

**Stochastic optimization of mining complexes
integrating capital investments and
operational alternatives**

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R. Dimitrakopoulos

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Stochastic optimization of mining complexes integrating capital investments and operational alternatives

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Abstract: Mining complexes are value chains where extracted material from different mines is transformed into sellable products through a set of processing streams. This value chain is governed by uncertainties in different aspects, from the pertinent geological attributes of the mineral deposits mined, to the different operational and processing components. Stochastic optimization formulations have been shown to maximize economic value and, at the same time, manage and reduce risk, thus providing reliable production plans and forecasts. However, related mine designs and production plans are static over the life of a mining complex and cannot include flexibility mechanisms to account for alternative, potential production and planning options. This paper presents a dynamic two-stage stochastic mixed integer non-linear programming formulation for modeling and optimizing a mining complex, including alternatives over capital expenditure investments and operational modes for different components of the value chain. More specifically, a dynamic decision-making mechanism is included, where mine production plans are allowed to branch, and parallel feasible plans are designed if a representative proportion of stochastically simulated scenarios of the mineral deposits mined conclude that it is profitable. This process generates new optimized plans that facilitate adaptation once more information is available. The practical implications of the proposed method are demonstrated through an application over a copper-gold mining complex comprised of one mine and six processing streams, where the dynamic model is compared to a traditional two-stage stochastic formulation, presenting a 10.5% increase in net present value.

1 Introduction

Mining complexes are mineral value chains where material flows from mines to customers through a set of interconnected components that can be optimized simultaneously to capitalize on the synergies that exist between them. These components include mines that are the source of raw materials, stockpiles, waste dumps, multiple processing streams that blend and transform the materials mined into sellable products, and transportation systems delivering products to customers. Mining complex components operate at a cost, related to a set of operating mode configuration alternatives used to meet targets and requirements. These targets are usually defined by available capacities, which are determined by capital expenditures (CAPEX). CAPEX investments are irreversible, of high magnitude, and have a limited lifespan, thus requiring extensive lead times to purchase equipment, acquire or build required infrastructure. For example, the project to build a new processing plant at the Escondida copper mining complex in Chile had a budget of US\$4.3 billion and required over four years to be completed (Mineria Chilena, 2015). Due to their effect and magnitude, investment decisions should be included in the strategic planning and optimization of a mining complex. The focus of strategic mine planning is to generate optimized mine production schedules that meet targets and contribute to maximizing discounted cash-flows over the life of the assets involved. A core aspect of optimizing mining complexes is the uncertainty and variability related to the supply of raw materials from the related mines, reflecting the uncertainty of the pertinent attributes (or geological uncertainty), including mineral grades, material types, tonnages, geometallurgical properties, etc. of the mineral deposits mined.

Conventionally, mining complexes are simplified and each component is optimized independently (Hustrulid et al., 2013). This practice ignores the synergies that exist between components, producing independent, overall sub-optimal production schedules and strategic mine plans. During the last years, new developments have advanced the conventional approaches to include several components and aspects of mining complexes, towards a more global or simultaneous optimization framework (Bodon et al., 2011; Hoerger et al., 1999; Pimentel et al., 2010; Stone et al., 2007; Whittle, 2007, 2018). Pimentel et al. (2010) introduce the concept where a mining operation is viewed as a supply chain served by logistic transportation channels and propose an integrated decision-support framework to address a global mining supply chain; however, while novel, their contribution is conceptual, and no specific method is provided. The same authors recognize the complexity of any real-world mining supply chain and suggest that heuristic approaches are the best alternative for optimizing them. Notable is also the work by Hoerger et al. (1999) who optimize material allocations in a gold mining complex with 90 metallurgical material types from 50 sources; Stone et al. (2007) formulate a model to optimize some elements and aspects of several mines to show improved performance compared to conventional optimization. Whittle (2007, 2018) introduces the global optimization tool Prober comprised of multiple interacting optimization components. Dagdelen and Traore (2018) consider a global analysis of open pit and underground mine production scheduling optimization in a group of mines.

While the above approaches capitalize on the synergies that exist between the various components of a mining complex, they have limitations in providing the complete simultaneous optimization of the corresponding mineral value chain, given the multifaceted complexity of the problem considered. Assumptions start from the consideration of only some of the components of a mining complex and the absence of a single mathematical programming formulation that accounts for all aspects of a mining complex; additionally, they all require predefined mine production schedules that avoid stockpiles, given their non-linear relations, they do not account for investment related options and use mining-block aggregations to larger volumes, ignoring mining selectivity. Another critical and major assumption of all previously mentioned studies is that uncertainty and variability related to the supply of raw materials extracted from the related mines are ignored. In other words, all pertinent geological characteristics of materials extracted from the mines of a mining complex are considered known, following the conventional mine design and planning optimization framework. It is well documented in the technical literature that geological uncertainty and variability are the main sources of risk affecting mining operations and has a strong effect over the operational feasibility of the mining schedule, preventing projects from meeting forecasted production targets and maximizing net present value (Dimitrakopoulos, 2011; Dimitrakopoulos et al., 2002; Dowd, 1997, 1994; Ravenscroft, 1992).

More recently, Dowd et al. (2016) discuss that quantifying and integrating geological, geometallurgical and operational uncertainties into the optimization process is a main challenge in strategic mine planning.

Stochastic simultaneous optimization of mining complexes has been introduced in the last few years (Goodfellow and Dimitrakopoulos, 2017, 2016; Montiel et al., 2016; Montiel and Dimitrakopoulos, 2015, 2017) to overcome the limits of past contributions noted above. The new framework starts with integrating geological uncertainty into the optimization model to manage technical risk, and considers a mineral value chain as an integrated engineering system that is driven by its sellable products over the life of the related mining assets, rather than the conventional economic values of materials extracted from mines. The new optimization framework involved generates the sequences of extraction for the mines involved, destination policies for different materials mined, deals with complex blending requirements and non-linear material transformations, considers operating alternatives for the processing streams and transportation needed, while generating production forecasts and financial evaluations along with their quantification of risk. Evidently, stochastic simultaneous optimization of a mining complex deals with tens of millions of binary variables and millions of constraints (Goodfellow, 2014; Lamghari and Dimitrakopoulos, 2015). As a result, different metaheuristic methods have been developed to deal with the related optimization complexity and have been documented in several studies showing their ability to produce good quality solutions (Goodfellow and Dimitrakopoulos, 2017; Lamghari et al., 2015; Lamghari and Dimitrakopoulos, 2016a, 2016b; Montiel and Dimitrakopoulos, 2017). An additional challenge, when modeling the stochastic simultaneous optimization of mining complexes, is the non-linear transformations that appear when integrating the different components, such as including blending constraints and stockpiles into the mathematical formulation. Goodfellow and Dimitrakopoulos (2017, 2016) deal with the non-linearities by treating variables as attributes and classifying them as primary or hereditary to model the flow of material through the mining complex. Primary attributes correspond to additive characteristics, such as metal content and tonnages, whereas hereditary attributes are derived from primary ones, such as recoveries and economic value, among others. Goodfellow (2014) includes the decision of investing in capital expenditures (CAPEX) and lets the optimizer define the fleet size and a fixed purchase plan. However, past work has not considered operational mode alternatives to deal with geometallurgical variables that affect the performance of a mining complex (Boisvert et al., 2013; Sepulveda et al., 2017).

Despite the contributions to simultaneously optimizing mining complexes, as mentioned above, all approaches are limited in the sense that they produce a static solution and strategic plan for the corresponding mineral value chain, which is similar to conventional practices and methods. As such, they are limited in terms of providing strategic plans with feasible alternatives for strategic plans that adapt to new information. Multistage stochastic optimization (Birge and Louveaux, 1997) offers a specific approach that aims to include dynamic decision-making into the optimization process, where uncertainty is also represented through a set of equally-probable scenarios of pertinent attributes. Multistage optimization uses non-anticipativity constraints to ensure that non-differentiated scenarios entail equal decisions, and thus, the solution is allowed to branch, that is divide, into parallel possible solutions like in a scenario tree, if scenarios appear to be sufficiently different. Boland et al. (2008) present a multistage stochastic optimization model under geological uncertainty for mine production scheduling, where the schedule is branched into parallel solutions as soon as blocks are found to be “differentiable.” However, the results are impractical in terms of actual mine design and planning requirements. Furthermore, the related branching mechanism produces schedule solutions that are over-fitted to the set of scenarios used and, thus, would have poor performance when tested against a different set of simulated scenarios. Additionally, the formulation would grow exponentially for real-size operations, rendering it unusable.

A new dynamic model for the stochastic simultaneous optimization of mining complexes is presented herein. The proposed model extends past approaches to a dynamic optimization method that provides information about the probability of occurrence of a set of feasible strategic planning alternatives that maximize net present value (NPV) and should be considered. The proposed dynamic model provides an optimized, feasible and flexible plan by generating parallel solutions that provide guidance and ease the transition to change for the current plan once more information is available. The proposed formulation aims to offer a dynamic evaluation of a set of high-impact CAPEX alternatives, providing a probabilistic analysis of the likelihood of investing in them and when, as well as the optimized mine design and production plans to

follow in each case. These alternatives are included as a way of increasing a mining complex's flexibility, which transforms the strategic plan into a dynamic mechanism that adapts to potential change. Three main considerations are included in the proposed formulation: (i) a dynamic investment schedule is developed, optimizing a set of CAPEX alternatives as a probability-based decision tree; (ii) operating mode alternatives are included in the mining complex to manage the effect of geometallurgical variables, specifically, rock hardness, throughput and recovery, at the mine and the processing levels; (iii) finally, as in previous work, supply (geological) uncertainty and inherent spatial variability is considered in the optimization through a set of equally probable simulated realizations of the pertinent attributes of the mineral deposits mined (Godoy, 2003; Goovaerts, 1997; Journel and Huijbregts, 1978).

In the following section, the proposed method for the dynamic stochastic simultaneous optimization of mining complexes and corresponding mathematical formulation are detailed. Next, the practical aspects of the proposed method are explored in an application at a copper-gold mining complex and results are compared to the corresponding two-stage stochastic approach. Finally, conclusions follow.

2 Proposed method

2.1 Problem description

In the proposed mathematical model, decision variables are grouped as (i) extraction, defining when a mining block is extracted; (ii) destination policy, setting where a block is sent once it is extracted; (iii) processing stream decisions, defining what percentage of material passes from one component of the mining complex to the next; (iv) operational mode, describing under which operational mode will the mine or processing stream operate; and finally (v) capital expenditures, defining which investments are acquired at a cost along the life of mine (LOM). The probabilistic analysis will be performed over a sub-set of these CAPEX decisions, which will be defined as branching decisions. These branching decisions correspond to big irreversible investments that have a decisive effect over the schedule (such as for example the investment in a new plant).

2.1.1 Generating the probability-based decision tree solution

The dynamic mine plan produced corresponds to a probability-based decision tree which branches according to investment decisions over high-impact CAPEX alternatives. These branching decisions have two available options, to invest in or not. Thus, the solution is represented as a scenario tree (Høyland and Wallace, 2001; Safavian and Landgrebe, 1991). To keep track of the branching solutions, traditional decision tree notation is used, where each node corresponds to the decisions taken on that given period; each possible solution is identified by a branch (which contains a root (ρ) and a leaf (l), and each node can have at most two to-the-power-of branching decisions alternative leafs. From this, it can be seen that branching alternatives increase exponentially with the number of branching decisions, for example, if two investments A and B are available, the partitions would be to invest in A but not in B, in B but not in A, in both, or in none.

To model this problem, an adapted multistage formulation is proposed. Some main differences of the proposed model compared to traditional stochastic multistage formulations (as per in (Birge and Louveaux, 1997)) correspond to the definition of a "stage", and the reason for branching onto parallel solutions. In conventional stochastic multistage formulations, stages are defined by specific time intervals, and, at each stage, decision variables are allowed to differ between scenarios (i.e. branch) if certain differences are encountered within the set of scenarios. This traditional formulation produces a set of parallel solutions over partitions of scenarios (Boland et al., 2008). In the proposed model, parallel solutions, or production plans, are generated depending on the value of a subset of decision variables, and not over differences between the actual individual scenarios. Thus, a stage is defined by the timing between investment decisions, rather than by specific time intervals. Together with this, as mentioned in the previous section, traditional multistage stochastic formulations have some strong limitations, mostly related to over-fitting the solutions obtained to the set of stochastic simulations used in the optimisation, as the solutions branch exponentially towards

later periods (where, by the end of the optimization, there might be as many solutions as scenarios used). This over-fit prevents the solution of being applicable if a different reality is encountered from the ones represented through the set of simulations. This occurs because simulations are used as possible realities, and not as a set which, as a whole, represent the probability distribution of the deposit’s spatial variability. To overcome these limitations, and to reduce the computational complexity of solving the model, the proposed method uses an iterative mechanism that quantifies the probability of executing these branching decisions, and controls the generation of branching solutions, ensuring that parallel plans are only generated if they have a representative probability of occurring.

This representativity is measured by setting a threshold R , where branching only occurs whenever the probability of investing in a CAPEX alternative during a given time window falls within this threshold. If the probability of investing is lower than the threshold, the solution does not branch and no investment is made. On the other hand, if the probability is higher than the threshold, there is also no branching, but the full model invests in the CAPEX alternative. The time window is defined here to provide some stabilizing lag within scenarios for the branching investment decision to be taken (i.e. invest at time $t + / - \omega$ periods). From here, the final branching period t^* is defined within the time window $t + / - \omega$, as the expected value of the period of investment (i.e. the period at which the branching decision is taken), within the scenarios that invest during that window.

To obtain the probability of investing, which is used to compare against the threshold R defined previously, a look-forward mechanism is used, where a set of sub-problems is iteratively solved. At each iteration, non-anticipative constraints are enforced over increasing time frames (from one period to the whole LOM). Through this algorithm, presented in Section 2.2, decision variables of later periods are initially left free, by dropping non-anticipative constraints which enforce decisions to be the same over all non-differentiated scenarios. This allows the algorithm to quantify the probability of branching at different periods. In turn, these values help the optimizer define its current design by setting future “differentiated” partitions of scenarios, where non-anticipativity constraints allow different decisions to be taken between partitions. Thus, providing a design that adapts to possible futures. Through this iterative process, the decision tree is created, where each branch corresponds to a unique production plan, with its corresponding investment schedule, which maximizes project value. Thus, the final solution provides a controlled set of possible mine design alternatives that have been probabilistically quantified to be worth considering.

An example of this branching mechanism for one branching decision is presented in Figure 1, where, if the branching decision is exercised over a representative number of scenarios, the solution branches and a unique mine plan is generated for each partition (referred to as *branch*). Each square (i.e. node) represents the decisions made on a given year (not necessarily at equal time intervals), where the optimizer decides to branch on period t^* , generating two parallel future solutions at that period, with and without investment. In turn, by period t^{**} , the top branch decides to do so, producing two parallel designs. Thus, the final solution of the optimization corresponds to three possible production schedules, with their corresponding probabilities of occurring. It must be noted that, even if the scenario tree presents varying time intervals, showing the decisions taken for the branching decisions, the global optimization is still performed over annual time intervals.

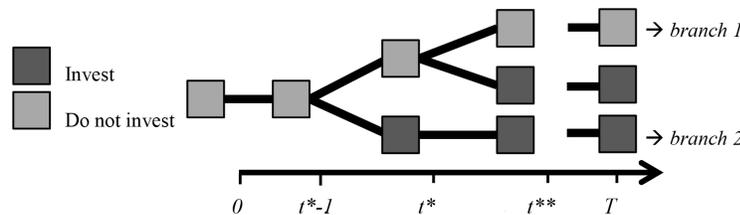


Figure 1: Branching mechanism of dynamic stochastic optimization

2.2 Mathematical formulation

The mathematical formulation presented is based on the two-stage stochastic model for mining complexes proposed by Goodfellow and Dimitrakopoulos (2017), with main adaptations to include operational and investment alternatives, as well as the dynamic branching mechanism described in Section 2.1. Similarly to Goodfellow and Dimitrakopoulos (2017), primary and hereditary attributes are used to model the mining complex, where there exists a function that transforms a primary attributes in a given component of the mining complex into a hereditary attribute. For example, the recovery in a plant, a hereditary non-linear attribute of a processing stream, is obtained by the grade of each individual mining block being fed to it at that period, calculated using metal tonnage and total tonnage, both simulated primary attributes. In addition, as in Goodfellow and Dimitrakopoulos (2017), blocks from each orebody are clustered by similarity of their simulated characteristics using a k-means++ algorithm and the destination policy decisions (defined as $z_{c,j,t,s}$ in Table 2) are set annually over each of these clusters, rather than at a block-level.

The different sets used in the mathematical formulation are defined in Table 1, followed by the list of decision and state-dependent variables in Table 2. Finally, Table 3 presents the general parameters, and the parameters used specifically for the flexibility alternatives considered. The full mathematical formulation follows.

State variables in Table 2 correspond mainly to the value of the different primary and hereditary attributes along the different components of the mining complex. As mentioned earlier, primary attributes correspond to additive simulated attributes of the rock (i.e. tonnage, metal content, etc), and hereditary attributes depend on these simulated primary-attribute values, and on the transformation function that defines them (presented in Table 3). Together with this, surplus and shortage variables ($d_{h,t,s}^+, d_{h,t,s}^-$) are defined to quantify and manage deviations from targets.

Table 1: Definition of sets used in the dynamic formulation

Sets and Indices	
P	Primary attributes that are tracked in the supply chain (e.g., metal content, tonnages)
H	Hereditary attributes (derived from primary attributes) that are tracked in the supply chain (e.g., grades, recoveries, economic values)
T	Time periods in the life of mine, indexed by $t = 1 \dots T$
Ω	Set of scenarios, indexed by $s = 1, \dots, S$. Where $\Omega_\rho \subseteq \Omega$ is the set of scenarios in branch ρ , and Ω_{p1}, Ω_{p2} are partitions of Ω_ρ , where $\Omega_{p1} \cup \Omega_{p2} = \Omega_\rho, \Omega_{p1} \cap \Omega_{p2} = \emptyset$
M	Set of mines, indexed by $m \in M$
B_m	Set of blocks in mine $m \in M$, indexed by $b \in B_m$
$O(b)$	Set of blocks that overlie block (b)
C	Clusters of blocks with similar attributes, indexed by $c \in C$
Sp	Stockpile destinations that can forward part or all their material to subsequent destinations, indexed by $sp \in Sp$
Pp	Processing stream destinations in the mining complex, indexed by $pp \in Pp$
D	Set of locations in the mining complex: clusters, stockpiles, processing streams, ($C \cup Sp \cup Pp$), $D_{op} \subseteq D$ Set of locations containing operational mode alternatives
K	Set of flexibilities and system alternatives, indexed by k . Where, $K^* \subseteq K$ Set of alternatives that allow branching over the design
Q_j	Set of operational alternatives in location $j \in D_{op}$, indexed by $q \in Q_j$
$\Theta(j)$	Set of locations which can receive material from location $j \in Sp \cup Pp$
$J(j)$	Set of locations which can send material to destination $j \in D$

With these variables and parameters, the branching threshold described in Section 2.1 is defined in Equation (1), where, for time window $\mathbf{t}^\omega = [t - \omega, t + \omega]$, the probability of branching must be within the threshold $\in [R, 1 - R]$.

$$\begin{cases} \text{branch at time window } \mathbf{t}^\omega & \text{if probability of investing in a decision } k^* \in [R, 1 - R] \\ \text{unique solution} & \text{otherwise} \end{cases} \quad (1)$$

Table 2: Variables used in the model

Decision Variables	
$x_{b,t,s}$	$= \begin{cases} 1, & \text{if block } b \text{ is extracted at period } t \in \mathbf{T} \text{ in scenario } s \in \Omega, \\ 0, & \text{otherwise} \end{cases}$
$z_{c,j,t,s}$	$= \begin{cases} 1, & \text{if cluster } c \text{ is sent to destination } j \in \Theta(c) \text{ in period } t \in \mathbf{T}, \text{ scenario } s \in \Omega, \\ 0, & \text{otherwise} \end{cases}$
$\mu_{q,j,t,s}$	$= \begin{cases} 1, & \text{if operational mode } q \in \mathcal{Q}_j \text{ is active in location } j \in D \text{ in period } t \in \mathbf{T}, \text{ scenario } s \in \Omega, \\ 0, & \text{otherwise} \end{cases}$
$u_{k^*,t}^{\rho l}$	$= \begin{cases} 1, & \text{if design branches over option } k^* \in K^* \text{ in node } \rho l, \text{ in period } t \in \mathbf{T}, \\ 0, & \text{otherwise} \end{cases}$
$y_{i,j,t,s}$	$\in [0, 1]$. Proportion of material sent from location $i \in Sp \cup Pp$ to $j \in Sp \cup Pp$ in period $t \in \mathbf{T}$, scenario $s \in \Omega$
$w_{k,t,s}$	$= \begin{cases} 1, & \text{if there is a purchase of investments on option } k \in K \text{ executed in period } t \in \mathbf{T}, \text{ scenario } s \in \Omega, \\ 0, & \text{otherwise} \end{cases}$
$\sigma_{k,t,s}$	$\in \{0\} \cup \{L_{k,t}, U_{k,t}\}$. Number of investments on option $k \in K$ executed in period $t \in \mathbf{T}$, scenario $s \in \Omega$
State Variables	
$v_{\varphi,j,t,s} \in \mathbb{R}$	Value of primary attribute $\varphi \in \mathbf{P}$, at location $j \in D \cup M$, period $t \in \mathbf{T}$, scenario $s \in \Omega$. Ex: $\varphi_1 = \psi$ tonnage; then: $v_{\psi,m,t,s}$ tonnage extracted from mine m , at $t \in \mathbf{T}$, scenario $s \in \Omega$
$v_{h,j,t,s} \in \mathbb{R}$	Value of attribute $h \in \mathbf{H}$, at location $j \in D \cup M$, period $t \in \mathbf{T}$, scenario $s \in \Omega$
$v_{h,t,s} \in \mathbb{R}$	Final value of attribute $h \in \mathbf{H}$, at period $t \in \mathbf{T}$, scenario $s \in \Omega$, where $v_{h,t,s} = \sum_{j \in D} v_{h,j,t,s}$
$t^* \in \mathbb{Z}^+$	Final branching period within time window (defined in Table 3), dependent on the investment decisions of branching alternatives $w_{k^*,t,s}$
$r_{\varphi,j,t,s} \in [0, 1]$	Recovery of attribute $\varphi \in \mathbf{P}$ at location $j \in Pp$, period $t \in \mathbf{T}$, scenario $s \in \Omega$
$d_{h,t,s}^+, d_{h,t,s}^- \geq 0$	Surplus or shortage variables (respectively), from deviations over targets of attribute $h \in \mathbf{H}$, period $t \in \mathbf{T}$, scenario $s \in \Omega$

Table 3: Set of parameters used in formulation

General Material Flow Parameters	
$\beta_{\varphi,b,s}$	Simulated value of primary attribute $\varphi \in \mathbf{P}$, for block $b \in B_m$ and scenario $s \in \Omega$. Ex.: $\beta_{\chi,b,s}$ = metal content $\chi \in \mathbf{P}$ of block b , in scenario s
$\theta_{b,c,s} \in \{0, 1\}$	Pre-defined cluster classification, = 1 if block $b \in B_m$ belongs to cluster $c \in C$, in scenario $s \in \Omega$, and 0 otherwise
$f_{h,j}(\varphi)$	Function that transforms primary attributes $\varphi \in \mathbf{P}$ into hereditary attribute $h \in \mathbf{H}$ in location $j \in D \cup M$ (defined by the modeller)
$U_{h,t,s}, L_{h,t,s}$	Upper and lower limit of attribute $h \in \mathbf{H}$, at period $t \in \mathbf{T}$, scenario $s \in \Omega$. Ex.: $U_{W,t,s}$ = Upper extraction capacity limit $W \in \mathbf{H}$, at period $t \in \mathbf{T}$, scenario $s \in \Omega$
$c_{h,t}^+, c_{h,t}^- \geq 0$	Unit cost of positive and negative deviation over targets of attribute $h \in \mathbf{H}$, at period $t \in \mathbf{T}$
$p_{h,t}$	Unitary price (or cost) of attribute $h \in \mathbf{H}$, at time $t \in \mathbf{T}$
Option-related parameters	
$\mathbf{N} \in \mathbb{Z}^+$	Min number of scenarios in a branch required to allow further branching in $t+1 \in \mathbf{T}$
R	$\in [0, 0.5]$, minimum proportion of scenarios (i.e. threshold) needed to branch design (threshold = $[R, 1-R]$)
$p_{k,t}$	Discounted purchase cost of option $k \in K$, period $t \in \mathbf{T}$
λ_k	Life of capital option $k \in K$
τ_k	Lead time before an option $k \in K$ is available (since the moment of decision)
$\kappa_{k,h}$	Per unit increment for constraints that investment option $k \in K$ has on attribute $h \in \mathbf{H}$
ψ_k	Allowed periodicity to take a decision over option $k \in K$
$\vartheta_q^{h,j}$	$\in [-1, 1]$. Effect/Adjustment factor over attribute $h \in \mathbf{H}$, at location $j \in D_{op}$ if option $q \in \mathcal{Q}$ is taken, where $\vartheta_k^{h,j} = 0$ if $j \notin D_{op}$
$\omega \in \mathbb{Z}^+$	Time lag to consider branching alternatives at time window $t' \in [t-\omega, t+\omega]$

If the probability of branching is within this threshold, then the final branching period $t^* \in \mathbf{t}^\omega$ is defined as the expected value of time of investment occurring within the time window, as defined in Equation (2).

$$t^* = \frac{\sum_{t' \in \mathbf{t}^\omega} \sum_{s \in \Omega} w_{k^*,t',s} \cdot t'}{\sum_{t' \in \mathbf{t}^\omega} t'}, \quad t^* \in \mathbf{t}^\omega = [t-\omega, t+\omega], \quad k^* \in K^* \quad (2)$$

Dynamic mining complex model

The dynamic mining complex model aims at maximizing the discounted profit obtained from processing the extracted material at the different processing streams, and minimizing the cost of different investments overtaken along the life of mine, as well as the deviations from production targets, which act to manage risk and defer it to later periods.

$$\max \frac{1}{S} \sum_{s \in S} \sum_{t \in T} \left(\underbrace{\sum_{i \in DUM} \sum_{h \in H} p_{h,t} \cdot v_{h,i,t,s}}_{\text{net profit from hereditary variables}} - \underbrace{\sum_{k \in K} p_{k,t} \cdot \sigma_{k,t,s}}_{\text{cost of investments}} \right. \\ \left. - \underbrace{\sum_{i \in DUM} \sum_{h \in H} (c_{h,t}^+ \cdot d_{h,i,t,s}^+ + c_{h,t}^- \cdot d_{h,i,t,s}^-)}_{\text{penalty for deviations}} \right) \quad (3)$$

Subject to:

1. *Mining Constraints* – Next, mining constraints are presented, which ensure that the extraction is geotechnically and operationally feasible (Equations (4)–(7)).

- (a) *Slope constraints* – ensure that a block b is only extracted once its predecessors $O(b)$ (i.e. its overlying blocks) have been extracted

$$x_{b,t,s} \leq \sum_{t'=1}^t x_{o,t',s} \quad \forall b \in B_m, m \in M, o \in O(b), t \in T, s \in \Omega_\rho \quad (4)$$

- (b) *Mine reserve* – a block b can be mined only once in the LOM

$$\sum_{t \in T} x_{b,t,s} \leq 1, \quad \forall b \in B_m, m \in M, s \in \Omega_\rho \quad (5)$$

- (c) *All extracted rock must be sent to a single destination* – this constraint ensures all clusters c have an assigned destination at every period. This destination defines the destination of each extracted blocks at that period (according to their cluster membership)

$$\sum_{j \in \Theta(c)} z_{c,j,t,s} = 1, \quad \forall c \in C, t \in T, s \in \Omega \quad (6)$$

- (d) *Mineability /Mining width* – these constraints ensure that the extraction sequence is smooth and continuous, penalizing the objective function (OF) through state variable $d_{h,m,t,s}^-$ if, for a given block, its neighbouring blocks nb are not extracted at or before that period. Here, i, j, k correspond to the coordinates of block b , and n is the number of surrounding blocks that must be also extracted on each direction to ensure mining equipment width requirements.

$$|nb| \cdot x_{b,t,s} \leq \sum_{b' \in nb} \sum_{\tau=1}^t x_{b',\tau,s} + d_{h,m,t,s}^- \quad \forall h \in H, m \in M, t \in T, s \in \Omega_\rho \\ nb = \text{neighbour blocks of block } b \quad (7)$$

$$b = \left\{ b'_{(i',j',k')} \in B_m : i' \in [i-n, i+n], j' \in [j-n, j+n], k' = k \right\}$$

2. Mining complex constraints

- (a) *Stockpile material balance between incoming and outgoing quantities* – these constraints ensure that there is a balance between incoming and outgoing material in the stockpiles of the mining complex. This balance is defined by the existing material, plus what is being fed to it, minus what is being taken from that whole amount to forward stages of the mining complex.

$$v_{\varphi,i,t,s} = \left(\underbrace{v_{\varphi,i,(t-1),s}}_{\text{Left-over in destination } j} + \underbrace{\sum_{c \in J(i) \cap C} \left(\sum_{m \in M} \sum_{b \in B_m} \beta_{\varphi,b,s} \cdot \theta_{b,c,s} \cdot x_{b,t,s} \right)}_{\text{Coming in from the mine to } i} \cdot z_{c,i,t,s} \right) \cdot \left(1 - \sum_{g \in \Theta(i)} y_{i,g,t,s} \right) \quad \forall \varphi \in P, i \in Sp, t \in T, s \in \Omega \quad (8)$$

- (b) *Processing material balance between incoming and outgoing quantities* – these constraints ensure that there is a balance between incoming and outgoing material in the different processing locations of the mining complex. Processors have no left-over material, thus current material is defined only by the period's feed from mines and stockpiles.

$$v_{\varphi,j,t,s} = \underbrace{\sum_{i \in J(j) \setminus C} v_{\varphi,i,t,s} \cdot y_{i,j,t,s}}_{\text{Coming in from other destinations to } j} + \underbrace{\sum_{c \in J(j) \cap C} \left(\sum_{m \in M} \sum_{b \in B_m} \beta_{\varphi,b,s} \cdot \theta_{b,c,s} \cdot x_{b,(t+1),s} \right)}_{\text{Coming in from the mine to } j} \cdot z_{c,j,t,s} \quad \forall \varphi \in P, j \in Pp, t \in T, s \in \Omega \quad (9)$$

- (c) *Capacity constraints for Mining/ Equipment* – these constraints are global for all components of the mining complex, and are affected by both operational and investment alternatives decisions.

$$v_{h,i,t,s} - d_{h,i,t,s}^+ \leq U_{h,i,t,\Omega_{\rho l}} \cdot (1 + \vartheta_q^{h,i} \cdot \mu_{q,i,t,s}) + \sum_{k \in K} \sum_{t' = t - \lambda_k - \tau_k}^{t - \tau_k} \kappa_{k,h} \cdot w_{k,t',s}, \quad (10)$$

$$v_{h,i,t,s} + d_{h,i,t,s}^- \geq L_{h,i,t,\Omega_{\rho l}} \cdot (1 + \vartheta_q^{h,i} \cdot \mu_{q,i,t,s}) + \sum_{k \in K} \sum_{t' = t - \lambda_k - \tau_k}^{t - \tau_k} \kappa_{k,h} \cdot w_{k,t',s} \quad \forall h \in H, i \in D \cup M, t \in T, s \in \Omega_{\rho l} \subseteq \Omega, q \in Q_i \quad (11)$$

For example, if the plant changes its operational mode to increase throughput (i.e. $\mu_{q,i,t,s} = 1$), the upper and lower capacity limits will increase by a factor of $\vartheta_q^{h,i}$. The same way, if the optimizer decides to invest in an extra crusher at the plant (i.e. $w_{k,t',s} = 1$), then (after the corresponding lead time has passed), the limits also increase by a quantity of $\kappa_{k,h}$. Note that investment alternatives increase the capacity only after the lead time has passed (τ_k), and only for the time defined by the life of the equipment purchased (λ_k).

3. Attribute calculation and state variable definition

The definition of the different primary and hereditary attributes is presented next. These definitions are thought out to be general in order to allow the model to adapt to the specific characteristics of different mining complexes, such as number of elements produced, set of possible processing streams, geometallurgical variables of interest, etc.

- (a) *Value of primary attributes* – (defined per scenario) these constraints ensure that the value of any primary attribute is only accounted for if the block is extracted on that period.

$$v_{\varphi,m,t,s} = \sum_{b \in B_m} \beta_{\varphi,b,s} \cdot x_{b,t,s}, \quad \forall m \in M, \varphi \in P, t \in T, s \in \Omega \quad (12)$$

- (b) *Final hereditary attribute value* – depends on the transformation function ($f_{h,j}(\varphi)$), which is affected by the selection of operational alternative ($\vartheta_q^{h,j}$), activated by the binary decision variable $\mu_{q,t,s}^j$.

$$v_{h,j,t,s} = f_{h,j}(v_{\varphi,j,t,s}) \cdot (1 + \vartheta_q^{h,j} \cdot \mu_{q,j,t,s}), \quad \varphi \in P, h \in H, j \in D \cup M, q \in Q_i, \quad \forall t \in T, s \in \Omega_\rho \quad (13)$$

For example, if operating alternative $q \in Q_j$ affects the recovery of element $\varphi \in Pat$ location $i \in Pp$, then the final recovery ($r_\varphi \in H$) is defined by $v_{r_\varphi,i,t,s} = f_{h,j}(v_{\varphi,i,t,s}) \cdot (1 + \vartheta_q^{r_\varphi,i} \cdot \mu_{q,t,s}^i)$.

4. Dynamic / option constraints

These sets of constraints enable the branching mechanism and ensure that equal decisions are taken over all scenarios if no branching has been defined.

- (a) *Non-anticipative constraints* - these constraints are enforced over a variable time frame T^α , which is iteratively augmented (as defined in algorithm in section 2.2). Within this time frame, these constraints are enforced always except if branching is “activated” (i.e. $u_{k^*,t}^\rho = 1$). Here, non-anticipative constraints are defined over extraction (14), destination (15), investment (16), and operational mode (17) decisions.

As there can be more than one branching decision in the model, branching can occur over any of them, and thus the value of $u_{k^*,t}^\rho$ is considered over all possible branching decisions available (set K^*). Note that if all $u_{k^*,t}^\rho = 0$, then all decisions must be the same within all scenarios (even branching investment decisions). Meaning the solution will remain unique (with or without investment).

Given the scenario partition $\Omega_{\rho 1} = \{s, w_{k,t,s} = 1, \forall s \in \Omega_\rho\}$, $\Omega_{\rho 2} = \Omega_\rho \setminus \Omega_{\rho 1}$, where $\Omega_{\rho 1} \cup \Omega_{\rho 2} = \Omega_\rho$; and $\Omega_{\rho 1} \cap \Omega_{\rho 2} = \emptyset$, the following set of constraints is defined for $s, s' \in \Omega_\rho, \forall s \in \Omega_{\rho 1}, \forall s' \in \Omega_{\rho 2}$, where, for ease of notation, the following substitution is done over Equations (14)–(17): $A = \left\lceil \frac{\sum_{k^* \in K^*} u_{k^*,t}^\rho}{|K^*|} \right\rceil$

$$(1 - A) (x_{b,(t+1),s} - x_{b,(t+1),s'}) = 0, \quad \forall t \in T^\alpha, b \in M \quad (14)$$

$$(1 - A) (z_{c,j,(t+1),s} - z_{c,j,(t+1),s'}) = 0, \quad \forall t \in T^\alpha, c \in C, j \in D \quad (15)$$

$$(1 - A) (w_{k,(t+1),s} - w_{k,(t+1),s'}) = 0, \quad \forall t \in T^\alpha, k \in K \quad (16)$$

$$(1 - A) (\mu_{q,j,(t+1),s} - \mu_{q,j,(t+1),s'}) = 0, \quad \forall t \in T^\alpha, q \in Q_j, j \in D_{op} \quad (17)$$

- (a) *Branching threshold constraint* – these set of constraints define the activation of branching in node ρ , which only occurs if the probability of branching during time window t^ω is within the threshold limits $\in [R, 1 - R]$. Here, two auxiliary variables $u1_{k^*,t}^\rho, u2_{k^*,t}^\rho \in \mathbb{R}$ are used to verify if the branching proportion is within the upper and lower limits of the threshold.

$$\frac{\sum_{t'=t-\omega}^{t+\omega} \sum_{\forall s \in \Omega_\rho} w_{k^*,t',s}}{|\Omega_\rho|} - m \cdot u1_{k^*,t}^\rho \geq 1 - R, \quad \text{with } m = \text{constant lower than } -(1 - R) \quad (18)$$

$$\frac{\sum_{t'=t-\omega}^{t+\omega} \sum_{\forall s \in \Omega_\rho} w_{k^*,t',s}}{|\Omega_\rho|} - M \cdot u2_{k^*,t}^\rho \leq R, \quad \text{with } M = \text{constant higher than } (1 - R)$$

- (b) *Linking auxiliary variables ($u1_{k^*,t}^\rho, u2_{k^*,t}^\rho$)* - the main branching decision variable $u_{k^*,t}^\rho$ is activated (i.e. $u_{k^*,t}^\rho = 1$) if the branching probability is within the upper and lower limits of the threshold (i.e. both auxiliary variables are active: $u1_{k^*,t}^\rho, u2_{k^*,t}^\rho = 1$) (Constraint (19)), and if there are enough scenarios in each possible partition (Constraint (20)).

$$\begin{aligned} u1_{k^*,t}^\rho + u2_{k^*,t}^\rho - 1 &\leq u_{k^*,t}^\rho, & u1_{k^*,t}^\rho &\geq u_{k^*,t}^\rho, & u2_{k^*,t}^\rho &\geq u_{k^*,t}^\rho \\ \frac{|\Omega_{\rho 1}|}{N} &\geq u_{k^*,t}^\rho, & \frac{|\Omega_{\rho 2}|}{N} &\geq u_{k^*,t}^\rho, & \forall k^* \in K^*, \Omega_\rho &\subseteq \Omega, t \in [T^\alpha, T] \end{aligned} \quad (19)$$

Where

$$\Omega_{\rho 1} = \{s; w_{k,t,s} = 1, \forall s \in \Omega_\rho\}, \quad \Omega_{\rho 2} = \{s; w_{k,t,s} = 0, \forall s \in \Omega_\rho\} \quad (20)$$

- (c) *Definition of branching period (t^*)* – Constraints 2.18 to 2.20 define if the system branches during time window \mathbf{t}^ω . If it does, Constraint (21) defines the actual branching period t^* , within time window $\mathbf{t}^\omega = [t - \omega, t + \omega]$, as the nearest integer value of the expected value of period of investment within this time window \mathbf{t}^ω . Note that t^* is only activated (i.e. $t^* > 0$), if $u_{k^*,t^*}^\rho = 1$.

$$t^* = \left\lfloor \frac{\sum_{t'=t-\omega}^{t+\omega} \sum_{\forall s \in \Omega_\rho} t' \cdot w_{k^*,t',s}}{\sum_{t'=t-\omega}^{t+\omega} t'} \cdot u_{k^*,t}^\rho + \frac{1}{2} \right\rfloor \quad \forall t \in [T^\alpha, T] \quad (21)$$

5. Operational constraints over investment alternatives

These constraints ensure that operational and purchase requirements over the set of investments available are respected.

- (a) *Periodicity of investments* – decision to invest in CAPEX alternative k is only allowed ψ_k periods after it was previously taken.

$$w_{k,t,s} + \sum_{\tau=t+1}^{t+\psi_k} w_{k,\tau,s} \leq 1, \quad \forall k \in K \setminus K_{op}, t \in T, s \in \Omega_\rho \subseteq \Omega \quad (22)$$

- (b) *Limits on purchases* – These constraints link the activation of the investment decision with the actual number of investments, which must be within the allowed upper and lower limits.

$$\sigma_{k,t,s} \leq U_{k,t} \cdot w_{k,t,s}, \quad \sigma_{k,t,s} \geq L_{k,t} \cdot w_{k,t,s} \quad (23)$$

- (c) *Limit on branching decisions* – as these decisions are defined as high-impact, high-cost decisions, they are allowed “only once in the LOM”. This constraint can be replaced by setting the allowed periodicity (ψ_k) in Constraint(22) big enough to forbid repeating that investment.

$$\sum_{\forall t \in T} w_{k^*,t,s} \leq 1, \quad \forall k^* \in K^*, \forall t \in T, \forall s \in \Omega_\rho \subseteq \Omega \quad (24)$$

Solution method

The proposed solution method consists of two main phases that are repeated iteratively. In the first phase, a sub-problem of the previous model is generated and solved, and in the second phase, information obtained from solving the sub-problem is used to update the scenario-partitions that define the branches of the decision tree solution. A step-by-step description of this solution method is provided in the algorithm that follows. Next, the solving mechanism used to solve each sub-problem of phase 1 is described.

The algorithm

The previous problem is iteratively solved as described in Algorithm 1. This is done to obtain the probability value of executing branching decisions, which is found by using a look-forward mechanism. The algorithm works by enforcing non-anticipativity constraints over an iteratively increasing time window. Starting from the first period, non-anticipativity constraints are set to work over an auxiliary time frame (T^α), which increases as a moving time window (i.e T^α equals the first period in the first iteration, and equals the whole LOM in the last). With this, at each iteration, the model is solved, and the partitions are updated.

Solving mechanism

Due to the complexity and number of variables in the formulation presented in the previous section, and considering that this model must be solved iteratively, a simulated annealing (SA) based metaheuristic algorithm is used to solve it. This algorithm is based on the metaheuristic described in Goodfellow and Dimitrakopoulos (2016), but, instead of integrating two different heuristic mechanisms (simulated annealing and particle swarm), an adaptive multi-neighbourhood simulated annealing is used.

Algorithm 1 Iterative solution mechanism**Initialization**

T and Ω , total the number of periods and number of simulations, respectively
 Ω_ρ partition of scenarios in branch ρ , initially equal to Ω
 FS final solution containing dynamic production schedule
 SP_i , sub-problem defined in Section 0, solved for the i -th iteration
 T^α auxiliary time which defines final period of enforcement of non-anticipativity constraints
 t^* period of branching, defined from Equations (18)–(20) (setting $u_{k^*,t^*}^\rho = 1$)

$t \leftarrow 1$, initial period of LOM to optimize

$T^\alpha \leftarrow t + 1$

Solve sub- problem SP_0 over Ω , as described in Section 2.2 for $\mathbf{T} = [t, T]$ with T^α defined

while $T^\alpha \leq T$ **do**

$t \leftarrow T^\alpha$

if $t^* > 0$ **then**

$T^\alpha \leftarrow t^*$

 Solve sub-problem SP_i for $\mathbf{T} = [t, T]$, setting $u_{k^*,t^*}^\rho = 1$ with corresponding partitions Ω_ρ calculated from SP_{i-1}

else do

$T^\alpha \leftarrow T^\alpha + 1$

 Solve sub-problem SP_i for $\mathbf{T} = [t, T]$

end else

 update t^*

$FS \rightarrow SP_i$

$i++$

end if

end while

return FS

Simulated annealing algorithm (Geman and Geman, 1984; Kirkpatrick et al., 1983) works by, starting from an initial solution, perturbing the solution, and accepting or rejecting the new solution depending on the annealing probabilities. adaptive multi-neighbourhood simulated annealing starts from the same basis, but each neighbourhood is visited, first randomly, and next according to an adaptive probability, which is updated according to the performance of that given perturbation in improving the current solution. Perturbations are, for example, changing the extraction period of a block, switching periods between two blocks, changing the proportion of material sent from the stockpile to the processing stream, changing the destination of a cluster on a given period, or swapping the destination of two clusters. Perturbations affecting the investment and operational mode decisions were added, considering the addition or removal of one or multiple investments at a given year, the swap of two different investments in two different periods, and the activation or deactivation of operational modes in different components of the mining complex. It must be noted that if a perturbation is chosen to modify the current solution, this modification must respect all the constraints of the model, for example, a block cannot be set to be extracted on a period where its predecessors haven't been extracted yet.

For the case study presented in the next section, the initial solution is generated using a greedy algorithm, and an annealing schedule is set, where the annealing temperature is repeatedly changed after a certain number of iterations, and the solving process stops once a total number of iterations are reached.

3 Case study

The proposed model is applied at an operating mining complex composed of one mine, and six possible processing streams (Figure 2): a sulphide mill with a stockpile, three heap leaches for sulphides, oxides and transition (SHL, OHL, and THL respectively), a sulphide dump leach (SDL) for sulphide low grade and waste, and an oxide dump, for oxide waste. The mining complex produces copper and gold, and the different processing streams have constraints over the type of material received, and the product produced. This is represented in Figure 2 by the squares with numbers beside each destination, which represent the type of material that is allowed in each (defined at the left side of the figure).

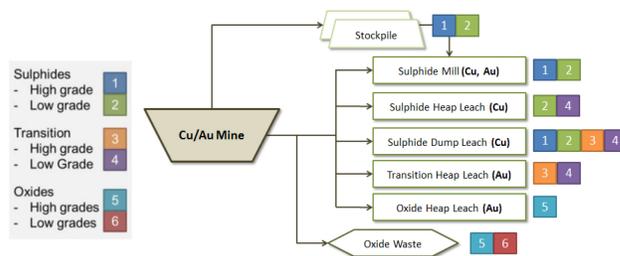


Figure 2: Mining complex layout with material allowed and metal produced per destination

The sulphide mill is the only processing stream which produces both gold and copper, and is the main source of profit of the mining complex. It has a production capacity of 2.4Mt per year, and the adjoint stockpile can store up to 1Mt. The sulphide heap leach also has a limited capacity of 6Mt, and it is assumed that all other destinations have no capacity restriction. Mining and economic parameters used are presented in Table 4. Values have been normalized by the mining cost for confidentiality reasons. In this case, fixed operating costs and commodity prices have been used. Table 4 also shows the branching parameters, which define an investment window (ω) of $+/- 1$ year, and a threshold parameter R_t , which means that if between 40% and 60% of scenarios decide to invest within a time window of $[t - 1, t + 1]$, then the design branches into parallel solutions.

Table 4: Mining and economical parameters of the copper/gold mine

Mining Complex Param.		Processing Costs		Economic Parameters	
Mining Cost	$\$1.0 * x$	Sulphide mill cost	$\$11.3 * x$	Copper Price	$\$3.9/\text{lb}$
Mining Capacity	6 Mt	Sulphide heap leach cost	$\$3.9 * x$	Gold Price	$\$1450/\text{oz}$
SHL Capacity		Sulphide dump leach cost	$\$1.9 * x$	Discount rate	10%
SM Capacity	3.4 Mt	Transition heap leach cost	$\$3.2 * x$	Other Parameters	
Mining width	100m	Oxide heap leach cost	$\$3.1 * x$	ω	1 year
				R_t	40%

3.1 Alternatives considered

Alternatives added to this case study are divided into “investment” and “operational”. Investment alternatives are included (i) in the sulphide mill, with the possibility of increasing the capacity by adding a secondary crusher to increase the production capacity, and (ii) at the mine, where the optimizer defines the truck fleet, and thus, the annual extraction capacity. Both alternatives are highlighted in Figure 2 by dotted lines. Two operational alternatives are included in this case study; (a) one that acts over the mine by adapting the blasting pattern to reduce mining cost, punishing grindability, and (b) an alternative over the sulphide mill’s processing configuration, which increases throughput by reducing the recovery. Operational details on each alternative are presented next.

Investment alternatives

An initial fleet of 2 trucks is assumed and is active for the first six years. However, the optimization process can increase this extraction capacity by purchasing additional trucks. Each additional truck has a life of equipment of six years, and once a truck is purchased, it is only available one year later. The decision to purchase a truck can be taken every 2 years, and the maximum purchase quantity is defined to be 5 trucks at a time. These and other operational details are presented in the first column of Table 5.

The second CAPEX alternative is the purchase of a secondary crusher at the sulphide mill, which increases the production capacity by 300kt per year. This investment decision is set to be a “branching alternative”, which means that the optimizer can branch and develop parallel mine design schedules if a representative

number of scenarios differ in this decision variable. These and other operational details are presented in the second column of Table 5.

Table 5: Purchasing details for the investment alternatives

	Truck (non-branching option)	Secondary Crusher (branching option)
Undiscounted cost (\$US)	3,800,000	25,000,000
Life of equipment	6 years	25 years
Periodicity of decision	2 years	-
Lead time	1 year	2 years
Maximum purchases per year	5 units	1 unit
Tonnage increment per unit	3,500,000 tpa	300,000 tpa
Initial Capacity available	7,000,000 tpa (2 trucks)	2,400,000 tpa

Operational alternatives

Mining mode The mining operational mode alternative works by reducing the number of blast-holes in the blasting pattern. This reduces the overall mining cost due to less drilling and a reduced amount of explosives, but at the same time produces a coarser blasted material, which requires additional work and energy from the system’s crusher, reducing the throughput (Figure 3). In this case, it is assumed that an 18 blast-hole net is reduced to a 16 blast-hole one (as shown in the left side of Figure 3), which reduces the mining costs by 8%, and in turn, reduces the crushing capacity by 3% ($\vartheta_q^{h,j}$ in the formulation in Section 2.2). These values were taken from the mine’s historical data.

These operational alternatives allow having a better control over the pertinent geometallurgical variables, as, for example, the optimizer may choose to concentrate blasting in areas with harder rock or where the grade is higher, ensuring that that material reaches the processing stream faster.

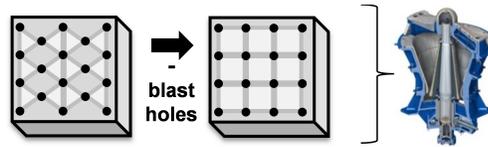


Figure 3: Blasting net alternative at the mine level

Processing mode The processing operational alternative at the sulphide mill is defined as the selection between a higher throughput with a lower recovery, or a lower throughput and an increased recovery (Figure 4). This metallurgical relation is well known, where the processing time can be reduced by shortening the feed’s time at the crusher by producing a coarser grinding or reducing the concentrate’s residence time in the different processing stages; however, this will have a negative effect on the metal recovered from processing the material at the sulphide mill.

In this case study, the “activated” operational mode increases the throughput by a 4.4% but reduces the plant’s recovery by 0.56% ($\vartheta_q^{h,j}$ in the formulation in Section 0). These values were also taken from historical data of the mine; the plant’s recovery curves presented in Figure 4 (values are not shown for confidentiality reasons).

3.2 Results

The following section presents the results obtained for the dynamic stochastic optimization method proposed are presented. Subsequently, these results are compared with the traditional two-stage stochastic formulation without alternatives are provided.

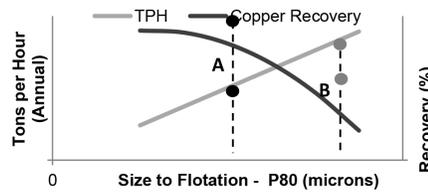


Figure 4: Relation of recovery/throughput alternatives (A and B) at the SM

Proposed dynamic two-stage stochastic formulation with alternatives

The solution of the formulation proposed in Section 2.2 shows that there is a 43% probability of investing on a secondary crusher in period 4; as 43% is within the threshold $R = 40\%$ defined in Table 2 ($43\% \in [40\%, 60\%]$), the design branches at that time. All results obtained from the branch without secondary crusher are presented on the left side of the figures, and the case with secondary crusher is presented on the right side. The crusher and truck investment plan is presented in Figure 5. The right side shows that, when a secondary crusher is purchased, the operation chooses to buy one extra truck by year 5, compared to the branch without secondary crusher, which is available on year 6 (also when the crusher is available), which allows to balance the extra mill feed required by the system. Together with this, in both branches trucks are purchased every two years, maintaining an average extraction capacity of 17.5Mt, with a five year ramp-up. This capacity increases further for the case with secondary crusher, reaching 20–25Mt capacity during years 6 to 13.

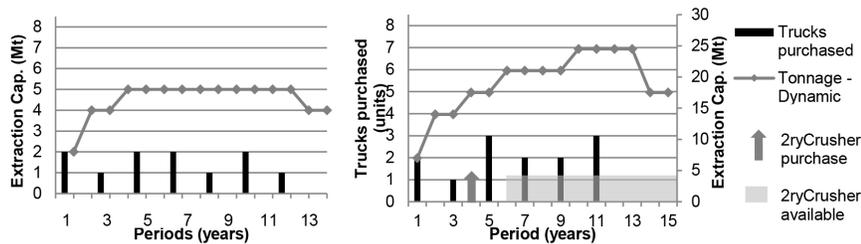


Figure 5: Trucks and secondary crusher purchase plan for each branch of the production plan solution, without secondary crusher (left) and with secondary crusher (right)

Operational alternatives for both branches are presented in Figure 6. It can be seen that, when the plan invests in a secondary crusher, the optimizer generally decides against increased throughput (mill mode alternative) and grindability (miner mode alternative), particularly on the last 3 periods, where, as there are 300kt extra of processing capacity, the optimizer chooses to maximize recovery and minimize mining costs by keeping both operational alternatives not active. Particularly in the case of operational mode alternatives, the time frame to change them can be considered shorter than a whole year. Because of this, and due to the flexibility of the proposed model, a mid-term analysis is performed by discretizing the first two years in three terms each, and defining the corresponding operational mode decision variables (blasting pattern mode and sulphide mill recovery mode) in that schedule’s time frame. This analysis provides a more realistic target to guide the short-term plan, considering the actual configuration flexibility that the processing streams have. Figure 7 shows the risk analysis over the mid-term feed plan for the sulphide mill, with the initial target in dotted red, and the target adapted by the sulphide mill’s operational alternatives in continuous blue. The percentiles 10, 50 and 90 are presented (P10, P50 and P90, respectively), which show there is a 10%, 50% and 90% probability of being under the values presented. It can be seen in the figure that the optimizer decides to apply the mill operational mode to increase the mill’s throughput in the last two terms of the first year, and on the last term of the second one, allowing the mining complex to increase the mill feed in those periods, and minimizing deviations from targets. It must be noted that, as the branching occurs on period 4, and the mid-term analysis is done only over the first two periods, this mid-term plan is common for both branches of the production plan (as are all other decisions for the first 3 years of production).

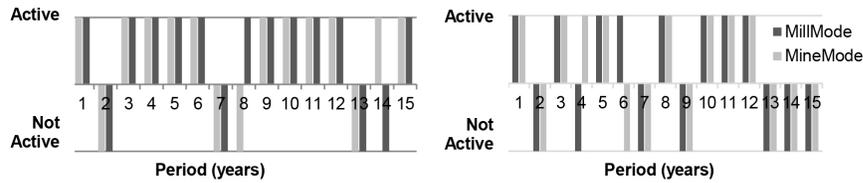


Figure 6: Operational mode decisions for the mine (grey) and the mill (black) for each branch of the production plan, without secondary crusher (left) and with secondary crusher (right)

These operational and investment alternative decisions result in the sulphide mill feed shown in Figure 8, where, as in Figure 7, the initial target is presented in dotted red line and the target adapted by the operational mode is presented in continuous blue line. The risk analysis (P10, P50, and P90 values) of the mill feed is also shown, which show that the optimizer does a good job at following the dynamic target in both cases, presenting a tight risk profile with minor deviations mostly on the last 5 years of life of mine. The left side of Figure 8 shows that, even though this branch did not invest in a secondary crusher, the production schedule is using the mill’s operational mode flexibility to increase its throughput in most periods, without any cost on investments. On the other hand, the branch with a secondary crusher (right of Figure 8) also decides to increase the mill’s processing capacity by using the operational modes in some periods, producing relatively tight risk profiles except on the final 3-4 years of production.

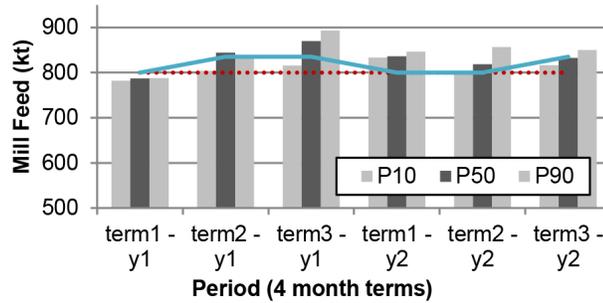


Figure 7: Risk analysis over SM feed for the mid-term discretization of the first two years with operational alternatives

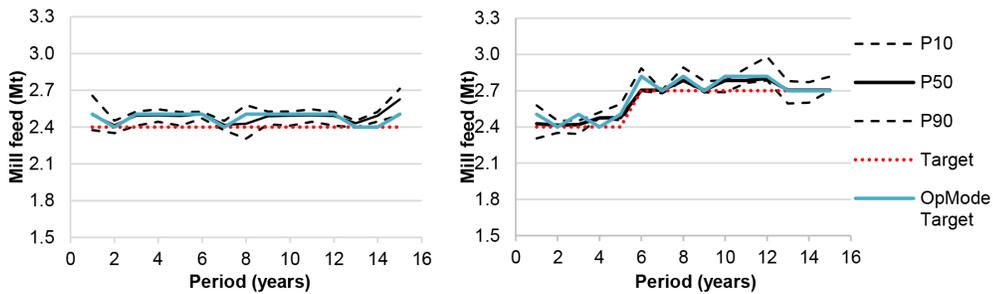


Figure 8: Annual SM feed for each branch of the production plan solution (without secondary crusher (left) and with secondary crusher (right))

The solution obtained from the proposed formulation presents an NPV with a P50 of MUS\$1460, a P10 of MUS\$1320 and a P90 of MUS\$1580. The full cumulative discounted cash flow distribution for the dynamic formulation is presented in Figure 9, together with the base case (presented next).

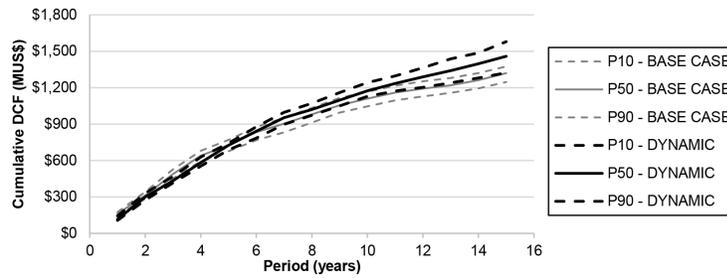


Figure 9: Cumulative net present value for the base case (light grey) and the proposed dynamic formulation (black)

Comparison to the two-stage stochastic formulation

Results for the two-stage stochastic formulation of the mining complex are presented in this section, where extraction decisions are first-stage decisions, and processing stream decisions are considered second-stage. This case not only ignores the dynamic algorithm presented in Section 0, but also removes all investment and operational mode alternatives from the model; in particular, this corresponds to the second term of the objective function (Equation (2)), as well as all investment and operating mode effects and decisions from the set of constraints.

As there are no CAPEX alternatives considered, it is assumed that the mine has a constant extraction capacity of 14Mt per year (i.e. a constant fleet of 4 trucks). The same way, the mill is assumed to have a constant processing capacity of 2.4Mt per year. Results from this optimization are presented in Figure 10, which shows the risk analysis of the annual sulphide mill feed (left), the extracted material (middle), and the cumulative discounted cash flow (right), with P10, P50, and P90 values presented for each case.

The mine production and extraction plans (left and middle graphs respectively) present very controlled risk, with minimal deviation from production and extraction targets (present mostly at the last three years). This shows that two-stage stochastic optimization is able to provide production plans that control and manage risk. The cumulative discounted cash flow presents a NPV distribution with a P50 of MUS\$1320, a P10 of MUS\$ 1,246, and a P90 of MUS\$1,374. Figure 10 compares these results, with the ones obtained on the proposed dynamic formulation.

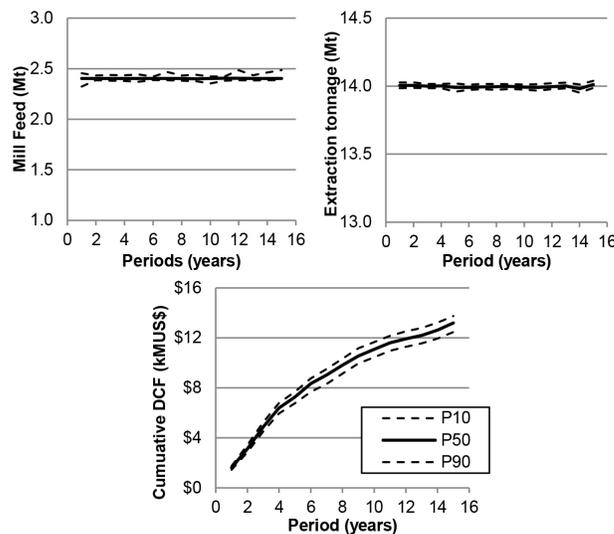


Figure 10: Sulphide mill feed (left), extraction tonnage (middle), and net present value (right) for the base case two-stage optimization without alternatives

Discussion

The two-stage approach presented in section 0 is able to manage and control risk. However, it assumes that the future is fixed, and does not capitalize in terms of value, or takes advantage for changing environments and new information. This value is accounted for in the dynamic formulation by allowing the optimization process to adapt to change by introducing flexibilities in the form of operational modes and dynamic investment alternatives. This can be seen by comparing the different NPV percentiles of both cases in Figure 10, where the dynamic model presents a 10.5% higher NPV in terms of P50, but almost a 15% higher P90. This shows that the dynamic formulation maximizes value, but, furthermore, provides a production plan that is able to take advantage from opportunities and capitalizes on the project's possibilities of adapting.

4 Conclusions

This paper presents a dynamic two-stage stochastic mixed integer non-linear programming formulation for modeling and optimizing a mining complex. Mining complexes are value chains where extracted rock from different sources is transformed into sellable products through a set of processing streams. This value chain is governed by uncertainties at different levels, from the geology of the orebody at the mine, to the different operational and processing components that lead the sellable products to the market. The model presented considers possible flexibilities in the mine production schedule by including alternatives over capital expenditure investments and operational modes at different levels of the value chain. More specifically, a dynamic decision-making mechanism is included, where the mine production plan is allowed to branch, and parallel solutions are designed if a representative proportion of geological stochastic simulations agree it is profitable. This model extends from a multistage formulation and prevents the model from producing over-fitted solutions to the set of stochastic simulations used. This is done by setting a representativity threshold that controls the branching mechanism, and thus, the final solution provides a controlled set of possible mine-plan alternatives that have been probabilistically quantified to be worth considering. This process generates new optimized plans that allow and ease the process of adapting, once more, the information available. Due to the size and complexity of the proposed formulation, exact solvers such as CPLEX are unable to provide any solution. Thus, an adaptive multi-neighbourhood simulated annealing metaheuristic is used, which is able to solve complex, non-linear problems, producing good solutions in a relatively short amount of time.

The practical implications of the proposed method are demonstrated through an application at a copper-gold mining complex comprised of one mine and six processing streams, where the proposed dynamic model is compared to a two-stage stochastic formulation, presenting a 10.5% increase in net present value in terms of P50, and a 15% higher NPV for the P90.

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