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J. A. Rodriguez, M. F. Anjos,
P. Côté, G. Desaulniers

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MILP formulations for generator maintenance scheduling in hydropower systems

Jesus A. Rodriguez^a

Miguel F. Anjos^a

Pascal Côté^b

Guy Desaulniers^a

^a GERAD & Department of Mathematics and Industrial
Engineering, Polytechnique Montréal (Québec) Canada,
H3C 3A7

^b GERAD & Rio Tinto, Saguenay, (Québec) Canada, G7S 4R5

jesus.rodriguez@gerad.ca
miguel-f.anjos@polymtl.ca
pascal.cote@riotinto.com
guy.desaulniers@gerad.ca

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Abstract: Maintenance activities help prevent costly generator breakdowns but because generators under maintenance are typically unavailable, the impact of maintenance schedules is significant and their cost must be accounted for when planning maintenance. In this paper we address the generator maintenance scheduling problem in hydropower systems. We propose a mixed-integer linear programming model that considers the time windows of the maintenance activities, as well as the nonlinearities and disjunctions of the hydroelectric production functions. Because the resulting model is hard to solve, we also propose an extended formulation, a set reduction approach that uses logical conditions for excluding unnecessary set elements from the model, and valid inequalities. We performed computational experiments using a variety of instances adapted from a real hydropower system in Canada, and the extended formulation with set reduction achieved the best results in terms of computational time and optimality gap.

Keywords: Hydroelectric power generation, integer linear programming, mathematical programming, optimal maintenance scheduling

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1 Introduction

In the power industry, preventive maintenance activities are carried out on a regular basis to prevent expensive equipment failures and to ensure a continuous operation within acceptable efficiency levels. As generators under maintenance are typically inactive, the maintenance scheduler should anticipate the impact of the maintenance plan on the system operation cost. However in hydroelectric systems these costs are difficult to estimate due to multiple interrelated physical variables. In particular, hydroelectricity production is a function of both the potential energy (the water head) and the kinetic energy of the water that drives the turbine-generators of the system. Formally [3],

$$p = \rho g \gamma q h \eta(q, h), \quad (1)$$

where p is the power output (MW), ρ the water density (kg/m^3), g the gravitational acceleration (m/s^2), γ the conversion factor (10^{-6}), q the turbine water discharge (m^3/s), h the net water head (m), and $\eta(q, h)$ the turbine-generator efficiency (%). For each turbine the efficiency η is a nonlinear function of the net water head and the water discharge of the turbine. Therefore, the efficiency factor η introduces further nonlinearities in the power production of the system. When the maximum discharge of each turbine is reached, water can be spilled to keep the reservoir within acceptable levels.

As the set of active generators affects the generation capacity as well as the optimal quantities of water spill and water discharge, the number of active generators has a nonlinear effect on the total power output. Figure 1 shows the power production as a function of water discharge and stored water in a reservoir for either four or five active generators. Rio Tinto computes these surfaces by solving a unit commitment problem via dynamic programming [9].

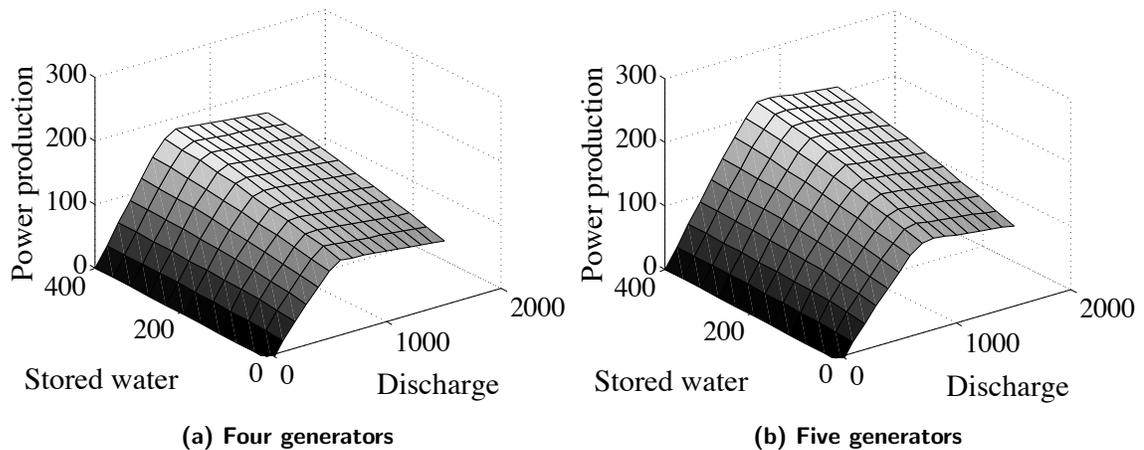


Figure 1: The maximum power output as a function of water discharge and stored water varies according to the number of active generators

Spatial and temporal inter-dependences should also be considered in the hydropower operation. First, because water discharges can feed downstream turbines in the current or in subsequent time periods, and second, because future operation costs are determined by present decisions, such as generator outages and water spills from reservoirs. All the aforementioned elements make the optimal planning of maintenance outages in hydropower systems a challenging endeavor.

In the electricity industry, the Generator Maintenance Scheduling Problem (GMSP) has been widely studied, see e.g. [4]. However, specific operating conditions of hydroelectric systems have been scarcely addressed. Feng et al. [7] represented the uncertainty of the power output with fuzzy variables, but omitted water storage levels and water head effects. Foong et al. [5] proposed a meta-heuristic for an oversimplified hydropower operation problem with constant power output in active units. Kuzle et al. [6] introduced transmission constraints in a simple GMSP where the nonlinearity of the production functions is neglected. Likewise, Perez-Canto [8] omitted relevant characteristics of hydropower systems, such as temporal and

spatial interdependencies, and nonlinearities of the power production. Clearly, a finer representation of the hydropower system's characteristics is necessary to achieve valid solutions to the GMSP for hydropower systems in practice.

Particular aspects of hydroelectric systems have been dealt with in works addressing the short-term operation, without incorporating maintenance scheduling decisions. For the day-ahead scheduling of generators, Conejo et al. [1] introduced piecewise linearization for representing the effects of the water discharge on the power production. The water head effect on the power output was estimated by interpolation among piecewise approximations for different stored water levels. Following a similar approach, Borghetti et al. [2] proposed a refined linearization for representing the water head effects. Due to the size of the resulting model, results were only reported for a single-reservoir system. More recently, Seguin et al. [9] approximated the power output with smoothing splines for the short-term scheduling of hydro units. These splines were fitted to a maximum power output surface computed by means of dynamic programming for different values of water discharge and stored water level in a reservoir.

In this paper, we propose a mixed-integer linear programming model for the GMSP in hydropower systems that accounts for the nonlinearities of hydroelectric operations via a convex hull approximation of the hydropower production function. Given the difficulty of the resulting optimization problem, we explore three approaches for strengthening the formulation: extended formulation, set reduction, and valid inequalities. The set reduction uses logical conditions for excluding superfluous set elements, in order to reduce the variables and constraints of the model. The possible combinations of these approaches led to eight formulations that we compared in terms of computational times and optimality gaps on test instances adapted from a real hydropower system in Canada.

This paper is structured as follows. Section 2 presents our basic mixed integer programming mathematical model. Section 3 describes the approaches to improve the formulation and the resulting alternative formulations. Section 4 reports our computational experiments for evaluating the different alternatives. Section 5 summarizes our findings and concludes the paper.

2 A basic mixed integer programming formulation

We consider the GMSP for hydropower systems in the general form

$$\min_{\substack{y \in \mathcal{Y} \\ x \in \mathcal{X}(y)}} \Psi(y) + \Phi(x, y),$$

where the variables in x represent operational decisions and those in y represent the maintenance decisions. The set \mathcal{Y} of possible maintenance decisions is defined by the time window constraints of the maintenance activities, the maximum number of simultaneous maintenance outages, and other logical constraints. The feasible set $\mathcal{X}(y)$ for the operation decision variables x is determined by the water balance constraints and the bounds of the hydropower operation, which are affected by the scheduled outages y . The functions $\Psi(y)$ and $\Phi(x, y)$ denote the maintenance cost and the hydropower operation cost, respectively. Note that $\Phi(x, y)$ is a function of the maintenance schedule y because the power production function is different for each set of active generators (Figure 1). The interdependency between the maintenance plan and the hydropower operation makes this a challenging nonlinear, nonconvex and combinatorial optimization problem.

In the next subsections we formulate in turn the hydropower operation, the linear approximation of the power production function, and the maintenance scheduling.

2.1 The hydropower operation

The hydropower operation problem optimizes the water discharges, water spills and stored water levels to maximize the total expected value of the electricity production, while respecting the physical constraints of the system and the target levels of the reservoirs at the end of the planning horizon. The physical constraints enforce the mass, energy and power balance, as well as the bounds of the variables, such as the water levels

in reservoirs. At each time period $t \in \mathcal{T}$, reservoirs can be fed by lateral inflows F_{it} from tributary rivers or snow-melt, or by turbine discharges u_{gt} and water spills v_{gt} from upstream reservoirs $g \in \mathcal{U}(i)$ (Figure 2).

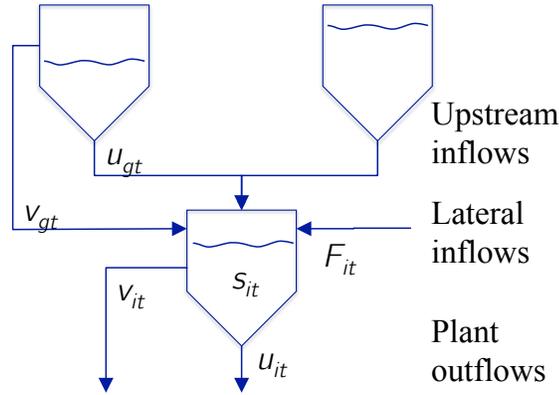


Figure 2: Decision variables, parameters and water balance in a hydropower system: water discharge (u), water spill (v), stored water (s) and lateral inflows (F). Adapted from [10]

At each powerhouse and time period, the mass balance (2) imply that the initial water volume $s_{i(t-1)}$ minus the water volume s_{it} at the end of the time period should be equal to the water inflows minus the total outflows, multiplied by the conversion factor Q . As it is customary, we assume that the outflows are equal to the total turbine discharge u_{it} and the water spill v_{it} of the reservoir.

$$s_{it} - s_{i(t-1)} = Q \left(F_{it} + \sum_{g \in \mathcal{U}(i)} [u_{gt} + v_{gt}] - u_{it} - v_{it} \right), \quad \forall t \in \mathcal{T}, i \in \mathcal{I}. \quad (2)$$

To ensure the consistency with the initial and the final water volume of the reservoirs, we define $s_{i(t-1)} = S_{i0}$ and $s_{it} = S_{iT}$ for $t = 1$ and $t = T$ in (2), respectively. In addition, (3)-(5) define the bounds on the water discharge, water spill and water volume.

$$0 \leq u_{it} \leq \bar{U}_i, \quad \forall i \in \mathcal{I}, t \in \mathcal{T}, \quad (3)$$

$$0 \leq v_{it} \leq \bar{V}_i, \quad \forall i \in \mathcal{I}, t \in \mathcal{T}, \quad (4)$$

$$S_i \leq s_{it} \leq \bar{S}_i, \quad \forall i \in \mathcal{I}, t \in \mathcal{T}. \quad (5)$$

The energy balance (6) requires that at each time period t , the total energy production plus the energy purchases equal the load d_t plus the energy sales:

$$\sum_{i \in \mathcal{I}} p_{it} + w_t^- = d_t + w_t^+, \quad \forall t \in \mathcal{T}. \quad (6)$$

2.2 Linearization of the power production function

For each powerhouse, the power output p_{it} is a nonlinear function Θ_i of the water discharge u_{it} and the net water head (which in turn is a nonlinear function of the stored water volume s_{it} and the total water discharge u_{it}). Since each generator may have a particular efficiency curve, the maximum power output in a powerhouse depends on the specific set of active generators. However, if the differences among power functions of individual generators are negligible, the power function in a powerhouse can be characterized by the number of active generators k_{it} , instead of the explicit set of active generators, that is, $p_{it} = \Theta_i(s_{it}, u_{it}, k_{it})$. This assumption significantly reduces the problem complexity, since otherwise a specific power function would be necessary for each combination of active generators.

Given a set of active generators with their respective efficiency curves, a dynamic programming algorithm can determine the maximum power output corresponding to each combination of water discharge and stored

water level [9]. For each number of active generators, a surface can represent the computed maximum power output (Figure 1). By definition, this set of points is contained in its convex hull, whose half-space representation can be obtained with a facet enumeration algorithm. Some implementations of this algorithm are freely available [11, 13].

The resulting polyhedron may contain a large number of hyperplanes, some of which should be dropped since they define the lower facets of the convex hull with respect to the power output p_{it} . The set can be additionally reduced by iteratively removing the hyperplane for which the remaining polyhedron has the smallest approximation error of the power output. This sequential elimination of hyperplanes is repeated until the target number of hyperplanes or a specified precision is reached. For each powerhouse i and number of active generators k , the resulting set of hyperplanes $\mathcal{H}(i, k)$ provides an outer approximation of the maximum power output, i.e.,

$$p_{it} \leq \beta_h^0 + \beta_h^u u_{it} + \beta_h^s s_{it} \quad \forall i \in \mathcal{I}, t \in \mathcal{T}, k \in \mathcal{K}(i, t), h \in \mathcal{H}(i, k).$$

At powerhouse i and time period t , if k^* is the number of active generators, power function constraints for $k \neq k^*$ can be relaxed by adding the bounding term $(1 - z_{itk})\bar{P}_i$ on the right hand side of (7), i.e.,

$$p_{it} \leq \beta_h^0 + \beta_h^u u_{it} + \beta_h^s s_{it} + (1 - z_{itk})\bar{P}_i, \quad \forall i \in \mathcal{I}, t \in \mathcal{T}, k \in \mathcal{K}(i, t), h \in \mathcal{H}(i, k), \quad (7)$$

where \bar{P}_i is the generation capacity of powerhouse i and the binary variables z_{itk} indicate if k generators are active at (i, t) . Since only one binary variable z_{itk} takes value 1 for each $(i, t) \in \mathcal{I} \times \mathcal{T}$,

$$\sum_{k \in \mathcal{K}(i, t)} z_{itk} = 1, \quad \forall i \in \mathcal{I}, t \in \mathcal{T}. \quad (8)$$

Thus, by (8) and the binary condition on z_{itk} , the power output p_{it} in (7) is bounded only by the hyper-plane set corresponding to the number of active turbines.

2.3 The maintenance scheduling problem

For each maintenance activity $m \in \mathcal{M}$, the interval between the earliest starting time period E_m and the latest starting time period L_m defines the set of time periods $\mathcal{T}(m)$ when the activity m can start: $\mathcal{T}(m) = \{t \in \mathcal{T} \mid E_m \leq t \leq L_m\}$. We assume that each activity can be completed within the planning horizon \mathcal{T} , i.e., $E_m \leq L_m \leq T - D_m + 1$, where D_m denotes the duration of the maintenance task m .

The definition of the binary variables y_{mt} representing the maintenance decisions (see Appendix C) avoids the definition of time window constraints since the set $\mathcal{T}(m)$ encodes the time window parameters of each activity. Unnecessary y_{mt} variables are excluded from the model because they are defined using $\mathcal{T}(m)$ instead of \mathcal{T} .

For the basic maintenance problem we consider only the constraints on: completion of maintenance tasks, maximum number of generator outages, and mapping the maintenance schedule to the number of active generators.

The task completion constraints (9) enforce each activity to start at one of the feasible time periods $\mathcal{T}(m)$. Constraints (10) compute for each powerhouse the number of maintenance activities r_{it} in execution at time period t , among the set of activities $\mathcal{M}(i)$ that must be completed at station i .

$$\sum_{t \in \mathcal{T}(m)} y_{mt} = 1, \quad \forall m \in \mathcal{M}. \quad (9)$$

$$\sum_{\substack{m \in \mathcal{M}(i) \\ t' \in \{\mathcal{T}(m) \mid (t - D_m + 1) \leq t' \leq t\}}} y_{mt'} = r_{it}, \quad \forall i \in \mathcal{I}, t \in \mathcal{T}. \quad (10)$$

Notice that at time period t an activity m is in execution if it starts between $t - D_m + 1$ and t . This is the interval of index t' on the summation term in (10).

The maximum number of outages O_{it} bounds r_{it} :

$$0 \leq r_{it} \leq O_{it}, \quad \forall i \in \mathcal{I}, t \in \mathcal{T}. \quad (11)$$

O_{it} depends on the maintenance resources. In addition, for a feasible operation, O_{it} cannot exceed the difference between the number of available generators \bar{G}_{it} and the minimum number of generators in service \underline{G}_i , i.e., $O_{it} \leq \bar{G}_{it} - \underline{G}_i, \forall i \in \mathcal{I}, t \in \mathcal{T}$. Notice that \bar{G}_{it} is a time varying parameter, since the number of available generators can be affected by existing generator outages or by previous maintenance scheduling decisions. On the other hand, the minimum number of generators \underline{G}_i is constant in time due to operational requirements.

Constraints (12) map the number of outages r_{it} into the variables z_{itk} . At each period and powerhouse, the maximum number of available generators \bar{G}_{it} equals the sum of the number of outages r_{it} plus the number of active generators k^* corresponding to $z_{itk^*} = 1$.

$$r_{it} + \sum_{k \in \mathcal{K}(i,t)} kz_{itk} = \bar{G}_{it}, \quad \forall i \in \mathcal{I}, t \in \mathcal{T}. \quad (12)$$

Constraints (14)–(13) specify the binary decision variables.

$$z_{itk} \in \{0, 1\}, \quad \forall i \in \mathcal{I}, t \in \mathcal{T}, k \in \mathcal{K}(i, t), \quad (13)$$

$$y_{mt} \in \{0, 1\}, \quad \forall m \in \mathcal{M}, i \in \mathcal{T}(m). \quad (14)$$

2.4 The complete basic model

The GMSP minimizes the sum of the maintenance costs plus the net cost of electricity trade calculated as the sum of the differences between electricity purchases and electricity sales. We refer to the resulting mixed-integer linear programming (MILP) problem as P_B :

$$\underset{\substack{w^+, w^-, u, v, s, \\ a, p, y, z}}{\text{minimize}} \quad \sum_{\substack{m \in \mathcal{M}, \\ t \in \mathcal{T}(m)}} C_{mt} y_{mt} + \sum_{t \in \mathcal{T}} (B_t^- w_t^- - B_t^+ w_t^+), \quad (15)$$

subject to constraints (2)–(14).

3 Tightening approaches

The formulation in Section 2.4 is difficult to solve for realistic instances. In this section we explore three approaches for tightening the formulation: extended formulation, set reduction and valid inequalities.

3.1 Extended formulation

The bound (7) can be very weak because it is valid for any operating condition and for any number of active generators k on the interval $(\bar{G}_{it}, \underline{G}_i)$. However, \bar{P}_{ik} and p_{itk} can be based on the actual number of active generators k and the specific operating conditions at each time period and powerhouse. Constraints (16) specify the power bound for each number of active generators, and (17) ensure the equivalence with the original variables p_{it} and in substitution of (7), constraints (18) define a linear approximation of the power function.

$$p_{itk} \leq z_{ik} \bar{P}_{ik}, \quad \forall i \in \mathcal{I}, t \in \mathcal{T}, k \in \mathcal{K}(i, t). \quad (16)$$

$$\sum_{k \in \mathcal{K}(i,t)} p_{itk} = p_{it}, \quad \forall i \in \mathcal{I}, t \in \mathcal{T}, \quad (17)$$

$$p_{itk} \leq \beta_h^0 + \beta_h^u u_{it} + \beta_h^s s_{it}, \quad \forall i \in \mathcal{I}, t \in \mathcal{T}, k \in \mathcal{K}(i, t), h \in \mathcal{H}(i, k), \quad (18)$$

Thus we have P_E as the MILP with the extended formulation:

$$\text{minimize (15) subject to (2)–(6), (8)–(14), (16)–(18).}$$

The bounds \bar{P}_{ik} for (16) can be obtained as the optimal values q_{ik}^* from maximizing the power output in (18) when the stored water level is maximum:

$$\underset{q,u}{\text{maximize}} \ q_{ik} \ \text{s.t.} \ q_{ik} \leq \beta_h^0 + \beta_h^u u_{itk} + \beta_h^s \bar{S}_i, \quad \forall h \in \mathcal{H}(i, k). \quad (19)$$

3.2 Set reduction

Next we exploit the time window parameters of the maintenance tasks in order to exclude unnecessary set elements. As a consequence, fewer constraints and variables are defined, leading to a tighter continuous relaxation and fewer choices for branching. We aim at reducing the set $\mathcal{K}(i, t)$ that determines both the number of binary variables z_{itk} and the degrees of freedom of the system (8) and (12).

A maintenance activity m beginning at E_m and with duration D_m spans the interval $\mathcal{T}^E(m) = \{t \in \mathcal{T}(m) \mid E_m \leq t < E_m + D_m\}$. Likewise, if activity m starts at L_m , it spans the interval $\mathcal{T}^L(m) = \{t \in \mathcal{T}(m) \mid L_m \leq t < L_m + D_m\}$. The overlap of the two intervals

$$\begin{aligned} \mathcal{T}^O(m) &\triangleq \mathcal{T}^E(m) \cap \mathcal{T}^L(m) \\ &= \{t \in \mathcal{T}(m) \mid L_m \leq t \leq E_m + D_m\}, \end{aligned}$$

defines the set of time periods when the activity necessarily will take place. Likewise, the span of a maintenance activity m is the interval $\mathcal{T}^S(m)$ where the activity can be in execution. Since activity m cannot start before E_m and it can finish no later than $L_m + D_m$, we define

$$\mathcal{T}^S(m) = \{t \in \mathcal{T}(m) \mid E_m \leq t \leq L_m + D_m\}.$$

These definitions are illustrated in Figure 3.

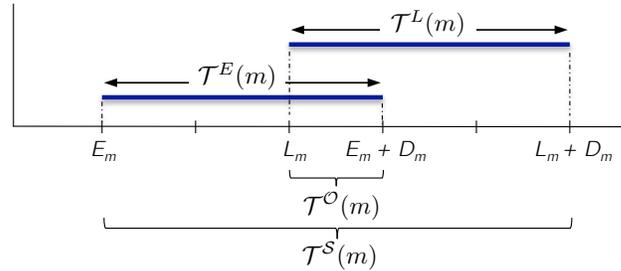


Figure 3: Timeline for a maintenance activity m

The number of maintenance activities that *can* be in execution at powerhouse i during time period t is the cardinality of the set of tasks whose spans $\mathcal{T}^S(m)$ intersect at time period t , that is,

$$\bar{R}_{it} = |\{m \in \mathcal{M}(i) \mid t \in \mathcal{T}^S(m)\}|.$$

Similarly, the set of activities that *must* be in execution at powerhouse i during time period t is,

$$\underline{R}_{it} = |\{m \in \mathcal{M}(i) \mid t \in \mathcal{T}^O(m)\}|.$$

Naturally, \bar{R}_{it} and \underline{R}_{it} bound the number of outages r_{it} :

$$\underline{R}_{it} \leq r_{it} \leq \bar{R}_{it}, \quad \forall i \in \mathcal{I}, t \in \mathcal{T}. \quad (20)$$

Proposition 1 *In formulations P_B and P_E , the feasible number of active generators k at period $t \in \mathcal{T}$ and powerhouse $i \in \mathcal{I}$ is in the set*

$$\mathcal{K}(i, t) = \{k \in \mathbb{Z} \mid \underline{K}_{it} \leq k \leq \bar{K}_{it}\}, \quad (21)$$

where

$$\underline{K}_{it} = \max\{\bar{G}_{it} - O_{it}, \bar{G}_{it} - \bar{R}_{it}\}, \quad (22)$$

$$\bar{K}_{it} = \bar{G}_{it} - \underline{R}_{it}. \quad (23)$$

See Appendix A for a proof of this proposition.

From (21–23) we see that the greater the difference between \bar{G}_{it} and \bar{K}_{it} , as well as between \underline{G}_i and \underline{K}_{it} , the greater the reduction in the number of variables and constraints with index $k \in \mathcal{K}(i, t)$.

3.3 Valid inequalities

Finally, we analyze the linear system formed by constraints (8) and (12), which in general is undetermined and has multiple non-integer solutions. We consider the case when $\underline{R}_{it} = 0$.

If $r_{it} = 0$, then from constraints (12), $\sum_{k \in \mathcal{K}(i, t)} z_{itk} k = \bar{G}_{it}$, which implies $z_{itk} = 1$ for $k = \bar{G}_{it}$, since by constraint (8) only one binary variable z_{itk} should be active for each $(i, t) \in \mathcal{I} \times \mathcal{T}$. On the other hand, if $r_{it} \geq 1$, then $z_{itk} = 0$ for $k = \bar{G}_{it}$ with $(i, t) \in \mathcal{I} \times \mathcal{T}$. By disaggregating r_{it} into the corresponding y_{mt} variables (see (10)), these logical implications are equivalent to

$$\sum_{t' \in \{\mathcal{T}(m) \mid (t - D_m + 1) \leq t' \leq t\}} y_{mt'} + z_{itk} \leq 1, \quad \text{for } k = \bar{G}_{it}, \quad \forall i \in \mathcal{I}, m \in \mathcal{M}(i), t \in \mathcal{T}, \quad (24)$$

which by the binary condition on z_{itk} and y_{mt} are facet defining inequalities.

Also, $r_{it} = 0$ implies $z_{itk} = 0 \forall k \in \{\mathcal{K}(i, t) \setminus \bar{G}_{it}\}$:

$$\sum_{k \in \mathcal{K}(i, t) \setminus \bar{G}_{it}} z_{itk} \leq r_{it}, \quad \forall i \in \mathcal{I}, t \in \mathcal{T}. \quad (25)$$

Next we show that constraints (24)–(25) allow relaxing the integrality of a subset of binary variables z_{itk} when $\bar{K}_{it} = \bar{G}_{it}$ and the number of degrees of freedom of the system (8), (12) is sufficiently small.

Proposition 2 *In models P_B and P_E with constraints (24)–(25) if for some $(i', t') \in \mathcal{I} \times \mathcal{T}$:*

- i) $\underline{R}_{i't'} = 0$,
- ii) $\bar{K}_{i't'} - \underline{K}_{i't'} \leq 2$,
- iii) *there exists an integer feasible solution,*

then the integrality condition (13) for $z_{i't'k} \forall k \in \mathcal{K}(i', t')$ can be relaxed as the variables $z_{i't'k}$ will be integer in any feasible solution.

See Appendix B for a proof of this proposition.

4 Computational experiments

In this section we report on our computational experiments to evaluate the eight formulations obtained starting from the basic model and including/excluding each of the three approaches in Section 3. The eight combinations are given in Table 2, where 1 indicates that a given approach is used in the corresponding formulation, and 0 indicates that the approach was not used.

We conducted two sets of experiments to determine the best combination. First, we solved smaller instances of GMSP and analyzed the computation times to selected a subset of formulations. Second, we evaluated this subset via experiments with larger instances.

Our test instances were adapted from a cascade 4-powerhouse system with 3100 MW generation capacity in the Lac Saint-Jean region in Quebec, Canada (see Table 1). For each powerhouse, we approximated

the hydropower production function with 30 linear inequality constraints (7) and (18). For each instance, maintenance requirements are specified with the following parameters for each activity: index, powerhouse, duration, earliest start time period, and latest start time period.

Table 1: Basic attributes of the power system used in the computational experiments

Powerhouse	System type	Number of generators	Installed capacity (MW)
1	Reservoir	5	205
2	Run of the river	5	210
3	Reservoir	12	402
4	Run of the river	17	1587
Total		39	2404

The powerhouses are ordered from upstream to downstream.

4.1 Computational results for all formulations

In this first set of experiments, we used two levels for each of the following factors of the instance size: number of maintenance tasks (8, 10), number of time periods (20, 25), time window length (5, 8), maximum number of outages in each powerhouse (2, 3), average duration of maintenance tasks (4, 5). For each of the $2^5 = 32$ combinations of these factors, we created two maintenance datasets, for a total of 64 test instances. The size of the MILP formulations ranged from 94 binary variables, 390 continuous variables and 4263 constraints, to 456 binary variables, 775 continuous variables and 12485 constraints. Because randomly generating instances for GMSP is prone to infeasibilities, we created new instances with random changes in a subset of parameters of initial feasible instances. When an infeasible instance was obtained by this procedure, we restored its feasibility by arbitrarily changing the instance parameters.

We ran the tests in a 24-processor server at 2.7 GHz with 32.9 GB RAM, with 4 cores dedicated for running the Xpress MIP solver. The models were coded in C++ with the Xpress BCL 8.1.0 callable library [12].

We chose CPU clock time as the basic performance metric, which allows to measure the actual computation time for solving the problem, without the effect of background processes. Given that the computation times increase significantly with the size of the instance and also differ between instances of similar size, we normalized for each instance the logarithmic CPU time according to the standard score

$$z_{jb} = (t_{jb} - \mu_j^t) / \sigma_j^t, \quad (26)$$

where t_{jb} is the logarithmic CPU time for solving instance $j \in \mathcal{J}$ with formulation $b \in \mathcal{B}$, and μ_j^t , σ_j^t are respectively the mean and the standard deviation of the logarithmic CPU times of the 8 models for solving instance j .

We report in Table 2 the mean \bar{z}_b and standard deviation σ_b^z of z_{jb} over the 64 test instances, for each formulation. The results show that the choice of formulation affects the computation times, as corroborated with a p -value of 0.005 for a one-way ANOVA, which for a significance level of $\alpha = 0.01$ indicates a significant effect of the selected formulation on the logarithmic CPU time.

While formulation 1 had the largest average normalized log CPU time, the smallest time was achieved by formulation 6 (basic model with set reduction and extended formulation). The latter also had the second smallest standard deviation. The maximum standard deviation corresponded to the formulation with only set reduction. Overall, the four formulations 2, 4, 6, and 8 give the best results in Table 2 and Figure 4.

The effect of the choice of formulation also shows in the performance profiles of Figure 5. A performance profile [14] gives the cumulative relative frequency $\rho_b(\tau)$ with which a formulation solves instances of the problem within a factor τ of the best possible value of $\log_2(r_{jb})$, where $r_{jb} = t_{jb} / \min_{b \in \mathcal{B}} t_{jb}$, and

$$\rho_b(\tau) = \frac{1}{n_j} \text{size}\{j \in \mathcal{J} : \log_2(r_{jb}) \leq \tau\}. \quad (27)$$

Table 2: Normalized log CPU times per instance, computed from 64 test instances

Formulation	Tightening approaches			Norm. log CPU time	
	Set reduc.	Valid ineq.	Extended formul.	Average \bar{z}_b	St. dev. σ_b^z
1	0	0	0	1.469	0.35
2	0	0	1	-0.849	0.40
3	0	1	0	0.790	0.38
4	0	1	1	-0.685	0.33
5	1	0	0	0.421	0.50
6	1	0	1	-0.880	0.34
7	1	1	0	0.511	0.39
8	1	1	1	-0.776	0.42

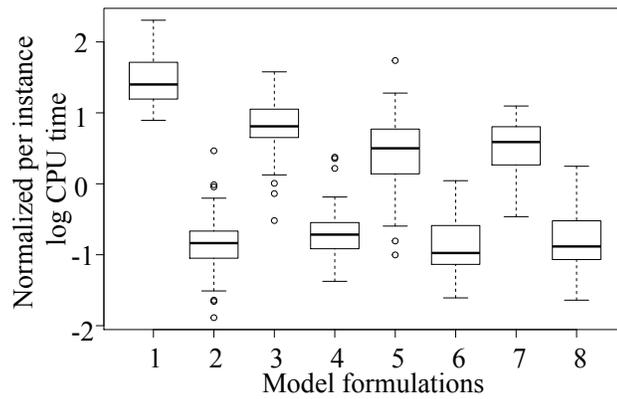


Figure 4: Boxplot of normalized logarithmic CPU times grouped by formulation

In summary, the curves closest to the top left corner correspond to the formulation with the best performance.

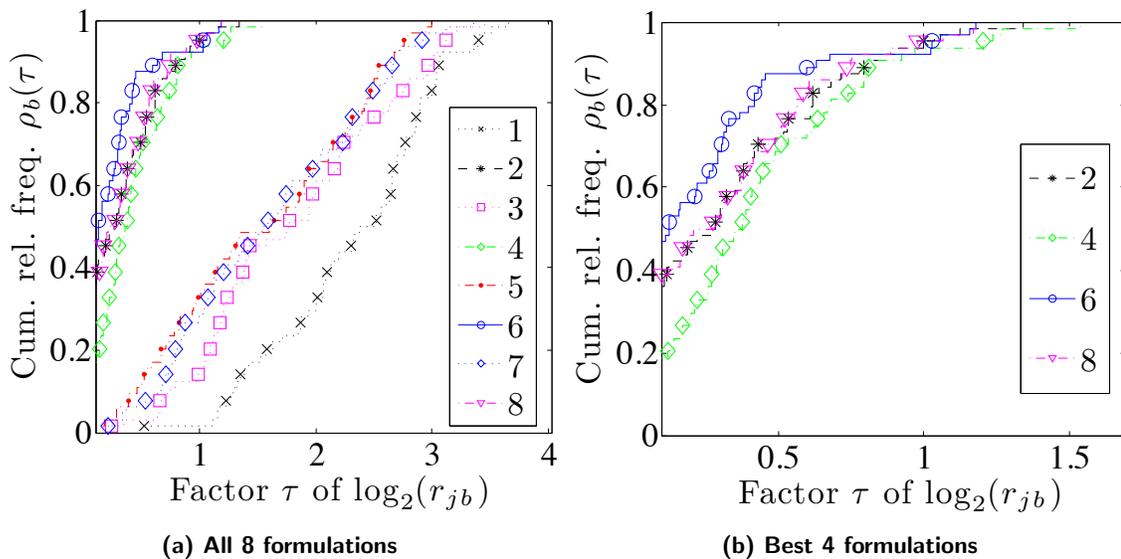


Figure 5: Performance profiles of the tested formulations

Figure 5a shows that the formulations with at least one tightening component perform better than the basic model (formulation 1). In Figure 5b, the performance profiles of the best 4 formulations indicate that formulation 6 is a clear winner for $\tau \leq 0.8$. In less than 10% of the instances, models 2 and 8 are a competitive choice.

The extended formulation is common to the 4 best-performing formulations in Figure 5a. The ANOVA results in Table 3 show that this approach, either alone or in combination with others, has a significant effect for arbitrarily small significance α levels (p -value = 2.36e-12). On the other hand, although formulation 3 (only valid inequalities) outperformed the basic model in Figure 5a, the effect of the valid inequalities is not statistically significant (p -value = 0.758). Finally, the effect of set reduction is only significant for $\alpha \geq 0.2$ (p -value = 0.181).

Table 3: p -values based on normalized log CPU time

Approach	p -value
Set reduction	0.181
Valid inequalities	0.758
Extended formulation	2.36e-12

4.1.1 Optimality gaps of the best formulations

For this second set of experiments, we work only with formulations 2, 6 and 8. These have the smallest average CPU times in Table 2, and clearly outperform formulation 4 in Figure 5b. Our focus is on the optimality gaps that these formulations can achieve for large instances of GMSP.

We tested these formulations on 16 instances with more maintenance tasks than the earlier instances. In particular, we specified: number of maintenance tasks (15, 20), average duration of maintenance tasks (4, 5) and time window length (5, 8). For each of the $2^3 = 8$ combinations of the levels of these factors, we created two datasets, for a total of 16 instances. We also defined a planning horizon with 25 time periods, and a maximum of 2 outages in each powerhouse. Table 4 reports the optimality gap statistics for the three formulations after 1,000 and 20,000 seconds of CPU time on each instance.

Table 4: Optimality gap statistics

Formulation	CPU time 20,000 s		CPU time 1,000 s	
	Mean	St. dev.	Mean	St. Dev
2	0.0144	0.0069	0.0295	0.0235
6	0.0144	0.0071	0.0229	0.0076
8	0.0151	0.0073	0.0273	0.0222

All three formulations reached average optimality gaps below 3 % within 1,000 s. Progress is substantially slower after that, and at 20,000 s (approx. 1 hour on an 8-core machine) the average optimality gap in all three formulations is close to 1.5%. Formulation 6 had the best overall performance after 1,000 s, and formulations 2 and 6 had similar average performance after 20,000 s.

Based on the overall results, we conclude that the most promising approach is the basic model augmented with the extended formulation, and possibly in combination with the valid inequalities. The resulting small optimality gaps for realistic instances of GMSP suggest that they are suitable for practical application.

5 Conclusions

We proposed a mixed-integer optimization model for the GMSP in hydropower systems, and three possible approaches to tighten its continuous relaxation: set reduction, valid inequalities, and extended formulation. Using a set of 64 realistic test instances, we found that the extended formulation had the most significant effect in decreasing the computational time, and that the combination of extended formulation and set reduction achieved the best average performance and small variability in computation time.

We proved that under some conditions, the valid inequalities allow relaxing the integrality condition on a subset of binary variables of the problem. Although this insight did not exhibit a statistically significant

effect in our tests, we consider that the mathematical result can be useful for developing heuristic solution methods for this problem as well as for other problems with similar integer-mapping constraints.

Because the GMSP is solved on a weekly basis, computational times can be higher than the 20,000 seconds used in this study, possibly leading to smaller optimality gaps, and perhaps optimal solutions for medium-size real instances. However, future research is necessary to exploit the structure of the problem for solving more complex real-size instances, such as when the uncertainty of water inflows is considered.

Appendix A Proof of Proposition 1

Proof. From constraints (11) and (20),

$$r_{it} \leq \min\{O_{it}, \bar{R}_{it}\}, \quad \forall i \in \mathcal{I}, t \in \mathcal{T}. \quad (28)$$

From constraints (12),

$$\begin{aligned} \sum_{k \in \mathcal{K}(i,t)} kz_{itk} &= \bar{G}_{it} - r_{it}, & \forall i \in \mathcal{I}, t \in \mathcal{T}, \\ &\geq \bar{G}_{it} - \max\{r_{it}\}, & \forall i \in \mathcal{I}, t \in \mathcal{T}, \\ &= \bar{G}_{it} - \min\{O_{it}, \bar{R}_{it}\}, & \forall i \in \mathcal{I}, t \in \mathcal{T}, & \text{(by Eq. 28)} \\ &= \max\{\bar{G}_{it} - O_{it}, \bar{G}_{it} - \bar{R}_{it}\}, & \forall i \in \mathcal{I}, t \in \mathcal{T}, \\ &\triangleq \underline{K}_{it}. \end{aligned}$$

Then, by constraints (8) and (12), $k \geq \underline{K}_{it}$, $\forall k \in \mathcal{K}(i,t)$. Similarly, from constraints (12),

$$\begin{aligned} \sum_{k \in \mathcal{K}(i,t)} kz_{itk} &= \bar{G}_{it} - r_{it}, & \forall i \in \mathcal{I}, t \in \mathcal{T}, \\ &\leq \bar{G}_{it} - \min\{r_{it}\}, & \forall i \in \mathcal{I}, t \in \mathcal{T}, \\ &= \bar{G}_{it} - \underline{R}_{it}, & \forall i \in \mathcal{I}, t \in \mathcal{T}, \\ &\triangleq \bar{K}_{it}, \end{aligned}$$

which also by constraints (8) and (12) implies $k \leq \bar{K}_{it}$, $\forall k \in \mathcal{K}(i,t)$. \square

Appendix B Proof of Proposition 2

Proof. To simplify the notation, we drop the indices $(i', t') \in \mathcal{I} \times \mathcal{T}$ from $\bar{K}_{i't'}$, $\bar{R}_{i't'}$, $\underline{R}_{i't'}$, $r_{i't'}$, $\mathcal{K}(i', t')$ and $z_{i't'k}$. In any feasible solution to P_B , P_E , variables y_{mt} are binary by (14) and r is integer by (10). By condition *i*), all available \bar{G} generators can be active, which implies $\bar{K} = \bar{G}$ according to (23). Condition *i*) also implies $r \geq 0$ by (20). On the other hand, by constraints (12) and Condition *ii*), $r \leq 2 = \bar{R}$. Therefore, for the analysis of the linear system with constraints (8) and (12), we consider three cases:

1. $r = 0$: By conditions *i*) and *ii*),

$$\mathcal{K} = \{\bar{G}, \bar{G} - 1, \bar{G} - 2\}. \quad (29)$$

Then, the linear system (8) and (12) can be written in extensive form as

$$z_{\bar{G}} + z_{\bar{G}-1} + z_{\bar{G}-2} = 1, \quad (30)$$

$$\bar{G}z_{\bar{G}} + (\bar{G} - 1)z_{\bar{G}-1} + (\bar{G} - 2)z_{\bar{G}-2} = \bar{G} - r. \quad (31)$$

By (25), $r = 0$ implies $z_k = 0 \forall k < \bar{G}$. Then, by (8) $z_{\bar{G}} = 1$. Therefore, the system (30)–(31) has a unique integer solution.

2. $r = 1$: By constraints (10) and (24), $r = 1$ implies $z_{\bar{G}} = 0$. Then, the system (30)–(31) reduces to

$$z_{\bar{G}-1} + z_{\bar{G}-2} = 1, \quad (32)$$

$$(\bar{G} - 1)z_{\bar{G}-1} + (\bar{G} - 2)z_{\bar{G}-2} = \bar{G} - 1, \quad (33)$$

with a unique integer solution $z_{\bar{G}-1} = 1, z_{\bar{G}-2} = 0$.

3. $r = 2$: By constraints (10) and (24), $r = 2$ implies $z_{\bar{G}} = 0$, and the resulting system of equations

$$z_{\bar{G}-1} + z_{\bar{G}-2} = 1, \quad (34)$$

$$(\bar{G} - 1)z_{\bar{G}-1} + (\bar{G} - 2)z_{\bar{G}-2} = \bar{G} - 2. \quad (35)$$

has a unique integer solution $z_{\bar{G}-1} = 0$ and $z_{\bar{G}-2} = 1$.

Therefore, in models P_B, P_E with equations (24) and (25) and conditions $i) - iii)$ satisfied for some $(i', t') \in \mathcal{I} \times \mathcal{T}$, the system (8) and (12) for (i', t') has a unique solution and this solution is integer even if the integrality condition on the $z_{i't'k}$ variables is relaxed for (i', t') and $\forall k \in \mathcal{K}(i', t')$. \square

Appendix C Notation

We denote decision variables and indices with lowercase, and parameters with uppercase.

Primary sets	
\mathcal{I}	Powerhouses
\mathcal{M}	Maintenance tasks
\mathcal{T}	Planning time periods, $t \in \mathcal{T} = \{1 \dots T\}$.
Parameters	
B_t^+	Electricity sale price in time period t , [\$/MWh].
B_t^-	Electricity purchase price in time period t , [\$/MWh].
C_{mt}	Total cost of maintenance task m started at time period t , [\$].
D_m	Duration of maintenance task m [day].
E_m	Earliest start time period of maintenance task m .
F_{it}	Lateral inflows to powerhouse i in period t , [m^3/s].
\bar{G}_{it}	Maximum number of available turbines in powerhouse i at time period t , [turbines].
G_i	Minimum number of available turbines in powerhouse i [turbines].
L_m	Latest start time period of maintenance task m .
O_{it}	Maximum number of turbine outages in powerhouse i at time period t , [turbines].
\bar{P}_i	Generation capacity in powerhouse i , [MWh/day].
\bar{P}_{ik}	Generation capacity in powerhouse i when k turbines are active, [MWh/day].
Q	Factor for conversion from flow per second in m^3 to flow per day in hm^3 [$0.0864 \cdot \text{s} \cdot \text{hm}^3 \cdot /(\text{day} \cdot \text{m}^3)$].
\bar{R}_{it}	Number of maintenance activities that <i>can</i> be in execution at powerhouse i in time period t .
\underline{R}_{it}	Number of maintenance activities that <i>must</i> be in execution at powerhouse i in time period t .
S_{0i}	Initial volume in reservoir of powerhouse i , [hm^3].
\bar{S}_i	Maximum volume in reservoir of powerhouse i , [hm^3].
\underline{S}_i	Minimum volume in reservoir i , [hm^3].
S_{Ti}	Target volume in reservoir of powerhouse i at the end of period T , [hm^3].
\bar{U}_i	Maximum discharge rate in powerhouse i , [m^3/s].
V_i	Maximum water spill in powerhouse i , [m^3/s].
Derived sets	
$\mathcal{T}(m)$	Time periods when maintenance task m can be initiated in order to be completed within \mathcal{T} .
$\mathcal{M}(i)$	Maintenance tasks m that should be executed in powerhouse i .
$\mathcal{M}(i, t)$	Maintenance tasks m that can be in execution in powerhouse i at time period t .
$\mathcal{U}(i)$	Powerhouses upstream of powerhouse i ($\mathcal{U}(i) \subset \mathcal{I}$).
$\mathcal{K}(i, t)$	Numbers of generators that can be active at time period t and powerhouse i .
$\mathcal{H}(i, k)$	Hyperplanes for approximating the maximum power output of powerhouse i when k turbines are active.

Parameters with indexes in derived sets

β_h^u	Coefficient of u_{it} in hyperplane $h \in \mathcal{H}(i, k)$ for bounding the power output of powerhouse i when k generators are active [MWh · s/(m ³ ·day)].
β_h^s	Coefficient of s_{it} in hyperplane $h \in \mathcal{H}(i, k)$ for bounding the power output of powerhouse i when k generators are active [MWh/(hm ³ ·day)].
β_h^0	Independent term of hyperplane $h \in \mathcal{H}(i, k)$ for bounding the power output of powerhouse i when k generators are active [MWh/day].

Decision variables

r_{it}	Number of maintenance activities in execution at powerhouse i and time period t .
p_{it}	Generation of powerhouse i during time period t [MWh/day].
p_{itk}	Generation of powerhouse i during time period t when k generators are active [MWh/day].
s_{it}	Content of reservoir in powerhouse i at the end of period t [hm ³].
v_{it}	Water spill of reservoir in powerhouse i at time period t [m ³ /s].
u_{it}	Water discharge of turbines in powerhouse i at time period t [m ³ /s].
w_t^+	Sale of electricity at period t [MWh].
w_t^-	Purchase of electricity at period t [MWh].
y_{mt}	Binary variable with value 1 if maintenance task m initiates at time period t , 0 otherwise.
z_{itk}	Binary variable with value 1 if k hydro-turbines are active in powerhouse i at time t , 0 otherwise.

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