

**Short term planning optimization model
for underground mines**

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Short term planning optimization model for underground mines

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Abstract: Scheduling activities in an underground mine is a very complex task. This paper presents an optimization model for short-term planning that takes into consideration all parts of the development and production as well as specific limitations on equipment and workers. To do so, a mix integer program is used in order to maximize mined tonnage on a given time horizon. Test results of the application to a mine are presented to confirm the model accuracy and solvability.

Keywords: Mixed-integer programming, mine planning, production scheduling

Résumé: Planifier les activités d'une mine souterraine est une tâche très complexe. Cet article présente un modèle d'optimisation pour la planification de la production à court-terme considérant la production, le développement et toutes les contraintes spécifiques aux équipements et travailleurs. Pour ce faire, un modèle en nombres entiers est utilisé afin de maximiser le tonnage extrait dans une période de temps donnée. Les résultats de l'application du modèle à une mine sont présentés afin de valider le modèle.

Mots clés: Programmation linéaire en nombres entiers, planification minière, ordonnancement de la production

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1 Introduction

Mining projects are made possible through the investment of massive funds. Initial capital costs are huge, running costs are high and risk is higher than in most other businesses. Nevertheless, when managed efficiently, these projects can become very profitable. In the optic of reaching profitability, an effective planning is an essential and powerful tool to get the most value out of a project. Activities all along the mine life are planned on different precision levels and time frames depending on the state of the project. This article presents a mathematical model that allows optimization and testing of different scenarios for short-term planning in underground mines.

The objective behind the development of this model was to make available a tool to facilitate the transition from medium- to short-term planning by first, allowing a quick testing of the feasibility of medium-term planning and then give an optimal dispatch of resources. The reason why a model like this is needed to reach these objectives is that short-term planning of activity in an underground mine is a very complex task that requires time and expertise.

First, even for a small-scale operation, there are numerous resources to manage from workers to equipment and ventilation, each of them having its own limitations. Then, in order to keep productivity high, many work sites must be active at any given time, which multiplies the possibilities of resource allocations. Then the transport of these resources in the limited space available underground creates even more limitations and complexity, particularly for mining equipment with low mobility like drills. Finally, the fact that a lot of work needs to be done before to be able to access the mineralized zones require to constantly prepare future work places to always have mineral resources accessible. For all these reasons, developing a precise short-term planning can quickly become a difficult task.

In the following sections, the data set used to test our model will be described, followed by a review of the currently available literature on the subject of underground mine planning. The model will then be presented and computational results of its application will follow. A brief discussion of the outcome and of future work will then conclude the article. But first, a short description of some of the terms and concepts used in the text will be given.

1.1 Terms and concepts

Three time frames are generally used in the mining industry when it comes to planning activities. The first, strategic or long-term planning, is used to describe objectives over periods of more than a year. It is a global estimation of the operations over the mine's life. Then the first periods of long-term planning are split in a tactic or medium-term planning. At this level, objectives and targets become more precise but are still estimates. Typical periods for medium-term planning are generally between three months and a year. Medium-term planning is finally separated in short-term or operational planning in periods ranging from hours to a month. At this level of planning resources are dispatched and precision is maximal.

As for the type of work, activities in an underground mine are typically separated in two categories: development and production. Development corresponds to all the excavations done in rocks that have no economic value, called waste. Developments are necessary expanses to reach and extract efficiently the rocks with economic value, called ore. Excavations in this type of rock are called production. Development and production are excavated using different equipment and techniques. The former usually aims at minimizing rock excavation for a given horizontal or vertical advance and the latter generally aims at maximizing the ore extracted from each blast. Developments also are normally more supported and secured than their production counterpart due to the fact that workers use these openings to perform different task, and thus, are exposed to risk.

1.2 Dataset

In order to test the model, a fictional mine plan was developed using Geovia Surpac to represent a small scale underground exploitation with typical values. These values and the general layout of the mine were based on data from an operating Canadian mine. A list of operations and equipment were created to fit as

closely as possible with real-world values. We will give here the reader a short description of the activities to be performed at the mine that are relevant to short-term planning.

The project starts with the excavation of the main shaft. Once the depth of the ore body is reached, stations are excavated horizontally as links between levels and the shaft. Then, drifts are developed between the stations and the ore body. Permanent openings such as garages and refuges are disposed along these drifts. Ventilation shafts are dug from the drifts to the surface to allow fresh air intake from the surface. Once the ore zones are reached, ramps are made to allow the development of drifts called sublevels at specified height along the veins. Ore and waste passes are excavated between levels and sublevels to transport ore and waste material respectively from the stopes to the shaft. From the sublevels, ore accesses are prepared depending on the mining method used. Figure 1 shows an isometric view of the mine layout with its developments in brown and stopes in various colors.

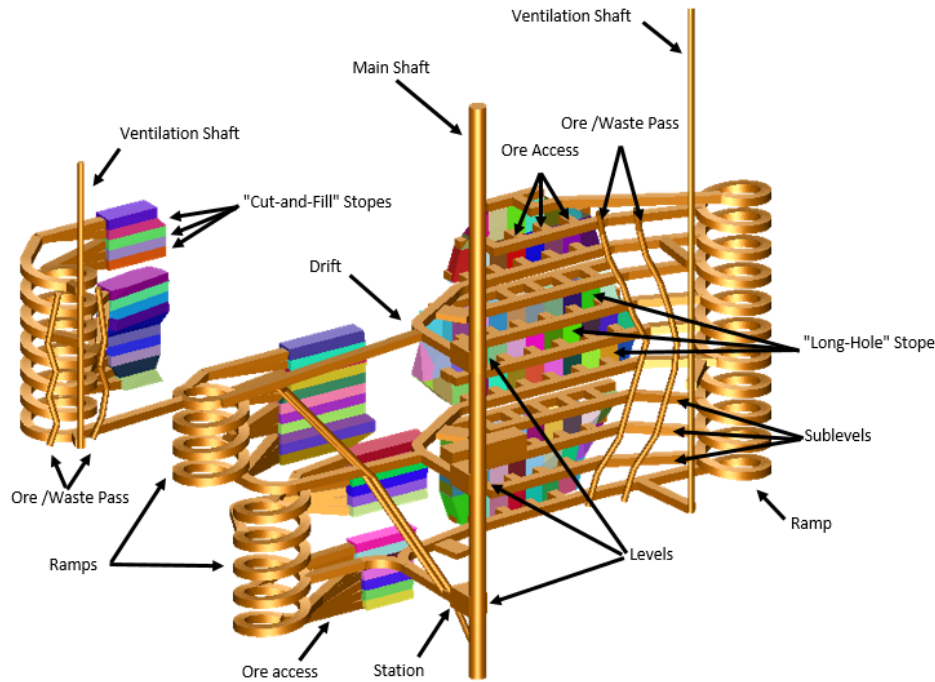


Figure 1: Mine layout

Two mining methods are used in the mine, backfilled Long-Hole and Cut-and-Fill. The Long-Hole method starts with the extraction of accesses over and under the section of the ore body targeted. Then, holes are drilled from these accesses to the surroundings of the ore body to insert cemented cables to provide additional support for the excavation. When the support is installed, a 30 inches wide hole is drilled using a what is commonly known as V-30 in order to create a space for broken rocks to expand during the main blast. Production holes are then drilled at regular intervals and filled with explosives. After the rock is blasted, the resulting fragmented rock is moved by haulage equipment to the closest ore pass.

The Cut-and-Fill method starts with the excavation of accesses from the side of the vein. A drift with variable dimensions is then extracted following the vein. When the total length is reached, the drift is filled with cemented backfill, and another drift is excavated over the last one. The full height of the vein is extracted by a series of superposing backfilled drifts.

The development part of the mine is made of 202 sites including drifts, ramps, ore and waste passes, ventilation shafts and ore access. As for the production, there are 110 stopes in total. Table 1 gives a summary of the quantity and total tonnage of the different type of excavations in the mine. Figures 2 and 3 show graphically the precedencies between the different part of the mine for the development of Cut-and-Fill and Long-Hole stopes. For Cut-and-Fill, a drift coming from the shaft is being dug first followed by the

Table 1: Mine summary

Site type	Quantity	Total Tonnage (tons)
Shafts	16	45352
Permanent Openings	7	24490
Drifts	34	96368
Ramps	7	117228
Ore/Waste Pass	35	13051
Ore Access	103	132205
Cut-and-Fill Stopes	39	186328
Long-Hole Stopes	71	216127
Total	312	831154

development blocks for each sublevel as presented in the Figure 2. A section of ramp and its corresponding ore and waste passes are excavated in order to reach a small drift near the vein. From this drift, ore accesses are dug to reach each of the sublevel's stopes. The development block for the following sub-level can be started as soon as the ramp and passes from the previous block are completed.

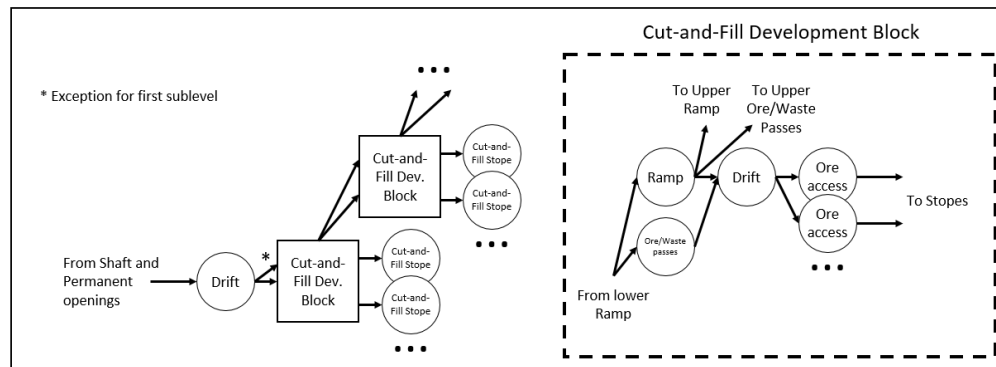


Figure 2: Cut-and-fill precedencies

For the long-hole stopes, a different model of development blocks is used as presented in Figure 3. For each sub-level, a drift is excavated from the ramp in a direction parallel to the vein length. From this, ore and waste passes can be excavated, along with the ore accesses for each of the stopes on the sub-level. But in order to start the extraction of a stope, the correspondent ore access from the following development block must also be completed.

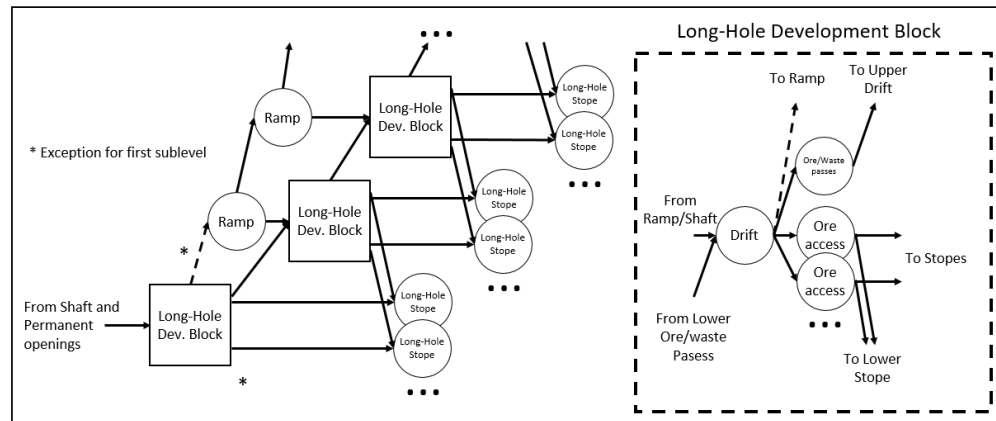


Figure 3: Long-hole precedencies

There are nine types of crew at work in the mine. What is understood as "crew type" is either specialized worker or equipment needed to perform a certain operation in the mine. Since these teams or equipment are

available in limited quantities and they have to visit multiple sites during a shift, their availability is often a limit on the productivity. A short description of each crew and their activities follows, and Table 2 shows the amount available and the number of work places through the mine where their presence is required.

- Horizontal Drilling: Drilling equipment used for blast holes in drifts, ramps, ore accesses and Cut-and-Fill stopes.
- Ground Support: Equipment specialized in installing ground support for drifts, ramps, ore accesses and Cut-and-Fill stopes.
- Services: Crew used to install necessary services to work faces in drifts, ramps, ore accesses and Cut-and-Fill stopes. e.g.: ventilation conduct, communication cables, water pipes
- Cable Drilling: Equipment specialized in installing cable support for Long-Hole stopes.
- V-30 Drilling: Drilling equipment used to drill an opening hole for Long-Hole stopes.
- Production Drilling: Drilling equipment used to drill blast holes for Long-Hole stopes.
- Haulage: Loading and hauling equipment used to remove blasted ore from every work sites except main shafts and transport it to appropriate passes.
- Backfilling: Specialized crew that supervise the backfilling of excavated Cut and Fill and Long-Hole stopes.
- Alimak: Specialized crew that work in passes and ventilation shafts.

Table 2: Crew summary

Type	Available	Work Places
Horizontal Drilling	2	190
Ground Support	2	190
Services	1	190
Cable Drilling	2	71
V-30 Drilling	1	71
Production Drilling	2	71
Haulage	3	308
Backfilling	1	110
Alimak	2	47

2 Literature review

Planning in the mining industry was traditionally based on planners experience and estimations from previous projects. In recent years though, more and more tools are being developed in order to automate and optimize this process. Since the first publication in the 1960s from [1], considerable progress was made, driven by developments in operation research and increase in computational power. For the most part, the application of optimization in the mining industry concerns open pit projects. One reason for that, as mentioned in [2], is that underground projects are constrained by many more factors than their surface counterpart. Moreover, applications of optimizations model to underground mines have to be site specific due to the numerous mining methods and rock haulage system used in the industry. The most recent literature review on the subject includes [3] that presents a review of optimization in natural resources with a section dedicated to mining [2], that discusses the advances of optimization in mine planning and [4] that gives a review specifically on the subject of optimization in underground mines.

2.1 Long-term planning

From the literature available on underground mine planning, the most part concerns long-term. A good example of a long-term application can be found in [5], where the authors present a model to optimize starting time in different parts of an underground coal mine. The instances are then solved using a method involving Benders' decomposition to find bounds on the solution and accelerate the solving of the problem. [6] and [7] then present methodologies to reduce computational time of general long-term planning models.

In the same optic, authors in [8] develop a "Greedy randomized adaptive search" procedure to accelerate the solve of a model developed for a copper mine.

Following these, [9] present a classical and improved formulation of the long-term planning model that considers the different resources and equipment needed for every site. The classical model simply assign a binary variable to every activity to be done in every stope and the variation uses a single binary variable for all activities under more restrictive hypothesis. [10] then propose a unified formulation for the long-term planning problem with simpler notation for resources and a modified version that optimizes with respect to block selection within the mine. On a larger scale, [11] gives a model of optimization with low resolution for a large mining complex including many open pit sections as well as underground parts. A year later, [12] creates another model of optimization for a mining complex with open pits and underground parts that maximize net present value with a variable cut off grade. In a subsequent article, [13] modify the model from [12] to consider geological uncertainty. [14] show the value of optimizing stopes shape in parallel with planning by providing a model that optimizes both with results proving an increase in value.

What comes out of these articles is that if earlier works were mostly considering low resolution mining units with constraints on global limitations the general trend in more recent work is now to either implement resources specific constraints like in [9] and [10] or to consider bigger problems involving multiple mines and processes as in [11] and [12].

2.2 Medium-term planning

Less work on medium-term planning is available than for long-term planning. Nevertheless, [15] present a model specific to the Stillwater mine for time units of three months. The model is then used to evaluate different investment scenarios by modifying values in the model. Then [16] and [17] give two adaptations of the long-term planning model developed for the Kiruna mine in Finland. Both model decision variables represent the option of starting or not the extraction of ore in all the possible extraction points although [17] allows more precision. [16] then use aggregation and a heuristic to solve the model and [17] use acceleration techniques to reach optimality in a reasonable time. Some years later, [18] explain an even more effective heuristic to solve [16] model that is based on multiple solves of the problem with different parts of the objective function.

With shorter periods, these models are all more precise than the ones for long-term planning. But the cost of this precision is that either heuristics have to be used to solve the models like in [18] or that the area of application must be limited to certain parts of the mine like in [17] where optimization is focused on the production.

2.3 Short-term planning

Very limited work exists in short-term underground mine planning. Some literature focus on real-time optimization as reviewed in [19], but these problems are more about equipment fleet dispatch and are very different from scheduling problems like ours. Still, [20] present an integrated short- and medium-term optimization model for production. Decision variables on start time for stope developments and excavations are used to smooth the variations on the mill feed and maximize net present value. It is tested on a conceptual 30 stopes model resulting in a small increase in net present value and less mill feed variations compared to separate short- and medium-term planning. Another model can be found in [21] with its application to short-term scheduling at the Lisheen mine in Ireland. Decision variables also dictate the starting time for the excavation of each part of the production. A heuristic is then used to solve the problem.

In all these models, from long- to short-term, the following hypothesis is used: once an activity is started at a location in the mine, it is executed for a fixed duration until it is over. This can be a good estimate when activity durations compared to period length are small as in long-term planning. It can also be applied when the emphasis is put on production where activity durations are similar as in cited short-term models but in our case, it can be problematic. The main reason for this is that when considering development, activity length can vary greatly, creating gaps between activities ends and starts.

The following figure gives a simple example of the type of time lost that can occur when preemption is not allowed. The illustration represents a case where five activities must be performed and a maximum of two activities can be performed at the same time. From these activities (2) and (4) cannot be done before (1) is completed and (3) and (5) are free to start at anytime. It can be seen that on the first line, where activity separation is not allowed, a gap occurs between the end of (3) and the start of (4). On the second line, where preemption is allowed, this gap is filled by a part of (5), which results in an economy of time for the completion of all activities.

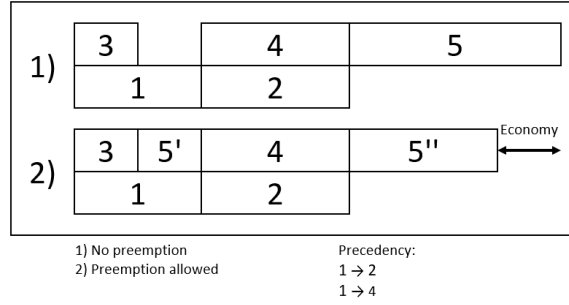


Figure 4: Planning example with and without preemption

3 Model

The model presented in this article addresses this problem particularly by using a mix of integer and continuous variables to create the activity schedule. It uses one-week periods to create a feasible short-term schedule based on medium-term objectives for planning horizons of 3 months to over a year. The idea used in [9] to regroup all the different activities occurring at one site under a single variable is also used in this model but with continuous variables. Also to reduce the size of the problem, \hat{Q}_{is} is used. Instead of separating drifts in small segments between each intersection, \hat{Q}_{is} allows to start a site if enough of its predecessor is completed. It is especially useful in the case of long drifts leading to multiple ore accesses. The drift is represented by a single variable instead of one for each segment, helping to reduce the size of the problem. Each location where extraction activities are planned is called site and given an index number s and each week over the planning period is given an index t . Veins, levels and crew types are also given an index number. Each site has a target parameter, corresponding to the total tonnage of the site if extraction was planned at this location in the medium-term planning, and zero if no extraction was planned. A list of indexes, sets, parameters variables and constraints used in the model follows with their definition.

3.1 Indexes

\mathcal{I}^S : Site index
 \mathcal{I}^T : Time index
 \mathcal{I}^V : Vein index
 \mathcal{I}^L : Level index
 \mathcal{I}^C : Crew index

3.2 Sets

\mathcal{E}_v^V : Set of sites located in vein v
 \mathcal{E}_l^L : Set of sites located in level l
 \mathcal{E}^B : Set of sites where backfilling is required
 \mathcal{E}^{Target} : Set of sites for which $Target_s > 0$
 \mathcal{P}_s^S : Set of sites preceding site s
 $\{end\}$: Last time period

3.3 Parameters

H : Number of work hours available per time period

C_{ct} : Available crews of type c at time t

R_{sc} : Number of work hours needed from crew type c to complete site s

R_{s0} : Number of hours needed to backfill site s

Q_s : Rock tonnage in site s

\hat{Q}_{is} : Fraction of site $i \in \mathcal{P}_s^S$ to complete before site s can start

B_s : Binary parameter indicating whether or not site s needs backfilling

$D_s^{kg/t}$: Rock density in site s

$D_s^{\%}$: Expected dilution in site s

T^{Mine} : Maximum possible tonnage extraction in the mine for one time period

T_l^{Level} : Maximum possible tonnage extraction in level l for one time period

T_v^{Vein} : Maximum possible tonnage extraction in vein v for one time period

T_s^{Max} : Maximum possible tonnage extraction in site s for one time period

T_s^{Min} : Minimum tonnage extraction in site s for one time period when extraction occurs at this site

$Target_s$: Target for site s , equals Q_s if extraction is planned at this site in the medium term planning and 0 if not.

3.4 Variables

c_{st} : Fraction of total work completed at site s during time period t

χ_{st} : Binary variable indicating whether or not work is performed at site s during time period t

q_{st} : Tonnage left in site s at the beginning of time period t

b_{st} : Fraction of backfilling completed at site s during time period t

β_{st} : Binary variable indicating whether or not backfilling is performed at site s during time period t

3.5 Objective

The model's objective is to maximize extracted tonnes from sites where extraction was planned. The objective value represents the total tonnage excavated from these sites.

$$Max \quad \sum_{s \in \mathcal{I}^S} \sum_{t \in \mathcal{I}^T} c_{st} Target_s \quad (1)$$

3.6 Constraints

$$\sum_{s \in \mathcal{I}^S} c_{st} R_{sc} \leq H C_{ct} \quad \forall c, t \quad (2)$$

$$\sum_{t \in \mathcal{I}^T} c_{st} \leq 1 \quad \forall s \quad (3)$$

$$\sum_{s \in \mathcal{I}^S} c_{st} D_s^{\%} Q_s \leq T^{Mine} \quad \forall t \quad (4)$$

$$\sum_{s \in \mathcal{E}_l^L} c_{st} D_s^{\%} Q_s \leq T_l^{Level} \quad \forall l, t \quad (5)$$

$$\sum_{s \in \mathcal{E}_v^V} c_{st} D_s^{\%} Q_s \leq T_v^{Vein} \quad \forall v, t \quad (6)$$

$$\sum_{c \in \mathcal{I}^C} c_{st} R_{sc} + b_{st} R_{s0} + \sum_{c \in \mathcal{I}^C} c_{s't} R_{s'c} + b_{s't} R_{s'0} \leq H \quad \forall s, t, s' \in \mathcal{P}_s^S \quad (7)$$

$$c_{st} Q_s - \chi_{st} T_s^{Max} \leq 0 \quad \forall s, t \quad (8)$$

$$c_{st} Q_s - \chi_{st} T_s^{Min} \geq 0 \quad \forall s, t \quad (9)$$

$$q_{s1} = Q_s \quad \forall s, t = 1 \quad (10)$$

$$q_{st} - q_{s(t-1)} + c_{s(t-1)}Q_s = 0 \quad \forall s, t \setminus 1 \quad (11)$$

$$\chi_{st}Q_{s'}\hat{Q}_{is'} + q_{s't} - c_{s't}Q_{s'} \leq Q_{s'} \quad \forall s, t, s' \in P_s^S \quad (12)$$

$$\beta_{st} - b_{st} \geq 0 \quad \forall s, t \quad (13)$$

$$\sum_{s \in \mathcal{I}^S} b_{st}R_{s0} \leq H \quad \forall t \quad (14)$$

$$\beta_{st}Q_s + q_{st} - c_{st}Q_s \leq Q_s \quad \forall s \in \mathcal{E}^B, t \quad (15)$$

$$\chi_{st} - \sum_{i=1}^t b_{s'i} \leq 0 \quad \forall s, t, s' \in P_s^S \cap \mathcal{E}^B \quad (16)$$

$$0 \leq c_{st}, b_{st} \leq 1 \quad (17)$$

$$q_{st} \geq 0 \quad (18)$$

$$\chi_{st}, \beta_{st} \in \{0, 1\} \quad (19)$$

Constraints 2 assure that for each period of time, crews cannot work more than the number of hours in a time period. Then, constraints 3 make sure all sites can only be excavated once. Constraints 4, 5, and 6 limit the amount of ore extracted in the mine, on every level and in every vein. Tonnage limits for the mine are usually dictated by the haulage capacity of the shaft. Limits on levels can be caused by ventilation or limited work space and limits on veins can be from the haulage or passes capacity. Constraints 7 limit the time spent in a site and its predecessor to the amount of hour in a period. These constraints are necessary since work at a predecessor and its successor is allowed in a single period, as long as the predecessor is completed at the end of the period. The effect of this constraints is to make sure that work at the successor is done after the excavation of the predecessor and not simultaneously.

Constraints 8 and 9 limit the extraction of rock for every site with a lower and an upper bound. The lower bound, T_s^{Min} is based on the amount of work necessary for one blast, since sending a team for less than one blast is considered impractical. The upper bound, T_s^{Max} , is the equivalent of 14 blasts since there are two possible blasts per day and seven work days in a week. Constraints 10 and 12 assure that variable q_{st} represent the tonnage left in each site at the beginning of a time period. Constraints 12 assure that precedence are respected and constraints 13 link variables β_{st} to b_{st} . Constraints 14 limit backfilling capacity for each period t . Finally, constraints 15 assure that backfilling can only start when the excavation is completed and 16 make sure the excavation of a site can only start if its predecessor is backfilled.

4 Results

The computational results of the application of our model to the mine described before and some randomly generated variations will be presented in this section. Three scenarios were created to simulate the different states in a mine life. The scenarios are made by varying the progress of operations in the mine. The first scenario is the equivalent of starting the operation from scratch, where all the sites are intact and a lot of development has to be done before to reach the ore zones. The second scenario simulates a more advanced state where 40 sites have already been excavated. Most of the main developments are completed but there are still local developments to excavate before production can start. The third scenario where 96 sites are considered done represent a state where some part of the mine have already started production and some development is left to do in further parts.

For each of these tests, the planning horizon was gradually increased from 12 to 60 weeks. Then, a sample of mines generated from random variations of the original were also tested for each scenario and time period to get more sampling data. The following tables show the results of our tests with their computation time in seconds and objective value in tons. Case 1 represents the basic mine model presented above and cases two to six are random variations. The first five cases are of similar sizes and case six is approximately three time bigger to test our model limits. The computation time was set to a limit of 600 seconds or ten minutes and

Table 3: Results: Scenario 1

Case		1		2			3		
Period	Obj	%	Time	Obj	%	Time	Obj	%	Time
12	62006	0	1.6	66043	0	0.2	74935	0	0.2
24	238826	0	1.4	250605	0	1.2	278051	0	2.0
36	487119	0	7.2	525077	0	9.4	566149	0	13.8
48	630063	0	24.4	683507	0	13.7	741918	0	24.1
60	816087	0	68.8	892174	0	380.9	956993	0	79.1
Case		4		5			6		
Period	Obj	%	Time	Obj	%	Time	Obj	%	Time
12	47695	0	1.5	59260	0	0.1	335539	0	2.3
24	185853	0	2.0	222476	0	1.0	637939	0	4.8
36	389815	0	7.9	489578	0	9.0	940339	0	33.5
48	519850	0	16.8	635266	0	41.4	0	-	600
60	20414	3118.1	600	631583	30.2	600	1274383	21.3	600

Table 4: Results: Scenario 2

Case	1			2			3		
Period	Obj	%	Time	Obj	%	Time	Obj	%	Time
12	208916	0	0.6	225770	0	0.8	240679	0	1.3
24	484703	0	6.6	520523	0	8.6	537138	0	11.6
36	571161	0	21.1	628253	0	17.9	672838	0	15.8
48	713840	0	574.3	784904	0	118.0	646193	29.5	600
60	713840	0	78.3	791760	0	103.4	858717	0	595.6
Case	4			5			6		
Period	Obj	%	Time	Obj	%	Time	Obj	%	Time
12	169605	0	1.3	210153	0	1.1	302400	0	3.4
24	391977	0	7.6	500242	0	7.8	604799	0	21.2
36	462887	0	24.7	585601	0	28.1	864303	5.0	600
48	564134	0	267.2	725047	0	173.1	0	-	600
60	564134	0	73.5	731757	0	78.7	0	-	600

the percentage column represent the gap between the best integer solution and the best upper bound found within the time limit. From the default settings of the solver engine used, a solution less than 0.1% from the upper bound was considered optimal.

The first thing that can be noticed from the results presented in the tables above is that solving times for 12 periods generally increase with the scenarios going from an average of one to ten seconds. This can be explained by the fact that in the first scenario, there are not that many options of sites to work at since the shaft and levels must be completed before to start any other development. This creates less possibility and makes it easier to find the optimal solution. It can also be observed from Table 5 that in most cases, the objective value caps after 36 weeks. This is because 36 weeks is enough to exploit the rest of the mine or meet the medium-term objective.

Another effect of this can be observed in scenario 2, case 1 with 48 periods and in scenario 3, case 2 with 36 periods where a sharp increase in computational time occurs, followed by a decrease for the following entry. Our explanation to this is that the amount of tons that can possibly be exploited for these instances is very close to the tonnage left in the mine. This creates a lot of complexity since almost all of the sites are accessible and scheduled, but the bound on the total tonnage cannot be reached to limit the expansion of the search tree. For the sixth case, it can be seen that most of the instances involving more than 36 periods could not be solved, but considering that this instance was approximately three times bigger than our other instances, these results are still considered acceptable.

Table 5: Results: Scenario 3

Case	1			2			3		
Period	Obj	%	Time	Obj	%	Time	Obj	%	Time
12	302400	0	16.6	302399	0	14.9	302400	0	6.0
24	494122	0	19.9	538139	0	19.0	574487	0	22.8
36	542515	0	35.4	593664	0	82.1	630985	0	37.6
48	542515	0	42.2	600216	0	48.5	650963	0	47.0
60	542515	0	53.5	600216	0	14.3	650963	0	75.0
Case	4			5			6		
Period	Obj	%	Time	Obj	%	Time	Obj	%	Time
12	302399	0	10.1	302399	0	11.8	302400	0	4.1
24	412721	0	18.4	506124	0	16.9	604800	0	9.4
36	434752	0	42.1	550640	0	179.9	904340	0.3	600
48	434752	0	61.7	556327	0	53.3	0	-	600
60	434752	0	97.0	556327	0	78.6	0	-	600

5 Conclusion

The motivation to develop the model presented in this article was to create a tool able to quickly verify the feasibility of a medium-term planning and to optimize scheduling of short-term activities. The result of the application to fictional but realistic mines prove that the model can be solved to optimality within less than ten minutes for instances representative of different medium-term periods. The next step in the project will be to complete more tests with real-world data in order to compare our model results to actual planning and make necessary adjustment if needed. The model in its current state can be seen as a fast way to get an upper bound on what can possibly be mined when considering development, production and resource constraints. Further testing will help tighten this bound.

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