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A robust optimization approach for the winner determination problem with uncertainty on shipment volumes and carriers' capacity

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Abstract: This paper addresses the winner determination problem (WDP) for TL transportation procurement auctions under uncertain shipment volumes and uncertain carriers' capacity. It extends an existing two-stage robust formulation proposed for the WDP with uncertain shipment volumes. The paper identifies and theoretically validates a number of accelerating strategies to speed up the convergence of a basic constraint generation algorithm proposed in the literature. Experimental results prove the high computational performance of the proposed new algorithm and the relevance of considering uncertainty on the carriers' capacity when solving the WDP.

Keywords: Winner determination problem, TL services, combinatorial auctions, uncertain shipment volumes, uncertain carriers' capacity, two-stage robust optimization, constraint generation

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1 Introduction

Combinatorial auctions are increasingly used market mechanisms for the strategic procurement of Total Truckload (TL) transportation services. Truckload procurement markets generally imply two main actors: (i) shippers who need to outsource all or a part of their transport operations, and (ii) carriers who possess the required assets to offer transport services (Caplice and Sheffi, 2006). A strategic procurement implies that the shipper seeks for a long-term engagement with a set of carriers (one to three years). In an auction context, the shipper is the auctioneer. It presents its transportation requests to a set of carriers that are invited to participate in the auction, possibly after a pre-selection phase. The participating carriers compete by submitting bids on the shipper requests (shipments). In a combinatorial auction, bids are permitted on a package of shipments. So, either all the shipments of the bid are allocated, or nothing at all. Combinatorial bidding enables thus carriers to optimize their network by minimizing empty routes and balancing loads, exploiting thus the economies of scale and scope characterizing TL markets (Lee et al., 2007; Song and Regan, 2005). This would result in interesting transportation rates for the shipper.

During the auction process, three main decisional problems are addressed: the bid construction problem (also called bid generation problem), the Winner Determination Problem (WDP), and the pricing problem (Abrache et al., 2007). The bid construction problem is faced by the carrier and consists in determining the set of profitable and promising bids it should submit to the auction. The WDP and the pricing problem must rather be solved by the shipper (or more generally, the auctioneer). The WDP consists in determining the set of winning bids that optimize the shipper objective(s). The pricing problem determines the price that should be allocated to each winning carrier. Our paper addresses the WDP in a combinatorial TL transportation procurement auction.

The majority of published papers dealing with WDP in TL transportation services procurement auctions assume a deterministic environment where all data is assumed known with certainty. However, such an assumption seems unrealistic: the auction is run at the strategic phase and intends to build long-term contracts with the winning carriers. Even efficient forecasting systems cannot predict the exact volumes to be shipped between different locations for the upcoming one to three years, nor the exact available carriers' capacity.

As will be pointed out by our literature review, the research on uncertain WDP for the procurement of TL transportation services is still embryonic. To the best of our knowledge, all the published papers address a unique uncertain parameter related to the shipment volumes requested by shippers. In the following, this problem is referred to as WDP-SD (WDP with Stochastic Demand). Moreover, the solution approaches proposed to date for WDP-SD still need to be improved to solve large instances in reasonable computing times. Our paper aims at filling some of these gaps by: (1) addressing a novel problem with an additional uncertain parameter: the carriers' capacity, (2) proposing a new algorithm to speed up the convergence of a basic constraint generation algorithm proposed in the literature for the WDP-SD and efficiently solve the new problem addressed, and (3) analyzing the relevance of considering uncertainty on carriers' capacity on the auction outcomes. To the best of our knowledge, this is the first time that uncertainty on both shipment volumes and carriers' capacity is addressed and analyzed.

In the following, the problem dealing with uncertainty on both shipper demand and carriers' capacity is referred to as WDP-SDC (WDP with Stochastic Demand and Capacity). Considering uncertainty on carriers' capacity is important. Indeed, carriers generally do not use very elaborated approaches when generating their package bids. Risk seeking carriers aim to win as much contracts as possible leaving the task of managing their capacity to the operational level. Other carriers, if less risky, have generally some difficulties to accurately predict their available capacity on a daily or weekly basis given the complexity of their transportation network and the existence of a variety of commitments with other shippers.

Our paper extends the two-stage robust formulation proposed by Remli and Rekik (2013) for WDP-SD. We investigate a number of strategies to accelerate the convergence of the exact solution algorithm presented therein so that the two uncertain parameters could be simultaneously addressed. Our experimental results clearly prove the efficiency of the proposed strategies. First, when compared to the basic constraint generation algorithm proposed in Remli and Rekik (2013), our new algorithm requires less than half of the time needed for 70% of the instances solved in Remli and Rekik (2013). It also solves the 15 instances that were not solved in Remli and Rekik (2013). Second, our algorithm performs very well for the 180 new instances we generate for WDP-SDC. The average computational time is about 1.73 hours and we were able to solve to optimality instances including up to 100 auctioned contracts, 40 carriers and 800 bids.

Our experimental study also analyzes the impact of considering uncertainty on carriers' capacity on the auction outcomes. This is firstly done by comparing transportation costs, winning bids and winning carriers under two contexts: a context where only demand on shipment volumes is uncertain and a context where both shipper's demand and carriers' capacity are uncertain. Our results prove that adding this new uncertain parameter results in a substantial change in the first stage decisions (winning carriers, contracts assigned to bidding carriers) and in transportation costs. So, addressing uncertainty on carriers' capacity with robust optimization, although making the problem harder to solve, results in first-stage decisions that would have been different if only uncertainty on demand was considered. We further investigate the latter observation by considering the optimal first-stage solutions obtained under the SD (when only demand is uncertain) and the SDC (when both demand and capacity are uncertain) contexts and computing the transportation costs yielded by the corresponding recourse problem for a set of randomly generated scenarios. Our results prove that the first-stage solution under the SDC context always yields monetary savings when compared to the cost resulting from the first-stage solution under the SD context. The relative monetary saving exceeds 41% for some instances.

The remainder of the paper is as follows. Section 2 is a literature review on recent research dealing with uncertainty in WDP for TL services procurement. Section 3 describes the two-stage robust formulation proposed for WDP-SDC. It briefly recalls the deterministic and the two-stage robust formulations proposed by Remli and Rekik (2013) for WDP-SD. Section 4 identifies and theoretically validates a number of accelerating strategies for the exact algorithm presented in Remli and Rekik (2013). Section 5 presents our computational results. Finally, Section 6 summarizes our findings and opens on future research avenues.

2 Literature review

Almost ten years ago, Caplice and Sheffi (2006) pointed out the uncertainty characterizing transportation services and called for "improved robustness in the WDP". While there has been a growing trend the last years to treat OR/MS problems in uncertain contexts, research in the field of transport procurement auctions and more particularly WDP problems remains limited. Our literature review identified only four recent papers dealing with stochastic WDP in TL transportation procurement: two papers employ stochastic programming concepts and the two others use the robust optimization paradigms.

Ma et al. (2010) were the first to propose a two-stage stochastic integer programming model with recourse for the WDP with uncertain shipment demands. The first-stage decision variables define the lanes won by each participating carrier (this decision is taken in an uncertain environment, before the actual volumes are known). The second-stage decision variables (or recourse variables) assign shipment volumes to each carrier on each lane won (volumes are computed once the demands are revealed). Ma et al. (2010) handle uncertainty by considering a finite number of scenarios. Each scenario is assumed to occur with a probability according to a discrete distribution. The proposed stochastic program determines winning carriers so that the total expected cost is minimized. An equivalent deterministic mixed integer programming (MIP) model is proposed in which recourse variables and demand constraints are replicated to take into account all the generated scenarios. This model is solved with the commercial solver CPLEX. The experimental study considers instances including up to 600 lanes, 50 bidders and 10 bids per bidder. The number of scenarios vary between 3 and 40 depending on the problem size. The authors observed that an increase in the number of scenarios yields a considerable increase in computing times. Solution times range from 7 seconds to 19 hours for the largest instance (150 lanes, 30 bidders, 5 bids per lane and 10 scenarios).

Zhang et al. (2014) extend the two-stage stochastic model of Ma et al. (2010) and propose what they call a refined formulation. Their model additionally considers a continuous decision variable to enable situations where the carrier is assigned a shipment volume lower than its minimum volume requirement. The proposed solution approach is based on a Monte Carlo procedure combined with the Sample Average Approximation (SAA) technique, as is common for two-stage stochastic models including a huge number of scenarios. The Monte Carlo approach (MCA) is employed to generate representative samples. The SAA technique consists in replacing the set of all plausible scenarios in the stochastic model by a sample of scenarios and solving the equivalent deterministic MIP. Shapiro (2008) observed that the quality of the approximation improves with the size of the sample. A trade-off should thus be managed between the model solvability and the solution quality. Based on this, Zhang et al. (2014) test their solution approach on a set of moderately sized instances including up to 300 lanes, 25 carriers and 10 bids per carrier. For all the instances, the stochastic solution is generated by solving the equivalent deterministic model for 10 samples of size 10 using CPLEX 12.4. Solution times range between 94 and 1761 seconds for the largest instance (300 lanes, 25 carriers and 10 bids per carrier).

Remli and Rekik (2013) consider almost the same problem setting as in Ma et al. (2010) but model it using robust optimization techniques. It is assumed that no probability distribution is available on the uncertain demands. Uncertainties are rather represented using interval numbers. Inspired by the work of Bertsimas and Sim (2003, 2004), the authors also consider the concept of budget of uncertainty to handle realistic contexts and avoid uncommon worst-case scenarios. The budget of uncertainty is a parameter pre-specified by the shipper that restricts the total deviation of demands from their nominal values (Gabrel et al., 2014). A constraint generation algorithm is developed to solve the two stage robust formulation. At each iteration, a master problem and a recourse problem, both modelled as MIPs, are solved using CPLEX 12.4. The experimental study considers instances including up to 600 lanes, 120 carriers and 10 bids per carrier. Solution times vary between 57 and 25065 seconds (almost 7 hours). 15 instances remain unsolved within 10 hours (these instances correspond to a problem setting with 200 lanes, 80 carriers and 20 bids per carrier).

Recently, Zhang et al. (2015) proposed a two-stage robust formulation for the WDP under uncertain shipment volumes. The problem setting is almost the same as in Remli and Rekik (2013) except that shortages in the volumes assigned to carriers are permitted but penalized (as in Zhang et al. (2014)) and no constraints on minimum and maximum volumes assigned to winning carriers are imposed by the shipper. Zhang et al. (2015) prove that their robust model remains valid if a lane can be attributed to more than one carrier. The authors apply a central limit theorem based approach to construct the demand uncertainty set where only the mean and the variance of the shipping demands are to be known. Their approach handles the cases where demand on lanes are either independent or correlated. Two solution approaches are presented and compared: (1) a constraint generation algorithm following the same principle as that proposed by Remli and Rekik (2013) but in which the recourse problem is more complex (it cannot be reduced to a MIP as in Remli and Rekik (2013) given the definition of the uncertainty set); and (2) the $B\&B$ procedure of CPLEX 12.4 applied to an equivalent MIP reformulation of the two-stage robust model. The experimental study considers five problem tests and the largest instance includes 180 lanes, 20 carriers and 10 bids per carrier. The reported results prove that the MIP reformulation based approach largely outperforms the constraint generation approach. It requires 12.38 seconds to solve the largest instance.

3 The stochastic winner determination problem

3.1 Context and assumptions

We consider a TL market where a single shipper (the auctioneer) has to outsource a number of its TL transportation operations to a set of external carriers (the bidders) under uncertain shipment volumes and uncertain carriers' capacity. A shipper request is defined by a lane (i.e., an origin-destination pair) and a volume to be shipped on it. The auction being run at the strategic phase, shipment volumes are not known with certainty at this step. As in Remli and Rekik (2013), we propose to represent this uncertainty by interval numbers while using the concept of budget of uncertainty introduced by Bertsimas and Sim (2003, 2004). The demand on a lane is thus assumed to lie within an interval, and the total deviation of uncertain demands from their nominal values is restricted to a pre-specified value, namely the budget of uncertainty.

The shipper submits its requests to the participating carriers. The latter make offers in forms of combinatorial bids. Each bid gathers the set of lanes the carrier offers to serve, the price asked for shipping one volume unit on each lane, and bounds on the minimum and maximum volumes to transport. The minimum volume restriction guarantees the winning carrier to be allocated a minimum volume at the proposed price. The maximum volume restriction translates the carriers' capacity. Uncertainty is thus added on this parameter. Following the same principle as for the demand, we represent this uncertainty by interval numbers. The carriers' capacity is assumed to lie within an interval and the total deviation of the uncertain capacity from its nominal value is restricted to a pre-specified value: the budget of uncertainty. Observe that budgets of uncertainty are constant parameters pre-fixed by the shipper. They are different depending on the nature of the uncertain parameter. The budget of uncertainty on the shipment volumes is closely related to the level of sophistication of the forecasting system used by the shipper. The budget of uncertainty on the carriers' capacity is based on the carrier reliability and its past performance with the shipper. As in Ma et al. (2010) and Remli and Rekik (2013), we associate a performance factor (taking a value in [-1,1]) with each participating carrier to model its service quality (on-time delivery, percentage of cancellation, for example). We propose to determine the budget of uncertainty based on this performance factor: the higher is the performance factor value, the less reliable is the carrier and the larger is the value of the corresponding budget of uncertainty.

The objective of the WDP is to select bids and associated volumes that minimize the shipper transportation costs, such that the worst demand and the worst capacity -delimited by the associated budget of uncertainty- are satisfied. In case winning bids are not able to meet all the demand or respect the carriers' capacity, the shipper has the possibility to call a carrier from the spot market to ensure the shipment of the remaining unsatisfied demands. We assume that the transportation price available at the spot market is always larger than the price asked by the carriers participating in the auction.

As in Ma et al. (2010) and Remli and Rekik (2013), we consider XOR bids (Nisan, 2006). That is, each carrier can submit any number of bids it wants but, in the final allocation, it can be awarded at most one bid. XOR bidding enables the carrier well exploiting and managing its available capacity when generating bids. Indeed, if OR bidding were to be permitted, the carrier should take into account the fact that two or more OR bids may win forcing it to divide its available capacity between them. As in Ma et al. (2010) and Remli and Rekik (2013), we also assume that each lane is restricted to be served by at most one winning carrier. Such a constraint forces the shipper to engage with a single strategic carrier on each lane. Relaxing such a constraint would probably, in some cases, result in lower transportation costs (by combining bids from different carriers). However, in practice, dealing with a single contract server is more easily manageable for both the shipper and the services in charge at pick-up and delivery locations. Finally, the shipper is assumed to set minimum and maximum volumes to allocate to each winning carrier as well as a minimum and a maximum values on the number of winners.

Observe that our problem context is the same as that considered by Remli and Rekik (2013) except that we add uncertainty on the carriers' capacity. This was done on purpose so that comparison between the basic constraint generation algorithm proposed in Remli and Rekik (2013) and our new algorithm is possible. Our paper also intends to study the effect of adding a new uncertain parameter on the new algorithm computational performance and on the auction outcomes. Studying the impact of relaxing some of the problem constraints (such as XOR bidding, or single lane contracting) is left for future work.

Hereafter, we present the notation and terminology used and that will be adopted throughout the paper.

Notation

T	set of carriers (bidders)	UV_{tb}	capacity of carrier t in bid b
L	set of lanes	$\overline{UV}_{tb}$	nominal value of the capacity of carrier t in bid b
d_l	demand on lane $l \in L$	$\widehat{UV}_{tb}$	maximum deviation on the capacity of carrier t in bid b
$\overline{d_l}$	nominal value of d_l	c_{tb}	price asked by carrier t in bid b for transporting one unit volume on each lane $l \in \mathcal{L}_{tb}$
$\hat{d_l}$	maximum deviation of d_l	ce_l	cost of shipping one unit volume on lane l by a spot carrier
Γ	budget of uncertainty	p_t	performance factor of carrier t
B_t	set of bids of carrier $t \in T$	q_t	minimum volume to allocate to carrier t if it wins
$\mathcal{L}_{tb}$	set of lanes that carrier t offers to serve in bid b	Q_t	maximum volume to allocate to carrier t if it wins
a^l_{tb}	a constant parameter: $a^l_{tb} = 1$ if $l \in \mathcal{L}_{tb}$; $a^l_{tb} = 0$, otherwise	N_{min}	minimum number of winning carriers
LV_{tb}	minimum volume guaranteed to carrier t if bid b wins	N_{max}	maximum number of winning carriers

3.2 Deterministic model

As in Remli and Rekik (2013), we propose to model the deterministic WDP using the following three sets of decision variables:

x_{tb} = 1 if bid b offered by carrier t wins; 0, otherwise.
y_{tb} = the volume assigned to carrier t on each lane covered by winning bid b.
e_l = the volume assigned to spot carriers on lane l.

The deterministic winner determination problem is thus formulated using model (W) as follows:

$$
(\mathrm{W}) \begin{cases}
\min & \displaystyle\sum_{t\in T}\sum_{b\in B_t}(1+p_t)c_{tb}\,y_{tb} + \sum_{l\in L} ce_l\, e_l & (1)\\
\text{s.t.} & \displaystyle\sum_{t\in T}\sum_{b\in B_t} a^l_{tb}\,y_{tb} + e_l \geq d_l, \quad l \in L & (2)\\
& \mathrm{LV}_{tb}\,x_{tb} \leq y_{tb} \leq UV_{tb}\,x_{tb},\ t\in T,\ b\in B_t & (3)\\
& \displaystyle\sum_{b\in B_t} x_{tb} \leq 1, \quad t \in T & (4)\\
& \displaystyle\sum_{t\in T}\sum_{b\in B_t} a^l_{tb}\,x_{tb} \leq 1, \quad l \in L & (5)\\
& \mathrm{N_{min}} \leq \displaystyle\sum_{t\in T}\sum_{b\in B_t} x_{tb} \leq \mathrm{N_{max}} & (6)\\
& \mathrm{q}_t \displaystyle\sum_{b\in B_t} x_{tb} \leq \sum_{b\in B_t} y_{tb} \leq \mathrm{Q}_t \sum_{b\in B_t} x_{tb}, \quad t\in T & (7)\\
& x_{tb} \in \{0,1\},\ y_{tb} \geq 0, \quad t\in T,\ b \in B_t & (8)\\
& e_l \geq 0, \quad l \in L & (9)
\end{cases}
$$

The objective function (1) minimizes the shipper transportation cost. Constraints (2) ensure that the volume requested by the shipper on each lane is satisfied either by the bids submitted in the auction or through the spot market. Constraints (3) translate the minimum and maximum volume restrictions on the volume allocated to a carrier if the corresponding bid wins. Constraints (4) model the XOR bidding. Constraints (5) ensure that each lane l is assigned to one participating carrier at most. Constraint (6) sets bounds on the number of winning carriers. Constraints (7) specify the minimum and maximum volume that each carrier $t \in T$ is allowed to ship if it wins. Constraints (8) and (9) are binary, respectively, non-negative, constraints on x_{tb}, respectively, y_{tb} and e_l variables.

3.3 Robust model

As in Remli and Rekik (2013), we propose to model the stochastic WDP as a two-stage robust formulation where variables x_{tb} representing the winning bids are the first-stage variables and variables y_{tb}, respectively, e_l representing the volumes of shipments allocated to winning, respectively, spot carriers are the second-stage variables (also called recourse variables).

We consider here uncertainty on both the demand and the carriers' capacity. Recall that uncertainty on these parameters is modelled by interval numbers. Namely, each demand d_l on lane $l \in L$ is known to belong to an interval $[\overline{d}_l - \hat{d}_l, \overline{d}_l + \hat{d}_l]$, where $\overline{d}_l$ is the nominal demand and $\hat{d}_l \geq 0$ is the maximum deviation. This is combined with the concept of budget of uncertainty Γ^d which restricts the total deviation of the demands from their nominal values to a prefixed value Γ^d. Remli and Rekik (2013) observed that when the budget of uncertainty Γ^d is integer -which we assume in the rest of the paper-, it represents the number of lanes for which the demand deviates from its nominal value and takes the worst value (i.e., the greatest one) $d_l = \overline{d}_l + \hat{d}_l$ (we refer the reader to Remli and Rekik (2013) for more details).

To address uncertainty on the carriers' capacity, we define an interval $[\overline{UV_{tb}} - \widehat{UV_{tb}}, \overline{UV_{tb}} + \widehat{UV_{tb}}]$, for each carrier $t \in T$ and each bid $b \in B_t$, where $\overline{UV}_{tb}$ is the nominal capacity and $\widehat{UV}_{tb} \geq 0$ is the corresponding maximum deviation. For each carrier t, a budget of uncertainty Γ_t is considered to restrict the total deviation of the capacities from their nominal values to Γ_t. As for the demand, when the budget of uncertainty Γ_t is integer -which we assume in the rest of the paper- it represents the number of bids submitted by carrier t for which the capacity deviates from its nominal value and takes the worst value (i.e., the lowest one) $UV_{tb} = \overline{UV_{tb}} - \widehat{UV_{tb}}$.

Hence, for given values of the vector $\Gamma = (\Gamma^d, (\Gamma_t)_{t\in T})$, the robust winner determination problem, denoted $\mathrm{W}_{rob}(\Gamma)$, consists in selecting the winning bids and the associated volumes at the minimum cost, such that the *worst demands* -delimited by Γ^d- and the worst capacities -delimited by $\Gamma_t, t \in T$- are satisfied. It is formulated as follows:

$$
\mathrm{W}_{rob}(\Gamma) \left\{ \begin{array}{ll}
\min & opt(R(x,\Gamma)) \\
\text{s.t.} & \sum_{b\in B_t} x_{tb} \leq 1, \quad t \in T \\
& \sum_{t\in T} \sum_{b\in B_t} a^l_{tb}\, x_{tb} \leq 1, \quad l \in L \\
& \mathrm{N}_{\min} \leq \sum_{t\in T} \sum_{b\in B_t} x_{tb} \leq \mathrm{N}_{\max} \\
& x_{tb} \in \{0,1\}, \quad t \in T,\ b \in B_t
\end{array} \right.
$$

where $opt(R(x,\Gamma))$ represents the optimum value of the recourse problem:

$$
R(x,\Gamma) \left\{ \max_{(d,UV)\in\mathcal{U}(\Gamma)} \min_{(y,e)\in\mathcal{Y}(x)} \sum_{t\in T} \sum_{b\in B_t} (1+p_t) c_{tb} y_{tb} + \sum_{l\in L} ce_l e_l \right.
$$

The uncertainty set $\mathcal{U}(\Gamma)$ is defined by:

$$
\begin{aligned}
\mathcal{U}(\Gamma) = \{ & d \in \mathbb{R}^{|L|} : d_l = \overline{d}_l + z_l\, \hat{d}_l,\ l \in L,\ z \in \mathcal{Z}(\Gamma^d), \\
& UV_{tb} \in \mathbb{R}^{|T|\times|B_t|} : UV_{tb} = \overline{UV}_{tb} - \zeta_{tb}\, \widehat{UV}_{tb},\ t \in T,\ b \in B_t,\ \zeta \in \mathcal{Z}'(\Gamma_t),\ t \in T\}
\end{aligned}
$$

where

$$
\mathcal{Z}(\Gamma^d) = \{ z \in \mathbb{R}^{|L|} : \sum_{l\in L} z_l \leq \Gamma^d,\ 0 \leq z_l \leq 1,\ l \in L\}
$$

and

$$
\mathcal{Z}'(\Gamma_t) = \{ \zeta \in \mathbb{R}^{|T|\times|B_t|} : \sum_{b\in B_t} \zeta_{tb} \leq \Gamma_t,\ t \in T,\ 0 \leq \zeta_{tb} \leq 1,\ t \in T, b \in B_t\}
$$

The feasible set $\mathcal{Y}(x)$ includes all vectors (y, e) satisfying the following constraints:

$$\sum_{t\in T}\sum_{b\in B_t} a^l_{tb}\, y_{tb} + e_l \geq d_l, \quad l \in L \tag{10}$$

$$y_{tb} \geq \mathrm{LV}_{tb}\, x_{tb}, \; t \in T, \; b \in B_t \tag{11}$$

$$y_{tb} \leq UV_{tb}\, x_{tb}, \; t \in T, \; b \in B_t \tag{12}$$

$$\sum_{b\in B_t} y_{tb} \geq \mathrm{q}_t \sum_{b\in B_t} x_{tb}, \quad t \in T \tag{13}$$

$$\sum_{b\in B_t} y_{tb} \leq \mathrm{Q}_t \sum_{b\in B_t} x_{tb}, \quad t \in T \tag{14}$$

$$y_{tb} \geq 0, \quad t \in T, \; b \in B_t; \quad e_l \geq 0, \quad l \in L$$

The problem $\mathrm{W}_{rob}(\Gamma)$, described above, is a min-max-min problem that is difficult to solve in its current form. Following the same steps as in Remli and Rekik (2013), it can be reformulated using the following MIP model. Details of the different steps are given in the appendix.

$$\mathrm{W}_{rob}(\Gamma)' \begin{cases} \min \quad A \\ \text{s.t.} \quad A \geq \sum_{l\in L} \overline{d}_l u^\sigma_l + \sum_{l\in L} \hat{d}_l s^\sigma_l + \sum_{t\in T}\sum_{b\in B_t} x_{tb}\mathrm{q}_t g^\sigma_t - \\ \qquad \sum_{t\in T}\sum_{b\in B_t} x_{tb}\mathrm{Q}_t h^\sigma_t + \sum_{t\in T}\sum_{b\in B_t} \mathrm{LV}_{tb} x_{tb} v^\sigma_{tb} - \\ \qquad \sum_{t\in T}\sum_{b\in B_t} \overline{UV}_{tb} x_{tb} w^\sigma_{tb} + \sum_{t\in T}\sum_{b\in B_t} \widehat{UV}_{tb} x_{tb} f^\sigma_{tb}, \quad \sigma \in \mathcal{S} & (15) \\ \sum_{b\in B_t} x_{tb} \leq 1, \quad t \in T & (16) \\ \sum_{t\in T}\sum_{b\in B_t} a^l_{tb}\, x_{tb} \leq 1, \quad l \in L & (17) \\ \mathrm{N}_{\min} \leq \sum_{t\in T}\sum_{b\in B_t} x_{tb} \leq \mathrm{N}_{\max} & (18) \\ A \geq 0, \; x_{tb} \in \{0,1\}, \quad t \in T, \; b \in B_t \end{cases}$$

where $\mathcal{S}$ is the set of the extreme points $(u^\sigma, s^\sigma, v^\sigma, w^\sigma, f^\sigma, g^\sigma, h^\sigma), \sigma = 1...|\mathcal{S}|$ of the recourse problem $Q'(x, \Gamma)$ formulated as :

$$Q'(x,\Gamma) \begin{cases} \max \quad \sum_{l\in L} \overline{d}_l u_l + \sum_{l\in L} \hat{d}_l s_l + \sum_{t\in T}\sum_{b\in B_t} \mathrm{LV}_{tb} x_{tb} v_{tb} - \sum_{t\in T}\sum_{b\in B_t} \overline{UV}_{tb} x_{tb} w_{tb} \\ \qquad + \sum_{t\in T}\sum_{b\in B_t} \widehat{UV}_{tb} x_{tb} f_{tb} + \sum_{t\in T}\sum_{b\in B_t} x_{tb}\mathrm{q}_t g_t - \sum_{t\in T}\sum_{b\in B_t} x_{tb}\mathrm{Q}_t h_t \\ \text{s.t.} \\ \qquad \sum_{l\in L} a^l_{tb} u_l + v_{tb} - w_{tb} + g_t - h_t \leq (1+p_t) c_{tb}, \quad t \in T, \; b \in B_t \\ \qquad u_l \leq ce_l, \quad l \in L \\ \qquad \sum_{l\in L} z_l \leq \Gamma^d \\ \qquad s_l \leq ce_l\, z_l, \quad l \in L \\ \qquad s_l \leq u_l, \quad l \in L \\ \qquad \sum_{b\in B_t} \zeta_{tb} \leq \Gamma_t, \quad t \in T \\ \qquad f_{tb} \leq M\, \zeta_{tb}, \quad b \in B_t, \; t \in T \\ \qquad f_{tb} \leq w_{tb}, \quad b \in B_t, \; t \in T \\ \qquad z_l \in \{0,1\}; s_l, \; u_l \geq 0, \quad l \in L \\ \qquad v_{tb}, \; w_{tb}, \; f_{tb}, \; g_t, \; h_t \geq 0, \zeta_{tb} \in \{0,1\} \quad t \in T, \; b \in B_t \end{cases}$$

The variables $u_l,\ v_{tb},\ w_{tb},\ g_t,\ h_t$ are the dual variables of the minimization problem associated with constraints (10)–(14). Observe that variables $s_l,\ l \in L$ and $f_{tb},\ t \in T,\ b \in B_t$ are introduced to linearize the recourse problem which is originally bilinear (as described in the appendix). Adding these variables requires defining a big constant M to link f_{tb} and ζ_{tb} variables. For, s_l and z_l linking constraints, we use ce_l as a constant big M as suggested in Remli and Rekik (2013). More details on the recourse problem linearization are given in the appendix.

4 Solution approaches

The solution approach we propose to solve WDP-SDC uses a constraint generation algorithm inspired by the algorithm of Remli and Rekik (2013). The experimental results reported in Remli and Rekik (2013) show that solution times become relatively huge for large instances. The authors explain this by an increase in the total number of iterations. They also noticed that most of the computing time is used to solve the master problem. Based on these observations, this section proposes a number of improvement strategies to speed-up the algorithm convergence. We first adapt the constraint generation algorithm proposed by Remli and Rekik (2013) to our new WDP-SDC problem. Then, we describe the acceleration strategies.

4.1 Basic constraint generation algorithm for WDP-SDC

Algorithm 1, hereafter, adapts the constraint generation algorithm, initially proposed by Remli and Rekik (2013) for WDP-SD, to solve WDP-SDC to optimality.

Algorithm 1 Basic constraint generation algorithm for WDP-SDC

Step 0: Initialization
Define and solve the problem $\mathrm{W}^0(\Gamma)$ containing no extreme point of the recourse problem (we suppose that $u^0 = v^0 = w^0 = g^0 = h^0 = z^0 = \zeta^0 = 0$).
Set $\mathrm{LB}^0 \leftarrow -\infty$, $\mathrm{UB}^0 \leftarrow +\infty$, $r \leftarrow 0$. Go to Step 1.

Step 1: Solve the master problem

$$\mathrm{W}^r(\Gamma)\begin{cases} \min & A \\ \text{s.t.} & A \geq \sum_{l\in L}\bar{d}_l u_l^i + \sum_{l\in L}\hat{d}_l\, u_l^i\, z_l^i + \sum_{t\in T}\sum_{b\in B_t} x_{tb}\mathrm{q}_t g_t^i - \sum_{t\in T}\sum_{b\in B_t} x_{tb}\mathrm{Q}_t h_t^i + \sum_{t\in T}\sum_{b\in B_t}\mathrm{LV}_{tb}x_{tb}v_{tb}^i - \\ & \quad \sum_{t\in T}\sum_{b\in B_t}\overline{UV}_{tb}x_{tb}w_{tb}^i + \sum_{t\in T}\sum_{b\in B_t}\widehat{UV}_{tb}x_{tb}w_{tb}^i\zeta_{tb}^i, \quad i = 0\ldots r \\ & \sum_{b\in B_t} x_{tb} \leq 1, \quad t\in T \\ & \sum_{t\in T}\sum_{b\in B_t} a_{tb}^l\, x_{tb} \leq 1, \quad l \in L \\ & \mathrm{N}_{min} \leq \sum_{t\in T}\sum_{b\in B_t} x_{tb} \leq \mathrm{N}_{max} \\ & A \geq 0,\ x_{tb} \in \{0,1\}, \quad t\in T,\ b \in B_t \end{cases}$$

and denote (x^r, A^r) its optimal solution. Update $\mathrm{LB}^r \leftarrow A^r$, and go to Step 2.

Step 2: For the fixed assignments x^r, solve the recourse problem $Q'(x^r,\Gamma)$ and denote $(u^{r+1}, v^{r+1}, w^{r+1}, g^{r+1}, h^{r+1}, z^{r+1}, \zeta^{r+1})$ its optimal solution. Set

$$\mathrm{UB}^r \leftarrow \min\{\mathrm{UB}^{r-1}, \sum_{l\in L}\bar{d}_l u_l^{r+1} + \sum_{l\in L}\hat{d}_l u_l^{r+1} z_l^{r+1} + \sum_{t\in T}\sum_{b\in B_t} x_{tb}^r \mathrm{q}_t g_t^{r+1} - \sum_{t\in T}\sum_{b\in B_t} x_{tb}^r \mathrm{Q}_t h_t^{r+1} + \sum_{t\in T}\sum_{b\in B_t}\mathrm{LV}_{tb}\, x_{tb}^r v_{tb}^{r+1} - \sum_{t\in T}\sum_{b\in B_t}\overline{UV}_{tb}\, x_{tb}^r\, w_{tb}^{r+1} + \sum_{t\in T}\sum_{b\in B_t}\widehat{UV}_{tb}\, x_{tb}^r\, w_{tb}^{r+1}\, \zeta_{tb}^{r+1}\}$$

if $\mathrm{UB}^r = \mathrm{LB}^r$ **then**
  return (x^r, A^r) as an optimal solution to the problem $\mathrm{W}_{rob}(\Gamma)$;
else
  $r \leftarrow r + 1$. Go to Step 1.
end if

The idea of Algorithm 1 is to start solving a relaxation of $\mathrm{W}_{rob}(\Gamma)'$ including none of constraints (15). Constraints (15) are then added iteratively (the iterator counter is denoted r), one at a time, until an optimal solution is found. Generating a constraint (15) at an iteration r implies finding an extreme point in $\mathcal{S}$. This is done by solving the recourse problem $Q'(x^r, \Gamma)$ to optimality, where x^r is an optimal solution of a relaxation of the master problem $\mathrm{W}_{rob}(\Gamma)'$ at iteration r, denoted $\mathrm{W}^r(\Gamma)$. At each iteration, a lower bound LB and an upper bound UB for the original problem $\mathrm{W}_{rob}(\Gamma)'$ are updated and compared. The algorithm terminates when $LB = UB$, which proves the optimality of the obtained solution.

At each iteration r, the lower bound LB is updated and takes the value, denoted A^r, of the optimal objective function of $\mathrm{W}^r(\Gamma)$. Obviously, A^r is a valid lower bound since $\mathrm{W}^r(\Gamma)$ is a relaxation of $\mathrm{W}_{rob}(\Gamma)$ (it includes less constraints). Besides, the optimal solution x^r of $\mathrm{W}^r(\Gamma)$ is a feasible first-stage solution (it satisfies constraints (16)–(18)). It can thus be used to solve the recourse problem $Q'(x^r, \Gamma)$ and obtain a feasible dual second-stage solution, denoted $(u^{r+1}, v^{r+1}, w^{r+1}, g^{r+1}, h^{r+1}, z^{r+1}, \zeta^{r+1})$. Recall that it is assumed that the recourse problem is always feasible. Hence, at iteration r, one can derive the value of the objective function of $\mathrm{W}_{rob}(\Gamma)'$ in a feasible solution, which is given by : $\sum_{l\in L} \overline{d}_l u_l^{r+1} + \sum_{l\in L} \hat{d}_l u_l^{r+1} z_l^{r+1} + \sum_{t\in T}\sum_{b\in B_t} x_{tb}^r \mathrm{q}_t g_t^{r+1} - \sum_{t\in T}\sum_{b\in B_t} x_{tb}^r \mathrm{Q}_t h_t^{r+1} + \sum_{t\in T}\sum_{b\in B_t} \mathrm{LV}_{tb}\, x_{tb}^r v_{tb}^{r+1} - \sum_{t\in T}\sum_{b\in B_t} \overline{UV}_{tb}\, x_{tb}^r\, w_{tb}^{r+1} + \sum_{t\in T}\sum_{b\in B_t} \widehat{UV}_{tb}\, x_{tb}^r\, w_{tb}^{r+1}\, \zeta_{tb}^{r+1}$. This constitutes an upper bound for the original minimization problem $\mathrm{W}_{rob}(\Gamma)'$.

4.2 Accelerating the basic constraint generation algorithm

Our main observation with regard to Algorithm 1 is that models $\mathrm{W}^{r+1}(\Gamma)$ and $\mathrm{W}^r(\Gamma)$ solved at iterations r and $r+1$, respectively, differ only by a unique constraint of type (15). This constraint is generated through solving the recourse problem $Q'(x^r, \Gamma)$ associated with the optimal solution x^r of $\mathrm{W}^r(\Gamma)$. Our main improvement strategy consists in generating multiple valid inequalities for $\mathrm{W}^r(\Gamma)$ within the same iteration. This would help improve the quality of the lower bounds (LB^r) and decrease the total number of iterations of the algorithm. One can also notice that model $\mathrm{W}^0(\Gamma)$ solved at the first iteration ($r = 0$) includes no cuts of type (15). We propose thus to add a valid cut when initiating the algorithm. Finally, we propose to bound the objective value of model $\mathrm{W}^r(\Gamma)$ at each iteration r by appropriate values. We roughly present hereafter the new constraint generation algorithm we propose (Algorithm 2). More details on each step are given in the following subsections.

Algorithm 2 Improved constraint generation algorithm

Step 0: Initialization
Set $\text{LB}^{-1} \leftarrow -\infty$, $\text{UB}^{-1} \leftarrow +\infty$, $\text{LB}^0 \leftarrow -\infty$, and $\text{UB}^0 \leftarrow +\infty$.
Set $r \leftarrow 0$, $D^0 = \emptyset$, $X^{0,LS} = \emptyset$.
Step 1: Generate an initial cut as described in Algorithm 3 (see Section 4.2.2). Go to Step 2.
Step 2: Solve the master problem

$$\text{W}^r(\Gamma)\begin{cases} \min & A \\ \text{s.t.} & A \geq \sum_{l\in L} \overline{d}_l u_l^{i^k} + \sum_{l\in L} \hat{d}_l\, u_l^{i^k}\, z_l^{i^k} + \sum_{t\in T}\sum_{b\in B_t} x_{tb} \text{q}_t g_t^{i^k} - \sum_{t\in T}\sum_{b\in B_t} x_{tb} \text{Q}_t h_t^{i^k} + \sum_{t\in T}\sum_{b\in B_t} \text{LV}_{tb} x_{tb} v_{tb}^{i^k} - \\ & \quad \sum_{t\in T}\sum_{b\in B_t} \overline{UV}_{tb} x_{tb} w_{tb}^{i^k} + \sum_{t\in T}\sum_{b\in B_t} \widehat{UV}_{tb}\, x_{tb}\, w_{tb}^{i_k}\, \zeta_{tb}^{i_k}, \quad i = 0\dots r; k = 0 \dots K^i \qquad (19) \\ & \sum_{b\in B_t} x_{tb} \leq 1, \quad t\in T \\ & \sum_{t\in T}\sum_{b\in B_t} a_{tb}^l\, x_{tb} \leq 1, \quad l \in L \\ & \text{N}_{min} \leq \sum_{t\in T}\sum_{b\in B_t} x_{tb} \leq \text{N}_{max} \\ & LB^{r-1} \leq A \leq UB^{r-1} \qquad (20) \\ & A \geq 0,\ x_{tb}\in\{0,1\}, \quad t\in T,\ b\in B_t \end{cases}$$

where $K^r = |D^{r-1}| + |X^{r-1,LS}| - 1$ if $r \neq 0$, and $K^0 = 0$. Denote (x^r, A^r) its optimal solution.
$D^r = \emptyset$. In set D^r, store all the intermediary feasible solutions encountered while solving $\text{W}^r_{rob}(\Gamma)'$. Solutions are placed in a descending order with respect to the corresponding objective value (see Section 4.2.3).
Update $\text{LB}^r \leftarrow A^r$, and go to Step 3.

Step 3: For the fixed assignments x^r, solve the recourse problem $Q(x^r,\Gamma)$. Denote $(u^{(r+1)^0}, v^{(r+1)^0}, w^{(r+1)^0}, g^{(r+1)^0}, h^{(r+1)^0}, z^{(r+1)^0}, \zeta^{(r+1)^0})$ its optimal solution and Θ^r the corresponding optimal objective function value.
Set $\text{UB}^r \leftarrow \min\{\text{UB}^{r-1}, \Theta^r\}$.
if $\text{UB}^r = \text{LB}^r$ **then**
  return (x^r, A^r) as an optimal solution to problem $\text{W}_{rob}(\Gamma)$
else
  go to Step 4.
end if

Step 4: Generate Local Search (LS) solutions as described in Algorithm 4 (see Section 4.2.4).
$X^{r,LS} = \emptyset$. Store LS solutions in set $X^{r,LS}$. Go to step 5.
Step 5:
for $\overline{x}_{r^k} \in D^r \setminus \{x^r\} \cup X^{r,LS}$ $(k = 1\dots|D^r| - 1 + |X^{r,LS}|)$ **do**
  Solve the recourse problem $Q(\overline{x}_{r^k}, \Gamma)$ and denote $(u^{(r+1)^k}, v^{(r+1)^k}, w^{(r+1)^k}, g^{(r+1)^k}, h^{(r+1)^k}, z^{(r+1)^k}, \zeta^{(r+1)^k})$ its optimal solution.
end for
$r \leftarrow r + 1$ and go to Step 2.

4.2.1 Valid inequalities

Theorem 1 *Any feasible solution $\tilde{x}^r$ of the restricted master problem $W^r(\Gamma)$ at an iteration r generates a valid cut for problem $W_{rob}(\Gamma)'$, of the form:*

$$A \geq \sum_{l\in L} \overline{d}_l \tilde{u}_l^{r+1} + \sum_{l\in L} \hat{d}_l \tilde{u}_l^{r+1} \tilde{z}_l^{r+1} + \sum_{t\in T}\sum_{b\in B_t} x_{tb}\, q_t \tilde{g}_t^{r+1} - \sum_{t\in T}\sum_{b\in B_t} x_{tb}\, Q_t \tilde{h}_t^{r+1} + \sum_{t\in T}\sum_{b\in B_t} LV_{tb} x_{tb} \tilde{v}_{tb}^{r+1} - \sum_{t\in T}\sum_{b\in B_t} \overline{UV}_{tb} x_{tb} \tilde{w}_{tb}^{r+1} + \sum_{t\in T}\sum_{b\in B_t} \widehat{UV}_{tb}\, x_{tb}^r\, \tilde{w}_{tb}^{r+1}\, \tilde{\zeta}_{tb}^{r+1}$$

where $(\tilde{u}^{r+1}, \tilde{v}^{r+1}, \tilde{w}^{r+1}, \tilde{g}^{r+1}, \tilde{h}^{r+1}, \tilde{z}^{r+1}, \tilde{\zeta}^{r+1})$ is an optimal solution of the recourse problem $Q'(\tilde{x}^r, \Gamma)$.

Proof. Let $\tilde{x}^r$ be a feasible solution of the restricted master problem $\text{W}^r(\Gamma)$ at iteration r. Clearly, $\tilde{x}^r$ is a feasible first-stage solution of the original (non-restricted) problem $\text{W}_{rob}(\Gamma)'$. As one can observe, the feasible set associated with the recourse problem $Q'(\tilde{x}^r, \Gamma)$ is independent of $\tilde{x}^r$ ($\tilde{x}^r$ appears only in the objective function). Hence, solving the recourse problem $Q'(\tilde{x}^r, \Gamma)$ yields an extreme point $\tilde{s} = (\tilde{u}^{r+1}, \tilde{v}^{r+1}, \tilde{w}^{r+1}, \tilde{g}^{r+1}, \tilde{h}^{r+1}, \tilde{z}^{r+1}, \tilde{\zeta}^{r+1})$ of $\mathcal{S}$ and thus an inequality of type (15) that is valid for $W_{rob}(\Gamma)'$. □

Based on Theorem 1, we propose to add an initial cut to initiate the algorithm (Step 1 of Algorithm 2) as well as a number of valid cuts at each iteration of the algorithm. This is done by considering intermediate feasible solutions at each iteration r that are either straightforwardly derived from the $B\&B$ procedure used for solving the restricted master problem, or constructed using local search techniques.

4.2.2 Initial cut

Recall that Algorithm 1 is used to solve $W_{rob}(\Gamma)'$ for a pre-fixed value of the budget of uncertainty $\Gamma = (\Gamma^d, (\Gamma_t)_{t\in T})$. In our case, Γ^d represents the number of lanes l for which the demand deviates from its nominal value and takes the greatest value $d_l = \overline{d}_l + \hat{d}_l$. Similarly, $\Gamma_t, t \in T$ represents the number of bids $b \in B_t$ submitted by carrier t for which the capacity deviates from its nominal value and takes the lowest value $UV_{tb} = \overline{UV_{tb}} - \widehat{UV_{tb}}$.

The objective of $W_{rob}(\Gamma)'$ consists in selecting the winning bids and the associated volumes at the minimum cost, such that the *worst demands* -delimited by Γ^d- and the worst capacities -delimited by $\Gamma_t, t \in T$- are satisfied. In Algorithm 1, the initialization step (step 0) starts with no feasible first-stage solution x and consequently with no cuts of type (15). We propose here to initiate the process with a first cut generated using Algorithm 3.

Algorithm 3 first defines a scenario of demands $\tilde{\omega}^d$ and carriers capacities $\tilde{\omega}^t, t \in T$ that are likely to be considered when solving the recourse problem (the highest Γ^d demands and the lowest $\Gamma_t, t \in T$ capacities). The deterministic WDP with these scenarios is then solved and the corresponding optimal first stage solution x^0 is retained. As stated in Theorem 1, any first-stage feasible solution (and so x^0) can be used to solve the recourse problem and generate a valid cut for $W_{rob}(\Gamma)'$.

Algorithm 3 Generating an initial cut

Step 1 : Generate a scenario $\tilde{\omega}^d = (\tilde{d}_l)_{l\in L}$ of demands as follows:

1. Place the lanes $l \in L$ in a descending order with respect to their worst demand $\overline{d}_l + \hat{d}_l$,
2. Define $\tilde{L}$ as the set of the Γ^d first lanes in the ordered set L,
3. $\forall l \in \tilde{L}$, set $\tilde{d}_l = \overline{d}_l + \hat{d}_l$,
4. $\forall l \in L \setminus \tilde{L}$, set $\tilde{d}_l = \overline{d}_l$.

Step 2 : For each carrier $t \in T$, generate a scenario $\tilde{\omega}^t = (\tilde{UV}_{tb})_{b\in B_t}$ of capacities as follows:

1. Place the bids $b \in B_t$ in an ascending order with respect to their worst capacity $\overline{UV_{tb}} - \widehat{UV_{tb}}$,
2. Define $\tilde{B}_t$ as the set of the Γ_t first bids in the ordered set B_t,
3. $\forall b \in \tilde{B}_t$, set $\tilde{UV}_{tb} = \overline{UV_{tb}} - \widehat{UV_{tb}}$,
4. $\forall b \in B_t \setminus \tilde{B}_t$, set $\tilde{UV}_{tb} = \overline{UV_{tb}}$.

Step 3 : Solve the deterministic WDP with demand scenario $\tilde{\omega}^d$ and capacity scenarios $\tilde{\omega}^t, t \in T$. Let x^0 its optimal solution.

Step 4 : Solve the recourse problem $Q(x^0, \Gamma)$ and let $(u^{0^0}, v^{0^0}, w^{0^0}, g^{0^0}, h^{0^0}, z^{0^0}, \zeta^{0^0})$ its optimal solution.

The initial cut is the one given by:

$$A \geq \sum_{l\in L} \overline{d}_l u_l^{0^0} + \sum_{l\in L} \hat{d}_l u_l^{0^0} z_l^{0^0} + \sum_{t\in T}\sum_{b\in B_t} x_{tb} \mathrm{q}_t g_t^{0^0} - \sum_{t\in T}\sum_{b\in B_t} x_{tb} \mathrm{Q}_t h_t^{0^0} + \sum_{t\in T}\sum_{b\in B_t} \mathrm{LV}_{tb} x_{tb} v_{tb}^{0^0} - \sum_{t\in T}\sum_{b\in B_t} \overline{UV}_{tb} x_{tb} w_{tb}^{0^0} + \sum_{t\in T}\sum_{b\in B_t} \widehat{UV}_{tb}\, x_{tb} w_{tb}^{0^0}\, \zeta_{tb}^{0^0}$$

4.2.3 Bundle cuts

Both the master and the recourse problems are solved using the branch-and-bound procedure of the commercial solver CPLEX. By tuning some parameters in CPLEX, one can simply collect the pool of all integer feasible solutions encountered during the $B\&B$ procedure. These solutions are referred to in the following as Cplex Intermediate or CI solutions.

As depicted in Step 2 of Algorithm 2, at each iteration r, when solving the restricted master problem $\mathrm{W}^r(\Gamma)$, all the feasible solutions encountered during the $B\&B$ procedure are stored in set D^r. These solutions are placed in a descending order with respect to the associated objective function value. The last element of D^r corresponds to the optimal solution x^r.

Based on Theorem 1, each feasible solution in D^r is used to generate a valid inequality that will be added to the restricted master problem at the next iteration $(r+1)$. Observe that in Algorithm 2, the optimal solution $x^r \in D^r$ is omitted at Step 5. This is done to avoid solving the recourse problem $Q(x^r, \Gamma)$ twice (Problem $Q(x^r, \Gamma)$ was already solved at Step 3 to test the solution optimality).

4.2.4 Local search based cuts

A more elaborated way to derive additional feasible first-stage solutions consists in using local search techniques. These are referred to in the following as Local Search Intermediate, or LS, solutions.

Assume that we are at iteration r and at Step 4 of Algorithm 2. The optimal solution x^r of $\mathrm{W}^r(\Gamma)$ is already known as well as the set D^r of all CI solutions. We aim to generate additional feasible first-stage solutions that: (i) are within a neighbourhood $\mathcal{N}(x^r)$ of the current optimal solution x^r, and (ii) have similarities with the best NB^{mem} first-stage solutions generated with the algorithm. NB^{mem} is a parameter to be fixed. Algorithm 4 describes the main steps used to generate LS solutions at an iteration r of Algorithm 1. To alleviate the algorithm description, let β denote the total number of bids submitted in the auction. That is, $\beta = \sum_{t \in T} |B_t|$.

Algorithm 4 Generating LS solutions at iteration r

Step 0: Initialization
$D^{r,mem} = D^{r-1,mem}$

Step 1: Update $D^{r,mem}$
if $|D^r| < NB^{mem}$ **then**
  Replace the first $|D^r|$ solutions of $D^{r,mem}$ with all the CI solutions of D^r
else
  Replace the solutions of $D^{r,mem}$ with all the last NB^{mem} solutions of D^r
end if

Step 2: Local search
Let α^r a β-vector such that $\forall t \in T, b \in B_t, \alpha^r_{tb} = \sum_{x \in D^{r,mem}} x_{tb}$.
Set $\phi^r = \frac{\sum_{t\in T}\sum_{b\in B_t} x^r_{tb}}{2}$.
$i \leftarrow 0$
while $i < NB^{LS}$ **do**
  $\mathcal{N}(x^r) = \emptyset$.
  Let $\overline{x}^r$ be a β-vector.
  Randomly select a subset of bids $\tilde{B}^r$ of size ϕ^r such that $\tilde{B}^r = \{(t,b), t \in T, b \in B_t : x^r_{tb} = 1\}$
  for $(t,b) \in \tilde{B}^r$ **do**
    $\xi \leftarrow random[0,1]$.
    if $\xi < \frac{\alpha^r_{tb}}{NB^{mem}}$ **then**
      $\overline{x}^r_{tb} = 1$
    else
      $\overline{x}^r_{tb} = 0$
    end if
  end for
  $\mathcal{N}(x^r) = \{x : \forall (t,b) \in \tilde{B}^r, x_{tb} = \overline{x}^r_{tb}\}$
  Solve $\mathrm{W}^r_{rob}(\Gamma)'$ on the restricted set $\mathcal{N}(x^r)$ and let $\overline{x}^{r*}$ its optimal solution.
  $X^{LS} = X^{LS} \cup \{\overline{x}^{r*}\}$.
  $i \leftarrow i+1$.
end while

Step 3: Update $D^{r,mem}$
if $|X^{LS}| < NB^{mem}$ **then**
  Replace the first $|X^{LS}|$ solutions of $D^{r,mem}$ with all the LS solutions of X^{LS}
else
  Replace the solutions of $D^{r,mem}$ with NB^{mem} solutions randomly selected in X^{LS}.
end if

Algorithm 4 first defines an ordered set $D^{r,mem}$ including the best NB^{mem} feasible solutions obtained until the end of iteration r. If the number of CI solutions obtained after solving $W^r(\Gamma)$ (i.e., $|D^r|$) is larger than NB^{mem}, $D^{r,mem}$ will include the best CI solutions in D^r. However, if $|D^r| < NB^{mem}$, only the first NB^{mem} solutions of $D^{r,mem}$ (the worst ones) are replaced with the newest solutions of D^r.

Based on set $D^{r,mem}$, we construct a $\beta - vector$ α^r such that: $\forall t \in T, b \in B_t, \alpha^r_b = \sum_{x\in D^{r,mem}} x_{tb}$. Vector α^r identifies the number of times a bid $b \in B_t, t \in T$ is a winning bid for the best NB^{mem} first-stage solutions encountered until iteration r. To define a neighbourhood $\mathcal{N}(x^r)$ of x^r, we force ϕ^r bids that were winning at iteration r (such that $x^r_{tb} = 1$) to be either winning or losing bids, depending on set $D^{r,mem}$. ϕ^r is a parameter set to $\frac{\sum_{t\in T}\sum_{b\in B_t} x^r_{tb}}{2}$. Hence, a neighbour $\overline{x}^r \in \mathcal{N}(x^r)$ of x^r will have 50% of its components with a value deriving from the current optimal solution x^r and from the best solutions in $D^{r,mem}$.

Formally, let $\tilde{B}^r$ be a randomly selected subset of ϕ^r pairs (t,b) such that $x^r_{tb} = 1$. Observe that $\tilde{B}^r$ can always be defined given our definition of parameter ϕ^r. For each pair (t,b) in $\tilde{B}^r$, a neighbour $\overline{x}$ will have its component $\overline{x}^r_{tb}$ fixed to either 0 or 1 depending on the probability of occurrences of 1 in the solutions of $D^{r,mem}$. More precisely, for each $(t,b) \in \tilde{B}^r$, a random number ξ is uniformly generated within the interval $[0,1]$. If $\xi < \frac{\alpha^r_{tb}}{NB^{mem}}$, then $\overline{x}^r_{tb} = 1$. Otherwise, $\overline{x}^r_{tb} = 0$. By this way, a winning bid $b \in B_t, t \in T$ in the current optimal solution x^r is more likely to remain winning in the neighbour $\overline{x}^r$ if it won in the majority of the best feasible solutions of $D^{r,mem}$. For example, assume that bid $b' \in B_t, t \in T$ is a winning bid in $\overline{x}^r$ and in all the feasible solutions of $D^{r,mem}$. Then, $\alpha^r_{tb'} = |D^{r,mem}| = NB^{mem}$. So, any randomly generated number ξ in $[0,1]$ will respect the condition $\xi < \frac{\alpha^r_{tb}}{NB^{mem}} = 1$ and $\overline{x}^r_{tb'} = 1$.

Finally, the restricted master problem $\mathrm{W}^r(\Gamma)$ is solved in the neighbourhood $\mathcal{N}(x^r)$ of x^r. Its optimal solution is an LS solution. The process is iterated until the number of desired LS solutions (referred to as NB^{LS}) is reached.

Generating an LS solution at an iteration r requires the resolution of a complex MIP but on a set of restricted variables (the variables within $\mathcal{N}(x^r)$). Fixing a number of variables x_b simplifies indeed the model resolution. For example, given the XOR restriction, each carrier can win at most one bid. Hence, if a bid b submitted by a carrier t wins ($x_b = 1$) then all the variables $x_{tb'}$ associated with all the other bids b' submitted by t will take a null value. Moreover, it is not rare that the same feasible solution appears several times in $D^{r,mem}$ which creates a redundancy in the cuts generated in the master problem. We tried to avoid such redundancy by testing the presence of a solution before adding the corresponding cut to the master problem. We noted that Cplex presolve made it as well and faster. Note however that one should keep redundant solutions in $D^{r,mem}$ when generating LS solutions to be consistent with the neighbourhood definition.

4.2.5 Bounding constraints

As depicted in Algorithm 2, we propose to add a new cut (constraint (20)) to bound the value of the objective function A at each iteration.

Proposition 2 *The following inequality is valid for the master problem $W^r(\Gamma)$ at iteration r: $LB^{r-1} \leq A \leq UB^{r-1}$.*

Proof. Observe that $LB^{r-1}{=}A^{r-1}$ where A^{r-1} is the optimal objective function of the restricted master problem at the previous iteration $(r-1)$. Problem $\mathrm{W}^{(r-1)}(\Gamma)$ is a relaxation of $\mathrm{W}^r(\Gamma)$ (it includes less constraints). Hence, $A^{r-1} = LB^{r-1}$ is a valid lower bound for $\mathrm{W}^r(\Gamma)$.

Besides, $\mathrm{UB}^{r-1} = \min\{\mathrm{UB}^{r-2}, \Theta^{r-1}\}$ where Θ^{r-1} is the optimal objective function of the recourse problem $Q(x^{r-1}, \Gamma)$ solved for the optimal solution x^{r-1} of $\mathrm{W}^{(r-1)}(\Gamma)$. It follows that UB^{r-1} corresponds to the best objective function value obtained for the unrestricted problem $\mathrm{W}_{rob}(\Gamma)$ up to iteration $(r-1)$. The optimum value will thus have a value lower than or equal to it. □

It is worth mentioning that in addition to the improvement strategies presented above, we tried a number of other heuristic and accelerating methods that have proven their efficiency for other problems as reported in the literature. For instance, we obtain no conclusive results when selecting a dominant cut in the recourse problem (Fischetti et al., 2010) or when using the Relaxed Induced Neighbourhood Search (Danna et al., 2005).

5 Experimental study

The objective of this section is threefold. First, we evaluate the impact of the improvement strategies we propose on the basic constraint generation algorithm performance. This is done by comparing the performance of our new algorithm to that proposed by Remli and Rekik (2013) for the instances of WDP-SD tested therein. Second, we analyze the computational performance of the new algorithm for WDP-SDC. Third, we study the relevance of adding uncertainty on the carriers' capacity, on auction outcomes and transportation costs.

5.1 Computational performance of Algorithm 2 for WDP-SD

In this section, we consider the same instances reported in Remli and Rekik (2013) for WDP-SD. Recall that in Remli and Rekik (2013), 360 instances were generated. These instances are grouped in eight instance sets $|L| - |T| - B$, where $|L|$ represents the number of lanes submitted by the shipper to the auction, $|T|$ is the number of participating carriers, and B is the number of bids offered by each carrier (it is assumed that all carriers submit the same number B of bids). For each instance set $|L| - |T| - B$ (45 instances in total), nine different values of the budget of uncertainty Γ^d are considered. These values range from 10% to 90% with a step of 10%. Finally, for each value of Γ^d, five instances are randomly generated within the corresponding instance set.

As in Remli and Rekik (2013), all the MIP and LP models are solved with CPLEX 12.4 (with its default parameters) on a 3.00 GHz Intel Core 2 Duo PC with a 4.00 Go RAM.

Table 1 gives for each value of the budget of uncertainty Γ^d, the average number of iterations (#*iter.*) and the average running time in seconds (Time) required by each algorithm. These averages are computed on the five generated instances of each instance set. A dash ($'-'$) in a cell indicates that no optimal solution was identified within a time limit of 10 hours. The last two columns of Table 1 report the savings in the number of iterations and computing times yielded by Algorithm 2 adapted to WDP-SD to Algorithm 1 (as proposed in Remli and Rekik (2013)). These savings are presented as ratios. For example, for the instance set $50-20-10$ and a budget of uncertainty $\Gamma^d = 10\%$, the average number of iterations required by Algorithm 1 is 3.59 times greater than that required by Algorithm 2. Similarly, the computing time needed by Algorithm 1 is 1.46 times larger than that required by Algorithm 2. Observe that the results reported for Algorithm 2 correspond to the case where the parameter NB^{mem} is set to 20 and NB^{LS} to 2. These values were set after a series of experiments.

Table 1: Results of Algorithm 2 vs Algorithm 1

		Algorithm 1		Algorithm 2		Ratio:Alg.1/Alg.2	
$\vert L\vert - \vert T\vert - B$	Γ^d(%)	# iter.	Time (s)	# iter.	Time (s)	# iter.	Time (s)
	10	130.6	57	36.4	39	3.59	1.46
	20	126.0	58	36.4	39	3.46	1.49
	30	114.8	63	33.2	33	3.46	1.91
	40	101.4	51	30.2	28	3.36	1.82
50-20-10	50	88.2	41	26.8	26	3.29	1.58
	60	82.6	36	26.4	24	3.13	1.50
	70	83.0	35	25.0	21	3.32	1.67
	80	81.2	33	25.6	22	3.17	1.50
	90	81.6	32	24.8	20	3.29	1.60
	10	285.8	572	75.8	355	3.77	1.61
	20	251	494	68.2	290	3.68	1.70
	30	226.4	448	64.2	259	3.53	1.73
	40	199.4	381	56.8	209	3.51	1.82
50-20-20	50	178.2	319	51.0	161	3.49	1.98
	60	175	307	49.2	156	3.56	1.97
	70	172.4	293	50.8	157	3.39	1.87
	80	173	308	49.0	156	3.53	1.97
	90	172.6	298	50.4	166	3.42	1.80
	10	303.6	556	68.4	205	4.44	2.71
	20	285	570	66.0	194	4.32	2.94
	30	259.8	594	57.6	151	4.51	3.93
	40	245	599	54.2	141	4.52	4.25
100-40-10	50	225.8	553	50.6	121	4.46	4.57
	60	206	510	46.0	100	4.48	5.10
	70	190.4	434	43.8	89	4.35	4.88
	80	185.4	411	40.6	83	4.57	4.95
	90	183.6	404	42.6	89	4.31	4.54
	10	1122.7	15623	183.6	5304	6.11	2.95
	20	1037	13888	168.4	4696	6.16	2.96
	30	946	11975	163.2	4212	5.80	2.84
	40	856	9710	149.8	3641	5.71	2.67
100-40-20	50	755.7	7816	135.2	2956	5.59	2.64
	60	660	7005	126.0	2448	5.24	2.86
	70	617.5	6703	119.8	2170	5.15	3.09
	80	613.7	6111	118.6	2154	5.17	2.84
	90	609.7	5666	137.0	2958	4.45	1.92
	10	632.4	2841	199.0	1075	3.18	2.64
	20	612.2	2471	193.0	1013	3.17	2.44
	30	593.8	2291	188.0	964	3.16	2.38

Table 1: Results of Algorithm 2 vs Algorithm 1

$\|L\|-\|T\|-B$	$\Gamma^d(\%)$	Algorithm 1		Algorithm 2		Ratio:Alg.1/Alg.2	
		# iter.	Time (s)	# iter.	Time (s)	# iter.	Time (s)
200-80-10	40	571.4	2029	181.8	914	3.14	2.22
	50	552	1890	176.8	846	3.12	2.23
	60	531.8	1706	169.2	772	3.14	2.21
	70	505.2	1599	161.0	700	3.14	2.28
	80	472.6	1461	156.8	645	3.01	2.27
	90	469.6	1571	157.0	636	2.99	2.47
200-80-20	10	–	–	437.8	9321	–	–
	20	–	–	401.2	7706	–	–
	30	–	–	379.2	6736	–	–
	40	869	24120	344.2	5438	2.52	4.44
	50	805	18818	319.4	4768	2.52	3.95
	60	754	12678	305.8	4020	2.47	3.15
	70	685	8840	247.2	3406	2.77	2.60
	80	623.5	5565	227.0	2756	2.75	2.02
	90	628.5	5253	223.4	2691	2.81	1.95
500-100-10	10	993	13334	292.3	3568	3.40	3.74
	20	987	13320	291.0	3524	3.39	3.78
	30	976.2	12996	287.5	3522	3.40	3.69
	40	958	11221	280.5	3365	3.42	3.33
	50	939	10454	274.3	3271	3.42	3.20
	60	915.5	9935	266.8	3180	3.43	3.12
	70	880.5	9452	259.5	3012	3.39	3.14
	80	834.5	8536	244.0	2744	3.42	3.11
	90	825.2	9395	241.3	2684	3.42	3.50
600-120-10	10	1196	25065	292.2	4355	4.09	5.76
	20	1189	22065	280.4	4150	4.24	5.32
	30	1174.5	20459	269.4	4066	4.36	5.03
	40	1156.5	17096	286.8	3882	4.03	4.40
	50	1125.5	15891	260.8	3957	4.32	4.02
	60	1098	16651	268.6	3600	4.09	4.63
	70	1067	16401	256.2	3597	4.16	4.56
	80	1006.5	12886	233.6	3584	4.31	3.60
	90	995.5	13335	229.6	3520	4.34	3.79

The results of Table 1 clearly show that Algorithm 2 largely outperforms Algorithm 1 assessing thus the efficiency of the improvement strategies proposed in this paper. First, observe that Algorithm 2 always requires less time than Algorithm 1. As depicted in the column *Ratio*, the time needed by Algorithm 1 is divided by more than 2 for 245 instances over the 345 (71%) that are solved by both algorithms. The computing time is divided by more than 4 (≥ 3.93) for 80 of these instances.

Second, when examining the 345 instances that are solved by both algorithms, the average computing time required by Algorithm 2 is 1857 seconds (almost 0.5 hour) with a maximum of 5438 seconds (1.5 hours) for the largest instance. However, 6443 seconds (1.8 hours) are required on average by Algorithm 1 and the largest instances takes almost seven hours on average. Finally, Algorithm 2 solves all the 15 instances, that were not solved by Algorithm 1, within almost 2.2 hours, on average. The time required by Algorithm 2 to solve all the 360 instances averages 2109 seconds (almost 0.6 hours).

The results of Table 1 confirm the observations made in Section 4.2 with regard to the sensitivity of Algorithm 1 computing times to the number of iterations. Indeed, our main improvement strategies consist in adding a multitude of valid inequalities to the master problem at each iteration so that the lower bound increases more rapidly and the total number of iterations decreases. The results of Table 1 clearly prove that these cuts do the work we want them to do. This can be observed through the values reported in the column *Ratio* (under $\#iter$).

Improving the algorithm performance by adding cuts to the master problem was also motivated by the results reported in Remli and Rekik (2013) regarding the percentage of time that was allocated to the master problem with respect to the recourse problem. Indeed, between 75% and 99% of the total time required by Algorithm 1 was dedicated to solve the master problem. For Algorithm 2, the percentage of total time allocated to the master problem slightly decreases but remains relatively large (between 74.1% and 98.8%). This was not totally expectable since with Algorithm 2, more recourse problems are solved at each iteration (one problem for each generated cut). This result however confirms that the recourse problem is much more easier to solve than the master problem. Table 5 in the appendix details all the results obtained for both algorithms (computing times, number of iterations, the percentage of time used to solve the master problems and the percentage of time used to solve the recourse problems).

5.2 Computational performance of Algorithm 2 for WDP-SDC

In this section, we generate 180 new instances to tackle the WDP-SDC. We use the same terminology as in Section 5.1 to represent an instance set. Four instance sets $|L|-|T|-B$ are generated by varying the number of contracts $|L|$ (50 and 100), the number of carriers $|T|$ (20 and 40) and the number of bids per carrier B (10 and 20). For all the instances, the bid price c_{tb} is uniformly generated within the interval $[10, 40]$ and the spot price ce_l within $[50, 100]$. Minimum and maximum volumes q_t and Q_t are uniformly generated within the intervals $[10, 15]$ and $[60, 75]$, receptively. The generated bids cover on average between 20% and 30% of the lanes. Regarding the uncertain parameters, the nominal demand $\overline{d}_l$ is uniformly generated within the interval $[10, 50]$ and the nominal capacity $\overline{UV}_{tb}$ within $[40, 75]$. The maximum deviation $\hat{d}_l$, respectively, $\hat{UV}_{tb}$, is set as $\hat{d}_l = \alpha \times \overline{d}_l$, respectively, $\widehat{UV}_{tb} = \alpha \times \overline{UV}_{tb}$, where α is randomly generated within $[0.1, 0.5]$. The minimum volume LV_{tb} is uniformly generated within the interval $[10, 20]$. For the new instances, we impose no limit on the maximum number of winning carriers. Five instances are generated in each set $|L| - |T| - B$. The new instances are available on https://drive.google.com/file/d/1I8mMjrsAhuQq_jxfn5bGYLkCeyVIWCLN/view?usp=sharing.

For each instance, nine different values of the budget of uncertainty of demand Γ^d are considered. These values are obtained by varying Γ^d from 10% to 90% with a step of 10%. The budget of uncertainty Γ_t of a carrier t is fixed to a unique value derived from the performance factor p_t. Recall that the performance factor p_t takes a value within $[-1, 1]$ and models the carrier t service quality as evaluated by the shipper. The higher is this value, the less reliable is the carrier, and the larger the value of Γ_t is. Formally, $\Gamma_t = 0.5(1+p_t)B, \forall t \in T$. Hence, when $p_t = -1$, the carrier t is reliable and the value of $\Gamma_t = 0$ meaning that all the capacity submitted by the carrier in all its B bids take their nominal values as initially proposed by the carrier. In the opposite, a value of p_t equal to 1 (the carrier is totally unreliable) results in $\Gamma_t = B$ meaning that for all the B bids, the capacity will take its worst value (the smallest one).

Table 2 gives for each value of the budget of uncertainty Γ^d, the average number of iterations ($\#iter.$) and the average running time in seconds (Time) required by Algorithm 2 to solve the new instances generated for WDP-SDC. These averages are computed on the five generated instances of each instance set. It also reports the average percentage of time required by the master and the slave problems, respectively.

The results of Table 2 show that Algorithm 2 performs well for WDP-SDC problems. An average time of 6243 seconds is required to solve the 180 instances. Computational times increase with the number of lanes, the number of carriers and the number of bids. Algorithm 2 requires only 115 seconds to solve small-sized instances. It solves instances including up to 100 new contracts, 40 carriers and 800 bids in less than 4.5 hours, on average with a maximum computational time of 9.27 hours. It fails however in solving problems including 200 contracts, 80 carriers and 10 bids to optimality -which was not the case for the WDP-SD- within a time limit of ten hours. This proves the complexity resulting from considering two uncertain parameters when solving the WDP and questions the relevance of considering uncertainty on carriers' capacity. The next section discusses this point.

Table 2: Results of Algorithm 2 for WDP-SDC

$\lvert L\rvert-\lvert T\rvert-B$	$\Gamma^d(\%)$	# iter.	Time (s)	% Master	% Slave
50-20-10	10	142.6	409.4	97.8	2.2
	20	126.4	285.2	98.2	1.8
	30	107.6	213.2	97.9	2.1
	40	88.4	179.6	96.2	3.8
	50	74.4	145.2	96.5	3.5
	60	68.8	125.8	96.9	3.1
	70	68.6	120.2	97.9	2.1
	80	68.2	126.6	97.2	2.8
	90	68.4	115.0	98.9	1.1
50-20-20	10	163.4	1015.4	99.1	0.9
	20	140.0	687.2	98.7	1.3
	30	130.6	571.0	98.2	1.8
	40	114.0	463.2	96.3	3.7
	50	92.4	319.6	98.9	1.1
	60	92.2	321.8	98.7	1.3
	70	92.0	318.0	99.0	1.0
	80	83.2	289.6	99.0	1.0
	90	86.0	331.6	99.2	0.8
100-40-10	10	813.0	11097.0	99.6	0.4
	20	685.8	8270.8	99.5	0.5
	30	527.2	6602.0	99.5	0.5
	40	393.6	5278.4	99.4	0.6
	50	357.4	3983.8	99.3	0.7
	60	336.2	6133.0	99.6	0.4
	70	286.8	13828.2	99.7	0.3
	80	312.2	11716.0	99.6	0.4
	90	304.8	12168.0	99.7	0.3
100-40-20	10	845.6	33401.8	99.7	0.3
	20	698.2	21542.2	99.6	0.4
	30	566.0	15021.2	99.6	0.4
	40	502.0	11607.4	99.5	0.5
	50	504.0	13616.0	99.5	0.5
	60	426.6	10285.0	99.6	0.4
	70	428.2	10646.4	99.6	0.4
	80	437.0	11720.0	99.6	0.4
	90	445.2	11779.0	99.6	0.4

5.3 Relevance of considering uncertainty on carriers' capacity

To the best of our knowledge, all published papers addressing stochastic WDP for TL transportation services procurement auctions consider uncertainty on shipment volumes only. As pointed out in Section 5.2, adding a second uncertain parameter makes the problem harder to solve. So, is it relevant to deal with this second parameter of uncertainty? Or would the uncertainty on demand be sufficient and yield almost the same auction outcomes? To answer these questions, we consider two contexts: a first context, denoted SD, corresponding to WDP-SD, where only demand is uncertain and the carriers' capacity takes its nominal value. The second context, denoted SDC, corresponds to WDP-SDC and assumes that both demand and capacities are uncertain. We consider the set of the new generated instances of Section 5.2 and compute the auction outcomes for each instance under each context with Algorithm 2.

Table 3 displays for each instance set, each value of Γ^d, and each context, the average objective function value (column *Obj.*), the average percentage of winning carriers (column $Win.$), and the average percentage of lanes allocated to the spot market. These averages are computed over the five instances of each instance set. Recall that each carrier is allowed to win at most one bid (given the XOR constraints). So the number of winning carriers represents also the number of winning bids. We also report in the last three columns more explicit comparative results between the SD and the SDC contexts. Adding this information enables highlighting the changes in the first-stage solutions between the two contexts. More specifically, for each instance, we first compute the relative difference (in percentage) between the total cost incurred under the SDC and the SD contexts ($\frac{Obj^{SDC} - Obj^{SD}}{Obj^{SD}}$). We report in Table 3 under the column $\neq Obj$, the average relative difference over the five instances of each instance set. The columns $\neq Win$ and $\neq Spot$ report the difference in the number of winning carriers, respectively, the number of lanes allocated to the spot market, between both contexts. These differences are normalized with regard to the number of participating carriers, respectively, the number of contracts of each instance set.

Table 3: Impact of adding uncertainty on the carriers' capacity on the auction outcomes

		Context SD			Context SDC			SDC vs SD		
$\|L\| - \|T\| - B$	$\Gamma^d(\%)$	*Obj.*	*Win.*	*Spot*	*Obj.*	*Win.*	*Spot*	$\neq$ *Obj.*	$\neq$ *Bids*	$\neq$ *Spot*
	10	27314.48	48.0%	20.0%	31048.02	46.0%	19.6%	13.9%	4.0%	7.6%
	20	29954.52	45.0%	23.6%	34804.60	44.0%	18.8%	16.5%	3.0%	8.8%
	30	31407.62	43.0%	21.6%	37086.92	45.0%	18.4%	18.4%	6.0%	4.8%
	40	31826.14	45.0%	18.0%	37881.38	41.0%	20.0%	19.3%	6.0%	7.6%
50-20-10	50	31865.34	41.0%	21.2%	37970.58	43.0%	17.6%	19.5%	10.0%	9.2%
	60	31865.34	42.0%	18.8%	37970.58	42.0%	18.8%	19.5%	4.0%	7.2%
	70	31865.34	39.0%	23.6%	37970.58	41.0%	19.2%	19.5%	8.0%	8.4%
	80	31865.34	36.0%	24.0%	37970.58	42.0%	18.4%	19.5%	10.0%	8.0%
	90	31865.34	40.0%	20.4%	37970.58	42.0%	18.4%	19.5%	6.0%	6.0%
	10	25238.76	36.0%	18.4%	29764.38	42.0%	18.4%	17.5%	10.0%	4.8%
	20	27471.14	43.0%	19.6%	33343.66	37.0%	16.0%	20.9%	10.0%	4.4%
	30	28678.66	40.0%	16.0%	35942.82	36.0%	16.8%	24.9%	8.0%	5.6%
	40	28983.38	39.0%	18.0%	37412.48	39.0%	16.0%	28.6%	4.0%	2.0%
50-20-20	50	28983.38	40.0%	18.8%	37597.62	42.0%	16.0%	29.2%	8.0%	5.2%
	60	28983.38	35.0%	19.2%	37597.62	43.0%	15.2%	29.2%	8.0%	7.2%
	70	28983.38	38.0%	15.6%	37597.62	42.0%	16.0%	29.2%	6.0%	2.0%
	80	28983.38	40.0%	20.0%	37597.62	39.0%	20.0%	29.2%	11.0%	6.4%
	90	28983.38	41.0%	16.4%	37597.62	41.0%	16.8%	29.2%	10.0%	6.0%
	10	60191.58	35.5%	32.0%	69740.26	37.5%	28.2%	15.9%	10.0%	6.2%
	20	65964.76	39.0%	26.2%	77694.50	35.0%	28.6%	17.8%	4.0%	7.6%
	30	67853.70	40.5%	29.2%	82007.92	36.5%	26.2%	21.0%	6.0%	8.6%
	40	69928.44	44.0%	28.6%	83945.36	31.0%	26.8%	20.1%	13.0%	7.0%
100-40-10	50	69231.30	37.0%	27.6%	85023.28	33.5%	23.2%	22.8%	5.5%	5.2%
	60	63948.96	40.0%	24.6%	85138.52	35.5%	25.6%	38.0%	15.5%	8.6%
	70	69145.08	36.5%	27.8%	84781.70	33.0%	23.2%	22.7%	5.5%	5.8%
	80	69829.40	44.5%	28.2%	84781.70	39.0%	27.4%	21.5%	12.5%	5.6%
	90	69115.74	37.0%	26.6%	85021.62	30.5%	24.8%	23.1%	7.5%	4.2%
	10	56187.96	44.0%	33.8%	65470.04	31.0%	34.0%	16.6%	13.0%	4.6%
	20	61389.04	36.0%	29.0%	73738.96	33.5%	26.2%	20.2%	8.5%	8.0%
	30	64098.72	30.5%	31.4%	75820.88	33.0%	25.4%	18.4%	6.5%	7.6%
	40	65209.86	35.0%	26.6%	77829.76	37.5%	21.8%	19.5%	5.5%	5.6%
100-40-20	50	65249.84	36.0%	24.8%	78357.48	37.5%	21.8%	20.2%	7.5%	5.0%
	60	65249.84	43.0%	27.4%	78541.42	36.5%	26.6%	20.5%	9.5%	4.0%
	70	65166.22	40.5%	29.6%	78074.82	36.0%	21.0%	20.0%	6.5%	8.6%
	80	65166.22	40.0%	29.6%	78721.00	35.0%	22.2%	21.0%	8.0%	7.4%
	90	65249.84	39.0%	26.6%	78963.76	33.5%	23.2%	21.2%	8.5%	3.8%

The results of Table 3 prove that adding uncertainty on the carriers' capacity results in a substantial change in the first stage solution when compared to the case where only uncertainty on demand is addressed. This can be deduced from the changes in the number of winning carriers and the number of contracts allocated to the spot market. Moreover, one should mention that the total cost resulting from considering uncertainty on both demand and capacity is always larger than that obtained with uncertain demand only. As depicted in Table 3, the relative deviation in percentage of this total cost is on average equal to 21% and reaches 38% for some instances. Hence, determining the winning carriers at the strategic level while ignoring the possible variation of their capacity would result in a considerable underestimation of the expected transportation cost.

To go more in deep with the latter observation, the rest of the section investigates the impact of considering uncertainty on carriers' capacity on the transportation costs under randomly generated scenarios. To this end, we consider for each instance, the optimal first-stage solutions (winning bids) obtained under the SD context, x^{SD}, respectively the SDC context, x^{SDC}, with a value of $\Gamma^d = 50\%$ (an intermediate value). Then, we randomly generate for each instance set $|L|-|T|-B$, 30 plausible scenarios $\omega \in \Omega$ of demand and carriers' capacity within the corresponding intervals $[\overline{d}-\hat{d}, \overline{d}+\hat{d}]$, and $[\overline{UV_t}-\widehat{UV_t}, \overline{UV_t}+\widehat{UV_t}], t \in T$. For each scenario $\omega \in \Omega$, we determine the transportation costs yielded by x^{SD}, denoted by C^{SD}, and that yielded by x^{SDC}, denoted by C^{SDC}. These costs are obtained by solving the deterministic model (W), fixing x variables to either x^{SD} or x^{SDC} values and considering the demands and the capacities of scenario ω. Table 4 reports for each instance the average deviation (Av.) in percentage of C^{SDC} with respect to C^{SD}, computed as $\frac{C^{SDC}-C^{SD}}{C^{SD}}$ and the corresponding standard deviation (Std). These averages and standard deviations are computed over the 30 scenarios considered for each instance set.

Table 4: Cost savings resulting from considering an SDC context versus an SD context

$\|L\|-\|T\|-B$	Instance	Av. (%)	Std
50-20-10	1	-21.64%	6.15%
	2	-6.87%	8.91%
	3	-31.14%	3.93%
	4	-2.39%	7.02%
	5	-34.56%	3.65%
50-20-20	1	-3.32%	4.04%
	2	-9.56%	6.77%
	3	-30.37%	5.56%
	4	-41.22%	6.74%
	5	-8.14%	6.20%
100-40-10	1	-19.17%	6.37%
	2	-28.35%	2.30%
	3	-25.97%	2.60%
	4	-21.78%	3.25%
	5	-12.76%	2.47%
100-40-20	1	-6.75%	6.21%
	2	-10.68%	4.54%
	3	-15.01%	2.68%
	4	-13.03%	6.73%
	5	-19.47%	1.76%

As one can see from Table 4, considering uncertainty on carriers' capacity yields important savings in transportation costs at the operational level when compared to the case where only uncertainty on demand is taken into account at the first-stage strategic level. Average savings exceed 10% for 20 instances over the 24 considered. A saving of 41% is obtained for instance 4 of $50-20-20$.

6 Conclusion

In this paper, we propose a number of improvement strategies to accelerate the convergence of the basic constraint generation algorithm proposed by Remli and Rekik (2013) to solve a two-stage robust winner determination problem with uncertain shipment volumes. Our experimental study clearly proves the efficiency of the proposed strategies. To the best of our knowledge, the proposed new algorithm shows the best computational performance to date in terms of computing times and instances size.

Our paper is also the first to consider two uncertain parameters when solving the WDP, namely shipment volumes and carriers' capacity. We propose a two-stage robust formulation extending that proposed by Remli and Rekik (2013). Our experimental results prove that our improved constraint-generation algorithm succeeds in solving small and medium sized instances of this new problem in a reasonable time. We also investigate the relevance of adding this second parameter of uncertainty. Our results are conclusive. Considering uncertainty on carriers' capacity when solving the WDP, although making the problem more complex, induces changes in the first stage decisions, avoids underestimating costs at the strategic level, and results in substantial savings at the operational level when compared to the case where only uncertainty on shipment volumes is considered.

Of course, a number of interesting research avenues remains to be investigated in combinatorial auction design for the procurement of transportation services under uncertainty. A first extension of this paper would be to study the impact of a number of business constraints and auction rules on the auction outcomes such as the XOR constraints, the maximum number of winning carriers, etc. Uncertainty should also be considered when solving the bid construction problem, a decisional problem that is faced by each participating carrier to help it decide on the lanes to submit in a bid and the corresponding ask price.

A Linearization of the recourse problem and reformulation of the master problem

This section gives details on how the final formulations of the master and the recourse problems presented in Section 3.3 were obtained. As already mentioned, this is based on the paper by Remli and Rekik (2013). In Section 3.3, the robust winner determination problem $\mathrm{W}_{rob}(\Gamma)$ is first formulated as:

$$\mathrm{W}_{rob}(\Gamma)\begin{cases} \min & opt(R(x,\Gamma)) \\ \text{s.t.} & \sum_{b\in B_t} x_{tb} \le 1, \quad t\in T \\ & \sum_{t\in T}\sum_{b\in B_t} a^l_{tb}\, x_{tb} \le 1, \quad l \in L \\ & \mathrm{N}_{\min} \le \sum_{t\in T}\sum_{b\in B_t} x_{tb} \le \mathrm{N}_{\max} \\ & x_{tb}\in\{0,1\}, \quad t\in T,\ b\in B_t \end{cases}$$

where $opt(R(x,\Gamma))$ represents the optimum value of the recourse problem:

$$R(x,\Gamma)\left\{ \max_{(d,UV)\in\mathcal{U}(\Gamma)} \min_{(y,e)\in\mathcal{Y}(x)} \sum_{t\in T}\sum_{b\in B_t}(1+p_t)c_{tb}y_{tb} + \sum_{l\in L} ce_l e_l \right.$$

The uncertainty set $\mathcal{U}(\Gamma)$ is defined by:

$$\mathcal{U}(\Gamma) = \{d\in\mathbb{R}^{|L|} : d_l = \overline{d}_l + z_l\,\hat{d}_l,\ l\in L,\ z\in\mathcal{Z}(\Gamma^d),$$
$$UV_{tb}\in\mathbb{R}^{|T|\times|B_t|} : UV_{tb} = \overline{UV}_{tb} - \zeta_{tb}\widehat{UV}_{tb},\ t\in T,\ b\in B_t,\ \zeta\in\mathcal{Z}'(\Gamma_t),\ t\in T\}$$

where

$$\mathcal{Z}(\Gamma^d) = \{z\in\mathbb{R}^{|L|} : \sum_{l\in L} z_l \le \Gamma^d,\ 0\le z_l\le 1,\ l\in L\}$$

and

$$\mathcal{Z}'(\Gamma_t) = \{\zeta\in\mathbb{R}^{|T|\times|B_t|} : \sum_{b\in B_t}\zeta_{tb}\le\Gamma_t,\ t\in T,\ 0\le\zeta_{tb}\le 1,\ t\in T, b\in B_t\}$$

As depicted in Section 3.3, the feasible set $\mathcal{Y}(x)$ includes all vectors (y,e) satisfying the constraints (10)-(14) as follows:

$$\sum_{t\in T}\sum_{b\in B_t} a^l_{tb}\, y_{tb} + e_l \ge d_l, \quad l\in L$$
$$y_{tb} \ge \mathrm{LV}_{tb}\, x_{tb},\ t\in T,\ b\in B_t$$
$$y_{tb} \le UV_{tb}\, x_{tb},\ t\in T,\ b\in B_t$$

$$\sum_{b\in B_t} y_{tb} \geq \mathrm{q}_t \sum_{b\in B_t} x_{tb}, \quad t\in T$$
$$\sum_{b\in B_t} y_{tb} \leq \mathrm{Q}_t \sum_{b\in B_t} x_{tb}, \quad t\in T$$
$$y_{tb}\geq 0, \quad t\in T,\ b\in B_t; \quad e_l \geq 0, \quad l\in L$$

The optimal solution of the recourse problem can be obtained by considering its dual (using the strong duality theorem). The dual of the inner minimization problem $R(x,\Gamma)$ is:

$$Q(x,\Gamma)\begin{cases}\max & \sum_{l\in L}\overline{d}_l u_l + \sum_{l\in L}\hat{d}_l u_l z_l + \sum_{t\in T}\sum_{b\in B_t}\mathrm{LV}_{tb}x_{tb}v_{tb} - \sum_{t\in T}\sum_{b\in B_t}\overline{UV}_{tb}x_{tb}w_{tb}\\ & +\sum_{t\in T}\sum_{b\in B_t}\widehat{UV}_{tb}x_{tb}w_{tb}\zeta_{tb} + \sum_{t\in T}\sum_{b\in B_t}x_{tb}\mathrm{q}_t g_t - \sum_{t\in T}\sum_{b\in B_t}x_{tb}\mathrm{Q}_t h_t\\ \text{s.t.} & \\ & \sum_{l\in L}a^l_{tb}u_l + v_{tb} - w_{tb} + g_t - h_t \leq (1+p_t)c_{tb}, \quad t\in T,\ b\in B_t\\ & u_l \leq ce_l, \quad l\in L\\ & \sum_{l\in L} z_l \leq \Gamma^d\\ & \sum_{b\in B_t}\zeta_{tb}\leq \Gamma_t, \quad t\in T\\ & z_l\in\{0,1\}; u_l\geq 0, \quad l\in L\\ & v_{tb},\ w_{tb},\ g_t,\ h_t\geq 0, \zeta_{tb}\in\{0,1\} \quad t\in T,\ b\in B_t\end{cases}$$

where the variables $u_l,\ v_{tb},\ w_{tb},\ g_t,\ h_t$ are the dual variables of the minimization problem associated with constraints (10)-(14).

Problem $Q(x,\Gamma)$ is bilinear. This problem can however be linearized given the assumption that Γ^d and $\Gamma_t, t\in T$ take integer values and that the recourse problem is feasible and bounded (Gabrel et al., 2014). Indeed, Gabrel et al. (2014) prove that under the latter assumptions, there is an optimal solution for the recourse problem such that z_l and ζ_{tb} variables are in $\{0,1\}$. Hence, the product $u_l z_l$ can be replaced by a new variable s_l and constraints must be added to enforce s_l to be equal to u_l if $z_l=1$ and 0, otherwise. Similarly, the product $w_{tb}\zeta_{tb}$ can be replaced by a new variable f_{tb} and constraints must be added to enforce f_{tb} to be equal to w_{tb} if $\zeta_{tb}=1$ and 0, otherwise. This results in the linear formulation $Q'(x,\Gamma)$ presented in Section 3.3:

$$Q'(x,\Gamma)\begin{cases}\max & \sum_{l\in L}\overline{d}_l u_l + \sum_{l\in L}\hat{d}_l s_l + \sum_{t\in T}\sum_{b\in B_t}\mathrm{LV}_{tb}x_{tb}v_{tb} - \sum_{t\in T}\sum_{b\in B_t}\overline{UV}_{tb}x_{tb}w_{tb}\\ & +\sum_{t\in T}\sum_{b\in B_t}\widehat{UV}_{tb}x_{tb}f_{tb} + \sum_{t\in T}\sum_{b\in B_t}x_{tb}\mathrm{q}_t g_t - \sum_{t\in T}\sum_{b\in B_t}x_{tb}\mathrm{Q}_t h_t\\ \text{s.t.} & \\ & \sum_{l\in L}a^l_{tb}u_l + v_{tb} - w_{tb} + g_t - h_t \leq (1+p_t)c_{tb}, \quad t\in T,\ b\in B_t\\ & u_l \leq ce_l, \quad l\in L\\ & \sum_{l\in L} z_l \leq \Gamma^d\\ & s_l \leq ce_l\, z_l, \quad l\in L\\ & s_l\leq u_l, \quad l\in L\\ & \sum_{b\in B_t}\zeta_{tb}\leq \Gamma_t, \quad t\in T\\ & f_{tb}\leq M\ \zeta_{tb}, \quad b\in B_t,\ t\in T\\ & f_{tb}\leq w_{tb}, \quad b\in B_t,\ t\in T\\ & z_l\in\{0,1\}; s_l,\ u_l\geq 0, \quad l\in L\\ & v_{tb},\ w_{tb},\ f_{tb},\ g_t,\ h_t\geq 0, \zeta_{tb}\in\{0,1\} \quad t\in T,\ b\in B_t\end{cases}$$

The optimal solution of the recourse problem $Q'(x,\Gamma)$ is reached at an extreme point of its feasible set (the problem is feasible and bounded). So, the robust problem $\mathrm{W}_{rob}(\Gamma)$ can be rewritten as:

$$
\mathrm{W}_{rob}(\Gamma)' \begin{cases}
\min \quad A \\
\text{s.t.} \quad A \geq \sum_{l\in L} \overline{d}_l u_l^\sigma + \sum_{l\in L} \hat{d}_l s_l^\sigma + \sum_{t\in T}\sum_{b\in B_t} x_{tb}\mathrm{q}_t g_t^\sigma - \sum_{t\in T}\sum_{b\in B_t} x_{tb}\mathrm{Q}_t h_t^\sigma + \sum_{t\in T}\sum_{b\in B_t} \mathrm{LV}_{tb}x_{tb}v_{tb}^\sigma - \\
\qquad \sum_{t\in T}\sum_{b\in B_t} \overline{UV}_{tb}x_{tb}w_{tb}^\sigma + \sum_{t\in T}\sum_{b\in B_t} \widehat{UV}_{tb}x_{tb}f_{tb}^\sigma, \quad \sigma\in\mathcal{S} \\
\sum_{b\in B_t} x_{tb} \leq 1, \quad t\in T \\
\sum_{t\in T}\sum_{b\in B_t} a_{tb}^l x_{tb} \leq 1, \quad l\in L \\
\mathrm{N}_{\min} \leq \sum_{t\in T}\sum_{b\in B_t} x_{tb} \leq \mathrm{N}_{\max} \\
A\geq 0, \ x_{tb}\in\{0,1\}, \quad t\in T, \ b\in B_t
\end{cases}
$$

where $\mathcal{S}$ is the set of the extreme points $(u^\sigma, s^\sigma, v^\sigma, w^\sigma, f^\sigma, g^\sigma, h^\sigma), \sigma = 1...|\mathcal{S}|$ of the recourse problem $Q'(x,\Gamma)$

B Detailed results for Algorithms 2 and 1 for WDP-SD

Table 5: Complete results of Algorithms 2 and 1 for WDP-SD

		Algorithm 1				Algorithm 2			
$\|L\|-\|T\|-B$	Γ^d(%)	# iter.	Time (s)	% Master	% Slave	# iter.	Time (s)	% Master	% Slave
50-20-10	10	130.6	57	83.6	16.4	36.4	39	74.3	25.7
	20	126.0	58	89.3	10.8	36.4	39	77.5	22.5
	30	114.8	63	91.4	8.6	33.2	33	76.6	23.4
	40	101.4	51	89.2	10.9	30.2	28	74.1	25.9
	50	88.2	41	85.2	14.8	26.8	26	78.1	21.9
	60	82.6	36	75.9	24.1	26.4	24	82.4	17.6
	70	83.0	35	93.0	7.0	25.0	21	84.9	15.1
	80	81.2	33	89.4	10.6	25.6	22	83.7	16.3
	90	81.6	32	81.5	18.5	24.8	20	76.5	23.5
50-20-20	10	285.8	572	98.0	2.0	75.8	355	93.3	6.7
	20	251.0	494	95.1	4.9	68.2	290	94.5	5.5
	30	226.4	448	95.8	4.3	64.2	259	92.4	7.6
	40	199.4	381	95.0	5.1	56.8	209	92.7	7.3
	50	178.2	319	97.5	2.5	51.0	161	94.0	6.0
	60	175.0	307	99.4	0.6	49.2	156	92.3	7.7
	70	172.4	293	99.2	0.8	50.8	157	93.0	7.0
	80	173.0	308	99.7	0.3	49.0	156	94.3	5.7
	90	172.6	298	99.3	0.7	50.4	166	93.8	6.2
100-40-10	10	303.6	556	98.0	2.0	68.4	205	92.0	8.0
	20	285.0	570	98.2	1.8	66.0	194	90.0	10.0
	30	259.8	594	98.1	1.9	57.6	151	93.2	6.8
	40	245.0	599	98.2	1.8	54.2	141	90.2	9.8
	50	225.8	553	98.0	2.0	50.6	121	88.3	11.7
	60	206.0	510	99.0	1.0	46.0	100	89.3	10.7
	70	190.4	434	98.2	1.8	43.8	89	87.1	12.9
	80	185.4	411	97.9	2.1	40.6	83	89.8	10.2
	90	183.6	404	98.5	1.5	42.6	89	82.5	17.5
100-40-20	10	1122.7	15623	99.7	0.3	183.6	5304	98.6	1.4
	20	1037.0	13888	99.7	0.3	168.4	4696	98.4	1.6
	30	946.0	11975	99.6	0.4	163.2	4212	98.4	1.6

Table 5: Complete results of Algorithms 2 and 1 for WDP-SD

		Algorithm 1				Algorithm 2			
$\|L\|-\|T\|-B$	$\Gamma^d(\%)$	# iter.	Time (s)	% Master	% Slave	# iter.	Time (s)	% Master	% Slave
	40	856.0	9710	99.6	0.4	149.8	3641	98.3	1.7
	50	755.7	7816	99.6	0.4	135.2	2956	98.3	1.7
	60	660.0	7005	99.7	0.3	126.0	2448	98.2	1.8
	70	617.5	6703	99.9	0.2	119.8	2170	98.3	1.7
	80	613.7	6111	99.8	0.2	118.6	2154	98.4	1.6
	90	609.7	5666	99.9	0.1	137.0	2958	98.8	1.2
200-80-10	10	632.4	2841	96.6	3.4	199.0	1075	95.1	4.9
	20	612.2	2471	96.5	3.5	193.0	1013	95.2	4.8
	30	593.8	2291	96.7	3.3	188.0	964	94.9	5.1
	40	571.4	2029	97.6	2.4	181.8	914	94.7	5.3
	50	552.0	1890	96.9	3.1	176.8	846	94.3	5.7
	60	531.8	1706	96.7	3.3	169.2	772	94.5	5.5
	70	505.2	1599	95.5	4.5	161.0	700	94.3	5.7
	80	472.6	1461	96.3	3.7	156.8	645	93.4	6.6
	90	469.6	1571	95.9	4.1	157.0	636	94.6	5.4
200-80-20	10	-	-	-	-	437.8	9321	98.7	1.3
	20	-	-	-	-	401.2	7706	98.7	1.3
	30	-	-	-	-	379.2	6736	98.2	1.8
	40	869.0	24120	99.9	0.1	344.2	5438	98.2	1.8
	50	805.0	18818	99.9	0.1	319.4	4768	98.2	1.8
	60	754.0	12678	99.8	0.3	305.8	4020	98.0	2.0
	70	685.0	8840	99.6	0.4	247.2	3406	98.0	2.0
	80	623.5	5565	99.8	0.2	227.0	2756	97.5	2.5
	90	628.5	5253	99.5	0.5	223.4	2691	97.8	2.2
500-100-10	10	993	13334	99.0	1.1	292.3	3568	96.7	3.3
	20	987.0	13320	99.4	0.6	291.0	3524	96.9	3.1
	30	976.2	12996	99.3	0.7	287.5	3522	96.8	3.2
	40	958.0	11221	99.3	0.7	280.5	3365	96.8	3.2
	50	939.0	10454	99.4	0.6	274.3	3271	96.6	3.4
	60	915.5	9935	99.1	0.9	266.8	3180	96.7	3.3
	70	880.5	9452	99.3	0.8	259.5	3012	96.6	3.4
	80	834.5	8536	99.2	0.8	244.0	2744	96.3	3.7
	90	825.2	9395	99.1	0.9	241.3	2684	96.6	3.4
600-120-10	10	1196.0	25065	99.6	0.4	292.2	4355	97.2	2.8
	20	1189.0	22065	99.7	0.3	280.4	4150	97.8	2.2
	30	1174.5	20459	99.7	0.3	269.4	4066	97.9	2.1
	40	1156.5	17096	99.7	0.3	286.8	3882	97.6	2.4
	50	1125.5	15891	99.7	0.3	260.8	3957	98.0	2.0
	60	1098.0	16651	99.7	0.3	268.6	3600	96.5	3.5
	70	1067.0	16401	99.6	0.4	256.2	3597	98.6	1.4
	80	1006.5	12886	99.7	0.3	233.6	3584	96.2	3.8
	90	995.5	13335	99.7	0.3	229.6	3520	92.6	7.4

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