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Integreted optimization of short- and medium-term planning in underground mine

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Abstract: This article describes a new model aiming at optimizing short- and medium-term underground mine scheduling. The complexity of the problem to solve and the frequency at which planners have to revise these schedules are among the main motivation for developing such a model. In order to address this problem, a Mixed Integer Programming model was developed with a variable time discretization to accurately represent both short- and medium-term operational constraints in a single model. Results of a preliminary model are presented with explanations an in-depth analysis. An improved formulation is also described with it's associated results and benefits. Further testing with scenarios similar to long-term planning show very promising results for the possible application of our modified formulation to existing long-term model.

Keywords: Mixed Integer Programming, underground mine, planning, scheduling

Résumé : L'article suivant décrit un modèle mathématique visant à optimiser la planification court- et moyen-terme des mines souterraines. La complexité du problème ainsi que la fréquence à laquelle ces planifications doivent être révisées par les planificateurs miniers sont parmi les principales justifications de la nécessité de développer un tel modèle. Pour ce faire, un modèle de programmation linéaire en nombres entiers basé sur une discrétisation variable du temps a été développé pour représenter adéquatement les contraintes opérationnelles de ces deux niveaux de planification en un seul modèle. Les résultats de la formulation initiale sont présentés ainsi que leur analyse. Puis, une version modifiée de la formulation du modèle est présentée avec ses justifications et avantages. Finalement, une dernière série de tests semblables à de la planification long-terme est présenté et démontre le potentiel pour cette nouvelle formulation d'améliorer, ou d'aider à la résolution de modèle long terme existant.

Mots clés : Programmation linéaire en nombre entier, mine souterraine, planification

1 Introduction

As for many other sectors, the mining industry has seen many changes in the recent years and many more seems to be coming in a near future. In order to increase profitability, more and more mines includes automation in their processes. From automated trucks to stope shape optimizers, a decreasing number of decisions are taken without the use of a computer. Among these, underground mine planning is a sector of activity that still lacks the tools to use the power of automation and optimization. This article aims at addressing this particular problem. In order to introduce the reader with all the necessary notions to the understanding of this article, we will first give a short description of some terms and concepts.

Figure 1 is a typical representation of an underground mine, illustrating many of the most important concepts needed here. The ore body represented is in the shape of a vertical vein. This type of narrow and highly concentrated ore deposits is very typical of Canadian underground mines. Mining operations often include many veins, either vertical or horizontal. These ore bodies can be extracted using two main categories of mining operations; open pit or underground mining. As seen on the figure, open pit mines are cone shaped excavations mined from the surface. Underground mines, on the other hand, are accessed by ramps or shafts or both, and their shape and size are dependent on many different factors like rock stability or ore deposit geometry. From these main accesses, many other excavations like haulage levels or ramps are excavated in order to reach the mineralized zone. All these activities needed to prepare for the extraction of the ore body are necessary costs regrouped under the term "developments".

When the ore body is reached, the extraction of stopes starts. Stopes are subdivisions of mineralized zone extracted with different mining methods. There exist many different mining methods, each of them with their own characteristics and suitable for different types of ore body. The following article will mainly focus on common mining methods used in vein-shaped ore body. The details of each of these methods not being essential to the understanding of this article, further precision on each of them can be found in [5]. Stopes are designed to be as large as possible to maximize profit and have to be extracted in a specific order dictated by the mining method in use and rock stability. Once extracted, it is often required for the stopes to be filled again with a mix of broken rocks and cement to maintain the stability of adjacent stopes. This activity is called backfilling. A delay of two to three weeks, depending on the material used, is then required for the backfill to solidify, during which all activities are forbidden in the adjacent stopes. This delay is called curing time. All of the activities related to the extraction of stopes are referred to as "production". Because of drilling costs and limitations on long distance drilling, stopes characteristics are subject to high uncertainty until developments have reached an area near the ore zone. As seen at the bottom of Figure 1, stopes characteristics are first estimated from a few holes made from distant developments or from the surface and then confirmed by exploration drilling made from close by developments. Thus, the precision in the description of the ore body increases with the progression of developments.

Planning in underground mines is made at four different levels. The first, long-term planning, is typically based on yearly time periods and defines general production objectives and development goals for the whole life of mine. The precision of the planning is very limited, considering that the time periods are very long and that the majority of the production planned is based on geostatistical models obtained from a few distant drilling holes. Stopes designs at this level of planning are often very generic, since the final shape of the vein is still uncertain. This planning is usually revised every year or two or as often as notable changes occur in the geological model. The second level, medium-term planning, is usually performed on time periods of one to three months. This planning is more precise than the long-term from its shorter time periods and better defined veins. This schedule is usually revised every three months or so to adjust previsions according to the actual advances in the mine. Short-term planning is the third level of this planning sequence and has a time horizon of one or two weeks. Short-term schedules are usually revised every week. Finally, the first week of the short-term planning is divided into shifts and revised at the end of every shift accordingly to the events of the day. This last level of planning is referred to as real time planning.

The following article will describe a model of mathematical programming designed to optimize simultaneously short- and medium-term planning. As will be demonstrated further, from the limited literature on the subject of underground mine planning, most articles are focusing on long-term planning. Very few

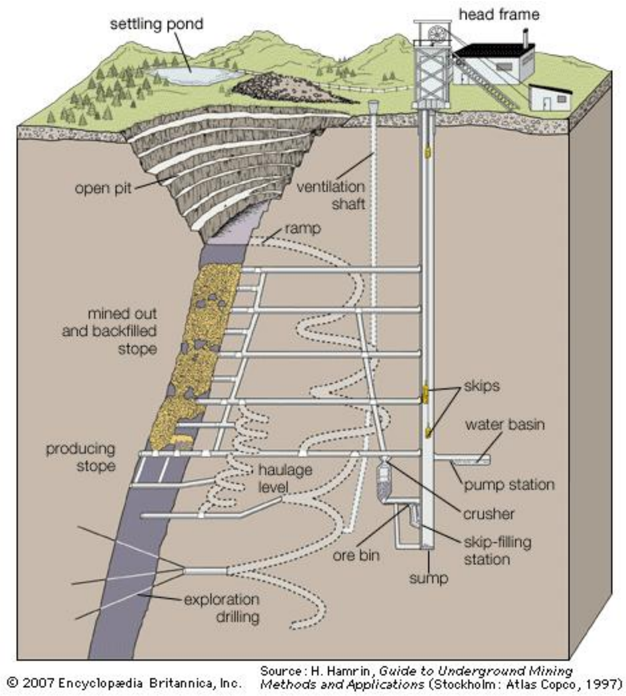

Figure 1: Typical representation of an underground mine [5]

models are available for lower planning levels, and there is a great need for more research to develop working models. Two major reasons make these levels of planning challenging. First, a mine is a work environment with many resources to allocate and many possible workplaces. This creates a large number of possibilities in scheduling and increases the planning problem's difficulty. Second, a typical mine layout involves many different ore zones spread across a large area. To reach these zones, series of developments must be excavated long before the stope's extraction can be started. This characteristic makes it even more difficult to plan and optimize activities in underground mines since development activities always have to be ahead of production activities in order to keep a constant production level.

The reason for the choice of an integrated model is that on the short-term level, decisions must take into account the medium-term objectives for development and production to be realistic. Thus, integrating both levels of planning in a single model guarantee that the short-term planning will take in consideration medium-term objective of development and production, and on that the medium-term planning will produce feasible and realistic planning for the short-term. Furthermore, it is well known that by solving a problem in separate parts instead of as a whole, optimality can be lost. The normal mine planning process involves the separation of the whole mine planning in four different levels at the expense of optimality. The integration of two of those levels together could create more planning potential.

The benefits from such a model are plenty. Even if the input of a mine planner would still be needed, it would greatly reduce the time taken to produce both short- and medium-term planning. This is even more emphasized by the fact that these two schedules have to be revised very frequently. Theses constant revisions also leave little time for the planners to optimize short- or medium-term planning. Therefore, the model would help optimize a part of planning that rarely is, by proposing optimal solutions and leaving more time for the planners to work on each schedule. Finally, mine planning is still mostly based on the planners experience and estimates. Our model can help normalize the process and make it less dependent on the planner.

In order to correctly address the problem of integrated short- and medium-term underground mine planning, the model will have to be precise on the short-term to define weekly objective, but with enough foresight to prepare for the development of distant ore zones. It will have to be solvable in less than an hour so that it can be used multiple time in the course of a planner's work shift. Finally, it will have to be able chose a subset of activities to be completed from the larger set of all of the mine's activities so that planners do not have to enter as input a list of activities to complete, which would reduce the optimization possibilities.

The following sections will introduce the reader to a literature review of the models already developed for underground mine planning with a focus on short- and medium-term. A model addressing all of the points presented above will then be introduced and a presentation of its improvement will follow. Results and comparison of the initial and improved model will then be shown and commented.

2 Literature review

Contrarily to its open-pit counterpart, optimization in underground mines is a much less documented subject. In its review of the current literature and opportunities in this field, [11] attributes this lack of interest to the greater complexity and specificity of underground mine problems. A theory supported by both [1] and [15] in their respective review of operation research in natural resources and in mine planning. In underground mine planning specifically, most of the mathematical models that have been developed have been designed to solve long-term planning problems. These problems are usually solved on a low resolution, i.e. long time periods and large mining units, to compensate for the considerable size of the problems.

[2] propose a standard and general formulation of the underground mine planning optimization model, summarizing it to precedencies, upper and lower bound on resources consumption and unique completion of each mining unit. A second formulation is then introduced to consider selectivity in the mining process, or the ability to select a variable grade from different stopes. A modified grade/tonnage curve taking into consideration the mining method is used in to model this selectivity while keeping a low level of resolution. Using a similar general definition of the planning problem, [8] present a method based on Lagrangian relaxation to accelerate the resolution of such problems. In the same year, [7] develop a heuristic to solve the long-term planning model of a group of open-pit and underground mines in Chile sharing multiple processes. The principle of the method is based on a rounding procedure of LP relaxations in order to achieve good solutions.

The idea of integrating multiple aspects of mining through a single formulation can also be found in [6]. Instead of integrating multiple mines, the model proposed integrates the selection of a variable cut-off grade to the optimization problem. [4] explore this approach further by adding geological uncertainty into the model solving it as a stochastic integer programming model. In a similar way, [9] integrate another aspect of mining to the planning problem by proposing a model considering variable stopes design. As for all other integration presented here (e.g. cut-off grade and scheduling, multiple mine site scheduling), results of the application proof that the integration of multiple aspect into a single problem yields sizable increase in the NPV of the solutions.

On a finer discretization level, [14] and [10] both propose models for optimization of underground mine schedules with monthly time periods. [14] present a standard and an improved model for underground mine scheduling that aims at maximizing NPV. The improvement is based on the aggregation of multiple binary variables into one for sequences of tasks that are known to follow one another without interruptions. Both models are tested on a conceptual 50 stopes operation over a period of 4 years with noticeable computational time reduction for the improved model. [10] on the other hand describes a heuristic to solve a known formulation of the underground mine scheduling problem that aims at minimizing the deviation from production targets. The heuristic is based on the successive relaxation of constraints related to the different parts of the objective. The information extracted from these relaxations is then used to fix certain variables before a final solve with all constraints is completed. The necessity of the heuristic is then proved by the comparison to the performances of the usual branch-and-bound on large problems where the former cannot find optimal solutions.

Table 1 and Table 2 present the short-term models available from the current literature. The first columns of each table refer to the articles in which the models are presented. In Table 1, The "Element" column refers to the main commodities being mined in the operation for which the model was developed. The "Mining Method" column list the mining methods considered by the model, where LH stands for Long-Hole, CF for Cut-and-Fill and RP for Room-and-Pillar. The "Data Source" column indicates from which source the testing dataset was taken.

In Table 2, the "Time Unit" column refers to the base unit for time discretization used by the models and column "Horizon" displays the longest time horizon for which the model was solved. The "Objective" column defines what is considered in the objective function of the models, where DE stands for discounted extraction, TD for target deviations and NPV for Net Present Value. What is understood as discounted extraction is any objective where a time-decreasing function of the tonnage of the site extracted is used. Target deviation includes all objective functions where a penalty is included for not reaching pre-defined targets on tonnage or ore quality and the net present value is the discounted value of all activities taking place in the schedule. The "Resource Constraints" column indicates whether the models have limitations on global or individual resources. That is, if similar crews or machines are considered as one resource with an equivalent capacity for all of them (Global) or if each of them is given its own capacity and assigned to specific tasks (Individual). The advantage of considering each machine or crew as an independent resource is that the resulting planning is directly applicable and does not need further processing in order to allocate each resource to work places. The downside of it being that it requires more variables to represent the same situation. The "Planning Variables" column indicates if the planning variables, i.e. the variables used to express the completion of a task, are binary or continuous. The advantage of using continuous variables to express task completion is that it creates more flexible models, where activities starting point and execution rates are not limited by the time discretization. But these additional possibilities also usually come with higher complexity and a need for larger computational resources.

Table 1: Short-term models contexts

Model	Element	Mining Method	Data Source
[3]	Au	LH, CF	Canadian Gold Mine
[13]	Cu	LH	Conceptual
[12]	Cu	LH	Conceptual
[16]	Pb, Zn	LH, CF, RP	Lisheen Mine

Table 2: Short-term models characteristics

Model	Time Unit	Horizon (Months)	Objective	Resource Constraints	Planning Variables
[3]	Week	6	DE+TD	Global	Continuous
[13]	Shift	2	TD	Individual	Continuous
[12]	Week+Month	18	NPV+TD	Individual	Con + Bin
[16]	Week	24	DE	Global	Binary

The first entry from [3] is a model applied to a Canadian gold mine dataset containing 385 workplaces and 6 equipment types. The continuous variables express the completion of different activity groups and a limit of 10 minutes is set for computation time. The second entry from [13], describes a model in which continuous variables are used to indicate how much ore has to be moved from different ore movement locations by specific machines. It is then applied to a conceptual operation of 60 ore movement locations with 8 machines and solved in less than a minute. [12] describes a model with similar concepts in which variables also dictate ore movements performed by specific machines but also includes binary variables to indicate the starts of development activities. The model is solved using two different time periods, week for the first 12 periods and months for the following. It is applied to a conceptual mine with 30 stopes, 21 developments and 5 machines, and solved to optimality. The model presented in [16] uses binary variables to denote the start of mining activities in workplaces. The model was developed for the Lisheen mine approximately two years before its closure, and plans the extraction of the 1193 workplaces left to mine in less than 20 hours of computations. [17] later described a heuristic to improve the tractability of this last model.

On a final note, there are also in the literature some articles covering real time planning in underground mine, like [18], but they will not be covered here since they are a rather different, being closer to job shop scheduling problems.

3 Model

From the review of the available models in the literature for short term scheduling, it can be noticed that none of the models are appropriate to solve our problem. The model presented in [3], although very well adapted to our dataset is lacking in foresight to completely cover a long-term period and optimize both short- and medium-term. A medium-term model that does not cover a full long-term period cannot assess the capacity of it's results to achieve long-term objectives.

The models from [13] and [12] being focused on ore movement and production, with little planning of the development would not fit with the reality of our problem. The size of the solvable problem may also be a problem. Finally, the model presented in [16], although working very well for its dataset, does not really fit our problem since the reality of a closing mine where all of the developments are already completed is very different than the reality we are trying to represent here.

Throughout the article, the word site will be used to describe any workplace where an activity has to be performed. The word crew will refer to any team of specialized workers or pieces of equipment. In each site, a specific sequence of crews must be followed in order to carry out all activities linked with this site. This sequence varies accordingly to the nature of the site and one crew must completely finish his activities in order for the next one to start his. The precedence graph $\mathcal{G}_s^{Crew}$ will be used in this article to describe the sequence of activities in a site s where any arc (i, j) indicate that activity i most be done before j. In the same fashion, sites are linked together by precedencies relations and all activities from a predecessor site must be completed before any activity of it's successor can be started. The precedence graph $\mathcal{G}^{Site}$ will be used to refer to these precedencies. Because of the mining method used, stopes are also linked to one another by precedencies, but contrarily to the site precedencies, only the first activity of the stope sequence needs to be completed before the successor stope can be started. The precedence graph $\mathcal{G}^{Stope}$ will be used to refer to these precedencies.

The model presented in this article, uses two different time discretizations to reach adequate precision for short-term planning and the foresight needed for medium-term planning while keeping it tractable. Thus the first 3-month period is divided into 12 weeks and the remaining of the planning horizon is divided in 3-month periods (ex: 1 year = 12 one-week periods + 3 three-months periods). Three main indexes are used through the model, s refers to sites, c to crews and t to the time periods. Two other indexes, v and l are used to designate the veins and levels respectively. A list of all sets, parameters, variables and equations follow.

3.1 Sets

$\mathcal{E}_{sct}^{Ex}$ and $\mathcal{E}_{sct}^{In}$ represent respectively the set of site/crew pairs that are impossible (exclusive) and possible (inclusive) to complete in the same period t than a given site/crew pair (s, c). These sets are computed through pre-processing procedures that return the minimum length of time between the end and start of all combinations of pairs sites/crew in the precedence graphs $\mathcal{G}_s^{Crew}$ and $\mathcal{G}^{Site}$.

$\mathcal{C}_s^H$	Set of crews performing haulage activity at site s
$\mathcal{C}_s^f$	Set of crews required to perform the first task at site s
$\mathcal{C}_s^e$	Set of crews required to perform the last task at site s
$\mathcal{C}_{sc}^P$	Crew preceding crew c in site s
$\mathcal{E}^S$	Set of all sites $\quad \mathcal{E}^S = \{1, ..., S\}$
$\mathcal{E}^C$	Set of all crew types $\quad \mathcal{E}^C = \{1, ..., C\}$
$\mathcal{E}^T$	Set of all time periods $\quad \mathcal{E}^T = \{1, ..., T\}$
$\mathcal{E}^L$	Set of all levels $\quad \mathcal{E}^L = \{1, ..., L\}$
$\mathcal{E}^V$	Set of all veins $\quad \mathcal{E}^V = \{1, ..., V\}$
$\mathcal{E}_l^S$	Set of sites located on level l
$\mathcal{E}_v^S$	Set of sites located in vein v
$\mathcal{E}_{sc}^D$	Set of pairs (s', c') that have to respect a certain delay after the completion of (s,c)
$\mathcal{E}_{sct}^{Ex}$	Set of pairs (s', c') that are not feasible in period t if (s,c) are active
$\mathcal{E}_{sct}^{In}$	Set of pairs (s', c') that are feasible in period t if (s,c) are active
$\mathcal{P}_s^P$	Set of sites preceding site $s \quad (s', s) \in \mathcal{G}^{Site}$
$\mathcal{S}_s^P$	Set of stopes preceding stope $s \quad (s', s) \in \mathcal{G}^{Stope}$
$\mathcal{S}_s^A$	Set of stopes adjacent to stope s

3.2 Parameters

Parameters $T^{D}_{scs'c'}$, $T^{S}_{scts'c't'}$ and $T^{SL}_{scts'c't'}$ are also obtained from a pre-processing using the precedence graphs $\mathcal{G}^{Crew}_{s}$ and $\mathcal{G}^{Site}$. Parameters T^{S}_{sc} and T^{C}_{sc} are both measures of time for a crew in a site but represent different values. T^{C}_{sc} is equivalent to the time actually spent by the specified crew in the site whereas T^{S}_{sc} is the minimum span between the start of the crew's activities in a site and its completion. The difference between these two values comes from the fact that some crew, like the jumbo drill for example, must visit more than one site per shift to be at full capacity. Thus, the actual time spent in each of these sites is lower than the span of the activity. Parameters T^{start}_{st} and T^{finish}_{st} are derived from these values and from the minimum rate. Finally, parameter R_t was used to lighten the formulation and is worth 1 for the short-term periods and 12 for the medium-term.

A_{ct}	Available crews of type c at time t (units)
C_{sc}	Cash flow associated with the activities of crew c at site s
T^{P}	Number of work hours available per time period (hours)
T^{Tot}_{s}	Maximum time span in weeks of all activities in site s (weeks)
$T^{D}_{scs'c'}$	Minimum time span in hours between the end of crew c activities at site s and the start of crew c' at site s' (hours)
$T^{S}_{scts'c't'}$	Minimum waiting time in hours for the start at site s' of crew c' at period t' if site s, crew c was active at period t (hours)
$T^{SL}_{scts'c't'}$	Minimum waiting time in hours for the start at stope s' of crew c' at period t' if stope s, crew c was active at period t (hours)
T^{S}_{sc}	Minimum time span in hours needed for crew type c to process it's activity at site s (hours)
T^{C}_{sc}	Number of work hours needed from crew type c to process it's activity site s (hours)
T^{start}_{st}	Minimum time period where activities at site s can be started and still be active at time t
T^{finish}_{st}	Maximum time period where activities at site s can still be active if started at time t
Q_s	Rock tonnage in site s (tonnes)
Q^{M}	Maximum possible tonnage extraction in the mine for one time period (tonnes)
Q^{L}_{l}	Maximum possible tonnage extraction in level l for one time period (tonnes)
Q^{V}_{v}	Maximum possible tonnage extraction in vein v for one time period (tonnes)
Q^{O}_{t}	Minimum ore tonnage to be extracted in period t (tonnes)
R_t	Length of period t (weeks)
δ_t	Discount factor for the objective at time period t

3.3 Variables

The planning variable x_{sct} is used to represent the non-cumulative completion of activities for each time period as a continuous variable. The two other binary variables χ_{sct} and γ_{st} are mostly used to enforce precedencies and limit activity duration.

x_{sct}	Fraction of work from crew c executed at site s during time period t
χ_{sct}	Binary variable indicating whether or not crew c is active at site s during time period t
γ_{st}	Binary variable indicating whether or not any of the crew is active at site s during time period t

3.4 Objective

The objective of the model presented in Equation (1) is to maximize the NPV of all activities executed over the horizon considered. Thus the objective function is simply the summation of the discounted values of the activities performed for each time period.

$$Max \qquad \sum_{s=1}^{S}\sum_{c=1}^{C}\sum_{t=1}^{T}\delta_t C_{sc} x_{sct} \tag{1}$$

3.5 Constraints

3.5.1 Modeling constraints

Constraints (2) limit the activity to be completed at most once. The inequality can be changed to an equality in cases where an activity has to be completed. Further discussion on this will follow in the result section.

Constraints (3) and (4) respectively link binary variables χ_{sct} to variables x_{sct} and γ_{sct}. Constraints (5) make sure that if an activity is started, it is completed within a certain number of periods, assuring a minimum completion rate. Constraints (6) forces any sites where activities are started to complete all of them unless it is started at the last period. The chosen interval of the summation helps in the branching process by making the model tighter. The constraints are also relaxed on the last period to allow the model to start activities in this last period that would in real-life be completed on the following weeks or months. Constraints (7) ensure that if an activity completion spans over more than 3 time periods, it is completed at the maximum rate between the beginning and end period.

$$\sum_{t\in\mathcal{E}^T} x_{sct} \leq 1 \qquad \forall s, c \tag{2}$$

$$x_{sct} - \chi_{sct} \leq 0 \qquad \forall s, c, t \tag{3}$$

$$\chi_{sct} - \gamma_{st} \leq 0 \qquad \forall s, c, t \tag{4}$$

$$\sum_{t\in\mathcal{E}^T} \gamma_{st} \leq T_s^{Tot} \qquad \forall s, c \tag{5}$$

$$\chi_{sc't} - \sum_{t'=T_{st}^{start}}^{T_{st}^{finish}} x_{sc''t'} \leq 0 \qquad \forall s, c' = \mathcal{C}_s^f, c'' = \mathcal{C}_s^e, t \in \mathcal{E}^T \setminus \{T\} \tag{6}$$

$$T^P \chi_{sct-1} - T_{sc}^S x_{sct} + T^P \chi_{sct+1} \leq T^P \qquad \forall s, c, t \in \mathcal{E}^T \setminus \{1, T\} \tag{7}$$

3.5.2 Tonnage constraints

Constraints (8) limits sum of haulage activities to the mine limit, often corresponding to the shaft or ramp haulage limit. Constraints (9) and (10) impose similar limits on haulage activities respectively for each level and vein. These level and veins limits usually correspond to the haulage capacity for certain sectors of the mine, or an estimate limit to avoid congestion delays in a section's operations. Constraints (11) enforces a lower bound on the amount of ore extracted for each time period. These lower bounds correspond to the minimum ore tonnage needed to keep the mill active during any time period.

$$\sum_{s\in\mathcal{E}^S} x_{sc't} Q_s \leq Q^M \qquad \forall c' \in \mathcal{C}_s^H, t \tag{8}$$

$$\sum_{s\in\mathcal{E}_l^S} Q_s x_{sc't} \leq Q_l^L \qquad \forall l, c' \in \mathcal{C}_s^H, t \tag{9}$$

$$\sum_{s\in\mathcal{E}_v^S} Q_s x_{sc't} \leq Q_v^V \qquad \forall v, c' \in \mathcal{C}_s^H, t \tag{10}$$

$$\sum_{s\in\mathcal{E}^S} x_{sc't} Q_s \geq Q_t^O \qquad \forall c' \in \mathcal{C}_s^H, t \tag{11}$$

3.5.3 Time constraints

Constraints (12) make sure that the sum of the time spend by all crews in a period is lower than the available time in the period. Constraints (13) make sure that the span associated with the completion of any activity in a time period is lower than the available time in the period. One may point out that Constraints (12) are included in Constraints (13), but the presence of Constraints (12) makes the formulation much tighter since it provides a stronger link between variables x_{sct} and χ_{sct} than Constraint (2) in cases where T_{sc}^S is greater than $T^P R_t$. Constraints (14) limits the amount of time needed by any crew type for a time period to the number of work hours available for the type.

$$T_{sc}^S x_{sct} - T^P R_t \chi_{sct} \leq 0 \qquad \forall s, c, t \tag{12}$$

$$\sum_{c\in\mathcal{E}^C} T_{sc}^S x_{sct} \leq T^P R_t \qquad \forall s, t \tag{13}$$

$$\sum_{s\in\mathcal{E}^S} T_{sc}^C x_{sct} \leq A_{ct} T^P R_t \qquad \forall c, t \tag{14}$$

3.5.4 Precedencies constraints

Constraints (15) and (16) enforce respectively the precedency relations between the activities of graphs $\mathcal{G}_s^{Crew}$ and $\mathcal{G}^{Site}$. Constraints (17) make sure that a pair of ancestor/descendant in $\mathcal{G}^{Site}$ that are separate by too much time cannot be completed in the same time period and help the branching process by linking integer variables together. Constraints (18) on the other hand make sure that the time spent on a pair of ancestor/descendant in $\mathcal{G}^{Site}$ in a time period is lower than the available time in the period minus the minimum time between them. Those are necessary to ensure that the model does not plan for the execution of an ancestor and its descendant simultaneously. Without this constraint a feasible solution could require to work the equivalent of a whole period in a site and it's successor in the same time period. Similarly, Constraints (19) ensure that the time distance between a pair of ancestor/descendant in $\mathcal{G}^{Site}$ is respected among activities completed in different time periods while tightening the formulation.

$$\sum_{t'=1}^{t} x_{sc't'} - \chi_{sct} \geq 0 \qquad \forall s, c, c' \in \mathcal{C}_{sc}^{P}, t \tag{15}$$

$$\sum_{t'=1}^{t} x_{s'c't'} - \chi_{sc''t} \geq 0 \qquad \forall s, s' \in \mathcal{P}_s^{P}, c' \in \mathcal{C}_s^{f}, c'' \in \mathcal{C}_{s'}^{L}, t \tag{16}$$

$$\chi_{sct} + \chi_{s'c't} \leq 1 \qquad \forall s, c, t, \{s', c'\} \in \mathcal{E}_{sct}^{Ex} \tag{17}$$

$$T_{scs'c'}^{D}\chi_{sct} + T_{scs'c'}^{D}\chi_{s'c't} + T_{sc}^{S}x_{sct} + T_{s'c'}^{S}x_{s'c't} \leq T^{P}R_t + T_{scs'c'}^{D} \qquad \forall s, c, t, \{s', c'\} \in \mathcal{E}_{sct}^{In} \tag{18}$$

$$T_{scts'c't'}^{L}\chi_{sct} + T_{scts'c't'}^{L}\chi_{s'c't'} + T_{sc}^{S}x_{sct} + T_{s'c'}^{S}x_{s'c't'} \leq T^{P}(R_t + R_{t'}) + T_{scts'c't'}^{L} \qquad \forall s, c, t, \{s', c', t'\} \in \mathcal{E}_{sc}^{D} \tag{19}$$

3.5.5 Stopes constraints

Constraints (20) make sure that the first activity of a stope can only start when the first activity of its predecessor in $\mathcal{G}^{Stope}$ is completed and Constraints (21) forbid any activity to happen in an adjacent stope during a stope's curing time.

$$\sum_{j=1}^{t} x_{s'c't} - \chi_{sc''t} \geq 0 \qquad \forall s, s' \in \mathcal{S}_s^{P}, c' \in \mathcal{C}_s^{f}, c'' \in \mathcal{C}_{s'}^{F}, t \tag{20}$$

$$T_{scts'c't'}^{SL}\chi_{sct} + T_{scts'c't'}^{SL}\chi_{s'c't'} + T_{sc}^{S}x_{sct} + T_{s'c'}^{S}x_{s'c't'} \leq T^{P}(R_t + R_{t'}) + T_{scts'c't'}^{SL} \qquad \forall s, c, t, \{s', c', t'\} \in \mathcal{S}_s^{A} \tag{21}$$

3.5.6 Definition constraints

Constraints (22) and (23) define the non-negative and binary nature of the variables.

$$x_{sct} \geq 0 \qquad \forall s, c, t \tag{22}$$

$$\chi_{sct}, \gamma_{sct} \in \{0, 1\} \qquad \forall s, c, t \tag{23}$$

As seen from the constraints, the model creates preemptive schedules with certain limitations for example on the duration of each activity. The continuous variables are mostly chosen to produce schedules that are less dependent on the time discretization used. The advantage of using continuous variables in situations similar to ours was demonstrated in [3]. The model also includes some constraints that are redundant with others, like Constraints (12) and (13), to make the model tighter. Those constraints were added following early tests that showed the complexity of the problem and their addition proved to speed up the resolution. The general formulation was made to be as tight as possible.

4 Results

In order to test the tractability of our model, a dataset based on values taken from an operating Canadian gold mine was used. It includes 338 possible workplaces where 842 activities must be completed by 10 types of specialized crew or equipment. Although the costs and profits used for the tests presented further in this article are not the ones actually used at the mine, they represent a realistic approximate. For a complete description of the dataset, the reader is referred to [3]. All of the tests presented below were implemented using IBM ILOG CPLEX Optimization Studio version 12.8.0.0 branch-and-cut algorithm with up to eight threads. The computer used to run the tests used an Intel(R) Core(TM) i7-4770 CPU @ 3.40GHz and 16 GB of RAM. The optimality precision tolerance, the maximum optimality gap at which a solution is considered optimal, was set to 0.1% and a time limit for all test was set to one hour. This section will first present the results of the application of our model to the original model and then improvements made to increase applicability and tractability.

4.1 Initial model

The datasets referred to here as scenarios one to five, in order of complexity, were used to test our model in its initial version. The unmodified dataset corresponds to Scenario 2, with Scenario 1 being a variation where development lengths were randomly increased by 0 to 40%. Scenarios 3, 4 and 5 are datasets where veins with stopes and developments were added to the initial dataset. Table 3 presents the result of the implementation of our model to the different scenarios in terms of variables and constraints. A procedure based on the computation of the earliest start of each task from precedence graph $\mathcal{G}^{Site}$ was used to reduce the size of the problems.

Table 3: Variables and Constraints of Scenarios 1 to 5

	S1	S2	S3	S4	S5
Continuous Variables	1314	1703	2877	3860	3981
Binary Variables	2392	3080	5231	7060	7286
Constraints	24323	31307	52048	68240	71033

Table 4 shows the numerical results of the implementations. The first and second lines of the table show the objective function value of the best integer solution found and the relative gap between this solution and the best upper bound available at the termination. The next two lines indicate the time needed to solve the initial linear relaxation at node zero of the branching tree and its objective value. The following three lines refer respectively to the number of nodes processed during the branching procedure, the number of nodes remaining at the end of it, and the number of integer solutions found along the way. The first three lines of the time section indicate how much time was needed to find a solution respectively within 5%, 1% and 0.1% of the optimal solution. The final lines then indicate the time needed to find the final solution and the total computation time.

From Table 4, it can be observed that scenarios 4 and 5 could not be solved to optimality. Scenario 4 was stopped by the time limit and Scenario 5 was stopped because the size of its branching tree exceeded the available memory after 40 minutes of computation. When looking at the LP Relaxation results, it can be observed that the LP relaxation of the problem is a bad estimate of our problem. If for the smaller datasets the integrity gap (the difference between the objective value and the LP relaxation value) is reasonable at less than 20%, it grows steadily with the size of the problem to reach near to 80%. [3] illustrates with an example how the relaxed solution of a problem with similar properties is far from the integer solution. The branching results show that many integer solutions are found along the branching tree, which shows that the feasibility is not the most constraining part of the problem. The time section also corroborates this theory with several good solutions found early in the resolution. In most cases, the longest time is spent proofing the optimality rather than finding the solutions. This combination of a great number of solutions found and a lot of time spend on optimality proofing is an indicator that there is a lot of symmetry in our problem, caused by many similar options in planning. CPLEX comes with algorithms to help break symmetry in problems, and even with the most aggressive symmetry breaking setting no difference in solving time was noticed. This is also why many efforts were put on trying to make the formulation as tight as possible.

Table 4: Initial results of scenarios 1 to 5

Value	**S1**	**S2**	**S3**	**S4**	**S5**
Objective	1.60E+07	2.19E+07	2.56E+07	2.54E+07	2.61E+07
Gap (%)	0.1	0.1	0.1	5.1	0.6
LP Relaxation					
Time	0.02	0.02	0.1	0.22	0.22
Value	1.94E+07	2.65E+07	3.84E+07	4.59E+07	4.68E+07
Branching					
Processed	5467	122305	1133175	615295	656620
Remaining	925	40386	325682	264139	295912
Nbr of Solution	31	30	61	41	23
Time (s)					
5%	0.76	1.96	12.7	-	-
1%	0.76	1.96	12.7	-	-
0.1%	0.76	2.21	12.8	-	-
Best Solution	5.06	12.0	103.6	251.5	71.1
Total	5.40	140.2	1946.4	3600	2360.4

4.2 Improved model

From the observation of the original results, many conclusions were made about possible improvements. First, in its original formulation, the problem grows quickly to become intractable with the addition of more zones and secondly, the schedules produced were not realistic. The reason is that in the solutions produced, the model would schedule all the possible stopes as early as possible and then makes the least possible development and push it as far as possible to maximize the NPV. This gave solutions where, in the first weeks, no activities are scheduled and then only the necessary developments are performed. This kind of schedule is for obvious reasons not realistic since the different equipment available in the mine are left idle for long periods of time whereas in real life, they would be assigned to developments. This aversion of the model to development also creates solutions where no developments are done in preparation for stopes to be extracted outside of the resolution time frame. Now the reason why the NPV works correctly for many of the available models in the literature and not for ours is that these models use as input a pre-defined set of tasks that have to be completed rather than a range of possible tasks to choose from, like ours. This forces the models to complete necessary developments even if they represent a loss on the short-term. In our case though, the model is not forced to complete these tasks.

To palliate these problems, a modification was made to the objective. Instead of using the NPV, the absolute values of the discounted profits and expanses were used. The idea behind it is that this change will mainly move the non-critical developments earlier in the planning while keeping the production, and the critical developments leading to it, mostly unchanged. The reasons for it are that first, in a precious metal mine, the production revenues are typically much higher than any development cost. In our dataset, the average cost of a development is less than 5% of the average profit of a stope. Moreover, development is the main limiting factor for production since any available production is extracted as soon as possible. Thus, the model tries to complete production activities as early as possible and since all development activities are predecessors to a production activity in $\mathcal{G}^{Site}$, all developments on the critical path to the production are also completed as early as possible. Using the absolute value of the NPV does not change this fact, and so, it will mostly affect the non-critical developments that will be completed earlier than later.

This effect is actually desirable for two main reasons. First, in underground mines, costs are computed in $\$/m$ and are mostly due to personnel, equipment and consumables. An exception to this rule would be for the main developments, like shafts for example, that are much more expensive to complete, but the scheduling of these developments are decided at the highest level of planning and thus are not included in our model. For a given crew, there is very little difference in cost between possible assignments. Thus, any

Table 5: Improved results of scenario 1 to 5

Value	S1	S2	S3	S4	S5
Objective	2.58E+07	3.16E+07	3.64E+07	3.62E+07	3.66E+07
Gap (%)	0.1	0.1	0.1	4.7	0.1
LP Relaxation					
Time	0.02	0.03	0.19	0.38	0.38
Value	2.84E+07	3.60E+07	4.95E+07	5.77E+07	5.84E+07
Branching					
Processed	3367	8704	16670	133239	13122
Remaining	134	1642	214	34834	1112
Nbr of Solution	74	63	65	162	46
Time (s)					
5%	1.38	3.25	214.9	-	395.7
1%	1.53	3.25	214.9	-	395.7
0.1%	1.62	6.26	252.3	-	395.7
Best Solution	4.98	26.4	252.5	3582.5	697.2
Total	4.98	26.5	255.6	3600	697.2

Table 6: Improved results of scenario 1 to 5 with parameter optimization

Value	S1	S2	S3	S4	S5
Objective	2.58E+07	3.16E+07	3.64E+07	3.63E+07	3.66E+07
Gap (%)	0.1	0.1	0.1	0.1	0.1
Total time (s)	3.39	15.8	216.9	3032.8	1177.5

planning with maximum equipment usage will produce the same development cost and there is no gain in delaying its execution. Secondly, activities in underground mines are subject to a lot of uncertainty and it is common practice for planners to start developments sooner than later to palliate to unpredictable delay that could happen in the execution of the activities. This practice helps produce more robust solutions. On a mathematical perspective, such a model should also be simpler to solve since instead of having two conflicting objectives; pushing development as far as possible and bringing production as early as possible, the objective consists of doing the maximum in the time horizon with a priority on the most valuable stopes.

Table 5 shows the results of the application of the modified model using the same format as used in Table 4. First, it can be noticed that Scenario 5 was solved to optimality in less than 700 seconds when it could not be solved with the previous formulation, and Scenario 4 reached a slightly smaller, but still not acceptable, optimality gap. The LP Relaxation section shows solving times of the same order but smaller integrity gaps, all of them under 60%. The number of branching necessary to proof optimality is also decreased in all of the scenarios. The most important result though is the decrease in total solving time for all scenarios. Many different combinations of parameters were tested in order to improve the solving efficiency and it was found that the "Hidden Feasibility" emphasis in branching implemented in CPLEX gave the best results. Table 6 gives an overview of the results of the implementation with the optimal parameters. Similar parameters were tested with the original model but did not improve the solving time, which is why the default parameters were used for the comparison. The main takeaway of this last table is that with the right objective and solving parameters, all of the scenarios could be solved to optimality within 1 hour of computations.

Figure 2 clearly illustrates the mathematical advantages of the modified objective. It is a graphical representation of values of χ_{sct} taken from the integer and relaxed solutions of the initial (NPV) and improved (ABS) model. The sites are all stopes following one another from the same vein and the crew chosen is a drilling team. The time periods represented by each column in the figure are the four last 3 months time periods. It can be noticed first that the integer solutions for both models are the same. The relaxation of the improved model is also very similar to the solution with all variables being already integers. Then the relaxation of the initial model shows many fractional values. This example clearly illustrates why the modified problem takes less branching to solve to optimality. These sites were chosen because they best express the advantages of the modified objective, but many other variables from the relaxation were fractional. Still, when looking at the whole problem, 7.2% of non-zero χ_{sct} variables were integers in the initial relaxation whereas 28.1% of them were integers in the modified relaxation, which clearly shows that the trend seen in the previous example can also be seen in the whole model.

	Integer (ABS)				Integer (NPV)				LP Relaxation (ABS)				LP Relaxation (NPV)			
Stope 1	1	0	0	0	1	0	0	0	1	0	0	0	0.4	0	0	0
Stope 2	1	0	0	0	1	0	0	0	1	0	0	0	0.4	0.3	0.1	0.1
Stope 3	1	1	0	0	1	1	0	0	1	0	0	0	0.4	0.4	0.1	0
Stope 4	0	1	0	0	0	1	0	0	0	1	0	0	0	0.7	0.1	0.1
Stope 5	0	1	0	0	0	1	0	0	0	1	0	0	0	0.7	0.1	0.1
Stope 6	0	0	1	0	0	0	1	0	0	0	1	0	0	0	0.9	0.1
Stope 7	0	0	1	0	0	0	1	0	0	0	1	0	0	0	0.9	0.1
Stope 8	0	0	1	0	0	0	1	0	0	0	1	0	0	0	0.9	0.1
Stope 9	0	0	0	1	0	0	0	1	0	0	0	1	0	0	0.1	0.9
Stope 10	0	0	0	1	0	0	0	1	0	0	0	1	0	0	0	0.9
Stope 11	0	0	0	1	0	0	0	1	0	0	0	1	0	0	0	0.9

Figure 2: Comparison of χ_{sct} values for initial and improved model

The advantages of the modified model in a computational perspective were proven in the previous paragraphs, and the practical advantages will now be covered. Table 7 indicates for each of the tested scenarios the practical results. The first section of the table shows the total NPV as well as the discounted value of the development and production part of the solution for the initial model. The number of completed sites is then shown on the last line. The second section shows first the value of the modified objective for each solution in line ABS. The solution was then used to compute the actual NPV and the results with the detailed value of discounted development and production are displayed. The number of completed sites is then presented on the last line. The final section presents the difference between the value of discounted productions in percentage and the difference in number of completed sites for the initial and modified models.

Table 7: Practical comparison of initial and modified objective

Initial	**S1**	**S2**	**S3**	**S4**	**S5**
NPV (M$)	16.03	21.87	25.62	25.42	26.11
Development (M$)	-3.26	-3.32	-3.53	-3.50	-3.32
Production (M$)	19.28	25.19	29.15	28.92	29.43
Nb completed	148.0	210.0	227.0	226.0	221.0
Modified					
ABS (M$)	25.74	31.64	36.39	36.30	36.63
NPV (M$)	12.82	18.74	22.57	21.72	22.24
Development (M$)	-6.46	-6.45	-6.91	-7.29	-7.19
Production (M$)	19.28	25.19	29.48	29.01	29.43
Nb completed	204.1	283.1	325.5	337.8	324.5
Comparison					
Production (%)	-0.0219	0.0002	1.4823	0.4221	0.0005
Δ Nb completed	56.1	73.1	98.5	111.8	103.5

When looking at the NPV of each scenario for the initial and modified model, it is clear that the NPV of the solutions of the initial model are always higher than the modified one. But when taking a closer look, one can notice that the amount of development made by the modified model is always much higher. This is also confirmed by the number of completed sites that is also higher for the modified model. As explained before, since the development costs are mostly fixed and would be spent in any way by keeping the equipment active, the modified model solutions are much closer to a realistic schedule by completing as much as possible during the time available. For the production part, where the profit is made, we notice similar numbers with the value of the modified model being slightly higher for most of the scenarios. This proof that using that absolute value of the objective creates solutions for production that are very similar to what the standard NPV would produce. The fact that most of the production value for the modified model are higher also show that by trying to avoid development expenses, the model can produce solutions that do not get the most production done during the time allowed. The only exception to this is for the first scenario, but the difference being smaller than the optimality gap, it can be considered as negligible.

4.3 Application-oriented model

In order to test the model with scenarios that are closer to what a normal usage would be by the planers of an underground mine, a new set of scenarios were developed. Scenarios 6 to 9 are all identical to the scenario (2), with the exception that the time horizon considered changed and different sites were forced to be completed. This is done by changing constraints (2) to an equality for the sites that must be completed. Scenario 6 use a time horizon of 1 year and 3 months, as scenario 2, but forces the completion of the development of a vein that was not included in the solution of scenario 2. Scenario 7 uses a time horizon of 1 year and 9 months (twelve week periods and six "3 months" periods) and Scenario 8 uses a time horizon of 2 years and 3 months (twelve week periods and eight "3 months" periods). Finally, Scenario 9 uses a time horizon of 3 years and 3 months (twelve-week periods and twelve "3 months" period) and forces the completion of all the sites in the dataset. Table 8 shows the characteristics in terms of variables and constraints of each scenario.

Table 8: My caption

	S6	S7	S8	S9
Continuous Variables	1714	2891	4401	7725
Binary Variables	3102	4875	7040	11710
Constraints	31402	72122	140971	320181

Table 9 shows the results of the application of the modified model to scenarios 6 to 9 in the same format as the one used in previous tables. The last column, "S9 (NPV)" being the application of the original model to Scenario 9. The results show that fixing the completion of certain sites greatly improve the solving time for the model. Even for scenarios that had horizons up to two times longer than the original one, the optimal solution could be found within 12 minutes. This is due in part to a smaller integrity gaps and good solution found earlier in the branching as the time section shows. As for Scenario 9, even if the optimality could not be reached in both cases, the modified objective produced much better results. This scenario is very similar to a long-term model considering that the time horizon covers more than three years and that it plans for the executions of all the activities in the dataset. First, it produced a solution very close from the optimality tolerance and even more important, it found 109 feasible solutions where the original version did not find any. Moreover the NPV value of the best solution found by our modified model was 1.01E+08, that is less than 1% away from the best-known bound on the original problem formulation. This comparison shows very promising results for our modified objective for an application to long-term models since our modified objective could produce solutions of good quality much faster than a regular formulation. These solutions could then be used as warm starts for branching or starting points for heuristics applied on other models.

Table 9: Improved results of scenario 6 to 9

Value	**S6**	**S7**	**S8**	**S9**	**S9 (NPV)**
Objective	2.94E+07	6.19E+07	8.62E+07	1.16E+08	-
Gap (%)	0.1	0.1	0.1	0.2	-
LP Relaxation					
Time	0.03	0.08	0.16	0.38	0.72
Value	3.49E+07	7.37E+07	1.03E+08	1.17E+08	1.02E+08
Branching					
Processed	1963	81425	20430	48325	5849
Remaining	18	8694	4770	17624	1400
Nbr of Solution	36	133	89	109	0
Time (s)					
5%	2.13	31.0	48.6	-	-
1%	2.13	185.1	129.9	-	-
0.1%	2.13	185.1	132.1	-	-
Best Solution	9.71	662.3	235.4	3584.7	-
Total	10.4	663.1	289.3	3600	3600

4.4 Integration comparison

Tries were made to measure the monetary benefits of using an integrated model instead of using two separate planning for short- and medium-term. To do so, a first solve was made using a time horizon of twelve one-week periods and using the absolute value objective. The solution of this first solve was then used to fix variables for the first twelve weeks in a second solve with 12 one-week periods followed by 4 three-months periods. The solutions from these partly fixed models were then compared to the solutions of the original models but the results were not as good as expected. For all of our scenarios, the difference in the objective values was between 0.2% and 1%. The small gain in value can probably be explained in part by our modified objective that diminishes the aversion of typical models to plan for extra developments. Nevertheless, the integration could yield larger benefits on other datasets and still represents a major benefit from the planners perceptive. Grouping two planning levels together reduces the time spent on each of them and does not require the planner spend time splitting medium-term objectives in smaller portions to use as an input for short-term planning.

5 Conclusion

This article presented a model for integrated short- and medium-term underground mine planning. It uses continuous variables to produce solutions that are realistic and not dependent on the time discretization used. A modified objective was then introduced to palliate to the flaws of the original one, namely the difficulty to solve and the low usage of equipment. The advantages and applicability of the modified objective were then demonstrated with mathematical and practical demonstrations. This modified objective showed promising results especially for potential applications to long-term models. A final set of scenarios was then tested with this modified objective to demonstrate the application possibilities of such a model. Many of the underground mine planning models share common characteristics like the long chains of precedence with profitable activities coming after many expanses. These similarities lead the authors to think that the application of the modified objective to existing long-term models could lead to improvement in their tractability. Further research will be needed to explore these possibilities.

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