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Heuristic method for the stochastic open pit mine production scheduling problem

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Abstract: Long-term open pit mine scheduling is generally assessed with a mixed integer programming (MIP) formulation which can be solved with different operations research techniques. To be closer to the reality of the exploitation, a model can, for instance, take into account a substantial number of blocks to represent the ore body, include several destinations, or consider the uncertainty of the geology with a stochastic formulation. The inherent complexity of such a model becomes too great to obtain an optimal solution or even a good feasible solution within a reasonable computational time. This paper first proposes several strategies to facilitate the resolution of such an MIP by reducing the number of binary variables. To do so, no assumptions are made over the final result; only a relaxation of binary constraints over a special pattern is considered. A fast heuristic method, defined as a stochastic topological sorting method, is also developed and provides a proof of optimality. The proposed methods are tested on a real case study and provide results within 2% of optimality after 12 minutes and down to 0.3% if a longer running time is allowed.

Keywords: Open pit mining, long-term production scheduling, stochastic optimization, topological sorting, partial relaxation, heuristic method

1 Introduction

A realistic forecast of the long-term production is required to evaluate the viability of an open pit mining project. Due to major costs of infrastructure and investments, lower revenues than expected or only delayed returns on investment can compromise all the project and even cause the bankruptcy of the company. Operations research is traditionally used to deal with all kinds of production prediction problems, and mine planning is one of them. Several MIP models have been developed over the last decades, see Ramazan and Dimitrakopoulos (2004) or Newman et al. (2010), for instance. In these models, the ore body is usually discretized into mining blocks and the objective of the optimizer is to schedule the extraction of the blocks in an optimal most profitable way, which is maximising the net present value over the life of mine. Johnson (1968) was among the first to propose a linear program to optimize the blocks extraction, but the linearity of its model resulted in partial extractions which could prove to be unfeasible. Gershon (1983) formulated a MIP model but the computational requirements limited its use. Dagdelen (1985) solved the MIP model with a Lagrangian relaxation but, again, could not guarantee feasibility. Since then, many improvements and good quality heuristic methods have appeared in the literature. Ramazan and Dimitrakopoulos (2004) successfully reduced the number of binary variables, major source of complexity, to facilitate the resolution. Cullenbine et al. (2011) proposed a sliding time window heuristic method which gave solutions within 2% of optimality. Several methods using aggregations of blocks have also been introduced. Ramazan et al. (2005) proposed what they call fundamental trees, which are aggregations of blocks with respect to the slope constraints and the similarities of the blocks' values. Boland et al. (2009) presented a method that schedules the aggregates but defines the processing at the block level. Although efficient, drawback of aggregating blocks is the way blocks are grouped, which is a strong assumption that constrains greatly the solution space, misrepresents mining selectivity and provides misleading results. Chicoisne et al. (2012) presented a topological sort based heuristic that is used as a basis of the final algorithm of this paper. The authors first efficiently solve the linear relaxation of a simplified model (without, for instance, blending constraints) with a decomposition method they call critical multiplier algorithm. Then, they use the obtained fractional schedule as an input for a topological sorting algorithm. Finally, they refine the solution with a local search algorithm defined as a descent method over subsets of blocks. All these methods are deterministic, as they do not account for uncertainty, but they are still widely used in the industry. Furthermore, many commercial schedulers use similar methods (GeoVia Whittle Strategic Mine Planning). Nonetheless, in reality the geological data is far from being known with certainty. It comes from drill holes which have to be sparse because of the dimensions of the deposits and the considerable drilling costs. From this data, average interpolations, named estimations (such as kriging), are conducted to attribute to each block its characteristics. These estimations tend to smooth the grades and do not represent the extreme values or the unavoidable uncertainty, which mislead the mine planning optimization. That's why a new stochastic approach has been developed for two decades; see a review in Dowd (1994); Ramazan and Dimitrakopoulos (2004); Dimitrakopoulos and Godoy (2006); and Dimitrakopoulos (2011). In this risk-based approach, equiprobable simulated realizations of the ore body are generated and used as an input for the mine scheduling. A stochastic conditional simulation can be defined as a "Monte Carlo technique which represents the in-situ ore body grade and material type variability", Dimitrakopoulos (1998). More details can also be found in Goovaerts (1997). The information provided by the simulations about the grade variability allows managing the risk while maximising the net present value (NPV). Dimitrakopoulos and Ramazan (2008) proposed a Stochastic Integer Programming (SIP) model for the open pit mine scheduling which was then used several times and has proven its efficiency. Compared to a deterministic approach, the stochastic approach consistently increases the NPV (up to 25% see Dimitrakopoulos 2011, Ramazan et al. 2013, Spleit 2014) and controls better the risk of getting a poor quality production over the set of simulations. Of course, solving such models is even harder than for the deterministic case, and exact methods become unpractical when dealing with instances of realistic size. Several methods have been developed. Among them, we find again the use of blocks aggregation (Menabde, 2004) as well as several metaheuristic methods. Metaheuristic methods do not rely on usual operation research solvers but generally start from an initial solution and modify it, allowing temporary deterioration of the objective function. Some metaheuristics have managed to tackle very large instances but their performance depends on the computational time allowed and the definition of many parameters as well as the generation of an initial solution. Lamghari et al. (2012) developed a tabu search method. Montiel et al. (2015) and Goodfellow et al. (2015) both used on simulated annealing, combined with particle swarm optimisation and differential evolution for the second to address stochastic mining complexes. Gilani and al. (2016) applied an ant colony optimization algorithm. Lamghari et al. (2016) presented a progressive hedging method, a scenario-based decomposition technique, hybridized with a sliding time window heuristic.

In this work, an SIP model is solved. To reduce the computational time, the solution approach takes advantage of the special structure of the problem. The optimisation of the problem is performed using the commercial solver Cplex (CPLEX User's Manual V12R6 2014, CPLEX Parameters References V12R6 2014) and does not require neither an

initial solution nor strong assumptions such as aggregates. The obtained intermediate solution, not fully binary, is then refined. The resulting global method aims to rapidly produce a near-optimal. In what follows, we first present the general Open pit Mine Planning Stochastic Integer Program (OMPSIP) based on Dimitrakopoulos and Ramazan's (2008, 2013) and Spleit's (2014) formulations. Then, two strategies of acceleration are proposed, both aiming to reduce the number of binary variables using the strong intercorrelations between the blocks. The first strategy relaxes variables over a special alternate pattern which simplifies the resolution, while still providing an almost binary solution. The second strategy iteratively selects exclusive sets of variables on which binary constraints are applied with the previous pattern, in order to converge toward a binary solution without any loss of optimality. Finally, a fully binary scheduling algorithm is presented as a stochastic topological sorting on the precedence digraph and weighted by the previous partial schedules obtained. The methods are tested on a case study, a real iron ore deposit owned by the industrial partner. Computational results are presented in Section 5.3 and are followed by conclusions.

2 Model

This part first presents the SIP model and two computational acceleration strategies. Then, the main algorithm is detailed.

2.1 Notation

Diverse sets, indices, and parameters, widely used in the proposed Open pit Mine Planning Stochastic Integer Programming (OMPSIP) formulation are described below.

Sets and corresponding indices

$\mathcal{B} = \{i = 1, \dots, N\}$	Set of blocks in the ore body;
$\mathcal{P} = \{p = 1, \dots, P\}$	Set of considered periods for the schedule;
$\mathcal{D} = \{0, 1\}$	Set of destinations available for the blocks where 0 represents the waste dump and 1 the mill;
$\mathcal{S} = \{s = 1, \dots, S\}$	Set of scenarios (equiprobable ore body stochastic simulations);
$\mathcal{C} = \mathcal{C}_1 \cup \mathcal{C}_2$	Set of blocks' characteristics, $\mathcal{C}_1 = \{c_1 = 1, \dots, C_1\}$ linear metallurgical characteristics (e.g. tonnages and trucks hours), $\mathcal{C}_2 = \{c_2 = 1, \dots, C_2\}$ nonlinear characteristics (grades);
$G(\mathcal{B}, A)$	Oriented graph representing the precedence relationships between blocks. On Figure 1, $(b, e) \in A$ which means that block $b \in \mathcal{B}$ is a predecessor of block $e \in \mathcal{B}$;
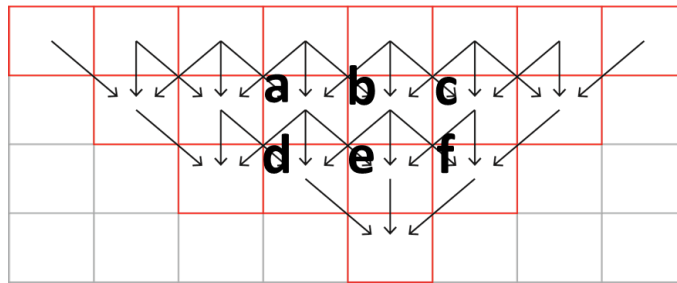
Figure 1 Precedence relationships between blocks

|	
$\Gamma_i^+ = \{b \in \mathcal{B}; (i, b) \in A\}$	Set of direct successors of block i. On Figure 1, $\Gamma_b^+ = \{d, e, f\}$;
$\Gamma_i^- = \{a \in \mathcal{B}; (a, i) \in A\}$	Set of direct predecessors of block i. On Figure 1, $\Gamma_e^- = \{a, b, c\}$;
Γ_i^{-Tot}	Set of the all cone of predecessors of block i. On Figure 1, $\Gamma_e^{-Tot} = \{a, b, c\} \cup \Gamma_a^{-Tot} \cup \Gamma_b^{-Tot} \cup \Gamma_c^{-Tot}$;
$\mathcal{N}(i)$	Set of neighbours of block i: typically blocks at the North, East, South and West on the same level and one block below;
$\mathcal{B}^{1/2}$	Subset of the ore body as a checked pattern defining one block on two toward each direction (Figure 2 and Figure 3);
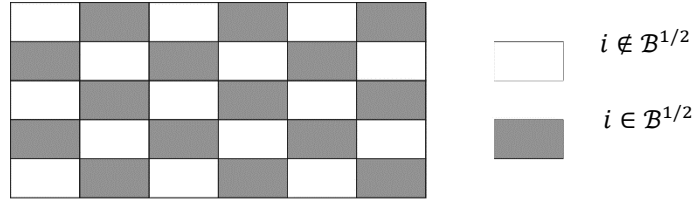
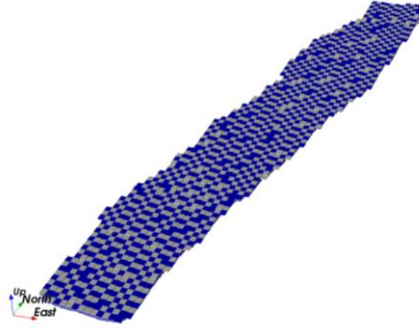Figure 2 Vertical section of the checked pattern ore body

$$i \notin \mathcal{B}^{1/2}$$

$$i \in \mathcal{B}^{1/2}$$

Figure 3 Case study checked pattern ore body

Parameters

$v_{i,d,s}$	Economic value of block i in scenario s if it is sent to destination d;

This economic value depends on several parameters:

$$v_{i,d,s} = \begin{cases} -E_{waste}^{cost}.t_{i,s} - TH^{cost}.TH_{i,d} & \text{if } d = 0 \Leftrightarrow \text{waste dump} \\ R_i - P_{conc}^{cost}.conc_{i,s} - E_{ore}^{cost}.t_i - TH^{cost}.TH_{i,d} & \text{if } d = 1 \Leftrightarrow \text{mill} \end{cases}$$

With:

R_i — Revenue from selling the metal content of block i;

$conc_{i,s}$ — Concentrate tonnes of block i in scenario s, $conc_{i,s} \in \mathcal{C}_1$;

Where:

$conc_{i,s} = t_{i,s}.Rec_{i,s}$

$Rec_{i,s}$ — Weight recovery of block i in scenario s, obtained from the simulation of the Davis Tube Weight Recovery (used in the case study);

P_{conc}^{cost}	Processing cost of concentrate material per tonne;
E_{ore}^{cost}	Extraction cost of ore material per tonne;
E_{waste}^{cost}	Extraction cost of waste material per tonne;
$TH_{i,d}$	Truck hours needed to send the material of block i to destination d;
TH^{cost}	Cost per truck hour;
$t_{i,s}$	Tonnes of block i in scenario s;
$q_{c_1,i,s}$	Quantity of characteristic c_1 of block i in scenario s;
$g_{c_2,i,s}$	Grade c_2 in scenario s of block i;
$target_{c,p}^{\pm}$	Minimum (-) and maximum (+) targets of quantity or grade c in period p;
$pen_{c,p}^{dev\pm}$	Penalty cost of deviation from the targets of quantity or grade c in period p (excess +, shortage -);
r	Discount rate taking into account the time value of money and the uncertainty of the future streams of cash flows;
$d_p = \frac{1}{(1+r)^{p-1}}$	Discount factor;

Variables

Binary variables

$x_{i,d,p} = \begin{cases} 1 \text{ if block } i \in \mathcal{B} \text{ is sent to destination } d \in \mathcal{D} \text{ by period } p \in \mathcal{P} \\ 0 \text{ otherwise} \end{cases}$;

To simplify the notation, we set $\boldsymbol{x_{i,d,p=0} = 0}, \ \forall \boldsymbol{i} \in \boldsymbol{\mathcal{B}}, \forall \boldsymbol{d} \in \boldsymbol{\mathcal{D}}$.

The expression "by period $p \in \mathcal{P}$" means that block i was extracted prior to or at period p, formulation used to facilitate the branching during the solving process (Caccetta and Hill, 2003).

Continuous variables

$\boldsymbol{dev^{\pm}_{c,p,s}} \in \mathbb{R}^+$ Deviations from the targets in terms of characteristics $c \in \mathcal{C}$ for scenario $s \in \mathcal{S}$, during period $\boldsymbol{p} \in \boldsymbol{\mathcal{P}}$ (excess +, shortage -);

2.2 General stochastic formulation

This section describes the OMPSIP formulation which will be used in the rest of the study.

Objective function

$$max\ Z = \overbrace{\frac{1}{S}\sum_{i\in\mathcal{B}}\sum_{d\in\mathcal{D}}\sum_{p\in\mathcal{P}}\sum_{s\in\mathcal{S}} d_p \cdot v_{i,d,s} \cdot (x_{i,d,p} - x_{i,d,p-1})}^{\textit{Part 1}} - \overbrace{\sum_{c\in\mathcal{C}}\sum_{p\in\mathcal{P}}\sum_{s\in\mathcal{S}} d_p \cdot \left(pen^{dev+}_{c,p} \cdot dev^+_{c,p,s} + pen^{dev-}_{c,p} \cdot dev^-_{c,p,s}\right)}^{\textit{Part 2}}$$

The objective function is a trade-off: $Part$ 1 aims at maximising the average profit, discounted cash flow (DCF), while $Part$ 2 minimizing the deviations, that is the risk associated with the geological uncertainty. Using this formulation, the expected result is a schedule robust to the set of simulations. This formulation accepts a lower average DCF to better control the risk. The application of the discount factor also delays the risk and favors the extraction of the most valuable blocks in the early periods. The latter is a key point for mining companies as they usually expect a fast return on their investment.

Constraints

Reserve constraints

(1) $x_{i,d,p} - x_{i,d,p-1} \geq 0$ $\forall i \in \mathcal{B}, \forall d \in \mathcal{D}, \forall p \in \mathcal{P}$

(2) $\displaystyle\sum_{d\in\mathcal{D}} x_{i,d,p} \leq 1$ $\forall i \in \mathcal{B}, \forall p \in \mathcal{P}$

The first set of constraints (1) specifies that a block extracted at a certain period is also defined as already extracted in the following periods. The set of constraints (2) states that a block can only be extracted once and sent to only one destination.

Slope constraints

(3) $\displaystyle\sum_{d\in\mathcal{D}} x_{i,d,p} \leq \sum_{d\in\mathcal{D}} x_{j,d,p}$ $\forall i \in \mathcal{B}, \forall j \in \Gamma^-_i, \forall p \in \mathcal{P}$

A block i is available for extraction only if all of its direct predecessors Γ^-_i have already been extracted or are extracted within the same period. This means that the block is reachable; i.e., without blocks above it and that the slope constraints for the stability of the walls are satisfied.

Capacities constraints

(4.1) Upper bound

$$\sum_{i\in\mathcal{B}}\left(q_{c_1,i,s} \cdot (x_{i,d,p} - x_{i,d,p-1})\right) - dev^+_{c_1,p,s} \leq target^+_{c_1,p} \qquad\qquad \forall c_1 \in \mathcal{C}_1, \forall p \in \mathcal{P}, \forall s \in \mathcal{S}$$

(4.2) Lower bound

$$\sum_{i\in\mathcal{B}}\left(q_{c_1,i,s} \cdot (x_{i,d,p} - x_{i,d,p-1})\right) + dev^-_{c_1,p,s} \geq target^-_{c_1,p} \qquad\qquad \forall c_1 \in \mathcal{C}_1, \forall p \in \mathcal{P}, \forall s \in \mathcal{S}$$

These two sets of constraints define soft constraints for the upper (4.1) and lower bound (4.2) on the quantities targets at each period and in each scenario. The variables $dev^\pm_{c_1,p,s}$ are used as buffers to allow deviations but are penalized in the objective function.

Grade quality constraints
(5.1) Upper bound

$$\sum_{i\in\mathcal{B}}\left(g_{c_2,i,s}\cdot t_{i,s}\cdot\left(x_{i,d,p}-x_{i,d,p-1}\right)\right)-dev^+_{c_2,p,s}\leq target^+_{c_2,p}\cdot\sum_{i\in\mathcal{B}}\left(t_{i,s}\cdot\left(x_{i,d,p}-x_{i,d,p-1}\right)\right)\quad\forall c_2\in\mathcal{C}_2,\forall p\in\mathcal{P},\forall s\in\mathcal{S}$$

(5.2) Lower bound

$$\sum_{i\in\mathcal{B}}\left(g_{c_2,i,s}\cdot t_{i,s}\cdot\left(x_{i,d,p}-x_{i,d,p-1}\right)\right)+dev^-_{c_2,p,s}\geq target^-_{c_2,p}\cdot\sum_{i\in\mathcal{B}}\left(t_{i,s}\cdot\left(x_{i,d,p}-x_{i,d,p-1}\right)\right)\quad\forall c_2\in\mathcal{C}_2,\forall p\in\mathcal{P},\forall s\in\mathcal{S}$$

Similar to the capacities constraints, constraints (5.1) and (5.2) penalize excess and shortage of the average grade c_2 within one period.

Extraction smoothing constraints

(6) $$\sum_{d\in\mathcal{D}}x_{i,d,p}\leq\sum_{d\in\mathcal{D}}x_{j,d,p}\qquad\qquad\qquad\qquad\qquad\forall i\in\mathcal{P}(\mathcal{B}),\forall j\in\mathcal{N}(i),\forall p\in\mathcal{P}$$

These operational constraints impose a continuous sequence of extraction in a way that the extracted blocks, at least within the same period, should be close to each other. We could penalize only nearby blocks not extracted together, but the case study considered in this paper justifies this stronger formulation in which we enforce the blocks on a checked pattern to be simultaneously extracted with its neighbours.

Earliest period of extraction constraints

(7) $$x_{i,d,p}=0\qquad\qquad\forall i\in\mathcal{B},\forall d\in\mathcal{D},\forall p\in\left\{\bigcup_{c_1\in\mathcal{C}_1}\left(t\in\mathcal{P};\ \forall s\in\mathcal{S},\sum_{t'=1}^{t}target^+_{c_1,t}\leq\sum_{j\in\Gamma_i^{-Tot}\cup\{i\}}q_{c_1,j,s}\right)\right\}$$

These last constraints, equivalent to an earliest start of a job, are optional. They eliminate variables to make the model easier to solve. The idea is that to reach a block i by period p, at least all its full cone of predecessors Γ_i^{-Tot} must be extracted. This cone plus the block i represent a certain tonnage or quantity which can be compared to the sum of the quantity targets from the first period to period p. If this last tonnage is less than the one of the cone, it is impossible to reach i by p, even in the most optimistic situation in which only the cone is mined. As a consequence, in such a case, the corresponding variables $x_{i,d,p}$ can be set to 0, which says that block i will not be extracted at period p, without any loss of optimality.

3   Acceleration strategies

3.1   Partial relaxation

The main issue when solving this kind of SIP problem with commercial solvers like Cplex is the required computational time. Indeed, the larger the number of variables and constraints is, the more complicated it is to obtain an optimal solution. When binary variables are considered, the complexity gets much larger and even obtaining a reasonably good solution may be hopeless, unless the problem is decomposed and solved sequentially. The ideas developed in this section aim to precisely reduce the amount of binary variables in order to accelerate the solution process. Of course, the inherent goal is also to obtain a final result close to the initial formulation; i.e., extraction variables which must be binary.

General assessment

The precedence relationship between two blocks i and j strongly links their extraction variables $x_{i,d,p}$ and $x_{j,d,p}$. Directly from the slope constraints (3), the following expression is obtained:

(8) $\quad \left(\forall i \in \mathcal{B}, \forall d \in \mathcal{D}, \forall p \in \mathcal{P}, \quad x_{i,d,p} = 1 \right) \Rightarrow \left(\forall j \in \Gamma_i^{-Tot}, \quad \sum_{d \in \mathcal{D}} x_{j,d,p} = 1 \right)$

This leads to the idea that if one block $\boldsymbol{i}$ is constrained to be binary then its extraction at period $\boldsymbol{p}$ enforces all its predecessors to be fully mined too: all the corresponding extraction variables will be binary since, in practice, a block is sent to only one destination.

Partial relaxation using an alternate checked pattern

For a given mining block i, the reserve constraints (1) and (2) tighten the possible values if for a given period the extraction variables have to be binary. From this assessment comes this idea of enforcing the binarity of only one block on two toward each direction with an alternation between two consecutive periods as shown on Figure .
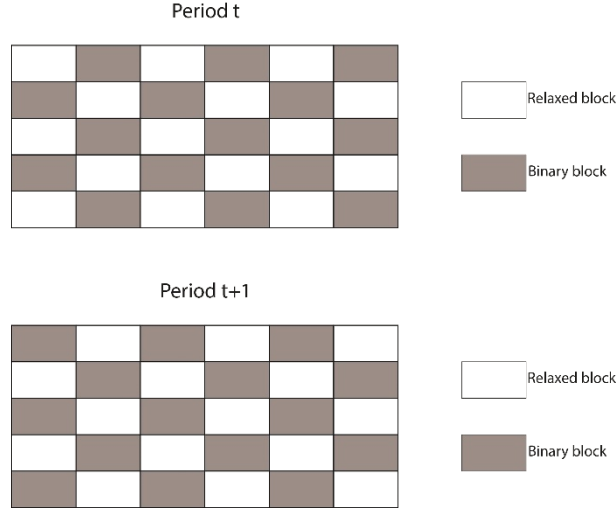
Figure 4 Alternate checked pattern relaxation

Formally, this can be defined as:

(9) $\quad x_{i,d,p} \in \begin{cases} \{0,1\} \text{ if } \left((i \in P(B) \text{ and } p \bmod 2 = 0) \text{ or } (i \notin P(B) \text{ and } p \bmod 2 = 1) \right) \\ [0,1] \text{ otherwise} \end{cases} \quad \forall i \in \mathcal{B}, \forall d \in \mathcal{D}, \forall p \in \mathcal{P}$

This relaxation divides the number of binary variables by two while leading toward an almost binary result and without adding any constraints.

Based on the above remarks, the following expression is obtained:

(10) $\quad \left(\forall i \in \mathcal{B}, \forall d \in \mathcal{D}, \forall p \in \mathcal{P}, \quad x_{i,d,p} > 0 \right) \Rightarrow \left(x_{i,d,p+1} = 1 \right)$

It means than whenever a block has begun to be extracted (partially or fully), it has to be fully extracted by the next period. The definition of the extraction variables by Equation (9) reduces quite substantially the solution space.

Then, the extraction variables corresponding to the waste destination $x_{i,d=0,p}$ can also be relaxed. A similar relaxation was proposed by Ramazan and Dimitrakopoulos (2004). The authors predefined the destination of each block based on its economic value and relaxed the extraction variables corresponding to blocks to be sent to the waste dump. The supportive idea is that a block sent to the waste dump is extracted only to access other profitable blocks below it. The solver has no interest in fractioning its extraction so binary values can be expected for these variables. When adding this new relaxation, the total amount of binary variables is divided by 4 compared to the initial model.

In order to test the efficiency of this partial relaxation, a set of tests has been run with three different integrality gaps (1%, 2% and 5%) and different numbers of periods (2, 3 and 4). The ore body used is the full ore body model presented later in Section 5. It contains 8223 blocks and 2 destinations. Table 1 presents, for different gaps and different number of periods, the results obtained for the partially relaxed model and the fully binary one. Columns 4 and 5 give the number of binary variables in the model before and after pre-processing in Cplex. Column 6 presents the computational time for the branch and cut algorithm, while column 7 shows the total computational time. Column 8 shows the computational efficiency of the partially relaxed model (PR) compared to the fully binary model (FM).

Column 9 shows the difference in the objective function of Cplex between the partially relaxed model and the full one; of course since the full model is more constrained its objective function is slightly lower. Columns 10 and 11 present the resulting number of fully extracted blocks and the remaining non-binary extraction variables.

Table 1 Comparison between the partially relaxed model and the full binary model

GAP	#PERIODS	MODEL	#binary variables	#binary variables Cplex	branch&cut (s)	time (s)	Comparison PR vs FM	Difference obj. cplex PR-FM	#fully extracted blocks	#non binary extraction variables
1%	2	PR	8223	6919	549	678	41.10%	0.15%	291	43
		FM	32892	9015	1018	1152			361	0
	3	PR	12334	10828	2253	2539	62.50%	0.08%	416	82
		FM	49338	14146	6477	6780			467	0
	4	PR	16446	14871	6858	7339	5.60%	0.07%	521	86
		FM	65784	19407	7318	7773			606	0
2%	2	PR	8223	6919	562	700	32.60%	0.15%	284	43
		FM	32892	9015	996	1039.2			361	0
	3	PR	12334	10828	2258	2560	62.10%	0.08%	416	82
		FM	49338	14146	6472	6756			467	0
	4	PR	16446	14871	6300	6776	13.90%	0.07%	521	86
		FM	65784	19407	7409	7874			606	0
5%	2	PR	8223	6919	13	142	10%	1.10%	484	30
		FM	32892	9015	13	158			494	0
	3	PR	12334	10828	64	358	66.70%	0.94%	595	105
		FM	49338	14146	750	1075			687	0
	4	PR	16446	14871	4354	4841	23.50%	0.40%	834	42
		FM	65784	19407	5834	6324			825	0

PR = Partially Relaxed Model (1on2)
FM = Full Model (all binary)
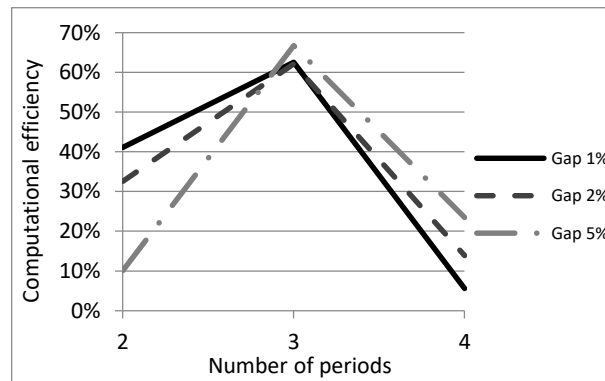
Figure 5 Computational efficiency: partial relaxation vs initial model
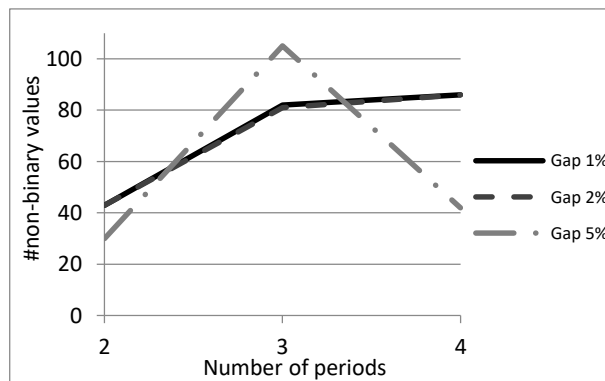
Figure 6 Number of non-binary values of the partially relaxed model

Even if it is difficult to define a general tendency about the efficiency of the partial relaxation (Figure), a computational gain of time is always present, from 5.6% and up to 66.7%. A similar effectiveness can be expected for

more periods. Of course, the trade-off of this gain is that some variables remain fractional (Figure), but they are very few: considering 4 periods those variables are between 42 and 86 (depending on the Gap) out of 65784 initially binary defined variables.

3.2   Binary convergence

The previous partial relaxation uses the strong relationship between blocks but these interconnections can be exploited further. Indeed, the optimization process is led by high value blocks which results in that their predecessors are extracted fast and completely. This idea is confirmed by the resolution of the Relaxed OMPSIP (R-OMPSIP) model (for which all the binary variables are linearized). The result of the relaxation is a fractional schedule but most of the blocks are either mined at once or within few consecutive periods. As an example, for the following case study, which consists of 8223 blocks, 10 periods and 2 destinations (164460 binary variables), the relaxed solution only presents 4819 fractional values, which corresponds to 1225 different blocks.

A sequential algorithm is proposed to decrease again these non-binary values. The general concept is to solve the relaxed model R-OMPSIP and to apply binary constrains only to a subset of variables, denoted Λ_k, at each iteration k, defined as:

$$\Lambda_k = \left\{ x_{i,d,p}; \ \exists k' \in [\![0, k-1]\!], \ x_{i,d,p}^{k-1} \notin \{0,1\} \right\}$$

More explicitly, this is the set of all the supposedly binary variables which have been attributed a non-binary value during any previous iteration. On this set is applied the partial relaxation defined in Section 3.1. Figure presents the result of the relaxed model. The blocks identified by a black cross are the partially extracted ones. In the next iteration (Figure), these same blocks are this time enforced to be binary (white cross), the new result presents other partially extracted blocks which once again will be binary constrained during the following iteration.
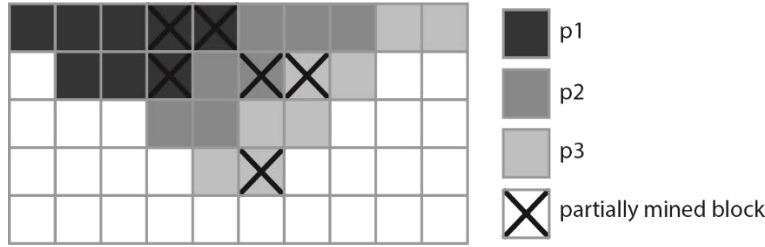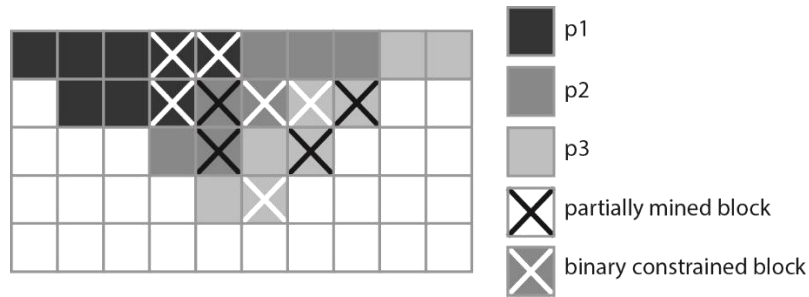

Figure 7 Binary convergence iteration 0

Figure 8 Binary convergence iteration 1

The expected result is a decreasing amount of non-binary variables. An interesting point is that, compared to the initial OMPSIP model, the space of search (feasible solutions) is only enlarged, so with the hypothesis that the algorithm converges to a fully binary solution, this solution would also be optimal for the initial model. Of course this is deceptive but a low number of fractional values would be totally satisfactory from the operational point of view.

The proposed binary convergence algorithm has been tested on the case study deposit with 8 iterations. The results are convincing, after 8 iterations of less than one hour each, the remaining number of fractional values drops to 348 (Figure), which corresponds to 202 different blocks. Of course, the number of binary constrained variables increases with the number of iterations but remains low: from 2372 at iteration 1 up to 6129 at iteration 7. Figure presents the

evolution of the number of remaining fractional values iteration after iteration (decreasing function) and the computational time they required.
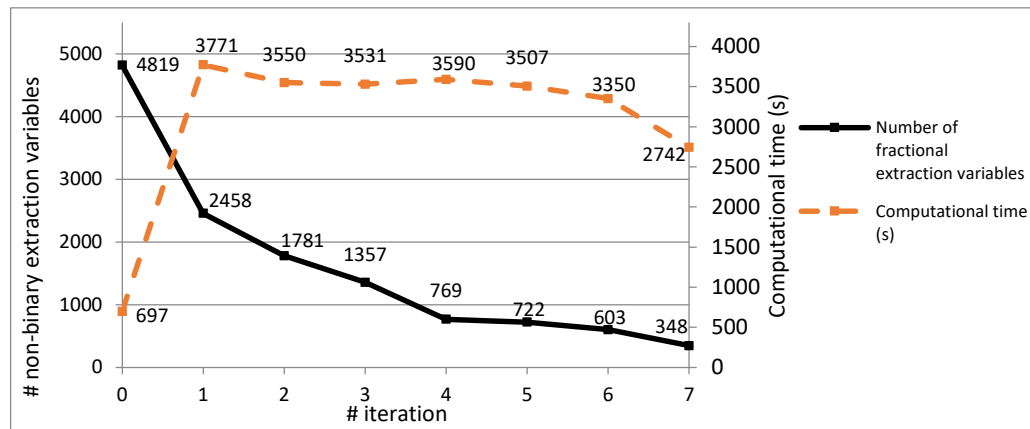

Figure 9 Binary convergence

Moreover, the associated risk is also well controlled. For example, Figure shows the Discounted Cash Flow (DCF) profiles for all the scenarios and their average. The scenarios do not present significant deviations from the average. Figure presents the profiles of the silica grade which is well in the range of tolerance, only less than 0.1% above for two scenarios and two periods among the late periods. This is acceptable and normal because the risk is differed to the latest periods with the applied discounted factor.
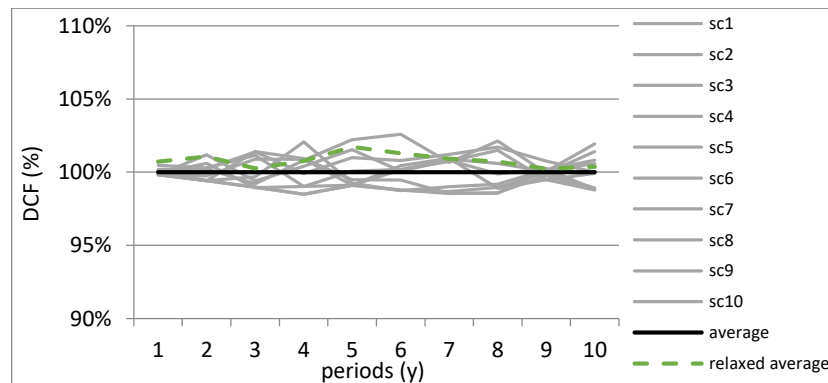

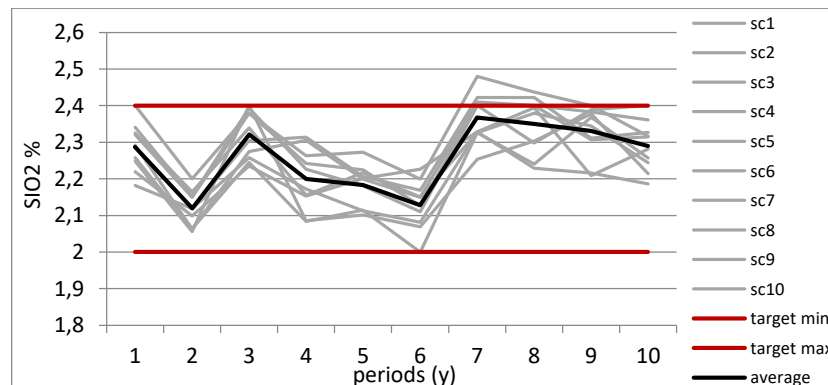**Figure 10 DCF per year of the binary convergence algorithm after iteration 7**

Figure 11 Silica grade from the binary convergence algorithm after iteration 7

4 Heuristic method

The previous algorithms facilitate the resolution of the OMPSIP model by reducing the number of binary variables, provide a final solution almost but not fully binary and require a reasonable computational time. The objective is now, still using strategies to reduce the computational time, to develop a heuristic method which is fast and which provides a full binary high quality solution. The method here was inspired from Chicoisne and Espinoza's work (2012). It was adapted to a stochastic formulation with several destinations and more block selectivity.

4.1 Linear relaxation

The first step is to solve the relaxed model R-OMPSIP described in Section 3.2. The result is a fractional schedule which will be used as an input for the main algorithm. Two options can be considered. The first one is to only use this relaxed solution which takes 11 minutes to obtain for the case study considered in this paper. The other option is to apply a few iterations of the binary convergence algorithm described in Section 3.2, for which each iteration takes around one hour. This second option gives a more binary input and is supposed to be closer to the optimal binary result. Depending on the time allowed for the total computational time, one option or the other can be used. Both are tested to study their impact over the final schedule.

4.2 Stochastic TopoSort Algorithm

The main idea of the algorithm is a topological sorting of the blocks based on pre-defined weights and under the condition that each block, when scheduled, is available; that is, its predecessors have already been extracted. The definition of the weights is essential as, for the assignment of a block to a period, the block having the highest weight among the available blocks is selected. More details are given after the formulation of the algorithm.

Global algorithm

Additional notation

$x_{i,d,p}^*$ Fractional value obtained from the relaxed model representing the percentage of block $i \in \mathcal{B}$ sent to destination $d \in \mathcal{D}$ at period $p \in \mathcal{P} \cup \{0\}$;

$d_{c_1,p,s}^*$ Value of the deviation of $c_1 \in \mathcal{C}_1$ from the quantity target in period $p \in \mathcal{P}$, for scenario $s \in \mathcal{S}$, obtained from the relaxed model;

(p_i, d_i) Pair of variables used to store the period of extraction and destination assigned to the block $i \in \mathcal{B}$;

$rcap_{c_1,p,s}$ Residual capacity of quantity $c_1 \in \mathcal{C}_1$ in period $p \in \mathcal{P}$ for scenario $s \in \mathcal{S}$; i.e., the quantity that can still fit into period p without exceeding the upper bound target;

Definition of the weights
From the relaxed or partially relaxed solution, two sets of weights $\{w1_i, \ \forall i \in \mathcal{B}\}$ and $\{w2_{i,d}, \forall i \in \mathcal{B}, \forall d \in \mathcal{D}\}$ are defined and will be used in the TopoSort algorithm.

$$(11) \qquad E1_i = \sum_{p=1}^{P} p. \sum_{d \in D} \left(x_{i,d,p}^* - x_{i,d,p-1}^*\right) + (P+1)\left(1 - \sum_{d \in D} x_{i,d,P}^*\right) \qquad \forall i \in \mathcal{B}$$

$$(12) \qquad w1_i = -E1_i \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \forall i \in \mathcal{B}$$

$$(13) \qquad w2_{i,d} = \sum_{p=1}^{P} \left(x_{i,d,p}^* - x_{i,d,p-1}^*\right) \qquad\qquad\qquad\qquad\qquad \forall i \in \mathcal{B}, \forall d \in \mathcal{D}$$

$E1_i$ can be defined as the expected value of block i's extraction period. Since the weight $w1_i$ is the opposite of $E1_i$, the higher the weight is the sooner the block is supposed to be extracted. The weight $w2_{i,d}$ represents the percentage of block i sent to destination d from which the most likely destination of the block can be directly determined. The objective of both sets of weights is to give to the main heuristic method an input that allows it to be as close as possible to the relaxed solution while respecting the various constraints.

Heuristic method: Stochastic TopoSort Algorithm (STA)

1. $\forall i \in \mathcal{B}, d_i \leftarrow arg\ max\{w2_{i,d};\ d \in D\}$
2. $j \leftarrow 1$
3. $p_{cu} = 1$
4. $\forall c_1 \in C_1, \forall p \in \mathcal{P}, \forall s \in \mathcal{S}, rcap_{c_1,p,s} \leftarrow target^+_{c_1,p}$
5. **while** $(j \leq N)$ **do**
6.   **if** $\big(\xi = \{i \in \mathcal{B}\ ;\ \delta^{-1}(i) = \emptyset\ and\ [(d_i = 1\ and\ \forall c_1 \in C_1, \forall s \in \mathcal{S}, rcap_{c_1,p_{cu},s} + d^*_{c_1,p_{cu},s} \geq q_{c_1,i,s}\big)\ or\ (d_i =$
    $0)]\} \neq \emptyset\big)$ **do**
7. $x \leftarrow arg\ max\{w1_y;\ y \in \xi\}$
8. $G \leftarrow G\backslash\{x\}$
9. $p_x \leftarrow p_{cu}$
10. $j \leftarrow j + 1$
11. $\forall c_1 \in C_1, \forall s \in \mathcal{S}, rcap_{c_1,p_{cu},s} \leftarrow rcap_{c_1,p_{cu},s} - q_{c_1,x,s}$
12.   **else do**
13. $p_{cu} \leftarrow p_{cu} + 1$
14. **return** $\{(p_i, d_i), \forall i \in \mathcal{B}\}$

Step 1 attributes the most probable destination to every block, using the relaxed solution obtained from the resolution of the R-OMPSIP model. Step 3 defines the current period as the first one. Step 4 states that for each scenario, each period, the initial residual capacity of quantity c_1 is the upper target. With Step 5 begins a loop over all the blocks to be scheduled. Step 6 defines, for each iteration, if it exists at least one block that is available from the predecessors' point of view and which fits into the current period. Here, a block "fits" into a period if it has to be sent to the waste dump or if for every scenario, all its linear characteristics c_1 are less than the remaining capacities of the period $rcap_{c,p_{cu},s}$ plus the corresponding deviation obtained from the relaxed solution. The cost and impact of this deviation were already considered when solving the relaxed model. If such blocks exist, the one with the highest weight $w1$ is selected in Step 7, it is removed from the graph G in Step 8 and assigned to the current period (Step 9). The number of already scheduled blocks and the residual capacities are updated in Steps 10 and 11. In the case where no block can fit anymore in the current period, the next period is considered (Step 13).

In the initial OMPSIP model, several grade quality or continuous extraction constraints are applied but are not taken into account in the STA. However, the algorithm relies on the relaxed result, which contains and respects, as much as possible, these constraints. As a consequence, it is expected from STA to obtain a satisfactory binary schedule with respect to these constraints.
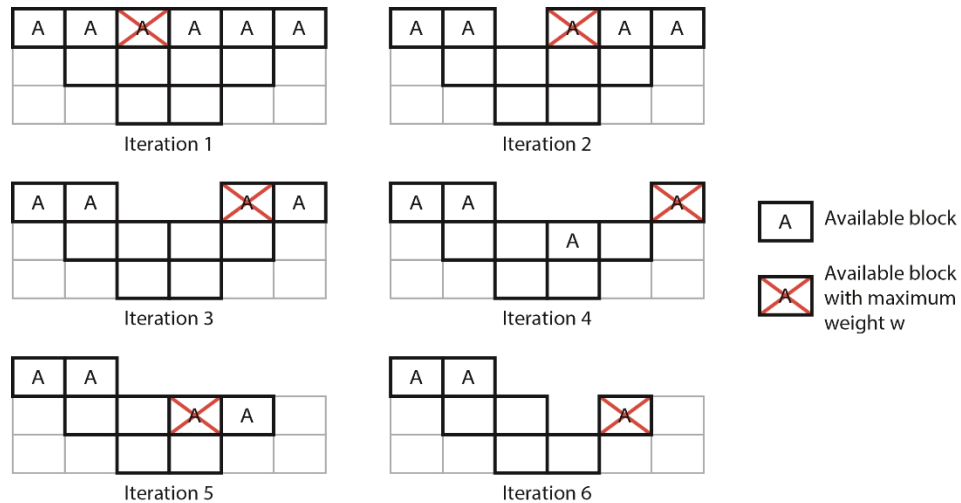

Figure 12 Topological sorting steps

5   Case study

The stochastic topoSort algorithm was tested on an iron ore deposit, owned by the industrial partner in Labrador, Canada.

5.1   Presentation of the deposit

The dimensions of the deposit are around 10 km long from South to North, 2.5km large from East to West and up to 180m deep. A particularity of this deposit is a low slope of around 6° East and the presence of several lithologies. The iron ore is of taconite type which is a sedimentary formation mainly composed of 25-30% magnetite. The typical method to extract the ore from the rock is a magnetic separation after fine crushing. A good estimate of the magnetic recoverable ore is the David Tube test.
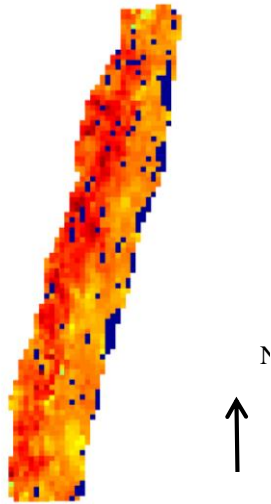
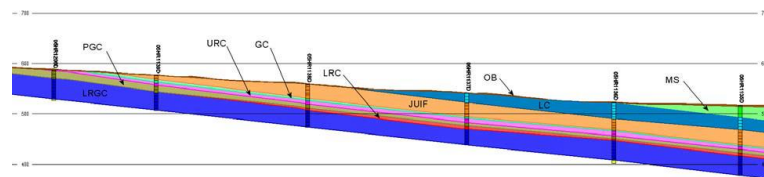**Figure 13 Plan view of the iron ore deposit, feh grades**

Figure 14 Cross section of the deposit with the low dip layers

5.2   Sensitive points and parameters

The geological data and uncertainty is based on a set of 10 stochastic conditional simulations provided by the company. The ore body model is composed of 8223 blocks of dimensions 100x100x15m. The method used to simulate, named DBMAFSIM for direct block minimum/maximum autocorrelation factors simulation (Spleit 2014), first simulates the lithology and then diverse grades: FeH (head iron grade), Fec (concentrate iron grade), DTWR (Davis Tube Weight Recovery) representing the recoverable iron grade, SiC (concentrate silica grade) for the blocks within each layer. In this case study, the only linear characteristic of interest is the amount of concentrate tonnes per period. Two non-linear characteristics are considered: the average DTWR grade per period and the average silica. The silica represents the main pollutant and is crucial for the quality of the production: a low grade assures a premium price on the market and provides a competitive advantage. Two destinations are considered: the mill to process the ore and the waste dump. The scheduling is done over 10 periods.

As previously mentioned, the low dip and great South-North extension of the deposit complicate the scheduling. Indeed, obtaining a continuous sequence of extraction is more delicate because of the amount of blocks without any precedence relationships. Moreover, the low depth does not allow defining an earliest period of extraction for many blocks (not many variables can be set to 0).

When solving the models, independently of the number of periods or blocks, a critical point for the required computational time is the definition of the deviations' costs $pen_{c_1,p}^{dev\pm}$ and $pen_{c_2,p}^{dev\pm}$. These artificial costs are only defined to allow flexibility to the solver in letting it go over the quality constraints but not too much to respect a tolerable margin from the contracts for instance. It was noticed that defining too high such costs not only leads to unnecessary strict constraints which result in a lower DCF but also increases considerably the computational time. This last point comes from the fact that with high penalties, a small modification in the result has a considerable impact on the objective function which forces to test much more combinations. Finding, by trial and error, the smallest cost possible to find a good quality solution (which respects well the targets of production) provides a considerable computational time gain.

With the previous parameters, the OMPSIP model contains 164460 binary variables (extraction variables $x_{i,x,p}$), 600 continuous ones (deviation variables $dev_{c,p,s}^{\pm}$) and around 900000 constraints. The earliest period of extraction constrains (7) fix 3780 binary variables to 0.

5.3   Results: Profits and robustness

5.3.1   Relaxed solution as input to the stochastic topoSort algorithm (STA)

In this section, the relaxed schedule is used as an input for the STA to calculate the most probable destination of each block and its expected period of extraction (weights). For this case study, which is of relatively small size, the relaxation took only 9 minutes to solve. For larger instances, a stochastic implementation of Bienstock and Zuckerberg's work (2010) on solving the relaxed model could be considered. The STA took 3min to run and the results are discussed below. Figure presents the obtained schedule, the blocks being identified with the label "-1" are those that are not extracted. We see the effect of the extraction smoothing constraints (6) with, especially in the Northern part of the deposit, blocks extracted close to each other. However, some isolated spots are also present. Those different zones are satisfactory considering the flatness of the deposit and the size of the blocks, excepting a 400m long zone, they are all more than 800m long.
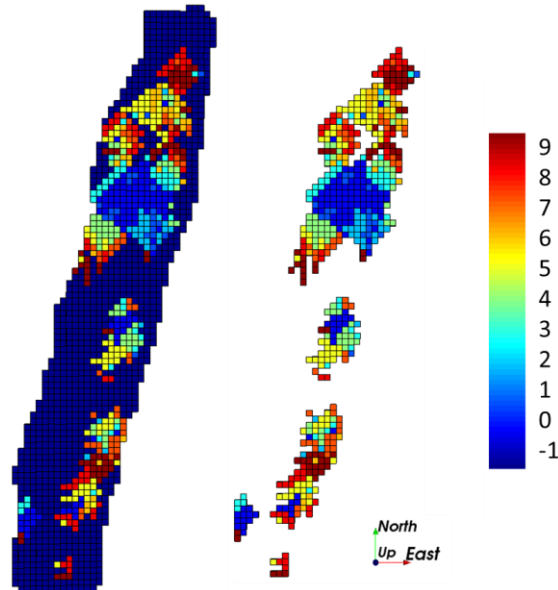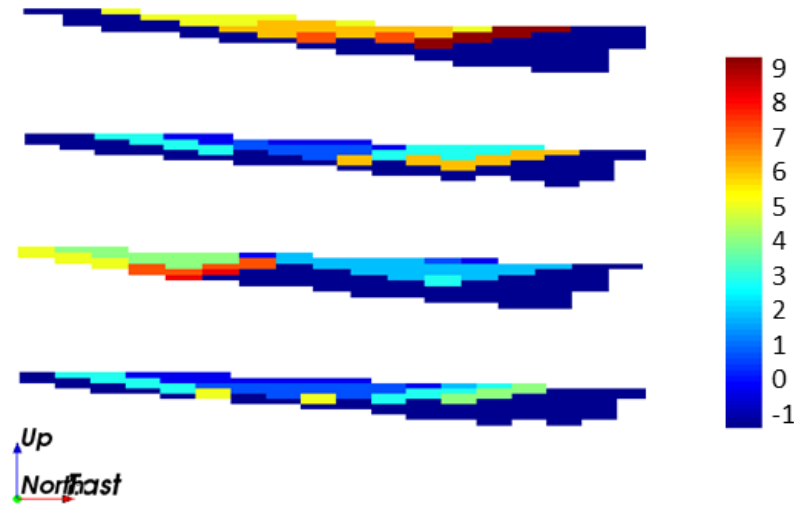

Figure 15 Schedule plan views

Figure 16 Typical schedule E-W sections

Figure presents four typical E-W sections of the schedule, we see the connectivity of the blocks extracted within a shared period and also the respect of the precedence constraints.

Figure shows the destination of the blocks, we can see that not a lot of waste is extracted.
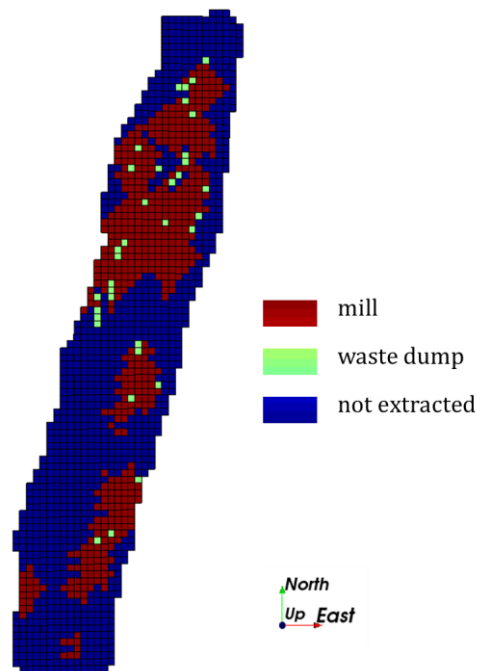

Figure 17 Destinations plan view

We know that the value of the relaxed solution represents an upper bound on the optimal value of OMPSIP, an ideal situation in which all blocks can be partially combined to give the highest profitability. To evaluate how close the solution from STA is to the relaxed one, we can calculate for each block the difference between the expected period of extraction (fractional values) obtained from the relaxed solution and the period of extraction from STA (binary values). The results are presented in Figure in which the abscissa represents the number of periods of differences and the ordinate the number of blocks that present this difference. The results are convincing, only 130 blocks are not scheduled within the same year and this number drops to 24 for a difference of at least two years.
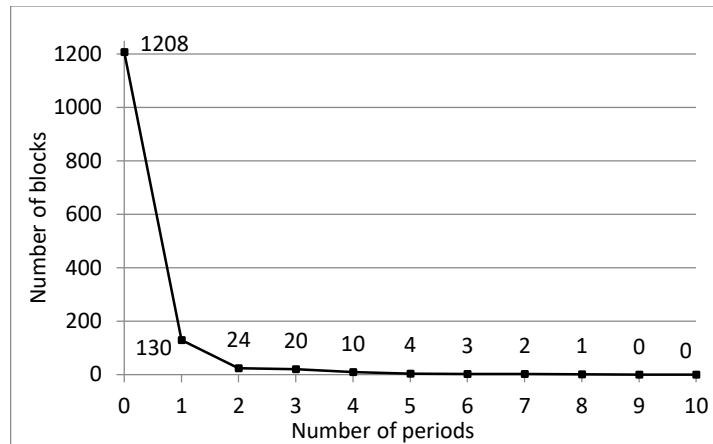
Figure 18 Differences between the relaxed and the binary solution

The profiles over the scenarios for the diverse quality constraints are very satisfactory. That is, an average close to the targets and a well-controlled risk distribution. The production concentrate material (concentrate of iron) is very well satisfied, on Figure we see that the average production is close the target. The run of mine (total amount of material sent to the mill), which was not constrained, is plotted in Figure to make sure the differences between scenarios are not too important. Great differences would require for instance different fleets of trucks and shovels, which is not desirable from the operational side. The silica (Figure) is also well controlled and the average DTWR grade (Figure), the recoverable iron grade, fluctuates around the upper target which is not surprising since the higher the average iron grade is, the more profitable the production is. We just want to make sure that not all the highest grade blocks are extracted together in the first periods.
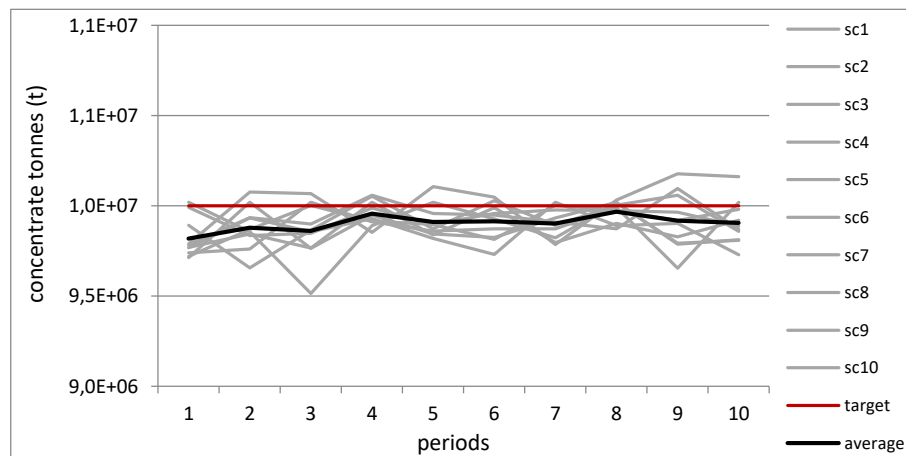
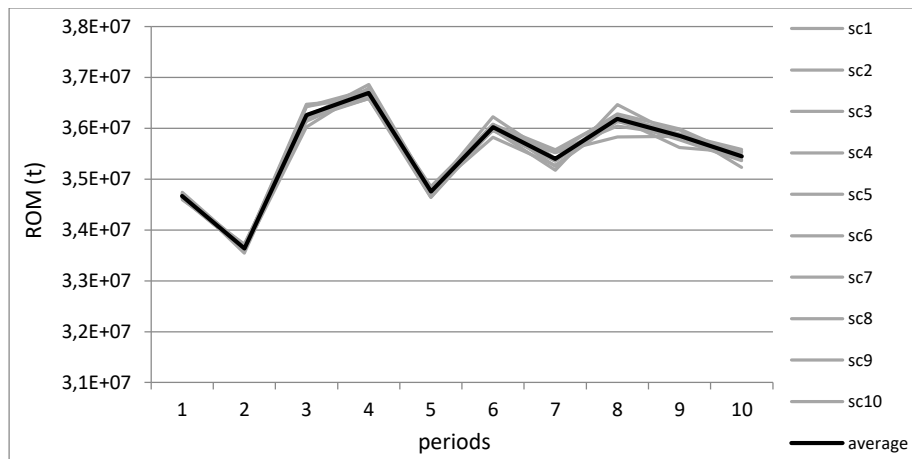

Figure 19 Concentrate tonnes (iron)

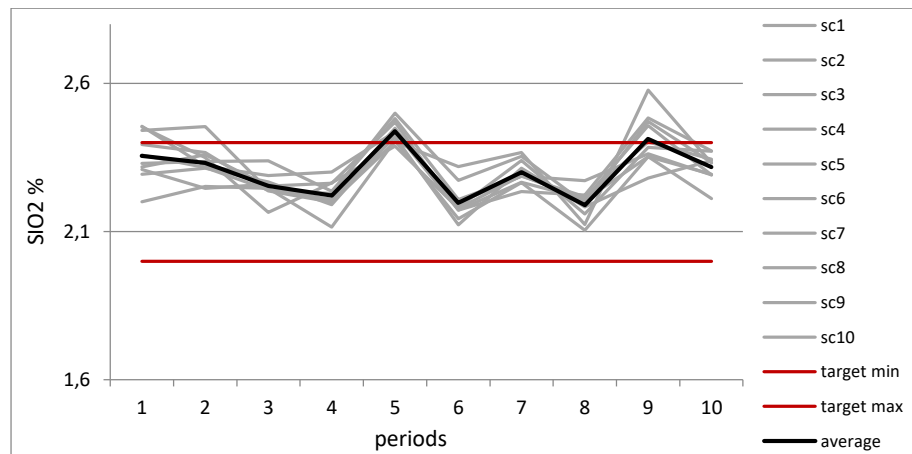Figure 20 Run of mine (material sent to the mill)
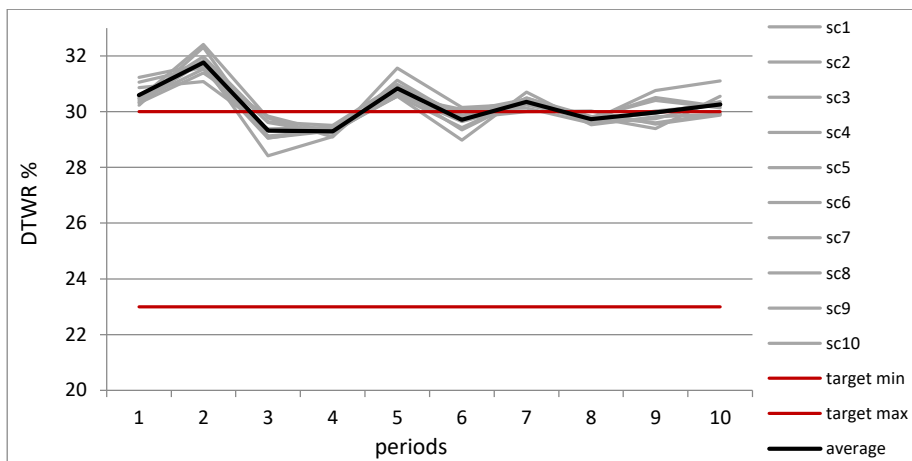

Figure 21 silica grade

Figure 22 DTWR grade (recoverable iron grade)

In terms of discounted cash flow, once again the schedule accounts for the uncertainty with DCF profiles close to their mean. The range of variability after 10 years corresponds to 1.45% of the expected DCF.
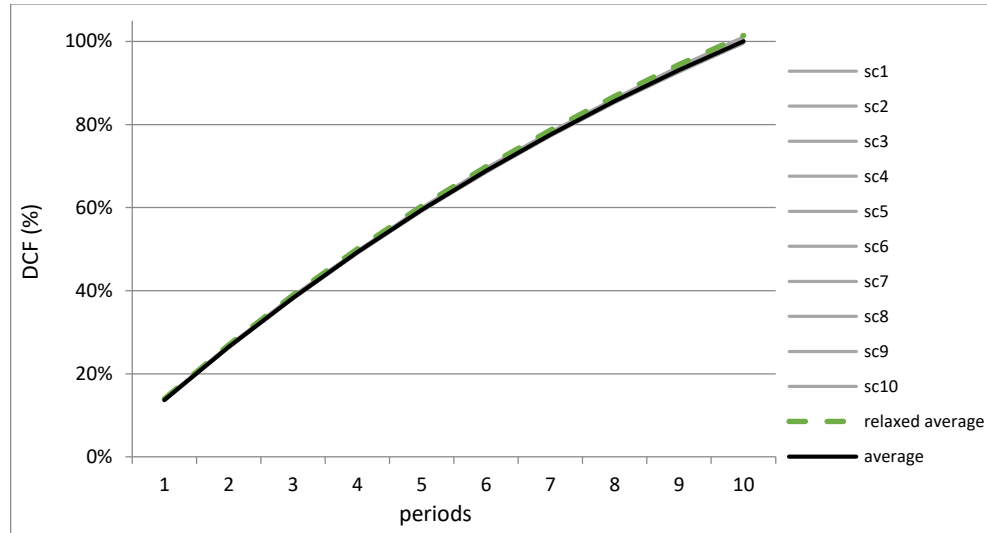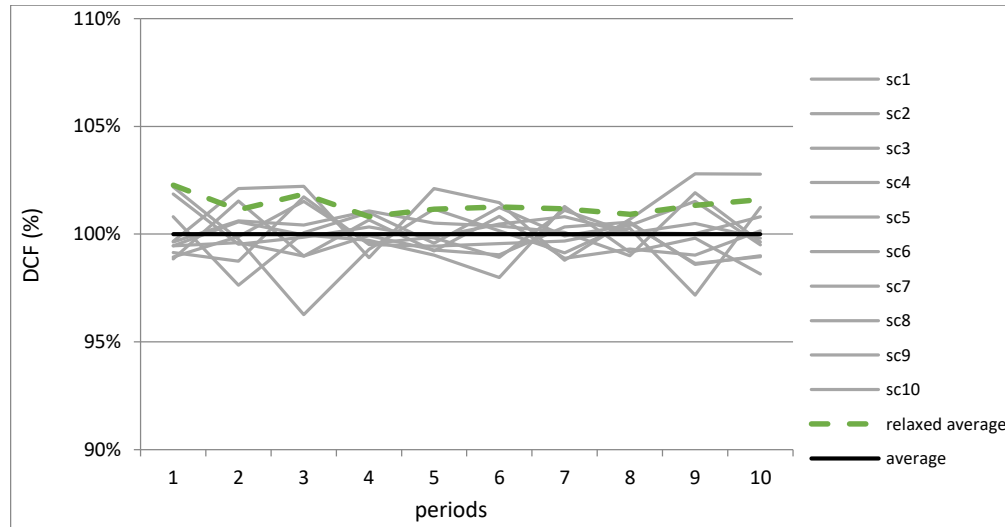
Figure 23 Cumulative DCF

Figure 24 Discounted cash inflow per period

The figures above only give information about the robustness of the schedule. To evaluate the quality of the DCF, we can, as previously mentioned, compare for each scenario the DCF of the relaxed solution (upper bound) and the binary solution. For each scenario s, we can calculate a gap defined as: $gap = \frac{DCF_s^{relaxed} - DCF_s^{STA}}{DCF_s^{relaxed}}$, for which the highest calculated value is 1.42%. This means that in the worst case, the schedule obtained from the STA is less than 1.42% away from the optimal solution.

Table 2 GAP between the relaxed model and the STA solutions

Scenario	1	2	3	4	5	6	7	8	9	10
Gap relaxed – STA (%)	1.38	1.36	1.36	1.31	1.37	1.38	1.33	1.36	1.42	1.39

These DCFs do not take into account the penalty costs ($Part 2$ of the objective function), essential in the resolution of the model. It is also interesting to compare the values of the objective function of Cplex with the new value $gap^{Cplex} = \frac{obj_{Cplex}^{relaxed} - obj_{Cplex}^{DCA}}{obj_{Cplex}^{relaxed}}$. This time the gap^{Cplex} is 2.291%. The larger value of here gap^{Cplex} compared to the gaps

calculated in term of DCF expresses that the deviations from the targets are larger for the solution from the STA than from the relaxed model, even if they remain moderate.

The obtained results are very satisfactory, especially being within 2.3% of the optimality and as the required computational time is only 629 seconds for the relaxed model and 87 seconds for the TopoSort algorithm, 11min56sec in total with Cplex v12.4 and computer equipped with a processor i7-2600S 2.8GHz and 8GB of RAM.

5.3.2   Partially relaxed solution as an input to the STA

In this section, instead of calculating the weights of the STA based on the relaxed model R-OMPSIP, the fractional schedule obtained after a few iterations of the binary convergence algorithm defined in Section 3.2 is used. The objective value of this schedule is a tighter bound on the optimal binary result. Indeed, since more extraction variables are constrained to be binary, the formulation of the last iteration is closer the initial OMPSIP model.

Four iterations of the binary convergence algorithm have been computed, the computational times required for each iteration and the number of remaining fractional variables is presented in Figure . We can see that this number of remaining fractional variables decreases fast and goes down to 166 after 4 iterations. As expected, since additional binary constraints are added at each iteration, the objective value of Cplex decreases up to 1% after three iterations. It is interesting to note that, since this objective value is a tighter upper bound on the optimal binary solution, we can recalculate the gap^{Cplex} of the previous STA solution obtained with the relaxed model as input and we obtain now a value of 1.9%.
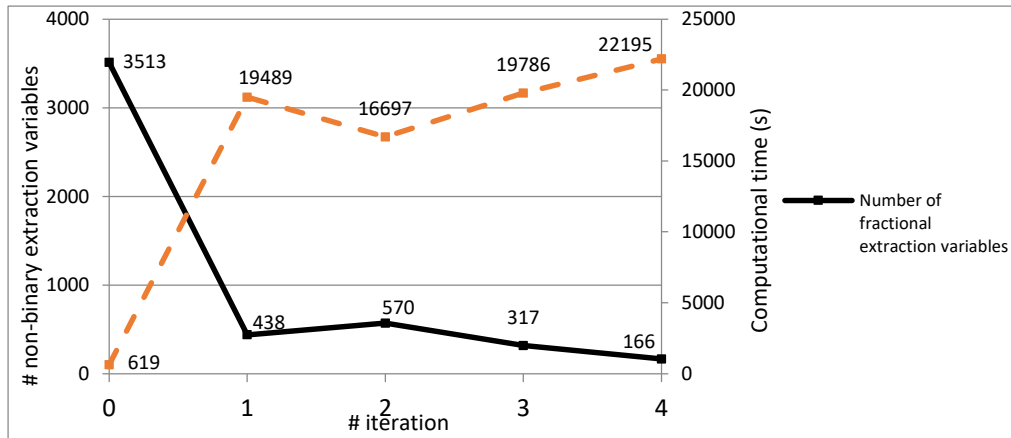

Figure 25 Binary converged schedule

The STA is applied using the fractional schedule provided by the 4th iteration (referenced as converged STA) and the results are convincing. First, the differences between the fractional input schedule and the final binary one are small as can be seen on Figure . Then, the quality constraints are even better respected with a concentrate production very close to the target (Figure 2), an average silica grade completely within the range of tolerance (Figure), and a DTWR grade closer to the upper bound (Figure).
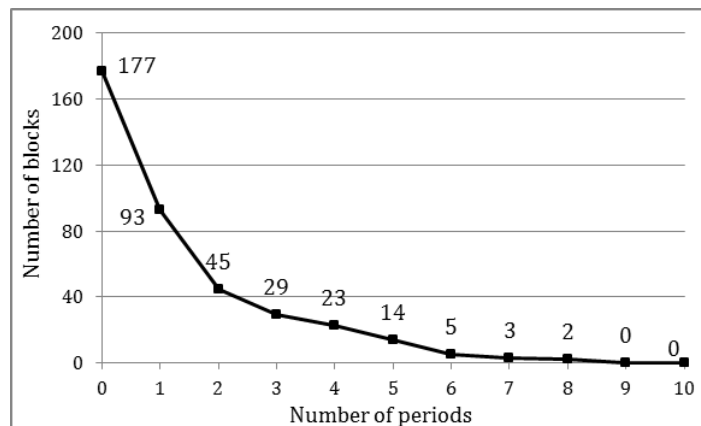

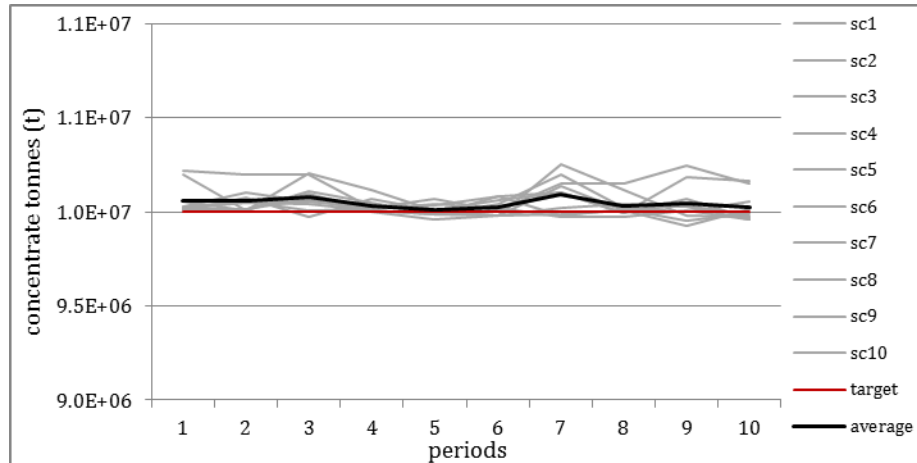**Figure 26 Differences partially relaxed vs converged STA solution**

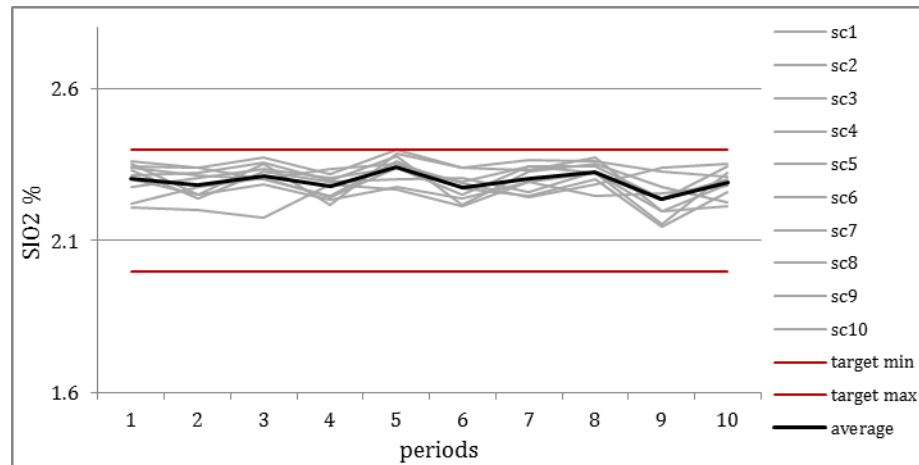**Figure 27 Concentrate tonnes, converged STA**

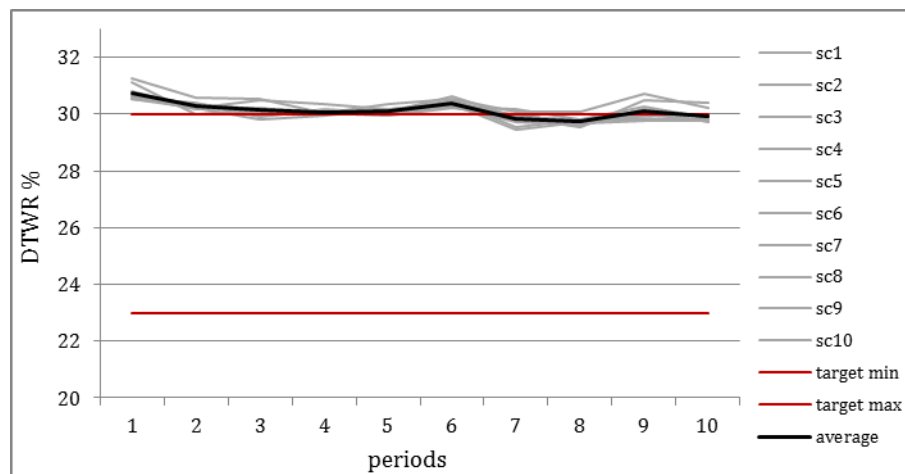**Figure 28 Silica grade, converged STA**

Figure 29 DTWR, converged STA

The DCF also presents less risk over the scenarios and its average is closer to the partially relaxed input (Figure 3), between 0.01 and 0.03% better (no costs of deviations considered here). When comparing the Cplex objective functions between the schedules obtained from the 4th iteration of the binary convergence algorithm and from the converged STA, a gap^{Cplex} of 0.3% is found. The solution is proven almost optimal.
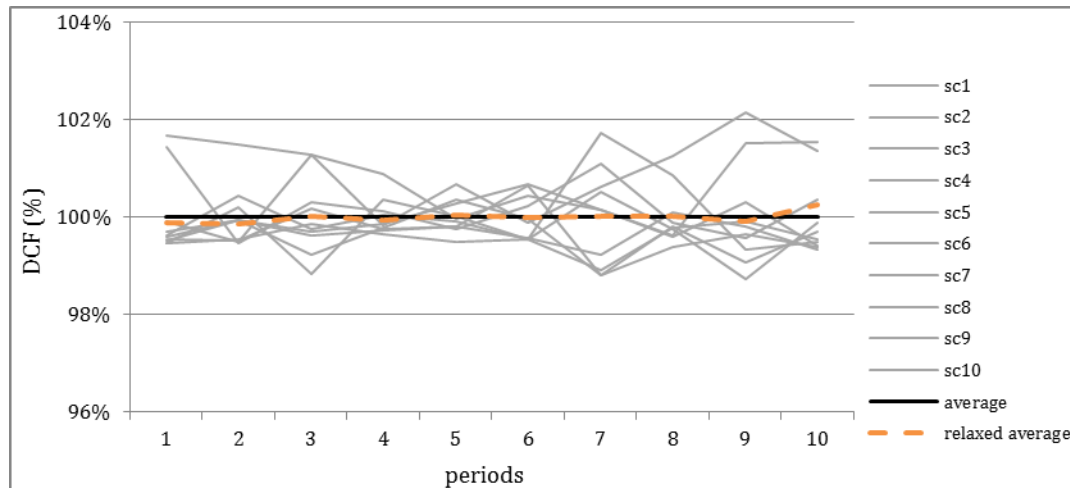
Figure 3 DCF per period, converged STA

Table 3 GAP between the partially relaxed and the converged STA solutions

Scenario	1	2	3	4	5	6	7	8	9	10
GAP partially relaxed – TopoSort (%)	-0.03	-0.02	-0.02	-0.02	-0.01	-0.02	-0.02	-0.02	-0.03	-0.02

The trade-off of a schedule closer to the optimal solution is a much higher computational cost, 21h53min instead of 12min, one can be more interested in one approach or the other.

6 Conclusions

A stochastic topological sorting algorithm was presented to solve the long-term open pit mine planning problem and gave good and fast results for the average discounted cash flow and the control of geological uncertainty. Two options are available as an input for the algorithm. The first option is to provide a fully relaxed solution of the production schedule. The advantage of this approach is the rapidity to obtain a solution, within 2% of optimality after 12min for the case study considered in this paper (more than 160 000 binary variables). The second option is to apply the two proposed acceleration strategies. This approach provides an almost binary optimal schedule in still a reasonable computational time. Depending on the requirement of the operations, the fractional schedule can be used as it is since after several iterations the number of remaining fractional variable is very low. In this case, the solution is optimal since no assumptions have been made except special patterned relaxations. If a fully binary schedule is required, the stochastic topoSort algorithm can be applied to the obtained fractional schedule. The result gets even closer to the optimality: less than 0.3% for the case study in less than 24h.

Extensions of this work could focus on make the proposed acceleration strategies even more aggressive to continue reducing the number of binary variables and to develop an algorithm similar to the one found in Bienstock and Zuckerberg (2010) to solve faster the relaxed model, which can be limiting with larger instances.

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