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M. Aziz, H. Dagdougui,
I. El Hallaoui

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Mohamad Aziz

Hanane Dagdougui

Issmail ElHallaoui

GERAD & Department of Mathematics and Industrial Engineering, Polytechnique Montréal (Québec) Canada, H3C 3A7

mohamad.aziz@polymtl.ca
hanane.dagdougui@polymtl.ca
issmail.elhallaoui@polymtl.ca

Abstract: Microgrid, a promising component of smart grid, will potentially yield a free electricity market. This paper proposes a novel construction for a community microgrid (MG) by deploying a virtual power bank which consists of household storage batteries and which mediates the communications between the MG and the macro-grid (MA). Households, representing the distributed energy resources (DER) and the demand side, are considered the MG prosumers (i.e. they are the consumers and the potential energy producers). In this paper, Nash Equilibrium strategies which minimize a linear combination of the households' energy generation cost, energy consumption cost and revenue of sold energy are found via an application of mean field control theory. The decentralized community microgrid optimization (MGO) problem via mean field control is configured for both flat rate and time of use macrogrid pricing mechanisms.

1 Introduction

The concept of microgrid (MG) has emerged as a promising platform that may integrate and coordinate a potentially large number of distributed energy resources (DERs) in a decentralized way [9]. MG provides a localized cluster of renewable energy generation, storage systems, distribution and local demand, grouped together within a limited geographical area to achieve reliable and effective energy supply with simplified implementation of smart grid functionality [6]. In addition to environmental benefits in terms of utilizing locally available resources, MG can reduce the transmission and distribution loss because of the physical proximity of DERs and loads. A concrete construction of MG that addresses all factors of energy exchanged has not been yet designed. In particular, the design of a decentralized autonomous MG where households are locally connected to the same MG, and are independently able to find their best responses has not been fully addressed in the current framework of smart grids. The notion of community energy storage has been addressed in litereture before where the game formed was non-cooperative [10] and where equilibrium prices for MGs and macro-grid (MA) are set by centralized units. Also in [12], Eduardo et al. proposed a hierarchical MG management system based on dynamic population games and a droop control.

Recent work in this field has models that are based on offline schemes where the objective is to find the day ahead actions. Chen et al. [4] proposed a real-time stochastic and robust optimization for a Monte Carlo price-based demand response management for residential appliances. Huang et al. [7] used Lyapunov optimization technique to derive an adaptive electricity scheduling algorithm by introducing the quality-of-service in electricity virtual queue and energy storage virtual queue to minimize the MG operation cost. Another online-convex-optimization programming for MG with a single turbine-boiler generator was proposed in [13] to minimize the production cost in each time step of a MG. An incentive-based game-theoretic automatic energy consumption scheduling (ECS) scheme for future residential smart grid with a nonrenewable energy generation was proposed in [11].

In this paper, we intend to optimize the energy exchange in a community MG that is connected to the MA. Here, we are formulating and optimizing the MG problem (MGO) via means of dynamical cooperative game theory. The MG will be composed of a sufficiently large number of households and a power bank. Households in the same MG are connected to each other and to the MA through the power bank. The objective of the MGO is to minimize a linear combination of each household's energy generation cost, energy consumption cost and revenue of sold energy, which in turn minimizes the aggregate total cost of the MG. The solutions to the MGO problem are found via means of decentralized MFG theory. By construction, each MG has a sufficiently large number of connected households. Hence, the finite problem converges to the infinite limit population problem with negligible error via MFG [2, 5].

1.1 Microgrid architecture

Figure 1 depicts a community MG where each household ha solar panel and is connected to the rest of the community MG households and to the MA through the power bank. In a MG, the exceeded generated energy is stored in batteries in the power bank or sold in the MG through the power bank. If the MG have no surplus, households communicate with the MA through the power bank and withdraw the needed energy to meet their current demand.

Each household in the MG has an account in the power bank whose balance represents its net amount of stored energy. Households can deposit into their accounts, withdraw from their accounts, or ask for a "energy loan" from the power bank. Households with a positive balance can sell their excess energy through the power bank. In case of a shortage in the power bank, MG will withdraw energy from the MA and meet the demand.

2 MGO: System dynamics

For the MG optimization problem (MGO) each household has the following set of time dependent variables: demand, generation (which depends on their location and technology deployed), and net storage or in other words the balance in the power bank account.
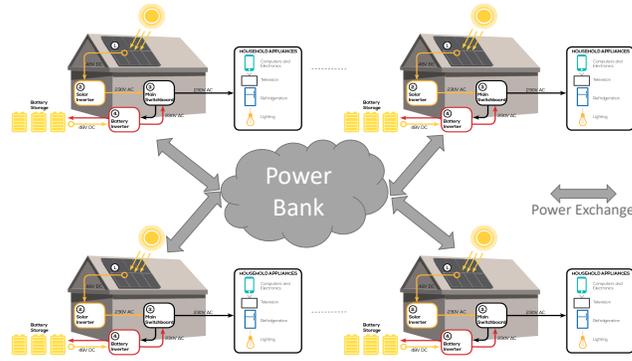
Figure 1: Community microgrid architecture with power bank

2.1 Household generation dynamics

In this paper, each household in the MG has its own coordinates abscissa and ordinate. Also, each MG has N housholds, where N is a sufficiently large number for the MFG theory to be applicable. Aziz et al. in [1] have shown that infinite control problem can be approximated by a decentralized MFG finite number problem for $N \geq 200$.

2.1.1 PV power generation model P_{PV}

In this framework we are considering the scenario where households are uniform (i.e. households deploy similar solar panel technology for generation). Using [8] and [14] the power generated by each PV at household i is given by:

$$P_{PV}^i = P_{STC}^i \times \frac{G_{ING}^i}{G_{STC}} \times \left(1 + k^i(T_c^i - T_r^i)\right) \tag{1}$$

where P_{PV} and P_{STC} are output power of the module at irradiance G_{ING} and at rated power G_{STC} respectively. T_c and T_r are cell and air temperature, respectively, and k is the maximum power temperature coefficient. Correspondingly, the generated energy Θ^i by household i at time t is governed by the following stochastic differential equation:

$$d\Theta^i = P_{PV}^i dt + \epsilon_\Theta^i dW_\Theta^i, \tag{2}$$

where W_Θ^i, $1 \leq i \leq N$, are N independent Wiener process (i.e. Brownian Motion).

Considering households are uniform, the stochasticisty in household power generation will be the result of $(G_{ING}^i, T_c^i, T_r^i)$ which are the results of air temperature and the position of the household. Here we assume that each household has adequate technology to determine T_c and T_r.

2.1.2 Household operation cost model

Dealing with PVs, solar radiation incurs zero fuel cost. Hence, the generation cost is mainly from the operation and maintenance costs (O&M). In this paper we are assuming the O&M is $\kappa = 0.1095(\$/kWh)$. Thus the cost of generation for household i is:

$$C_{O\&M}^i(t) = \kappa \times \Theta^i(t) \tag{3}$$

2.2 Demand management

Demand is the amount of electricity the household consumes (i.e. load). The load is met by withdrawing from their storage, from the MG through the power bank (i.e. from other households) or from the MA through the power bank.

For simplicity, we are assuming that household i, $1 \leq i \leq N$ has a time varying fixed load denoted by $y(t)$ which has to be met at each time.

2.3   Household power bank account

Each household has an account at the power bank. By analogy to financial banks, each household has chequing, investment, and credit accounts which are denoted by $b^i(t)$, $r^i(t)$ and $\gamma^i(t)$ respectively. $\Theta^i(t)$ and $y^i(t)$ are the amount of energy generated by household i and load of household i at time t respectively. The set of decision variables, $u_r^i(t)$ and $u_\gamma^i(t)$, represent the amount of energy sold through or withdrawn from the MG power bank, respectively. Define $\delta^i(t) := \Theta^i(t) + b^i(t) - y^i(t)$ as the net energy after meeting current demand. The dynamics of $b^i(t)$, $r^i(t)$ and $\gamma^i(t)$ are as follows :

$$b^i(t + dt) = \delta^i(t) - r^i(t) + \gamma^i(t) \tag{4}$$

$$r^i(t) = \begin{cases} u_r^i(t)\delta^i(t) \\ \text{s.t. } u_r^i(t) = 0 \text{ when } \delta^i(t) \le 0 \ \& \ 0 \le u_r^i(t) \le 1 \end{cases} \tag{5}$$

$$\gamma^i(t) = \begin{cases} u_\gamma^i(t)C_{max}^i + \mid \delta^i(t) \mid & \text{if } \delta^i(t) \le 0 \\ u_\gamma^i(t)\left(C_{max}^i - \delta^i(t)\right) & \text{if } \delta^i(t) > 0 \\ \text{s.t. } 0 \le u_\gamma(t) \le 1 & \text{for all } 0 \le t \le T \end{cases} \tag{6}$$

where $0 \le b^i(t) \le C_{max}^i$ for all t and where C_{max}^i is the storage battery capacity for household i. The dynamics are formulated such that household i will not contribute to the MG power bank at time t when household i has shortage (i.e. $\delta^i(t) \le 0$) and that $y^i(t)$ is always met for all time t and for all i $1 \le i \le N$. Using (4), (5) and (6) the dynamics of $b(t)$ can be derived to the following set of differential equations. For simplicity we will drop the household subscript i. The observations can be categorized in two scenarios; (i) scenario one (SC1) where $\delta(t) \le 0$ and (ii) scenario two (SC2) where $\delta(t) \ge 0$. In SC1, we have $\delta(t) \le 0$ then $u_r^* = 0$, $0 \le u_\gamma^*$ and:

$$\partial b = C_{max}u_\gamma^* dt \tag{7}$$

SC2 where $\delta(t) > 0$ will be divided into two cases:

- Case 1: It is optimal to sell, then $0 < u_r^*$, $u_\gamma^* = 0$ and

$$\partial b = \frac{1 - u_r^*}{1 + u_r^*}(P_{PV} - \partial y)dt \tag{8}$$

- Case 2: It is optimal to recharge, then $u_r^* = 0$, $u_\gamma^* > 0$ and

$$\partial b = \frac{1 - u_\gamma^*}{1 + u_\gamma^*}(P_{PV} - \partial y)dt \tag{9}$$

Assuming P_{PV} and y are piece wise continuous and differentiable and using (7), (8) and (9) the readers can prove that ∂b is piece wise continuous and differentiable and that u_γ and u_r are continuous.

3   MGO: Dynamical game formulation

The objective of each household is to minimize its cost function over the time period $0 \le t \le T_f$. Treating households as prosumers, household i at time t can either withdraw from or sell through the power bank. Thus in case of energy surplus at time t (i.e. $\delta^i(t) > 0$), household i can either sell their surplus to the MG through the power bank (i.e. $u_r^i(t) > 0$) or buy from the power bank to fill its battery (i.e. $u_\gamma^i(t) > 0$). On the other hand, in the case of energy shortage at time t (i.e. $\delta^i(t) \le 0$) household i will withdraw energy from the power bank to meet its current load and optimally fill its battery; thus, $u_r^i(t) = 0$, $u_\gamma^i(t) \ge 0$ and $\gamma^i(t) \ge \mid \delta^i(t) \mid$. The state variables of each household at time t are: energy generated $\Theta(t)$, energy balance in the power bank $b(t)$, demand $y(t)$, $\gamma(t)$ and $r(t)$. Denote by $\mu_y(y, t)$ and $\mu_\Theta(\theta, t)$ the probability density function for y and Θ at time t respectively.

3.1 MG equilibrium price: P_{MG}

The overall objective of the MGO is to minimize the aggregate cost. Thus regarding the energy equilibrium price in the MG, we will adopt a pricing mechanisim that maximizes the social welfare (i.e. households will minimize their individual costs which in turns minimize the aggregate cost and maximize the social welfare in the MG). For this reason we will use Walrasian equilibrium theory to find the MG equilibrium price [15, 3]. In essence, the key result of the Walrasian Equilibrium theory is the fundamental first welfare theorem.

For the simplicity of the model, we will assume household i is buying energy from itself. Thus the aggregate demand in the MG at time t is the sum of the households' demands at time t i.e. $\sum_{i=1}^{i=N} y^i(t)$. Assuming that the retail price of energy is proportional to the first order derivative of the time-dependent generation cost and using the fundamental first welfare theorem, the pareto-optimal pricing mechanism denoted by P_{MG}^* in the MG is proportional to the aggregate cost of meeting the total load in the MG. Thus, $P_{MG}^*(t)$ is proportional to $\bigtriangledown C_u(G_{MG}(t))$ where C_u is the price function which is concave with respect to $G_{MG}(t)$ where $G_{MG}(t)$ is the aggregate load in the MG i.e.

$$G_{MG}(t) = \sum_{i=1}^{i=N} y^i(t) = N \times E\{y(t)\} = N \int_{\Omega_y} y\mu_y(y, s)dy \tag{10}$$

where Ω_y is the range of $y(t)$ for all $0 \le t \le T$.

The MA energy pricing mechanisms considered here are of two types: (i) flat rate (i.e. P_{MA} is constant over time) and (ii) time of use (TOU) (i.e. $P_{MA}(t)$ is time dependent). Households in the MG know which mechanism is applied and have access to P_{MA} at any t, $0 \le t \le T$.

3.2 Decentralized mean field optimal control

The MFG approach is a decentralized control theory where each agent finds the expected value of the mass and finds its best response accordingly. The result of such approach is proven to be the Nash Equilibrium i.e. households have no incentive to deviate. Each household action has a negligible influence on the aggregate value of the energy produced or demanded in the grid. Hence, household individual action has negligible effect on P_{MG} i.e.

$$\frac{P_{PV}^i}{\sum_{1 \le i \le N} P_{PV}^i} \sim \frac{y^i}{\sum_{1 \le i \le N} y^i} \sim \epsilon.$$

The state variables are $\{\Theta^(t), y^i(t), \gamma_i(t), r_i(t)\}$. The decision variables are: $u_r^i(t)$ and $u_\gamma^i(t)$. Each agent knows the probability density functions for $y(t)$ and $\Theta(t)$ for all t, $0 \le t \le T$.

3.3 System performance and cost functions

The cost function $L(b, y, \Theta, \gamma, r, t)$ is given by

$$L(\bullet) := E \int_0^{T_f} (\gamma(t)\,(w_1(t)P_{MG}(t) + w_2(t)P_{MA}(t))$$
$$- r(t)P_{MG}(t) + \kappa\Theta)dt \tag{11}$$

where w_1 and w_2 denote the fraction of demand consumed from the MG and the MA respectively. Assuming fairness in the MG, i.e. households are incurred the same ratio of consumption from the MG and the MA, then

$$w_1(t) = \min\left\{1, \frac{\sum_i \Theta^i(t)}{\sum_i y^i(t)} = \frac{E\{\Theta(t)\}}{E\{y(t)\}} = \frac{\int_{\Omega_\Theta} \Theta\mu_\Theta d\Theta}{\int_{\Omega_y} y\mu_y dy}\right\} \tag{12}$$

and $w_2 = 1 - w_1$ where Ω_Θ is the range of $\Theta(t)$ for all t, $0 \le t \le T$. Recall the P_{MG} in (10), the expected cost function can be written as:

$$l(\bullet) := \kappa\Theta + N\nabla C_u\left(\int_{\Omega_y} y\mu_y dy\right)(\gamma w_1 - r) + \gamma(t)w_2 P_{MA}$$

Hence the cost function in (11) becomes:

$$L(\bullet) = \int_0^{T_f} (\kappa\Theta + N\nabla C_u \left(\int_{\Omega_y} y\mu_y dy \right) (\gamma w_1 - r)$$
$$+ \gamma(t) w_2 P_{MA}) dt \tag{13}$$

In (13), coupling occurs in the cost function where the aggregate demand and aggregate generated energy affects the decision of the household, particularly in P_{MG}. Following the cost function $L(\bullet)$ in (13) the cost to go $J(b, y, \Theta, \gamma, r, s)$ is given by:

$$J(\bullet) = E\left[\int_s^{T_f} L(b_t, y_t, \Theta_t, \gamma_t, r_t) dt \right] \quad s.t. \tag{14}$$
$$b_s = b, y_s = y, \Theta_s = \Theta, \gamma_s = \gamma, r_s = r$$

and accordingly the value function $v(\bullet)$ is given by:

$$v(b, y, \Theta, \gamma, r, s) = \inf_{u_\gamma, u_r} J(b, y, \Theta, \gamma, r, s) \tag{15}$$

Assuming that all the functions are sufficiently smooth then the mean field Hamilton-Jacobi-Bellman equations (MF-HJBs) for each scenario are:

- SC1: $\delta(t) \leq 0$, $u_r^* = 0$ and the MF-HJB-SC1 is:

$$-\frac{\partial v}{\partial t} = -\kappa\Theta + P_{PV}\frac{\partial v}{\partial \Theta} + \frac{\epsilon_\Theta^2}{2}\frac{\partial^2 v}{\partial \Theta^2} \tag{16}$$
$$+ \inf_{u_\gamma} \left\{ \gamma\left(w_1 P_{MG} + w_2 P_{MA}\right) - rP_{MG} + C_{max}u_\gamma\frac{\partial v}{\partial b} \right\}$$

and thus:

$$u_\gamma^* = \inf_{u_\gamma} \left\{ C_{max}u_\gamma\frac{\partial v}{\partial b} \right\} \Rightarrow u_\gamma^* = \begin{cases} 0 & \text{if } \frac{\partial v}{\partial b} \geq 0 \\ 1 & \text{if } \frac{\partial v}{\partial b} > 0 \end{cases} \tag{17}$$

- SC2: $\delta > 0$ i.e. household can either sell the surplus, fill the battery or do nothing. The MF-HJB-SC2 is:

$$-\frac{\partial v}{\partial t} = -K\Theta + P_{PV}\frac{\partial v}{\partial \Theta} + \frac{\epsilon_\Theta^2}{2}\frac{\partial^2 v}{\partial \Theta^2} \tag{18}$$
$$+ \inf_{u_\gamma, u_r} \left\{ \gamma\left(w_1 P_{MG} + w_2 P_{MA}\right) - rP_{MG} \right\}$$
$$+ \inf_{u_\gamma, u_r} \left\{ \left(\frac{1 - u_\gamma}{1 + u_\gamma} + \frac{1 - u_r}{1 + u_r} \right) (P_{PV} - dy)\frac{\partial v}{\partial b} \right\}$$

using thus:

$$\{u_\gamma^*, u_r^*\} = \inf_{u_\gamma, u_r} \left\{ \left(\frac{1 - u_\gamma}{1 + u_\gamma} + \frac{1 - u_r}{1 + u_r} \right) (P_{PV} - dy)\frac{\partial v}{\partial b} \right\} \tag{19}$$

Using (19) and taking into account that (i) P_{PV} represents the rate of change in energy production Θ (i.e. supply), (ii) dy represents rate of change in load y (i.e. consumption) and (iii) the term $\frac{\partial v}{\partial b}$ represents the rate of change in cost to go function v with respect to household balance b i.e.

$$\frac{\partial v}{\partial b} = \frac{v(t + dt) - v(t)}{b(t + dt) - b(t)}$$

the optimal solutions for the MF-HJB-SC2 equation in (18) are derived. The optimal solutions are presented in Table 1.

Table 1: Optimal Solution considering SC2

Scenario	$P_{PV} - dy$	$\dfrac{\partial v}{\partial b}$
$u^*r = 1$ $u^\gamma = 0$	≥ 0	≥ 0
$u^*r = 0$ $u^\gamma = 0$	> 0	< 0
$u^*r = 0$ $u^\gamma = 0$	< 0	≥ 0
$u^*r = 0$ $u^\gamma = 1$	≤ 0	> 0

The existence and uniqueness of the solution is dependent on the existence and uniquenss of the price in the MG. For the existence and uniquenss of the equilibrium price in MG we refer the readers to [3, 9] and for the existence and uniqueness of the MF-MGO solution we refer the readers to [1].

4 Concluding remarks

The work in this paper presents a game theoretic analysis for a community microgrid. The novel architecture of such MG is constructed by the deployment of a virtual power bank which (i) contains the households' batteries and (ii) mediates the communications between the MG households and the communication between the MG and the MA. This paper presents a formulation for the decentralized MGO problem for the constructed community MG via mean field game theory. We propose a framework that solves both the MF-HJB-SC1 and the MF-HJB-SC2 equations in (16) and (18) respectively. The solutions for the MGO are proven to be Nash Equilibrium strategies that minimize household individual cost functions and in turn the aggregate cost in the MG.

For future investigations:

- Computational investigation of the decentralized MGO problem will be held.
- Extending the framework presented in this paper to solve the general MGO problem where the MA electricity market is free (i.e. P_{MA} is based on the demand and supply solely) and households possess shiftable demand instead of just a specified load.

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