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B. Vardar,
G. Zaccour

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GERAD HEC Montréal
3000, chemin de la Côte-Sainte-Catherine
Montréal (Québec) Canada H3T 2A7

Tél. : 514 340-6053
Téléc. : 514 340-5665
info@gerad.ca
www.gerad.ca

Strategic bilateral exchange of a bad

Baris Vardar [a]
Georges Zaccour [b]

[a] GERAD & HEC Montréal, Montréal (Québec) Canada H3T 2A7

[b] Chair in Game Theory and Management, GERAD & HEC Montréal, Montréal (Québec) Canada H3T 2A7

baris.vardar@gerad.ca
georges.zaccour@gerad.ca

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Abstract: A private bad is a commodity that causes its owner disutility. This article studies the bilateral exchange of a bad for a good that provides utility. Considering the price of exchange to be fixed, we first characterize the first-best choice of a single agent and study its properties, then investigate the equilibrium strategies of the two-player game in cooperative and non-cooperative settings. The non-cooperative solution is characterized by the normalized equilibrium à la Rosen (1965). In the equilibrium, the agents are assigned an exogenous r-weight, and their weighted dissatisfactions resulting from the exchange are equalized. We show the relationships between the first-best, cooperative and non-cooperative solutions. We then study market-based policy instruments such as taxes or subsidies that aim to direct agents to the cooperative outcome.

Keywords: Exchange of bads, coupled constraints, normalized equilibrium, first-best solution, joint welfare optimization

Résumé : Un produit dont la possession induit une désutilité à son propriétaire est appelé une mauvaise commodité (par exemple des déchets). Cet article s'intéresse aux échanges d'un tel produit. La raison qui peut amener un agent à acheter une telle commodité est la fabrication d'un produit ayant une utilité positive (produit obtenu par recyclage de déchets). Dans un premier scénario, on suppose que le prix est donné par le marché et l'agent décide combien échanger à ce prix. Dans un deuxième scénario, on suppose que deux agents font le commerce de cette commodité sous une contrainte jointe et on détermine un équilibre de Rosen. Dans le dernier scénario, les joueurs peuvent collaborer en vue de la maximisation jointe de leur profit. Ces solutions sont ensuite comparées et on discute des possibilités d'implémentation de mécanismes règlementaires qui induisent les agents à coopérer.

Mots clés : Échange de mauvaises commodités, contraintes jointes, équilibre normalisé, solution coopérative.

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1 Introduction

A private bad is a commodity that causes its owner disutility. Garbage, a local pollutant and a hazardous material are examples of private bads. Some of these commodities can be exchanged, with the receiving agent being compensated in return. Agents will engage in this bad-good exchange when their disutilities for bad and/or their utilities for good differ among themselves. An illustrative example of this type of exchange is the trade of solid municipal waste (SMW) between cities and countries. According to UN Comtrade data on municipal waste (6-digit HS code 382510), countries reported 1.12 million tons of municipal waste exports worldwide in 2015. These deals cost the waste exporter less than it would to stash its waste in landfills or to use other disposal methods. Kellenberg (2012) provides an extensive empirical analysis of the waste trade around the world.

There exist a number of theoretical studies on the management of waste through trade between countries. Some examples are Copeland (1991), Fullerton and Kinnaman (1995), Cassing and Kuhn (2003), and Dubois and Eyckmans (2015). These studies focus on issues of waste management efficiency, recycling, illicit dumping or burning, and on the analysis of various policy instruments to incentivize the first-best or the second best outcomes. The frameworks in these articles consider the price of exchanging waste to be endogenous and determined by the trading countries' waste supply and demand. However, there are cases in which the price of exchange is determined by a policy maker, and the trade partners must decide and agree only on the quantity. Price-taking behavior is studied in another stream of literature focusing on whether or not a competitive equilibrium can exist when bads are in play (see, e.g., Hara (2005, 2008), and Hirai et al. (2006)), but to the best of our knowledge none of the literature has looked at strategic non-cooperative or cooperative behavior in a bilateral exchange.

The present work focuses on bilateral exchanges of bads between agents, using a simple and general framework. First, we investigate the outcomes of the first-best scenario (a single agent vs. the market) and next, the outcomes in a two-player cooperative and non-cooperative game, respectively. We provide results on the relationships between these scenarios as well as on the policy instruments that direct the agents to attain the cooperative outcome.

In our framework, we consider the quantity of exchange (q) to be a decision variable, with no assumption on its sign (thus, the agent can be a buyer or a seller of bads) and we suppose that the price to pay for a unit of bad export (p) is exogenously given. Agent i's decision depends on the utility of good from exchange $u_i(pq_i)$ and the disutility of bad from exchange $d_i(q_i)$. In the first-best scenario, there is a market for what the welfare-maximizing agent wants to buy or sell, and the optimal quantity is obtained by equalizing the marginal utility and marginal disutility resulting from the exchange. In a bilateral exchange, the quantity traded must suit the interest of both agents, not only the seller's.

An important issue is how to make sure that both parties reach an agreement on the quantity to exchange. There may exist situations in which the seller's supply is greater than the buyer's demand. Also, it may well be the case that both agents are sellers of bads. If one assumes that the two agents can cooperate, then a joint optimization solution can be implemented using, e.g., the Nash bargaining solution. The present work takes another approach to the agreement problem by using the normalized equilibrium (also known as generalized Nash equilibrium) introduced by Rosen (1965).

Rosen's work investigates the equilibrium points in concave games in which the joint strategy space of agents is constrained. He then introduces the normalized equilibrium in which player i is assigned an exogenous weight (r_i) and maximizes her payoff while satisfying the coupled constraint. Rosen shows that the equilibrium exists and is unique in games that satisfy certain conditions. The theorems in Rosen's work have been applied to various subjects in environmental economics and operations research. Some examples in environmental economics are Krawczyk (2005), Tidball and Zaccour (2005, 2009), Boucekkine et al. (2010), and Bahn and Haurie (2016), and a survey of applications in operations research is provided by Facchinei and Kanzow (2007).

In our analysis, we consider an agreement to be a constraint in the joint strategy space of agents facing that exchange. The agreement takes place when both parties want to exchange the same quantity but in opposite signs. For instance, if agent i wants to import quantity q of bad ($q_i = q$) and agent j wants to export the same quantity ($q_j = -q$), then we say that both parties agree on making that exchange. Therefore, the coupled constraint $q_i + q_j = 0$ represents the agreement points in the joint strategy space of agents i and j. Then, all agents maximize their payoffs by taking into account that constraint, and we do not consider an outside option. This means that the agents either want to agree, are compelled to agree, or are incentivized to agree. When one of these is the case, then, by applying Rosen's theorems, we show that the game of exchanging a bad has a unique agreement point that is a normalized equilibrium. A similar application of a normalized equilibrium to a bilateral exchange problem can be found in Flåm (2016).

In the cooperative solution (q_i^c), all agents are assigned a weight (ω_i), and the sum of weighted payoffs (joint welfare) is maximized given that both agents agree. In the non-cooperative solution (q_i^0), the normalized equilibrium is characterized such that all agents are assigned a weight (r_i) and each agent's weighted penalty from being constrained is equalized with the other's. The equilibrium consists of an agent who is willing to buy more (or sell more) bad and a counterpart who is willing to sell less (or buy less) bad.

The results show that both the cooperative and non-cooperative solutions belong to the interval bounded by the first-best choices of agents i and j, namely, $q_i^c, q_i^0 \in (q_i^*, -q_j^*)$. The position of the agreement point with respect to the first-best quantities is determined by the relative weights of agents (r_i/r_j). Moreover, the non-cooperative solution coincides with the cooperative one if the $\mathbf{r}$-vector is the inverse of the agents' weights under joint welfare ($\mathbf{r} = 1/\boldsymbol{\omega}$). The implication of these results are twofold. First, if the agents can choose their r-weights, then they can attain the cooperative outcome without the existence of a market or the presence of a regulator. Second, if there is a policy maker that can influence the agents' r-weights, then she can incentivize them to attain the cooperative outcome. When the policy maker uses market-based instruments such as taxes and subsidies, we determine the optimal levels of regulation for each agent that lead to the cooperative level of exchange, and show their relationships to the weight parameters in the cooperative and non-cooperative solutions.

The article is organized in the following order: Section 2 describes the framework with the assumptions made on the functional forms for utility and disutility. Section 3 presents the first-best scenario, in which a single agent exchanges a bad in a market that accepts any quantity that she is willing to trade. Section 4 studies the two-player game in cooperative (4.1) and non-cooperative (4.2) settings. Section 5 concludes.

2 The framework

Consider a commodity that is a bad. Agents, which can be regions, countries, municipalities, or individuals, can exchange it among themselves. The agent's endowment in bad is b. We denote by q the quantity of exchange and by $d(q)$ the corresponding disutility. The exchange can take place in both directions, that is, $q \in [-b, +\infty)$, which means that the agent can be a buyer or a seller of a bad. When $q < 0$, the disutility that the agent suffers is less than in the no-exchange case.

We make the following assumptions on the disutility function:

Assumption 1 *$d : \mathbb{R} \mapsto \mathbb{R}$ is increasing and strictly convex ($d' > 0, d'' > 0$) with $d'(0) = \varphi \in (0, +\infty)$ and $d'(-b) = 0$.*

The convexity of the disutility function is common in the literature dealing with pollution or other bads. The statement that the marginal disutility is positive and finite in the absence of exchange follows from the fact that the agent already owns some bad. Another way of interpreting this assumption is saying that the agent can live without making this exchange. This assumption is crucial in this setup since it enables the study of an intermediate case in which an agent can be the buyer or a seller of bad. Finally, $d'(-b) = 0$, meaning that the agent sells all the bad in her possession, ensures that the first-best quantity will always be interior.

The bad can be exchanged with other agents at price $p > 0$ per unit, which denotes the quantity of good to be given in exchange for a unit of bad. The agent selling a quantity q of a bad pays the buyer pq quantity of goods (considered the numéraire), which provides utility $u(pq)$. Note that, as for the disutility function, the utility function can also have a negative input ($pq < 0$) when the agent sells bads and pays the buyer goods in return, taking away from her utility. Along with the concavity assumption, we suppose that the marginal utility is positive and finite when there is no exchange.

Assumption 2 *$u : \mathbb{R} \mapsto \mathbb{R}$ is increasing and strictly concave ($u' > 0, u'' < 0$) with $u'(0) = \psi \in (0, +\infty)$.*

The shape of the utility function we assume here has some similarities to and differences with the standard utilitarian framework. Since the currency of exchange is a generic good, the assumption of the utility function's concavity is consistent with the common approach of continuous and strongly monotonic consumer preferences. Yet, we will see there are interesting consequences to utility depending on the revenue (or cost) of exchange. The motivation behind the finiteness of the marginal utility at the no-exchange state is similar to the one for disutility: one can suppose that the agent is initially endowed with some good and thus, that her marginal utility is positive and finite, and that also exchange is not an essential need for the agent.

The payoff (or welfare) of the agent is given by

$$v(q) = u(pq) - d(q). \tag{1}$$

This payoff function is similar to the one commonly used in models of public bads, e.g., the transboundary pollution game and the economics of climate change (see the surveys in Missfeldt (1999) and Tol (2009)), and other classical problems, e.g., production of a firm. However, our formulation has two distinctive features with respect to the literature, namely, the payoff depends on revenue, and we allow for negative-value decisions.

In the next section, we explore the first-best scenario. Then, we proceed to the game between two agents in both cooperative and non-cooperative settings.

3 First-best scenario: Single agent vs. the market

The objective of this section is to study the first-best choice of a single agent facing the decision of whether or not to exchange a bad. Suppose there is a market for this commodity and its price p is exogenously given. The agent's optimization problem is

$$\max_q v(q) = u(pq) - d(q). \tag{2}$$

Maximization yields

$$pu'(pq^*) = d'(q^*),$$

that is, the agent is willing to exchange $q^*(p)$ quantity of the bad that equalizes marginal utility to marginal disutility. As both $u(\cdot)$ and $d(\cdot)$ are strictly increasing, then, for any given price of exchange, there exists a unique $q^*(p)$ that satisfies the first-order condition.

Let $\tilde{p} = d'(0)/u'(0) = \varphi/\psi$, which is the price level that equalizes the marginal utility of an additional good to the marginal disutility of an additional bad when the quantity of exchange is null. By Assumptions 1 and 2, it is a given positive constant. The agent is satisfied with not trading when the price of the bad is at that level. For any other price $p \neq \tilde{p}$, the agent will buy or sell a quantity of bad. Comparative statics on $q^*(p)$ show that its change with respect to p is ambiguous and depends on the level of p. The following proposition shows the properties of a single agent's offer curve for exchanging a bad:

Proposition 1 *The first-best quantity of exchange that maximizes the agent's welfare ($q^*(p)$) is such that $pu'^*(p) = d'^*(p)$ holds. It can be positive, null, or negative, depending on the price level.*

$$q^*(p) \begin{cases} < 0 & \text{if } p < \tilde{p} \\ = 0 & \text{if } p = \tilde{p} \\ > 0 & \text{if } p > \tilde{p} \end{cases} \tag{3}$$

The change in the welfare-maximizing quantity with respect to price depends on the price level and the shape of the utility function.

$$\frac{dq^*(p)}{dp}\begin{cases} > 0 & \textit{if } \sigma(p) > 1 \\ = 0 & \textit{if } \sigma(p) = 1 \\ < 0 & \textit{if } \sigma(p) < 1 \end{cases} \tag{4}$$

where

$$\sigma(p) = -\frac{u'^*(p)}{pq^*(p)u''^*(p)},$$

is the endogenous relative risk-aversion coefficient.

Proof. The first part is obtained by the first-order condition ($pu'^*(p) = d'^*(p)$). For the second part, applying implicit differentiation to the first-order condition and rearranging terms leads to

$$\frac{dq^*(p)}{dp} = \frac{u'^*(p) + pq^*(p)u''^*(p)}{-p^2u''^*(p) + d''^*(p)}. \tag{5}$$

The denominator in (5) is clearly positive. Then, the sign of $\frac{dq^*(p)}{dp}$ depends on the sign of the numerator, which gives the result of Proposition 1. $\square$

Proposition 1 reveals some first insights about the first-best choice for an agent. When the price is sufficiently low ($p < \tilde{p}$), the agent is willing to give some of it up to reduce the amount of bad she owns. The agent would thus sell some of the bad and pay the buyer some good in return. On the contrary, when the price is sufficiently high ($p > \tilde{p}$), the marginal utility that could be gained through additional good outweighs the disutility that additional bad would cause. The agent is then willing to buy some bad from the market to benefit from the utility of the good taken in return. Clearly, the value of p is a crucial parameter in determining the first-best choice.

The second part of Proposition 1 shows how the first-best quantity of exchange varies with the price of the good. It shows that the agent's willingness to sell less (or more), or buy more (or less) of a bad when p rises depends on the shape of the utility function, and more specifically on the agent's relative risk-aversion ($\sigma(p)$). The first observation from condition (4) is that, when the agent is a seller (that is, $q^*(p) < 0$), a higher p implies a higher value of $q^*(p)$ (a lower absolute value).[1] The second observation is that, if the level of price is $p = \tilde{p}$, yielding $q^*(\tilde{p}) = 0$, then an increase in the price of good always makes the agent a buyer of bad $\left(\frac{dq^*(\tilde{p})}{dp} > 0\right)$.[2] Accordingly, the agent's offer curve of exchange crosses $q^*(p) = 0$ at the single point $p = \tilde{p}$ with a positive slope; therefore it intersects only once the $q = 0$ axis on the (q, p) plane.

When the agent is a buyer ($q^*(p) > 0$), the effect of an increase in the price of the good is not as straightforward. It is not certain that the condition in (4) that ensures a decrease in quantity with price ($\sigma(p) < 1$) could ever hold true. To understand this, we check whether the endogenous relative risk aversion coefficient can ever be equal to 1. The following corollary presents the result.

Corollary 1 *If the marginal utility is concave* $\left(u''' \leq 0\right)$*, then there exists* $\hat{p} > \tilde{p}$ *with* $q^{*\prime}(\hat{p}) = 0$*, such that, for any* $p > \hat{p}$*, we have* $q^{*\prime}(p) < 0$ *as long as* $q^*(p) + pq^{*\prime}(p) > 0$*.*

[1] Condition $\sigma(p) > 1$ holds if

$$u'^*(p) > -pq^*(p)u''^*(p). \tag{6}$$

When the first-best quantity of exchange is negative ($q^*(p) < 0$), an increase in p will always increase $q^*(p)$ (making it less negative) because the term on the right-hand side is always negative ($-pq^*(p)u''^*(p)) < 0$) and the left-hand side is always positive ($u'^*(p)) > 0$).

[2] This is because the right-hand side in condition (6) is null but the left-hand side is positive when $q^*(p) = 0$.

Proof. We previously showed that ${q^*}'(p) > 0$ when $q^*(p) \leq 0$. Now we consider the case in which the agent is a buyer $q^*(p) > 0$. From condition (4) we have ${q^*}'(p) = 0$ iff $\sigma(p) = 1$, which is true when

$$u'^*(p) = -pq^*(p)u''^*(p). \tag{7}$$

We study whether condition (7) holds true for a price level $p > \tilde{p}$. When $p = \tilde{p}$ we have $q^*(\tilde{p}) = 0$, which implies that $LHS(7) = \psi$ and $RHS(7) = 0$. Then, whether condition (7) holds true for a level of p depends on how LHS and RHS change with p.

The change of $LHS(7)$ with respect to p is written as $u''^*(p))\left[q^*(p) + p{q^*}'(p)\right]$, which is negative, and hence, $LHS(7)$ is decreasing in p. The change of $RHS(7)$ writes as $-(q^*(p)+p{q^*}'(p))u''^*(p) - pq^*(p)(q^*(p)+p{q^*}'(p))u'''^*(p)$. The first element is positive. So $RHS(7)$ is certainly increasing in p when $-pq^*(p)(q^*(p)+p{q^*}'(p))u'''^*(p) \geq 0$. This is true if $u'''^*(p) \leq 0$. In this case, it is certain that (7) will hold true for a price level $p = \hat{p} > \tilde{p}$. For $p > \hat{p}$, the first-best quantity of exchange declines (${q^*}'(p) < 0$) as long as $q^*(p) + p{q^*}'(p) > 0$. □

The shape of the exchange offer curve depends on the convexity-concavity of the marginal utility, the third derivative of the utility function. If the marginal utility is concave, then it decreases more rapidly as more of the good is received in exchange (a higher pq). Because of this effect, the agent is willing to buy less of the bad after a certain price threshold. However, when the marginal utility is convex, the reduction in marginal utility decreases with the quantity of the good. Then, it is ambiguous whether a certain price threshold like that exists: it may well be the case that the agent would always buy more of the bad as the price increased.

The results presented above show the first-best outcome for an agent. In this analysis there is always a counterpart (the market) that enables what the agent is willing to buy or sell. The more interesting case is how two agents facing such an exchange, both having the first-best choices presented as above, can achieve an agreement cooperatively and non-cooperatively. The following section studies both situations.

4 Two-player game

There are two agents, indexed by $i = 1, 2$, each possessing b_i amount of a bad. The disutility for bad (d_i) and the utility of good (u_i) satisfy Assumptions 1 and 2. As in the first-best case, q quantity of bad could be exchanged in return for pq quantity of good, with the price p being an exogenously given constant. The strategy of player i is the amount of the bad that she is willing to exchange, $q_i \in E^i = [-b_i, +\infty)$. The vector $\mathbf{q} = \{q_1, q_2\}$ denotes the pair of strategies and belongs to the joint strategy space E ($\mathbf{q} \in E = E^1 \times E^2$).

In the previous scenario, the agent could find a taker (the market) for what she wanted to exchange. Now we have two agents engaged in a bilateral relationship, and what one proposes may not correspond to what the other wants. Consequently, a solution materializes only if the agents are willing to exchange the same quantity in opposite signs ($q_i = -q_j$). Therefore, $R = \{\mathbf{q} \in E |\ q_i + q_j = 0\}$ denotes the set of agreement points in the joint strategy space of agents i and j. The set R is convex, closed, and bounded.

The payoff function of each player is denoted as $v_i(\mathbf{q}) = u_i(pq_i) - d_i(q_i)$. By Assumptions 1 and 2, $v_i(\mathbf{q})$ is concave in q_i and does not depend on the other player's strategy q_j. However, to execute an exchange, the agents need their counterpart to agree. Hence, this is a class of game in which the players' objective functions are decoupled, but the strategy sets are coupled due to the dependence on the counterpart's will.

We first investigate how the agents reach an agreement in a cooperative way, and next we study the non-cooperative game.

4.1 Cooperative exchange

The joint welfare is given by $\sum_i \omega_i v_i(\mathbf{q})$, where $\omega_i > 0$ is the weight assigned to player i, with $\omega_1 + \omega_2 = 1$. The optimization is carried out subject to the constraint $q = q_1 = -q_2$. Consequently, the optimization problem can be written as follows:

$$\max_q \left\{\omega_1 \left(u_1(pq) - d_1(q)\right) + \omega_2 \left(u_2(-pq) - d_2(-q)\right)\right\}. \tag{8}$$

The following proposition presents the cooperative solution.

Proposition 2 *For any given weight vector $\boldsymbol{\omega} = \{\omega_1, \omega_2\}$, there exists a unique cooperative quantity of exchange $\mathbf{q}^c = \{q^c, -q^c\}$ satisfying*

$$\omega_1 \left(pu_1'^c - d_1'^c\right) = \omega_2 \left(pu_2'^c - d_2'^c\right). \tag{9}$$

For two given weight vectors $\boldsymbol{\omega}^A = \left\{\omega_1^A, \omega_2^A\right\}$ and $\boldsymbol{\omega}^B = \left\{\omega_1^B, \omega_2^B\right\}$ with $\omega_1^A > \omega_1^B$ and $\omega_2^A = \omega_2^B$ (which means that agent 1 has more weight in $\boldsymbol{\omega}^A$), the cooperative solution A is closer to the first-best choice of agent 1 compared to B, that is, $\left|q^A - q_1^*\right| < \left|q^B - q_1^*\right|$.

Proof. Equation (9) is the first-order condition associated to program (8). Let $\omega_i v_i(q) = \omega_i \left(u_i(pq) - d_i(q)\right)$. By Assumptions 1 and 2, we have $\omega_i v_i'(q) \geq 0$ or $\omega_i v_i'(q) \leq 0$, so the terms on the left-hand side and right-hand side can be either both positive or both negative. We also have $\omega_i v_i''(q) < 0$ and $-\omega_i v_i''(-q) > 0$. Therefore, in Equation (9), the term on the LHS is decreasing in q whereas the RHS is increasing in q. Therefore, a solution to (8) is guaranteed and is unique.

For the second part of the proposition, let q_i^* be such that $v_i'(q_i^*) = 0$ (as in the first-best case), and denote q^A and q^B the cooperative solutions corresponding to weight vectors $\boldsymbol{\omega}^A$ and $\boldsymbol{\omega}^B$. There can be two cases: $\omega_1^B v_1'^B\left(q^B\right) < 0$ and $\omega_1^B v_1'^B\left(q^B\right) > 0$.

First consider $\omega_1^B v_1'^B\left(q^B\right) < 0$. This is the case when $q^B > q_i^*$. By (9), we have $\omega_1^B v_1'^B\left(q^B\right) = \omega_2^B v_2'^B\left(-q^B\right)$. Now replace ω_1^A with ω_1^B ($\omega_1^A > \omega_1^B$), which yields $\omega_1^A v_1'^B\left(q^B\right) < \omega_2^B v_2'^B\left(-q^B\right)$. To satisfy the equality, $v_1'^B\left(q^B\right)$ must increase and thus q^B must decrease, yielding $q^A < q^B$. Moreover, q cannot decrease until q_i^* because it requires $v_i'(q_i^*) = 0$ but the RHS can have a zero value only if $q_j^* = -q_i^*$. Therefore, when $q_j^* \neq -q_i^*$ we have $q_i^* < q^A < q^B$.

Now consider the second case, $\omega_1^B v_1'^B\left(q^B\right) > 0$. In this case, $q^B < q_i^*$. Replacing ω_1^A in Equation (9) for $\boldsymbol{\omega}^B$ writes as $\omega_1^A v_1'^B\left(q^B\right) > \omega_2^B v_2'^B\left(-q^B\right)$. As $v_1'(q)$ is decreasing, q^B must decrease to satisfy the equality. Therefore, $q^A > q^B$ and $q^B < q^A < q_i^*$. Figure 1 illustrates this graphically. □

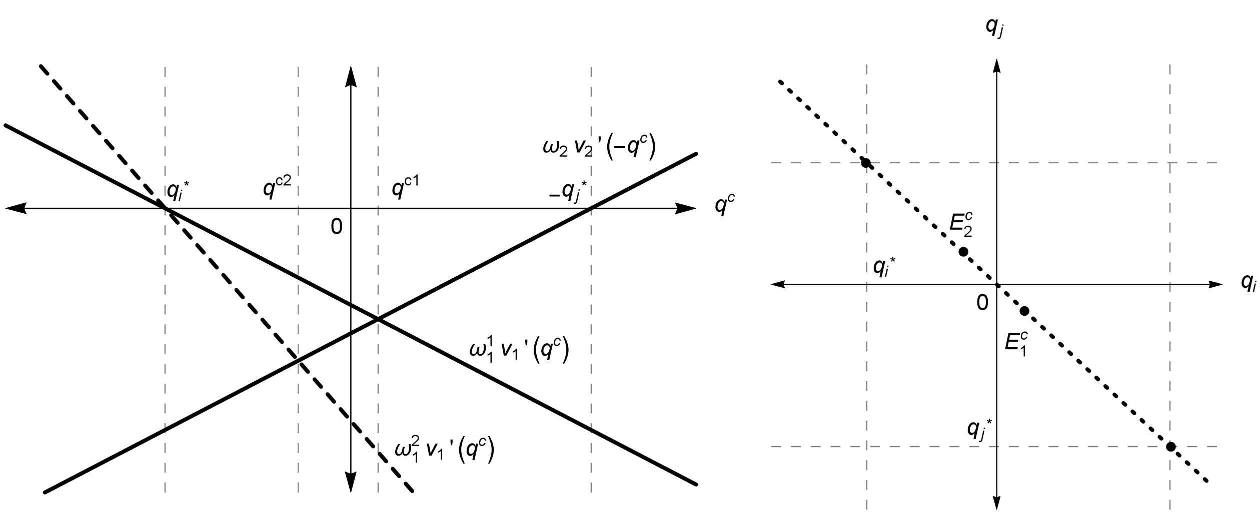


Figure 1: Sample equilibrium points in the cooperative game ($\omega_1^1 < \omega_1^2$)

Figure 1 illustrates the results of Proposition 2. We see that the weighted marginal gain from exchange is equalized between the two agents in the cooperative solution. Indeed, the cooperative solution depends on a range of factors including the agents' utility and disutility as well as their weights in joint welfare. For instance, in Figure 1, suppose that two agents have equal weights ($\omega_1^1 = \omega_2^1$), and each of them would be willing to sell some of their bad if there was a counterpart that would accept ($q_1^* < 0$ and $q_2^* < 0$). In that case, the cooperative solution favors the agent who is willing to sell more (agent 2 in the figure). The agents cooperatively agree on the exchange in which agent 2 sells q^{c1} quantity of the bad to agent 1 (E_1^c). Furthermore, consider the case in which agent 1 has more weight in joint welfare than agent 2 ($\omega_1^2 > \omega_2^2$). When this is the case, the cooperative solution favors agent 1 and gets closer to the first-best choice of agent 1. Then, agent 2 buys q^{c2} quantity of the bad from agent 1, which increases agent 1's welfare with respect to agent 2 (E_2^c).

To sum up, the joint-welfare maximizing solution equalizes the marginal gain from exchange, and it favors the agent with a greater payoff weight. In a bilateral monopoly context, there is no reason to assume different weights for the agents. Another interpretation of cooperation could be that a policy maker assigns different weights to the players and determines the optimal exchange.

4.2 Non-cooperative exchange

Our aim is to characterize an equilibrium point where two agents exchange in a non-cooperative way and to study its links with the first-best and cooperative solutions. We then analyze potential instruments that incentivize the agents to reach an agreement on the cooperative joint-welfare maximizing quantity of exchange.

In the non-cooperative game, each agent aims to maximize her individual payoff. Unlike in the first-best or cooperative cases, the counterpart's agreement is not taken as given. The assumption is that the agents do not have any outside option and that the exchange can take place only if two agents want to exchange the same quantity in opposite signs ($q_i = -q_j$). Therefore, a non-cooperative solution exists only when the pair of strategies belongs to the set of agreement points ($\mathbf{q} \in R$), which represents the constraint in the joint strategy space of agents i and j. The agents' payoffs are concave and there is a coupled constraint that restricts the agents' actions, therefore the game at hand is a concave game as in Rosen (1965). The non-cooperative equilibrium point of this two-player game is given by a point $\mathbf{q}^0$ that satisfies the following:

$$v_i(\mathbf{q}^0) = \max_{q_i} \left\{ v_i(q_i, q_j^0) \;\middle|\; (q_i, q_j^0) \in R \right\} \text{ for } i = 1, 2. \tag{10}$$

Rosen introduced the notion of normalized equilibrium for the class of N-person concave games. In this type of equilibrium, all agents maximize their payoffs while satisfying the coupled constraint, leading to having a multiplier associated to this constraint (μ_i) that reflects the constraint's effect on the payoff of agent i. Consider an exogenous vector $\mathbf{r}$ that assigns each agent a weight r_i. The normalized equilibrium is defined as the point where the weighted values of coupled constraint multipliers are equalized among all agents (here $\mu_i/r_i = \mu_j/r_j$). Rosen provides the necessary conditions for the existence and uniqueness of a normalized equilibrium for any given positive $\mathbf{r}$-vector ($\mathbf{r} > 0$). In the following lemma, we verify that these conditions are satisfied for the game we address; hence there is a unique quantity of the bad such that both agents agree to an exchange in the non-cooperative game.

Lemma 1 *For any given $\mathbf{r} > 0$, the two-player game of exchanging a bad attains a unique normalized equilibrium à la Rosen.*

Proof. Using the notation of Rosen (1965), let $\sigma(\mathbf{q}, \mathbf{r}) = r_1 v_1(\mathbf{q}) + r_2 v_2(\mathbf{q})$, where $\mathbf{r} > 0$ denotes the vector that assigns r_i weight to player i's payoff. Furthermore, let $g(\mathbf{q}, \mathbf{r})$ be the pseudogradient of $\sigma(\mathbf{q}, \mathbf{r})$, that is, $g(\mathbf{q}, \mathbf{r}) = [r_i \nabla_i v_i(\mathbf{q})]$. The Jacobian of g with respect to $\mathbf{q}$ is written:

$$G(\mathbf{q}, \mathbf{r}) = \begin{bmatrix} r_1(p^2 u_1''(pq_1) - d_1''(q_1)) & 0 \\ 0 & r_2(p^2 u_2''(pq_2) - d_2''(q_2)) \end{bmatrix}. \tag{11}$$

The Jacobian G is negative-definite since each diagonal element is negative ($G_{ii} < 0 \ \forall i$). The payoff functions are diagonally strictly concave, and the coupled constraint R is convex, closed, and bounded; therefore the game attains a unique equilibrium as shown in the uniqueness theorem of Rosen (1965). □

The existence and uniqueness of a normalized equilibrium is guaranteed by the decoupled structure of the payoff functions and the coupled constraint R being convex, closed, and bounded. Because of the decoupled concave payoff functions, the Jacobian of the marginal payoff matrix is ensured to be negative-definite since it is a diagonal matrix with all negative elements. In effect, this is Rosen's diagonal strict concavity condition, which ensures the uniqueness of the equilibrium.

We proceed with the characterization of the unique normalized equilibrium. The Lagrangian of agent i associated to (10) is as follows:

$$\mathcal{L}_i(\mathbf{q}, \mu_i) = u_i(pq_i) - d_i(q_i) - \mu_i(q_i + q_j). \tag{12}$$

First-order conditions write as

$$pu_i'(pq_i) - d_i'(q_i) - \mu_i = 0 \text{ for } i = 1, 2, \tag{13}$$

$$q_1 + q_2 = 0. \tag{14}$$

The variable μ_i in condition (13) denotes the multiplier of agent i associated to the coupled constraint $q_1 + q_2 = 0$. It plays a crucial role in the characterization of the non-cooperative equilibrium, and it can be interpreted as the dissatisfaction or penalty that agent i experiences for being constrained by the counterpart's will on the quantity of exchange.

Let q_i^* be the quantity that maximizes the payoff of agent i without any constraint, $pu_i'(pq_i^*) - d_i'(q_i^*) = 0$, which is the level agent i would choose in the absence of the constraint, as analyzed in Section 3. If the counterpart's choice is the same quantity but in opposite sign ($q_j = -q_i^*$), then the payoff of agent i is already at its maximum on the coupled constraint. In that case, the value of the multiplier is null ($\mu_i = 0$). The agent exchanges her first-best quantity, and being constrained by the need for the counterpart's agreement does not cause any dissatisfaction or penalty.

When the counterpart's choice is different than the first-best choice of agent i ($q_j \neq -q_i^*$), the value of the multiplier is going to be non-zero ($\mu_i \neq 0$). The agent is willing to increase or decrease the quantity of exchange but is bound by her counterpart. To satisfy the coupled constraint, agent i's choice is going to be different than the first-best quantity ($q_i = -q_j \neq q_i^*$). This is why the agent will experience a dissatisfaction about the exchange, represented by the magnitude of the multiplier μ_i. The difference in the exchange quantity from the first-best quantity of agent i increases as the absolute value of μ_i becomes larger, and vice versa.

The sign of μ_i can be positive or negative, as the constraint can be binding from below or above. For instance, if $-q_j = q_i < q_i^*$ then agent i would like to increase the quantity being exchanged but is restricted by her counterpart. This case corresponds to $\mu_i > 0$ as $q_i < q_i^*$ leads to $pu_i'(pq_i) - d_i'(q_i) > 0$. In this case, agent i would like to buy more (or sell less) of the bad, but agent j does not agree to it. In the contrary case, that is, if $-q_j = q_i > q_i^*$, then the agent is willing to decrease the quantity of the exchange but her counterpart does not allow it. Then, we have $pu_i'(pq_i^*) - d_i'(q_i^*) < 0$ and $\mu_i < 0$, corresponding to the situation in which the agent is either willing to buy less of the bad or sell more of it.

For a given vector of weights $\mathbf{r} = \{r_1, r_2\}$, the unique normalized equilibrium is defined as the quantity of exchange such that the weighted value of each agent's multiplier is equal to the other's. This is to say that each agent's weighted dissatisfaction from the exchange is equalized in the equilibrium, that is,

$$\frac{\mu_1}{r_1} = \frac{\mu_2}{r_2}. \tag{15}$$

The equality in (15) must hold; therefore μ_i is either positive or negative for both agents. Therefore, the equilibrium consists of either an agent willing to buy more and its counterpart willing to sell less, or an agent willing to sell more and its counterpart willing to buy less of the bad. By conditions (13), (14), and (15), the equilibrium quantity of exchange (q_0) must satisfy

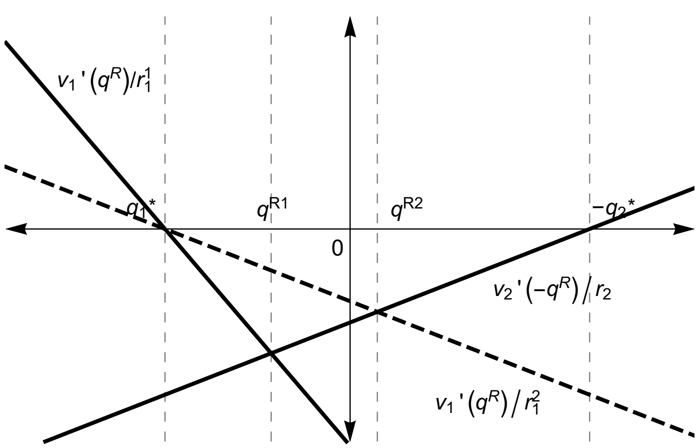
$$\frac{pu_1'(pq_0) - d_1'(q_0)}{r_1} = \frac{pu_2'(-pq_0) - d_2'(-q_0)}{r_2}. \tag{16}$$

The following propositions characterize the unique normalized equilibrium of the two-player game:

Proposition 3 *For a given $\mathbf{r} > 0$, the unique normalized equilibrium of the two-player game of exchanging a bad is $(q_i^0, q_j^0) = (q_0, -q_0)$ that satisfies (16).*

Proposition 4 *For two given weight vectors $\mathbf{r}^A = \{r_1^A, r_2^A\}$ and $\mathbf{r}^B = \{r_1^B, r_2^B\}$ with $r_1^A > r_1^B$ and $r_2^A = r_2^B$ (which means that agent 1 has more weight in $\mathbf{r}^A$), the non-cooperative solution B is closer to the first-best choice of agent 1 compared to A, $|q^A - q_1^*| > |q^B - q_1^*|$.*

Proof. The proof of uniqueness is identical to the one for the cooperative solution in Proposition 2. The proof for the effect of r follows the same methodology as the proof for the role of ω in Proposition 2. The result is inverse since r and ω have opposite effects on the equilibrium condition. A graphical illustration is presented in Figure 2. $\square$



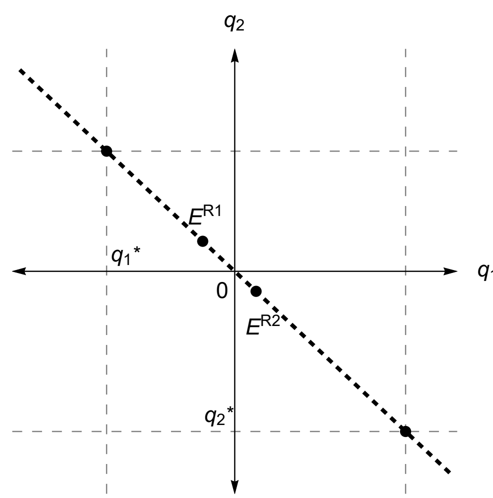


Figure 2: Sample equilibrium points in the non-cooperative game ($r_1^1 < r_1^2$)

Figure 2 illustrates the results given in Propositions 3 and 4. To facilitate comparison, it depicts two agents who are willing to sell a bad ($q_i^* < 0$ for $i = 1, 2$) as in Figure 1 in the cooperative case. The normalized equilibrium takes place when the weighted dissatisfaction from the exchange is equalized among all agents.

Remark 1 *The equilibrium quantity of exchange always lies within the interval of first-best choices for agents i and j, $q_i^0 \in (q_i^*, -q_j^*)$.*

Remark 1 implies that the quantity of exchange can be null only if the first-best levels of all agents share the same sign, meaning that the two agents must be either both sellers (as in the case depicted in the figure), or both buyers. In that case, for a given r-vector, if the price of exchange satisfies

$$p = \frac{(r_2\varphi_1 - r_1\varphi_2)}{(r_2\psi_1 - r_1\psi_2)},$$

then there is no exchange in the equilibrium. For all different values of p and $\mathbf{r}$ there will be an exchange. In the other case, if an agent would like to buy and the other would like to sell, then the equilibrium quantity of exchange is always non-zero. One can illustrate that case in Figure 2 with both marginal payoff curves crossing the horizontal axis at the same side of the origin.

Whether the quantity of the exchange is close to an agent's first-best choice depends on the exogenous weight vector $\mathbf{r}$. One can observe that having a higher r-weight for agent 1 while keeping r_2 constant changes the slope of the weighted marginal payoff of 1 to be more horizontal (the dashed curve), which means that the equilibrium point moves closer towards the first-best choice for agent 2 (from E^{R1} to E^{R2}), and agent 1's dissatisfaction from exchange increases with r_1. Hence, the weight vector $\mathbf{r}$ affects the quantity of the exchange in an inverse way compared to the effect of $\boldsymbol{\omega}$ weights in the cooperative case. The following corollary presents this result formally:

Corollary 2 *Consider that the weights of agent 1 and 2 in joint welfare are $\{\omega_1, \omega_2\}$. The non-cooperative solution coincides with the cooperative one if*

$$\{r_1, r_2\} = \left\{\frac{1}{\omega_1}, \frac{1}{\omega_2}\right\}. \tag{17}$$

Proof. Follows directly from (9) and (16). □

The relation between the non-cooperative and cooperative outcomes is shown in Corollary 2. Accordingly, a higher agent weight in joint welfare requires a lower r-weight assigned to that agent in the non-cooperative game in order to achieve the level of cooperative joint-welfare maximizing exchange. This leads us to make the following remarks:

Remark 2 *If the agents can choose their r-weights, then they can attain the cooperative outcome without the existence of a market or the presence of a regulator, by setting their weights as in Corollary 2.*

Remark 3 *If the policy maker can influence the agents' r-weights, then it can incentivize them to attain the cooperative outcome.*

Remark 2 emphasizes that the cooperative solution is included in the non-cooperative one for $\mathbf{r} = 1/\boldsymbol{\omega}$. Therefore, if the agents wish to exchange such that joint welfare is maximized, they can do so by choosing their r-weights according to the weights in the joint welfare. Remark 3 refers to policy intervention. For the game we address, some policy tools that could influence the r-vector are market-based instruments such as taxes or subsidies on per-unit of exchange, or regulations that enforce a certain quantity of exchange. In effect, for the case of market-based instruments, the magnitude of the multiplier of agent i (μ_i) reflects a per-unit tax or subsidy imposed on agent i. This can be seen from (13). The optimal tax or subsidy rates for each agent are

$$\tau_1 = pu_1'(pq^c) - d_1'(q^c), \tag{18}$$

$$\tau_2 = pu_2'(-pq^c) - d_2'(-q^c), \tag{19}$$

where q^c is given by Proposition 2, and $\tau_1/\tau_2 = \mu_1/\mu_2$. In the following, we consider agent 1 as the buyer of a bad ($q^c > 0$), and thus, agent 2 as the seller of a bad. When $\mu_1, \mu_2 > 0$, which corresponds to the case in which agent 1 would like to buy more and agent 2 is willing to sell less, the instruments τ_1 is a per-unit tax and τ_2 is a per-unit subsidy. Accordingly, to incentivize the agents to agree on the jointly optimal level of exchange, the agent with a lower weight in joint welfare (a lower ω_i) has to be assigned with a higher per-unit tax or subsidy (a higher r_i) compared to the other agent. More precisely, the ratio of taxes and subsidies has to be the inverse of the ratio of the agents' weights in joint welfare ($\tau_1/\tau_2 = \omega_2/\omega_1$).

And when $\mu_1, \mu_2 < 0$, which is the case where agent 1 is willing to buy less and agent 2 is willing to sell more, the instrument τ_1 is a per-unit subsidy and τ_2 is a tax. In this case, the agent with a lower weight in the joint welfare is given a higher subsidy to accept the jointly optimal level. The joint optimum requires the transfer of a bad to an agent who is not willing to buy it; thus the policy instrument corrects the disutility caused by the buyer's additional quantity bad, making her agree to buy.

Lastly, the following corollary presents the effect of an increase in price (p) in the non-cooperative game:

Corollary 3 *The effect of an increase in the price of exchange depends on the differences in weighted marginal utilities* $\left(\frac{u_1'(pq^0(p))}{r_1} - \frac{u_2'(-pq^0(p))}{r_2}\right)$.

Proof. Considering the equilibrium quantity of exchange $q_0(p)$ in (16), and applying implicit differentiation, and then rearranging yields:

$$\frac{dq^0(p)}{dp} = \frac{r_2u_1'(pq^0(p)) - r_1u_2'(-pq^0(p)) + pq^0(p)\left[r_2u_1''(pq^0(p)) + r_1u_2''(-pq^0(p))\right]}{r_2d_1''(q^0(p)) + r_1d_2''(-q^0(p)) - p^2(r_2u_1''(pq^0(p)) + r_1u_2''(-pq^0(p)))}. \tag{20}$$

The denominator is positive. We investigate two possible cases: $q^0(p) > 0$ and $q^0(p) < 0$.

When $q^0(p) > 0$, hence when agent 1 is the buyer and agent 2 is the seller, the third term in the nominator is negative. Then the sign of the nominator depends on the sign of $r_2u_1'(pq^*(p)) - r_1u_2'(-pq^*(p))$. If $r_2u_1'(pq^*(p)) - r_1u_2'(-pq^*(p)) < 0$ then $\frac{dq^0(p)}{dp} < 0$. In the second case ($q^0(p) < 0$), the third term in the nominator is positive. This implies that $\frac{dq^0(p)}{dp} > 0$ when $r_2u_1'(pq^*(p)) - r_1u_2'(-pq^*(p)) > 0$ and vice versa. □

Corollary 3 implies that there is an ambiguity about the sign of $\frac{dq^0(p)}{dp}$. However, even though it is possible to have $\frac{dq^0(p)}{dp} > 0$ when $q^0(p) > 0$, the sign is going to change to negative after a level of p because the first term in $r_2u_1'(pq^*(p)) - r_1u_2'(-pq^*(p))$ is decreasing, whereas the second term is increasing in p. Thus, for high prices, we have $q^0(p)$ moving towards zero. Similarly, when $q^0(p) < 0$, we have the inverse result, that is, after a level of p, the equilibrium quantity of the exchange must increase and thus moves towards zero.

5 Conclusion

This article characterizes the first-best, cooperative and non-cooperative outcomes of an exchange in which agents buy (or sell) a bad in return for receiving (or paying) a good with an exogenously given price. The framework of the analysis is simple and general, and differs from the related literature in various ways. We consider agents who can be both buyers and sellers and who are independent in their decision-making but constrained by the need for their opponent's agreement on the quantity to be exchanged. The non-cooperative solution is characterized by a normalized equilibrium à la Rosen (1965) that assigns exogenous r-weights to agents.

The results show that agents' first-best outcomes determine the boundaries of the interval in which cooperative and non-cooperative solutions can exist. Moreover, the non-cooperative solution is inversely related to the cooperative one in terms of exogenous r-weights and agents' weights under joint welfare. We show that the agents can attain the cooperative solution in the non-cooperative game if they choose their r-weights accordingly, even in the absence of a regulator or a market structure. Moreover, the policy maker can direct the agents to the cooperative solution if it can influence the r-weights, or by using market-based instruments such as taxes and subsidies.

One can verify that the result of existence and uniqueness in Lemma 1 can be generalized to the case of N-players that engage in a good-bad exchange among themselves. The exchange is organized in such a way that all agents decide and agree on the quantity to be exchanged and then there is a distribution of goods and bads among agents according to the agreed-upon quantities. Therefore, an agent can be transferring bads or goods from/to multiple agents. A framework consisting of a network of agents could be used in order to study a purely bilateral exchange.

Further research directions include the extension to a dynamic framework where the quantity of the bad accumulates with some decay. The present work considers the price of exchange to be exogenous, but results would differ if there were a bidding mechanism or a reference price that the agents can manipulate over time.

In addition, our framework abstracts from some important determinants of bads exchanges, such as transfer costs and reasons to exchange for recycling. Including these features would make it possible to analyze their roles in the outcomes of different scenarios.

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