r\delta + 2\delta^2 - r\sqrt{(r+2\delta)^2 - \frac{2\rho^2}{2\beta-\theta^2}} \right)}{2r\,(2\beta - \theta^2)\left(r - \sqrt{(r+2\delta)^2 - \frac{2\rho^2}{2\beta-\theta^2}} \right)^2}, \\
M_{3-} &= \frac{(\alpha - c_N\beta)^2\left(r^2 + 2r\delta + 2\delta^2 + r\sqrt{(r+2\delta)^2 - \frac{2\rho^2}{2\beta-\theta^2}} \right)}{2r\,(2\beta - \theta^2)\left(r + \sqrt{(r+2\delta)^2 - \frac{2\rho^2}{2\beta-\theta^2}} \right)^2}.
\end{aligned}$$

Before finding the coefficients of the value function associated to the retailer's problem, let's observe that the motion equation depends only on the manufacturer's coefficients, more precisely only on M_1 and M_2, in fact:

$$G'(t) = (M_1 - \delta)\ G(t) + M_2.$$

The stability of the steady state depends on the sign of $(M_1 - \delta)$. Observing that $M_{1+} - \delta > 0$ for all n-uples of parameters values, we can infer that any solution obtained with the triplet (M_{1+}, M_{2+}, M_{3+}) is unstable, and therefore, we'll continue our analysis with the (M_{1-}, M_{2-}, M_{3-}) triplet. The stability condition $M_{1-} - \delta < 0$ holds if and only if $2\delta(\delta + r) - \frac{\rho^2}{2\beta-\theta^2} > 0$. The coefficients of the retailer's value functions associated to such a triplet are $R_1 = \frac{\rho^2}{4(2\beta-\theta^2)(r+2\delta-2M_{1-})}$, $R_2 = \frac{\rho(\rho M_{2-} + (\alpha-\beta c_N)(2(\delta-M_{1-})+r))}{4(2\beta-\theta^2)(r+2\delta-2M_{1-})(r+\delta-M_{1-})}$ and $R_3 = \frac{(\alpha-c_N\beta)^2}{8r(2\beta-\theta^2)} + \frac{\rho M_{2-}(\alpha-c_N\beta)}{4r(2\beta-\theta^2)^2(r+\delta-M_{1-})} + \frac{\rho^2 M_{2-}^2}{4r(r+2\delta-2M_{1-})(r+\delta-M_{1-})}$.

Finally, plugging the derivatives of the values functions into the expressions $w(G)$ and $a_N(G)$ provides the channel members' pricing strategies at the equilibrium displayed in Proposition 1. □

Appendix B

We follow the same steps as in Appendix A. The retailer's HJB equation is

$$\begin{aligned} rV_R^{II}(G) = \max_{\substack{a_{NR}, a_{PR} \\ p_N, p_R}} \quad & \{(p_N - w)(\alpha - \beta p_N + \gamma p_P + \theta a_{NR} - \tau a_{PR} + \rho G) \\ & (p_P - c_P)(\alpha - \beta p_P + \gamma p_N + \theta a_{PR} - \tau a_{NR}) + \\ & -\frac{1}{2}a_{NR}^2 - \frac{1}{2}a_{PR}^2 + \frac{dV_R^{II}(G)}{dG}(a_N - \delta G)\Big\} \end{aligned}$$

Solving the FOC with respect to the control variables p_N, p_R, a_{NR}, and a_{PR}, we obtain the retailers' reaction functions:

$$\begin{aligned} \bar{p}_N(w, G) =& \frac{\alpha\left(2(\beta+\gamma) - (\theta+\tau)^2\right) + c_P\left(2\beta\theta\tau - \gamma\left(\theta^2+\tau^2\right)\right)}{S} + \frac{Qw}{S} + \frac{\rho\left(2\beta - \left(\theta^2+\tau^2\right)\right)}{S}G \\ \bar{p}_P(w, G) =& \frac{2\alpha}{4\beta - 2\left(2\gamma + (\theta-\tau)^2\right)} + \frac{Q}{S}c_P + \frac{2\beta\theta\tau - \gamma\left(\theta^2+\tau^2\right)}{S}\ w + \frac{2(\gamma - \theta\tau)}{S}\rho\ G \\ \bar{a}_{NR}(w, G) =& \frac{\alpha(\theta-\tau)}{2(\beta-\gamma) - (\theta-\tau)^2} - \frac{c_P\left(2\left(2\beta^2\theta - \beta\theta\left(\theta^2-\tau^2\right) - \gamma\left(2\gamma\theta - \theta^2\tau + \tau^3\right)\right)\right)}{2S} \\ &+ \frac{\left(-2\beta^2\theta + \beta\theta\left(\theta^2-\tau^2\right) + \gamma\left(2\gamma\theta - \theta^2\tau + \tau^3\right)\right)}{S}w + \frac{\left(\rho\left(2\beta\theta - \theta^3 - 2\gamma\tau + \theta\tau^2\right)\right)G}{S} \\ \bar{a}_{PR}(w, G) =& \frac{\alpha(\theta-\tau)}{2(\beta-\gamma) - (\theta-\tau)^2} - \frac{c_P\left(2\left(\beta^2-\gamma^2\right)\theta + (-\beta\theta + \gamma\tau)\left(\theta^2-\tau^2\right)\right)}{S} \\ &+ \frac{\left(-\gamma\theta^3 + 2\beta^2\tau - 2\gamma^2\tau + \beta\theta^2\tau + \gamma\theta\tau^2 - \beta\tau^3\right)}{S}w - \frac{\rho\left(-2\gamma\theta + 2\beta\tau + \theta^2\tau - \tau^3\right)}{S}G \end{aligned}$$

where

$$S = \left(2\beta + 2\gamma - \theta^2 - 2\theta\tau - \tau^2\right)\left(2\beta - 2\gamma - \theta^2 + 2\theta\tau - \tau^2\right) \neq 0$$

and

$$Q = 2\beta^2 - 2\gamma^2 - 3\beta\theta^2 + \theta^4 + 6\gamma\theta\tau - 3\beta\tau^2 - 2\theta^2\tau^2 + \tau^4.$$

The second–order optimality conditions claim that $S > 0$. Let us substitute the reaction functions into the manufacturer's HJB equation:

$$rV_M^{II}(G) = \max_{w, a_N}\left\{(w - c_N)(\alpha - \beta\bar{p}_N + \gamma\bar{p}_P + \theta\bar{a}_{NR} - \tau\bar{a}_{PR} + \rho G) - \frac{1}{2}a_N^2 + \frac{dV_M^{II}(G)}{dG}(a_N - \delta G)\right\}$$

and compute $w^{II}(G)$ and $a_N^{II}(G)$ by solving the partial derivatives of its RHS:

$$w^{II}(G) = \frac{\alpha K + c_N U + 2c_P Z}{2U} + \frac{\rho\left(U+\gamma\left(-2\beta\theta\tau+\gamma\left(\theta^2+\tau^2\right)\right)\right)}{2\beta U}G, \tag{15}$$

$$a_N^{II}(G) = \frac{dV_M^{II}(G)}{dG}. \tag{16}$$

where

$$K = 2\beta^2 - \beta(\theta+\tau)^2 + \gamma\left((\theta+\tau)^2 - 2\gamma\right),$$
$$U = 2\beta^3 - \beta^2\left(\theta^2+\tau^2\right) - 2\beta\gamma(\gamma - 2\theta\tau) - \gamma^2\left(\theta^2+\tau^2\right),$$
$$Z = \beta^2(\gamma+\theta\tau) - \beta\gamma\left(\theta^2+\tau^2\right) + \gamma^2(\theta\tau - \gamma).$$

Given $Y = 2\gamma\theta\tau - \frac{U+\gamma^2(\theta^2+\tau^2)}{\beta}$ and $T = \alpha K + c_N U + 2c_P Z$, (15) and (16) give the feedback strategies $w^{II}(G)$ and $a_N^{II}(G)$ in Proposition 2.

The second–order optimality conditions claim that $S > 0$ and $U > 0$. Now we substitute strategies $w^{II}(G)$ and $a_N^{II}(G)$ into Equation (15) and conjecture that $V_R^{II}(G)$ and $V_M^{II}(G)$ are quadratic and given by the expressions in proposition 2. Under this assumption (16) becomes $a_N^{II}(G) = N_1 G + N_2$. The coefficients $N_1, N_2, N_3,\ Z_1, Z_2, Z_3$ can be obtained by identification after replacing $V_R^{II}(G)$ and $V_M^{II}(G)$ as well as their first derivatives into the HJB equations.

For the manufacturer's value function we obtain the two triplets of coefficients (N_{1+}, N_{2+}, N_{3+}) and (N_{1-}, N_{2-}, N_{3-}), where

$$N_{1+} = \frac{1}{2}(r+2\delta+R), \quad N_{1-} = \frac{1}{2}(r+2\delta-R), \quad N_{2+} = \frac{F2Y\rho}{SU(r-R)}, \quad N_{2-} = \frac{F2Y\rho}{SU(r+R)}.$$

$$N_{3+} = \frac{1}{4rSU}\left((2c_P Z - c_N U + K\alpha)^2 + \frac{2F2^2Y^2\rho^2}{SU(r-R)^2}\right)$$

and

$$N_{3-} = \frac{1}{4rSU}\left((2c_P Z - c_N U + K\alpha)^2 + \frac{2F2^2Y^2\rho^2}{SU(r+R)^2}\right)$$

with

$$R = \sqrt{(r+2\delta)^2 - \frac{2Y^2\rho^2}{SU}}, \qquad Y = 2\gamma\theta\tau - \frac{U+\gamma^2\left(\theta^2+\tau^2\right)}{\beta}$$

and

$$F2 = c_N U - \alpha(\beta-\gamma)\left(2\beta+2\gamma-(\theta+\tau)^2\right) + 2c_P\left(\gamma^2(\gamma-\theta\tau) - \beta^2(\gamma+\theta\tau) + \beta\gamma\left(\theta^2+\tau^2\right)\right).$$

In order to study the stability of the solution, we substitute the reaction function $\bar{a}_N^{II}(G)$ in the state equation and obtain

$$\dot{G}(t) = (N_1 - \delta)G(t) + N_2. \tag{17}$$

It is a first–order linear differential equation, thus its solution is stable if and only if $(N_1 - \delta) < 0$. Observing that $N_{1+} - \delta > 0$ for all n-uples of parameters values, we can infer that any solution obtained with the triplet (N_{1+}, N_{2+}, N_{3+}) is unstable, and therefore, we'll continue our analysis with the (N_{1-}, N_{2-}, N_{3-}) triplet. The stability condition $N_{1-} - \delta < 0$ holds if and only if $\delta(\delta+r) - \frac{Y^2\rho^2}{2SU} > 0$.

Analogously for the manufacturer's HJB equation, we obtain the feedback strategies in Proposition 2. with the following coefficients for the value function:

$$C_1 = 2U\rho\left(2\beta\theta - \theta^3 - 2\gamma\tau + \theta\tau^2\right) - Y\rho A, \qquad D_1 = \frac{2KU\alpha(\theta-\tau)}{\beta-\gamma} + 2c_P UB + TA,$$

$$C_2 = 2U\rho\left(2\gamma\theta - 2\beta\tau - \theta^2\tau + \tau^3\right) - Y\rho B, \qquad D_2 = \frac{2KU\alpha(\theta-\tau)}{\beta-\gamma} + 2c_P UA + TB,$$

$$C_3 = 4U\rho\left(\gamma - \theta\tau\right) + Y\rho\left(\gamma\theta^2 + \gamma\tau^2 - 2\theta\tau\beta\right), \qquad D_3 = 2U\alpha\left(2\beta + 2\gamma - (\theta+\tau)^2\right) + 2c_P UQ - T\left(\gamma\theta^2 + \gamma\tau^2 - 2\theta\tau\beta\right),$$

$$C_4 = -2U\rho\left(-2\beta + \theta^2 + \tau^2\right) - Y\rho Q, \qquad D_4 = 2U\alpha\left(2\beta + 2\gamma - (\theta+\tau)^2\right) + 2c_P U\left(\gamma\theta^2 + \gamma\tau^2 - 2\theta\tau\beta\right) + TQ$$

and

$$A = \gamma\tau^3 - \beta\theta\tau^2 - \gamma\theta^2\tau + \beta\theta^3 + 2\gamma^2\theta - 2\beta^2\theta, \qquad B = -\gamma\theta^3 + \beta\tau\theta^2 + \gamma\tau^2\theta - \beta\tau^3 - 2\gamma^2\tau + 2\beta^2\tau.\square$$

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