Two-stage stochastic surface constrained mine production scheduling with pit discretization

A. Marinho
R. Dimitrakopoulos

Two-stage stochastic surface constrained mine production scheduling with pit discretization

Alexandre Marinho[a]

Roussos Dimitrakopoulos[a,b]

[a] COSMO – Stochastic Mine Planning Laboratory, Department of Mining and Materials Engineering, McGill University, Montreal (Quebec) Canada H3A 2A7

[b] and GERAD

alexandre.almeida@mcgill.ca
roussos.dimitrakopoulos@mcgill.ca

Abstract: This paper addresses the optimization of long-term mine production schedules, building upon a previous stochastic integer programming (SIP) formulation based on surfaces, whereby the uncertain supply of metal is described by a set of equally probable orebody model representations. Surfaces are defined as limits that separate mining blocks assigned to two consecutive mine production periods. The proposed formulation maximizes discounted cash flows and minimizes the risk of deviating from production targets. The sequential implementation of previous work is considered herein for a pit space discretization followed by a yearly-based mine production scheduling, controlling the number of binary variables involved in the optimization processes and facilitating production scheduling for relatively large mineral deposits. The application of the sequential implementation of the proposed mathematical formulation in a relatively large gold deposit shows efficiency and improved results if compared with industry best practices, with a 43% higher ore production and a 19% increase in project net present value expectations.

1   Introduction

Mining is an activity based on extracting underground materials in a given sequence, so as to maximize the net present value (NPV) of the project. Planning the sequence of extraction, by the definition of a life-of-mine production schedule, requires understanding the uncertain metal quantities available throughout the deposit. Conventional estimation methods do not capture the intrinsic geological variability and uncertainty, returning single and possibly misleading production forecasts (Dimitrakopoulos et al., 2002; Dowd, 1997), which may underestimate potential metal production and project value (Godoy, 2003). A review of these issues as well as an overview of recent developments in dealing with uncertainty (stochasticity) in optimizing mine design and production scheduling can be found in Dimitrakopoulos (2011). Stochastic optimization for long-term mine production scheduling using simulated annealing was a concept introduced by Godoy and Dimitrakopoulos (2004) and was further explored in Leite and Dimitrakopoulos (2007) and Albor and Dimitrakopoulos (2009) and is currently being applied to scheduling with multiple rock types and processing streams (Montiel and Dimitrakopoulos, 2012). Stochastic integer programming with recourse is introduced in Ramazan and Dimitrakopoulos (2005, 2012) to maximize total discounted cash flows while minimizing deviations from production targets (ore tonnage, grade and metal), as well as deferring risk to latter production periods considering the concept of orebody risk discounting introduced by Dimitrakopoulos and Ramazan (2004), which penalizes deviations from production targets differently over mining periods. The SIP framework considers stockpiles and allows for controlling grades, ore and metal productions. The well-known drawback of any mixed-integer programming based approach in the mine scheduling context however is computational time due to the number of binary variables (Hustrulid and Kuchta, 2006). Notable variations of the SIP framework include: long- and short-term mine production scheduling based on simulated future grade control data (Jewbali, 2006); use of the SIP formulation for pushback design, demonstrating that stochastically generated pit limits are larger than the corresponding conventional ones (Albor and Dimitrakopoulos, 2010); the proposition of an alternate formulation that uses a variable cutoff grade and relies on aggregations of blocks to ensure the problem is computationally tractable (Menabde et al. 2007) and; a multi-stage stochastic programming approach that considers both processing and mining decisions (Boland et al. 2008). To address the computational and size limits of SIP mine scheduling formulations, Lamghari and Dimitrakopoulos (2012) introduced Tabu and Variable Neighborhood Search, bypassing the need to solve SIP formulations with conventional integer programming solvers and assisting computationally efficient solutions. Regarding stochastic pit space discretization, something which can facilitate the scheduling of large deposits, Asad and Dimitrakopoulos (2013) proposed a graph structure to consider geological and market uncertainties and solve the problem using a parametric maximum flow algorithm integrated with Lagrangian relaxation and the subgradient method. Goodfellow and Dimitrakopoulos (2012) proposed two formulations to modify a given pushback design while keeping original pushback sizes using the simulated annealing metaheuristic. The approach considers grade and material uncertainty in order to reduce risks of misclassification over different processes. Marinho and Dimitrakopoulos (2012) introduced a SIP formulation based on mining surfaces, building upon previous work from Goodwin et al. (2005), where the objective is to maximize discounted cash flows and control the risks of not achieving ore production targets. The work proposed a hybrid approach combining the mathematical formulation with a sequential implementation. Each mining period is subdivided and the schedule is sequentially optimized until an initial solution for the entire mining period is found, which is included in the long-term schedule and optimized jointly with periods previously defined. This formulation does not consider recourse actions for risk management which is included herein.

The concept of surfaces in mine production schedule optimization, first defined in Goodwin et al. (2005), is based on the fact that mining blocks describing a deposit are not independently distributed in space and can be grouped into vertical columns. Surfaces are defined as sets of elevations in which mining periods in the production schedule are divided. Each column of blocks can be partitioned by T surfaces into $T + 1$ groups of blocks. Surfaces are divided into small pieces called cells. For each surface (or period), a cell is defined as a linear variable carrying the elevation associated with a fixed pair of coordinates (x, y). Block attributes are accumulated starting from the topography down to the last block over each column, with cumulative values being stored at each level, which allows for calculations by taking differences between surfaces. A key aspect of this approach is the need to associate blocks with surface cells and it is performed by comparing their elevations in space, as discussed herein. The present paper builds upon previous work

by considering the modelling with surfaces and the sequential approach from Marinho and Dimitrakopoulos (2012) and a two-stage SIP formulation with recourse actions as in Ramazan and Dimitrakopoulos (2012). In the next section, the mathematical formulation is proposed with a review of the sequential implementation. Then, a case study in a gold deposit shows the application of the method to a highly variable and relatively large deposit. The approach is applied first to discretize the pit space and, later, to define the yearly-based production schedule. Conclusions follow.

2   Two-stage stochastic surface based scheduler

2.1   Notation

- M: number of cells in each surface.

- Z: number of levels in the orebody model.

- T: number of periods over which the orebody is being scheduled.

- S: number of simulated orebody models considered.

- E_c^z: elevation of the centroid for a given block (c, z).

- H_x: maximum difference in elevation for adjacent cells in contact laterally in the x direction, calculated by $H_x = \Delta x \times \tan(\theta)$, where Δx is the block size in x and θ is the maximum slope angle.

- H_y: maximum difference in elevation for adjacent cells in contact laterally in the y direction, calculated by $H_y = \Delta y \times \tan(\theta)$, where Δy is the block size in y.

- H_d: maximum difference in elevation for adjacent cells in contact diagonally, calculated by $H_d = \sqrt{(\Delta x)^2 + (\Delta y)^2} \times \tan(\theta)$.

- X_c, Y_c and D_c: equivalent to H_x, H_y and H_d concept. The sets of adjacent cells, laterally in x, in y and diagonally, for a given cell c, respectively.

- T_c^z: cumulative tonnage of block (c, z) and all blocks above it.

- $O_{c,s}^z$: cumulative ore tonnage of block (c, z) and all blocks above it in scenario s.

- $\underline{T_t}$ and $\overline{T_t}$: lower and upper limits, respectively, in total tonnage to be extracted during period t.

- $\underline{O_t}$ and $\overline{O_t}$: lower and upper target limits, respectively, in ore tonnage to be extracted during period t.

- C_t^- and C_t^+: costs associated to unit shortage and surplus, respectively, in tonnage of ore processed over period t.

- $V_{c,t,s}^z$: cumulative discounted economic value of block (c, z) and all blocks above it in scenario s and period t.

- $e_{c,t}$: continuous variables associated with each cell c for each period t, representing cell elevations.

- $x_{c,t}^z$: binary variables that assumes 1 if block (c, z) is the last block being mined in period t over c, and 0 otherwise. $x_{c,0}^z$ is defined as constant equal to 0, $\forall(c, z)$.

- $d_{t,s}^-$ and $d_{t,s}^+$: deviation variables measuring shortage and surplus, respectively, in the tonnage of ore processed over period t under scenario s.

2.2   Mathematical model

The mathematical model proposed in Marinho and Dimitrakopoulos (2012) is extended herein to a risk management framework with a two-stage SIP formulation with recourse actions (Ramazan and Dimitrakopoulos,

2012). The objective function (1) maximizes the expected net present value from mining and processing selected blocks over all considered mine production periods, and manages the risk of not achieving ore production targets through the definition of a risk profile:

$$\max \frac{1}{S} \left(\sum_{s=1}^{S} \sum_{t=1}^{T} \sum_{c=1}^{M} \sum_{z=1}^{Z} V_{c,t,s}^{z} \left(x_{c,t}^{z} - x_{c,t-1}^{z} \right) - \sum_{s=1}^{S} \sum_{t=1}^{T} \left(C_{t}^{-} d_{t,s}^{-} + C_{t}^{+} d_{t,s}^{+} \right) \right) \tag{1}$$

The constraints presented in Equations (2) to (8) are scenario-independent, while constraints in Equations (9) and (10) are scenario-dependent stochastic constraints.

Surface constraints: the following set of constraints (2) guarantee that each surface t has, at maximum, the same elevation as surface , which is used to avoid crossing surfaces and blocks being mined more than once. $e_{c,0}$ are constant elevations defined by the actual topography of the deposit:

$$e_{c,t-1} - e_{c,t} \geq 0 \qquad c = 1, \ldots, M; \quad t = 2, \ldots, T \tag{2}$$

Slope constraints: the maximum surface slope angle is guaranteed herein by Equations (3) to (5). Each cell elevation is compared to the elevation of the 8 adjacent cells, which therefore represents a set of $8 \times nx \times ny \times T$ linear constraints. Note that adjacent cells are compared twice, guaranteeing upward and downward slopes. The number of slope constraints controlled by surface relations does not depend on slope angles and requires fewer constraints than conventional formulations. This is noted as follows:

$$e_{c,t} - e_{x,t} \leq H_x \qquad c = 1, \ldots, M; \quad t = 1, \ldots, T; \quad x \in X_c \tag{3}$$

$$e_{c,t} - e_{y,t} \leq H_y \qquad c = 1, \ldots, M; \quad t = 1, \ldots, T; \quad y \in Y_c \tag{4}$$

$$e_{c,t} - e_{d,t} \leq H_d \qquad c = 1, \ldots, M; \quad t = 1, \ldots, T; \quad d \in D_c \tag{5}$$

Link constraints: mining blocks and surfaces are linked in the formulation by comparing the elevation of each block centroid with the elevation of each surface. Variables $x_{c,t}^{z}$ will assume value 1 only for block centroids that are in the same elevation or exactly above the correspondent surface (index t). Constraints (6) guarantee this link and constraints (7) guarantee that there is only one block defining the end of each period t over each column c of blocks z:

$$0 \leq \sum_{z=1}^{Z} \left(E_c^z x_{c,t}^z \right) - e_{c,t} \leq \Delta z \qquad c = 1, \ldots, M; \quad t = 1, \ldots, T \tag{6}$$

$$\sum_{z=1}^{Z} x_{c,t}^z = 1 \qquad c = 1, \ldots, M; \quad t = 1, \ldots, T \tag{7}$$

Mining constraints: constraints (8) ensure ore and waste production requirements are respected during each mining period:

$$\underline{T_t} \leq \sum_{c=1}^{M} \sum_{z=1}^{Z} T_c^z \left(x_{c,t}^z - x_{c,t-1}^z \right) \leq \overline{T_t} \qquad t = 1, \ldots, T \tag{8}$$

Constraints (7) and (8) measure deviations in processed ore tonnages, considering upper and lower bounds, $\overline{O_t}$ and $\underline{O_t}$, in order to penalize in the objective function.

$$\sum_{c=1}^{M} \sum_{z=1}^{Z} O_{c,s}^z \left(x_{c,t}^z - x_{c,t-1}^z \right) - d_{t,s}^+ \leq \overline{O_t} \qquad s = 1, \ldots, S; \quad t = 1, \ldots, T \tag{9}$$

$$\sum_{c=1}^{M} \sum_{z=1}^{Z} O_{c,s}^z \left(x_{c,t}^z - x_{c,t-1}^z \right) + d_{t,s}^- \geq \underline{O_t} \qquad s = 1, \ldots, S; \quad t = 1, \ldots, T \tag{10}$$

The variables involved in the formulation are defined as the following. Note that elevations are linear and can be initially constrained by the limits of the orebody models:

$$e_{c,t} \in \Re \qquad c = 1, \ldots, M; \quad t = 1, \ldots, T \tag{11}$$

$$x_{c,t}^z \in \{0,1\} \qquad c = 1, \ldots, M; \quad t = 1, \ldots, T; \quad z = 1, \ldots, Z \tag{12}$$

$$d_{t,s}^-, d_{t,s}^+ \in \Re^+ \qquad s = 1, \ldots, S; \quad t = 1, \ldots, T \tag{13}$$

2.3 Sequential implementation

It was shown in Marinho and Dimitrakopoulos (2012) that solving a similar mathematical formulation based on surfaces using an exact method is impractical. The sequential implementation proposed therein is also considered herein, replacing the mathematical formulation by the one presented in Section 2.2. The approach is used first to discretize the pit space into phases of similar ore tonnages, where the formulation forces the scheduling of more profitable materials to initial stages of development. These defined phases are used later in order to reduce the complexity of the yearly-based production scheduling, which is optimized using the same approach, but following the pre-defined phases. The sequential implementation works as follows, where periods can be considered as years or phases. The case study to be presented in Section 3.2 solved more than 100 smaller optimization processes before finding the final solution, therefore steps are explained here in general terms and schematic illustrations can be found in Marinho and Dimitrakopoulos (2012).

Periods are included in the process in a stepwise fashion. A new period $t+1$ is added to the optimization process only after the method returns its best solution for a mining schedule of t periods. The proposed formulation is executed successive times and results for one optimization process are used as limiting assumptions for subsequent processes. The steps of the sequential implementation are:

1. Take the actual topography of the deposit as a top limiting surface; no other surface can go above this limit.
2. Similarly, define a bottom limit by eliminating external waste volumes, according to slope angles, but guaranteeing that every block with some probability of being ore is above this limit. No surface in any period can go deeper than such limits and blocks below are not considered in optimization processes.
3. If no phases are provided, this step should be skipped; otherwise, each period must be associated with one phase beforehand. Each surface cannot go deeper than its phase limits and blocks below are not considered in optimization processes for this period.
4. Find an initial solution for period 1 with successive runs of the mathematical formulation proposed in Section 2.2, given pertinent operational considerations, i.e., fractional periods, bench and maximum bench limits.
5. Iteratively improve the initial solution, using the same formulation in a Local Search approach with neighborhood definition.
6. Find an initial solution for period 2, as in Step 4. The results of Steps 5 and 6 will give a feasible schedule of 2 mining periods.
7. Improve this schedule considering the same Local Search approach from Step 5 but for all mining periods jointly.
8. Loop over Steps 6 and 7, including new periods, until there is no more profitable material available.
9. Freeze all periods except for the last and look for extra deeper blocks to be included, still respecting the limits defined in Steps 2 and 3.

In order to use this approach for pit discretization, targets and limits required for each phase should be defined, including a rescaled economical discount rate per phase. The approach is then performed including phases until there is no more profitably mineable ore available. Finally, Step 9 of the implementation is performed, guaranteeing all profitable material is included. The schedule of phases is performed more efficiently, due to the reduced number of phases, if compared to the number of mining periods. The economical discounting per phase forces the method to mine the best material available under physical constraints in the

first phase, and the same occurs sequentially for the next phases. The resulting surfaces for this schedule of phases represent an optimized discretization of the pit space into pieces with controlled conditions in terms of ore tonnages. These surfaces are later used to limit the complexity of the yearly-based mine production schedule optimization. Each mining period is associated to one phase; for example, if two phases were defined with ~100Mt of ore production and the schedule has to provide mining periods with ~40Mt of ore, Periods 1 and 2 can be assigned to Phase 1 and the remaining periods to Phase 2. The scheduling method limits the bottom of each mining period surface (Step 3), eliminating blocks below the assigned phase prior to the optimization processes, which improves efficiency and allows for the scheduling of larger deposits with longer life-of-mine.

3   Case study at a gold deposit

The conventional approach and the proposed method are applied to a gold orebody, represented by 98,081 mining blocks of $20{\times}20{\times}20\text{m}^3$ size, to demonstrate the performance of the proposed formulation and its sequential implementation in comparison to industry best practices. Table 1 shows the parameters considered for the yearly-based schedule, which were taken as approximated values from actual large gold projects for testing purposes that illustrate the method.

Table 1: Economic and technical parameters for the case study

Gold price, $/g	40.0
Mining cost, $/t	6.0
Processing cost, $/t	20.0
Recovery, %	90.0
Discounting rate per period, %	10.0
Expected ore production, Mt/year	15.0
Slope angle, degrees	54.0

3.1   Conventional mine production scheduling

The Whittle Software was used for mine production scheduling according to industry best practices based on a single estimated model (deterministic approach). In order to assess the geological uncertainty involved, this schedule was evaluated for each simulated orebody model considered, generating the risk profiles presented over Figures 1 to 3.

All profiles illustrate that the answers provided by the deterministic approach are not likely to be achieved in practice when evaluated over equally probable scenarios. Figures 1 and 2 show that a representative amount of ore is being misclassified as waste, with consequent overestimation of the fleet required to handle inexistent extra quantities of waste. This schedule, if performed in practice, would require a stockpile with capacity above 15Mt to handle the ore not forecasted by the plan that has to be reclaimed in one extra year of production. Figure 3 shows the respective metal content available in each scheduled period with the risk profile also above deterministic expectations. Net present value profiles are not comparable as the provided schedule is not realistic in terms of processing capacities.

3.2   Stochastic pit discretization

The method proposed in Section 2 has no requirements in terms of time frame for each period being scheduled; hence, it can be used to schedule weeks or decades of production and conclusions will be valid for the time frame chosen and the economic discounting rate considered. As the time frame increases, more material is smoothed out inside periods and the economic discounting rate should be rescaled. For the same reason that yearly-based schedules are considered as a fair pit discretization to allow for later scheduling of months of production, the scheduling of phases is a fair discretization of the pit space to constrain a yearly-based schedule. The advantage of this kind of pit discretization is that geological uncertainty is taken into account while controlling ore tonnages per phase, which are more suitable limiting assumptions for a stochastic mine
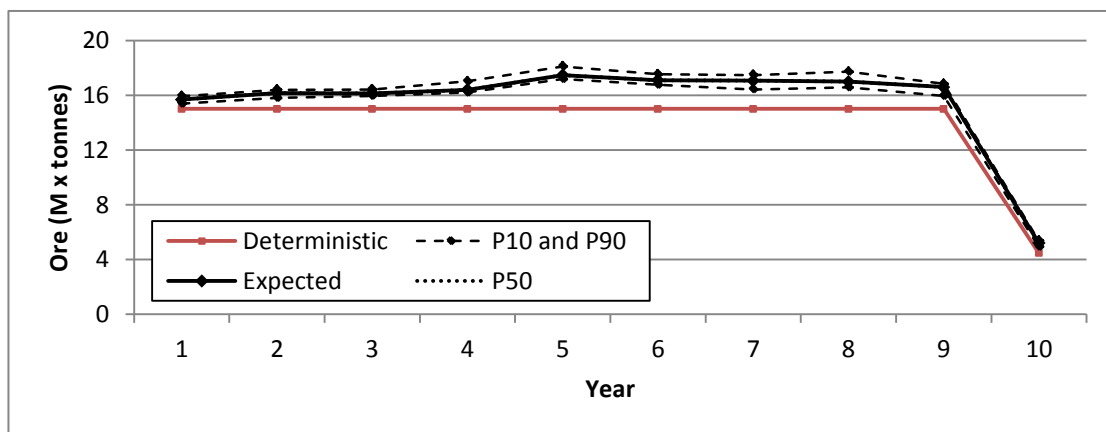
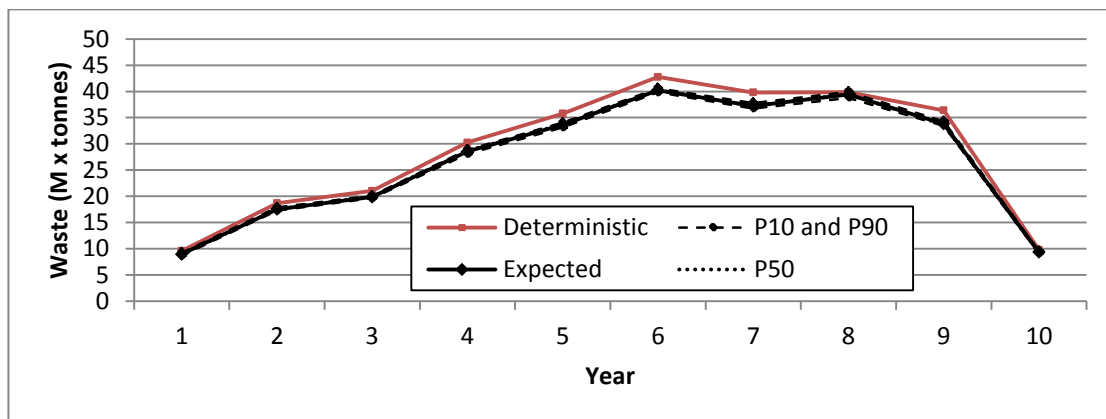Figure 1: Risk profile for ore production in the conventional schedule

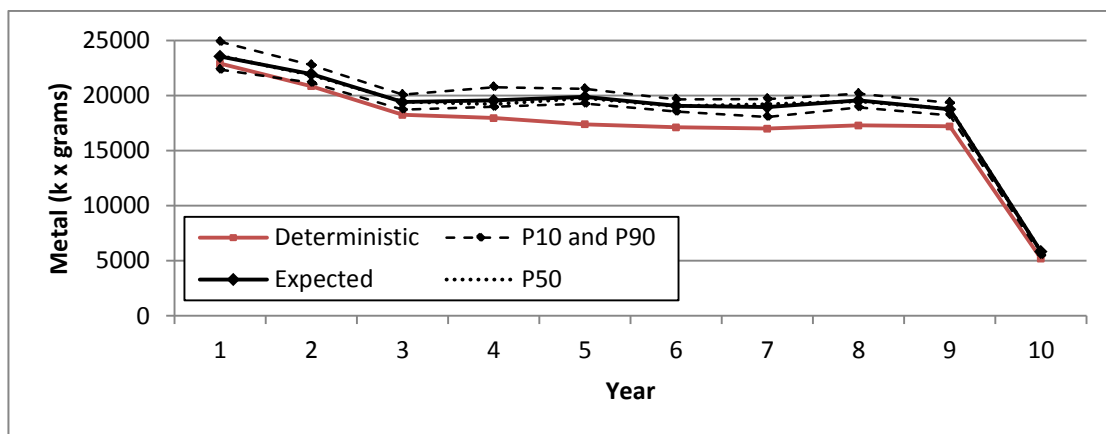Figure 2: Risk profile for waste production in the conventional schedule

Figure 3: Risk profile for metal production in the conventional schedule

scheduler. Note that the concept here is different than the conventional definition of pushbacks where nested pits are defined by maximizing the undiscounted cash flow for a set of increasing metal prices. The effect of economic discounting considered here forces blocks with higher value to be mined as soon as possible with opposite effect for blocks with lower values; working with undiscounted values is just a shortcut for methods that are incapable to work within time frames.

The sequential algorithm was first applied to discretize the pit space into phases with similar ore production, but there is no operational parameter being controlled, such as minimum mining width, for example. The target ore production was setup to 50Mt in order to allow for 3 years of schedule (15Mt each) inside the first phase, leaving 10% (or 5Mt) of flexibility for risk management. Figures 4 and 5 show sections of the resulting 4 phases, presenting shapes of increasing size with more waste being mined in later phases.
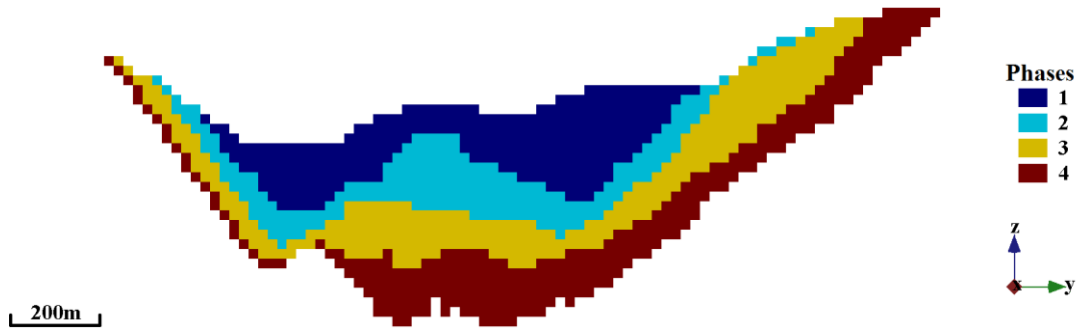

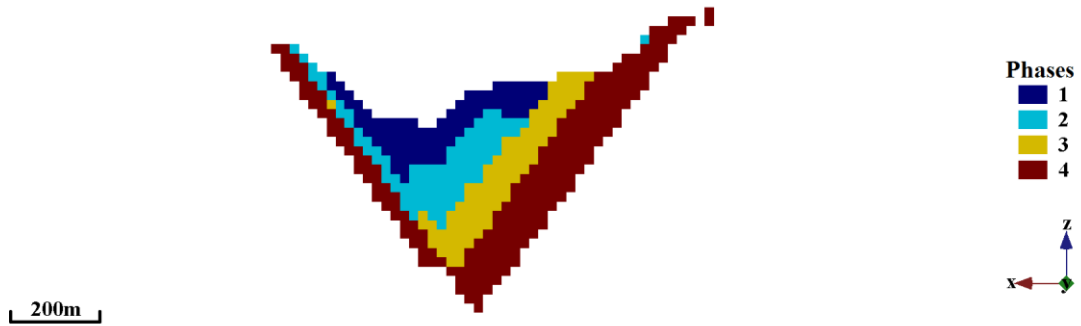Figure 4: North-South vertical section E1030 for the stochastic mining phases

Figure 5: East-West vertical section N2250 for the stochastic mining phases

The associated risk profiles over cumulative NPV, ore, waste and metal productions, along with the corresponding deciles P10, P50 and P90, are shown over Figures 6 and 7. The ore production profile shows higher production over the last phase, as the material of Phase 5 (~10Mt of ore) was incorporated into Phase 4. The waste production profile shows increasing behavior throughout phases. Metal production is always between 55 and 70 tonnes per phase and all production profiles demonstrate low uncertainty around expected values. The profile over cumulative NPV shows increasing behavior until the last phase and a small increment given by Phase 4; this profile cannot be evaluated in terms of absolute values, as the discounting rate applied is a long term approximation.

The amount of ore contained up to Phase 4 can be schedule at most in 14 periods. Hence, mining periods 1 to 3 are assigned to Phase 1; 4 to 6 to Phase 2; 7 to 9 to Phase 3; and the remaining to Phase 4. The surface of each period cannot cross below the surface of its assigned phase during the mine production schedule optimization.
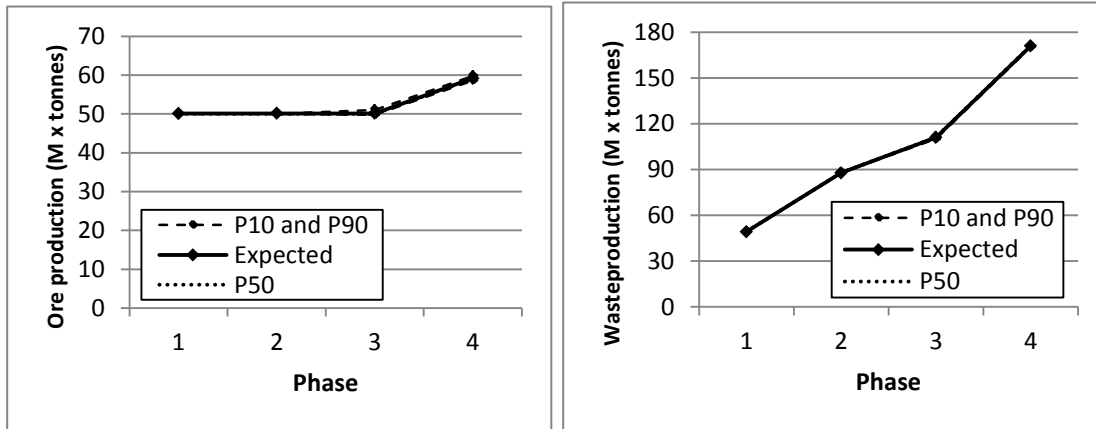
Figure 6: Risk profiles for ore and waste production for the stochastic mining phases
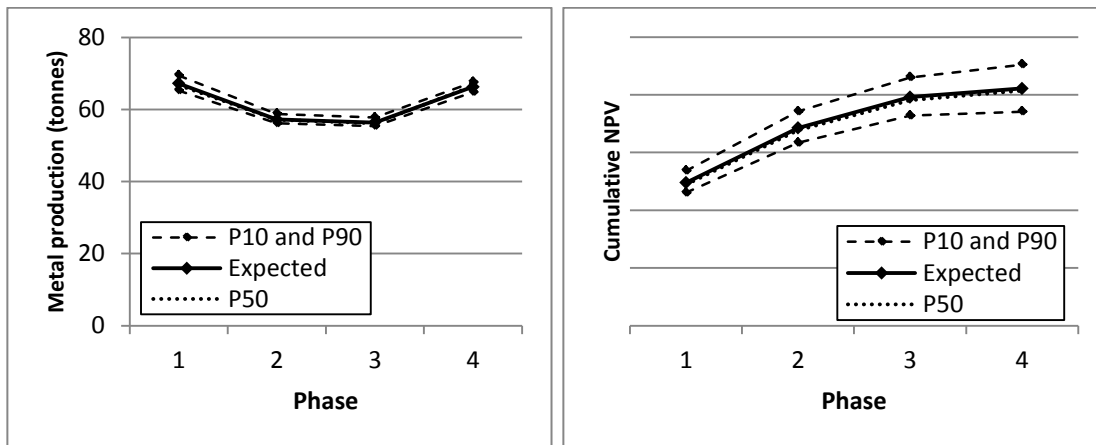

Figure 7: Risk profiles for metal and cum. NPV for the stochastic mining phases

3.3   Stochastic mine production scheduling

The resulting physical schedule is presented over Figures 8 and 9 with mining periods respecting phases previously defined. Note that results are not expected to be smoothed out as operational constraints are not taken into account in the mathematical formulation.

The associated risk profiles over cumulative NPV, ore, waste and metal productions, along with the corresponding deciles P10, P50 and P90, are shown over Figures 10 to 13, compared with the results obtained by running Whittle Software using the e-type model (deterministic). When considering the stochastic optimizer, four extra mining periods are scheduled. Figure 10 shows production controlled close to the target, except by a 10% shortfall in Year 9, but stable production over the remaining years. The shortfall represents a situation where it was worthy to pay for the incurred penalties instead of mining more ore to complete the capacity, probably due to the amount of overlaying waste. 43% higher production of ore for the stochastic solution is expected. The profile shows lower risk in initial periods if compared to later periods, as proposed by the formulation. Figure 11 shows more waste being produced by the deterministic solution, which is due to the ore misclassified as waste by the estimation method. The profile has increasing behavior, similarly to predefined phases, and almost no risk throughout the whole life-of-mine. Figure 12 shows similar expectation in terms of metal production for both cases up to Year 9 with higher metal production during the first two
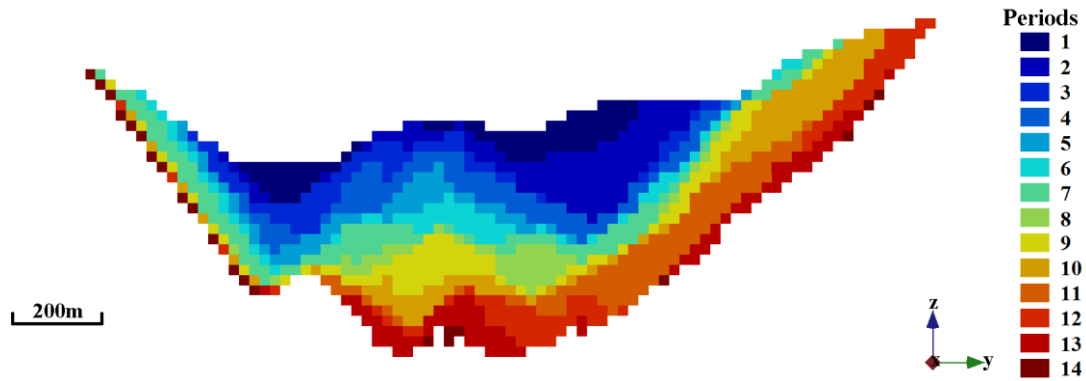
Figure 8: North-South vertical section E1030 for the stochastic yearly-based schedule
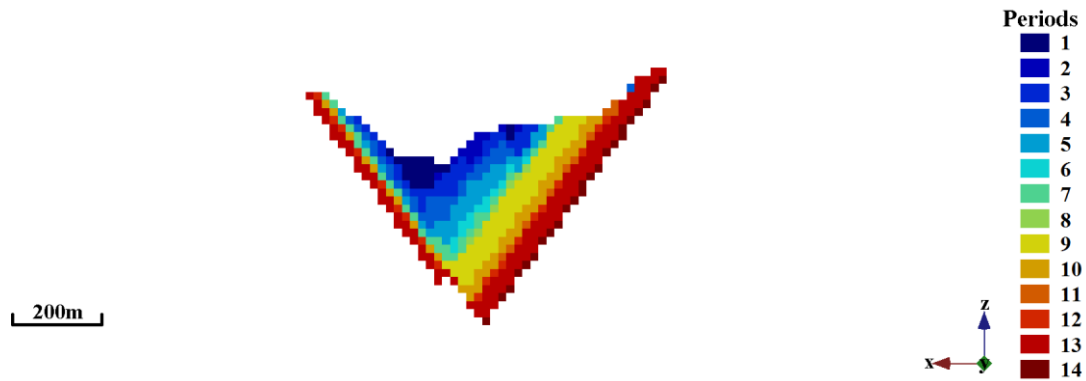

Figure 9: East-West vertical section N2250 for the stochastic yearly-based schedule

years, stabilizing after that. Section 3.1 has shown that the deterministic expectations are misleading. There is a low risk associated and it does not vary throughout the life-of-mine. Figure 13 presents the risk profile for the cumulative NPV with a total expected value of 1.35 billion dollars with 90% chances of being above 1.18 billion dollars. There is less than a 3% increment in expected NPV after period 10 for the stochastic case. Figure 13 also present similar expected NPV for both approaches until Year 3 and an overall 19% higher expected NPV for the stochastic solution.

The extra four years of production added (+43% of ore) are justified by misclassification errors and extended ultimate pit limits given by the stochastic approach, as shown in Figures 14 and 15. Stochastic approaches always return solutions greater or equal to the deterministic ones in terms of value (Birge and Louveaux, 1997). Notice, in Figure 13, that most of the value added is not related to extra ore included but to a different schedule defined up to Year 10.

4   Conclusions

The present work proposes a new mathematical programming formulation that makes use of limiting surfaces in the context of SIP for mine production scheduling optimization, adding the benefits of easier and general slope angle management with simultaneous maximization of discounted cash flows and minimization of risks of not achieving production targets. The sequential implementation considered divides the scheduling problem into sub-problems, using surfaces based on relevant engineering aspects of multiple-period scheduling. Periods are included in a stepwise fashion by first defining one initial solution and later improving this solution using a Local Search strategy based on the same mathematical formulation. The approach is first applied to discretize
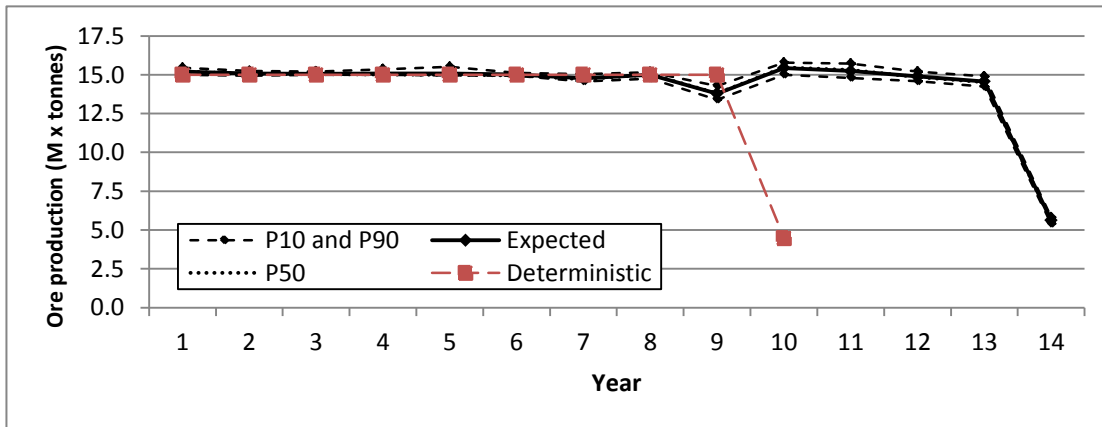
Figure 10: Risk profile for ore production for the stochastic yearly-based schedule
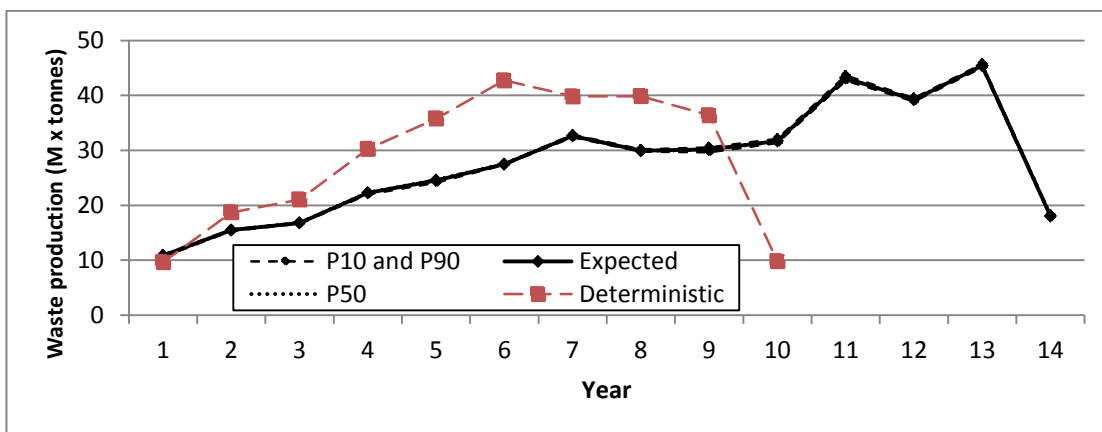

Figure 11: Risk profile for waste production for the stochastic yearly-based schedule
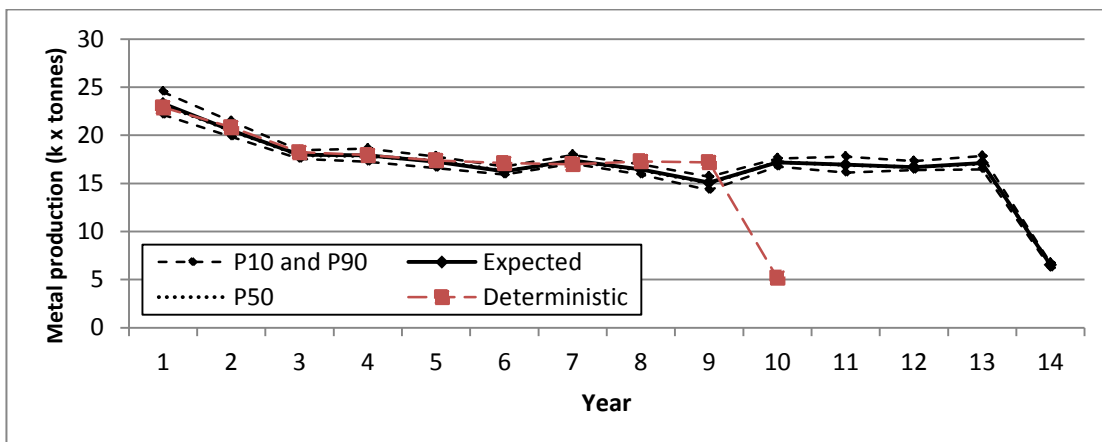

Figure 12: Risk profile for metal production for the stochastic yearly-based schedule
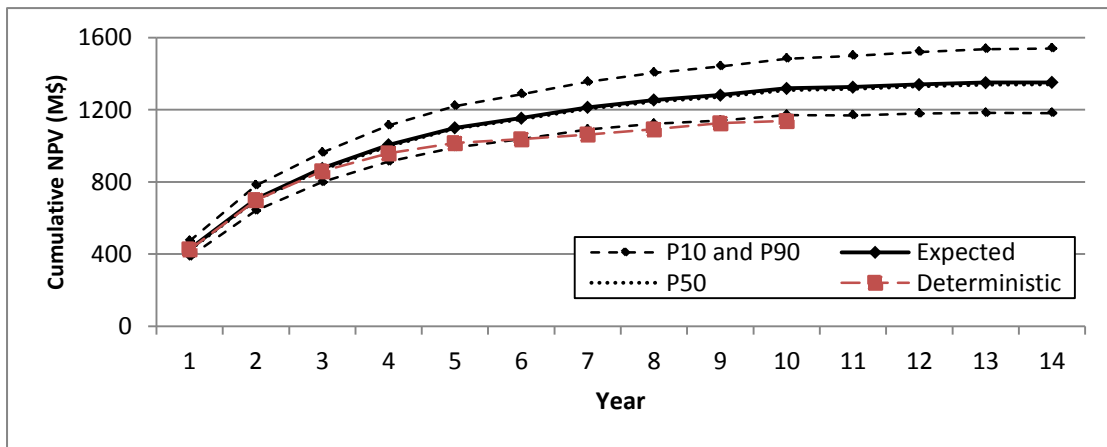
Figure 13: Risk profile for cumulative NPV for the stochastic yearly-based schedule
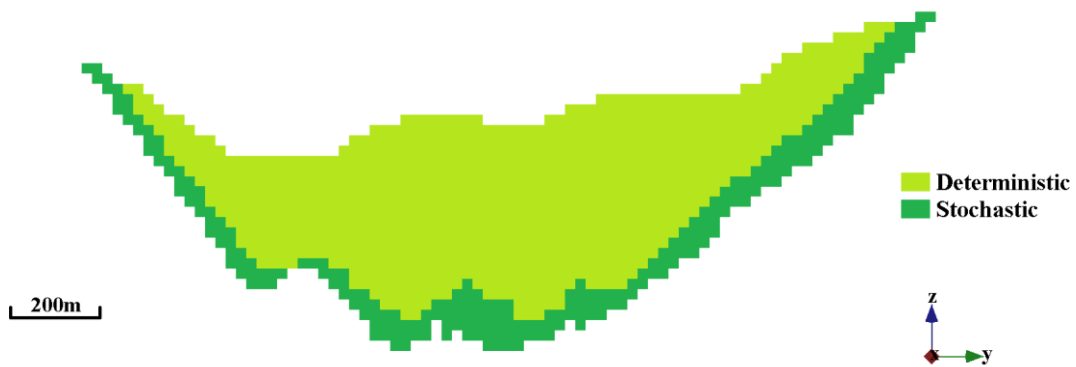

Figure 14: North-South vertical section E1030 for deterministic x stochastic pit limits
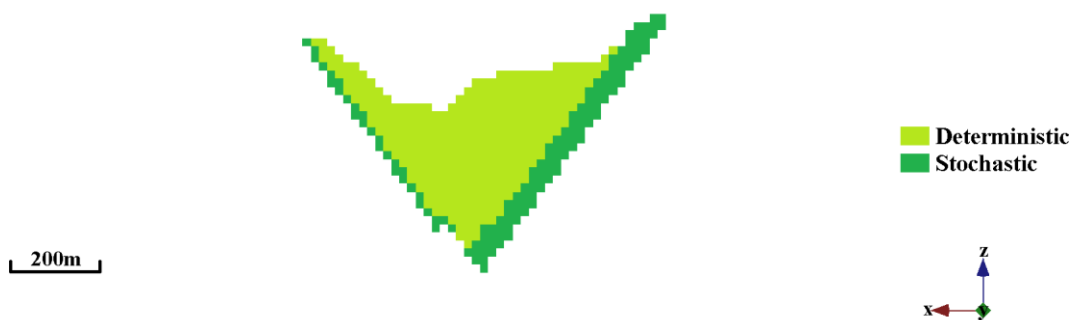

Figure 15: East-West vertical section N2250 for deterministic x stochastic pit limits

the pit space into mining phases, which are then considered to reduce the complexity of the yearly-based mine production schedule optimization. A case study over a full field gold deposit addresses the question of size and returns 19% higher expected NPV with 43% more ore processed, if compared to industry best practices.

References

Albor F, Dimitrakopoulos R (2009) Stochastic mine design optimization based on simulated annealing: Pit limits, production schedules, multiple orebody scenarios and sensitivity analysis. IMM Transactions, Mining Technology, 118(2), 80–91.

Albor F, Dimitrakopoulos R (2010) Algorithmic approach to pushback design based on stochastic programming: method, application and comparisons. IMM Transactions, Mining Technology, 119(2), 88–101.

Asad MWA, Dimitrakopoulos, R (2013) Implementing a parametric maximum flow algorithm for optimal open pit mine design under uncertain supply and demand. Journal of the Operational Research Society, 64, 185–197.

Birge JR, Louveaux, F (1997) Introduction to Stochastic Programming, Springer Verlag.

Boland N, Dumitrescu I, Froyland G (2008) A multistage stochastic programming approach to open pitmine production scheduling with uncertain geology. Optimization Online.

Dimitrakopoulos R, Farrelly C, Godoy MC (2002) Moving forward from traditional optimisation: Grade uncertainty and risk effects in open pit mine design. Transactions of the IMM, Section A Mining Industry, 111, A82–A89.

Dimitrakopoulos R, Ramazan S (2004) Uncertainty based production scheduling in open pit mining. SME Transactions, 316, 106–112.

Dimitrakopoulos R (2011) Stochastic optimization for strategic mine planning: A decade of developments. J Mining Science, 84(2), 138–150.

Dowd PA (1997) Risk assessment in reserve estimation and open-pit planning. Transactions of the Institution of Mining and Metallurgy, 103, A148–A154.

Godoy M (2003) The efficient management of geological risk inlong-term production scheduling of open pit mines, PhD thesis, University of Queensland, Brisbane.

Godoy M, Dimitrakopoulos R (2004) Managing risk and waste mining in long-term production scheduling. SME Transactions, 316, 43–50.

Goodfellow R, Dimitrakopoulos R (2012) Algorithmic integration of geological uncertainty in pushback designs for complex multiprocess open pit mines. Submitted to IMM Transactions, Mining Technology.

Goodwin GC, Seron MM, Middleton RH et al. (2005) Receding horizon control applied to optimal mine planning. Automatica, 42(8), 1337–1342.

Hustrulid W, Kuchta M (2006) Open pit mine planning and design. AA Balkema, Rotterdam, 636 p.

Jewbali A (2006) Modeling Geological Uncertainty for Stochastic Short-Term Production Scheduling in Open Pit Metal Mines. PhD Thesis, School of Engineering, University of Queensland.

Lamghari A, Dimitrakopoulos R (2012) A diversified tabu search approach for the open-pit mine production scheduling problem with metal uncertainty. European Journal of Operational Research, 222, 642–652.

Leite A, Dimitrakopoulos R (2007) A stochastic optimization model for open pit mine planning. Application and risk analysis at a copper deposit. IMM Transactions, Mining Technology, 116(3), 109–118.

Marinho A, Dimitrakopoulos R (2012) Surface constrained mine production scheduling with uncertain metal available. To be submitted to Optimization and Engineering.

Menabde M, Froyland G, Stone P, Yeates G (2007) Mining schedule optimisation for conditionally simulated orebodies. The Australasian Institute of Mining and Metallurgy, Spectrum Series, 14.

Montiel L, Dimitrakopoulos R (2012) An Extended Stochastc Optimization Method for Multi-Process Mining Complexes. COSMO Research Report No. 6.

Ramazan S, Dimitrakopoulos R (2005) Stochastic optimisation of long-term production scheduling for open pit mines with a new integer programming formulation. In: Orebody modelling and strategic mine planning. Spectrum Series, vol. 14, 353–360. The Australasian Institute of Mining and Metallurgy (AusIMM), Victoria.

Ramazan S, Dimitrakopoulos R (2012) Production scheduling with uncertain supply: a new solution to the open pit mining problem. Optimization and Engineering, 13, 1–20.