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F. Gilbert, M.F. Anjos,
P. Marcotte, G. Savard

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GERAD HEC Montréal
3000, chemin de la Côte-Sainte-Catherine
Montréal (Québec) Canada H3T 2A7

Tél. : 514 340-6053
Téléc. : 514 340-5665
info@gerad.ca
www.gerad.ca

Bilateral contract optimization in power markets

François Gilbert[a]
Miguel F. Anjos[a]
Patrice Marcotte[b]
Gilles Savard[a]

[a] *GERAD & Département de mathématiques et de génie industriel, Polytechnique Montréal, Montréal (Québec) Canada H3C 3A7*

[b] *Département d'informatique et de recherche opérationnelle, Université de Montréal, Montréal (Québec) Canada H3C 3J7*

francois.gilbert.fg@gmail.com
miguel-f.anjos@polymtl.ca
marcotte@iro.umontreal.ca
gilles.savard@polymtl.ca

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Abstract: In this paper, we propose a model for an energy broker who acts as a third party between the grid and its clients, through the maintenance of a two-sided portfolio of bilateral contracts. These contracts are positioned for roles involving a finite number of pointwise interventions within specified availability periods, and are well suited for clients such as distributed generators or ancillary services, on the grid side. The setting is otherwise quite general and may for instance involve bilateral contracts on wholesale markets. While bilateral contracting hedges the parties involved against the volatility of energy prices, the management of the broker portfolio raises a number of modelling and computational issues, which stem from the aggregation of disparate resources. To address these challenges, we devise an innovative algorithmic framework that involves robust optimization with respect to short term decisions, factoring in long term information obtained from a secondary model that embeds the full extent of all contracts' durations.

Key Words: distributed energy resource, co-generation, bilateral contract, energy market, dynamic resource allocation.

1 Introduction

Due to opportunities related to restructured energy networks, recent years have witnessed the arrival of new players to the field: aggregators [1], microgrids [2], virtual power plants [3] and various coalitions [4–8]. The aim of the present work is to analyze the operations of an *energy broker*, who acts as a third party between the grid and distributed energy resources, and provides offers to the grid via aggregation. We are in particular concerned with combined heat and energy units [9–11]), fuel cells [12], resources aggregated in virtual power plant or micro grids, and which may involve renewable resources [13, 14] or responsive loads [15, 16].

The broker is endowed with a two-sided portfolio of internal and external bilateral contracts with its clients and the grid, respectively. Internal bilateral contracting serves as a flexible tool to adjust any on client participation to the broker's additional activities on the grid, so that they be tailored to the client's financial goals and specific processes. External operations may be positioned on a multiplicity of levels, such as ancillary services [17, 18], wholesale market [3, 14] or load curtailment [19].

The bilateral contracts are designed for the pointwise involvement of the broker clients in the delivery of resources to the grid. At the beginning of each week, the broker broadcasts a schedule that assigns resources to grid contracts on weekly times slots, where mobilized resources remain available for grid requests. Each contract is endowed with a fixed cost for the total amount of power to be delivered, on multiple occasions and in response to grid requests, and with variable mobilization costs, which are paid on a weekly basis.

The current study focus on medium term strategic planning. The broker portfolio is fixed, and we assume that all fixed costs have been paid in advance. It then remains for the broker to allocate his resources so to complete all grid contracts without reaching outside the contracted resources, while minimizing mobilization costs. The optimization tools we devise in the sequel provide efficient mobilization policies and yields incentive for contract construction.

Notwithstanding the fact that the settlement of bilateral contracts can be tightly coupled to the production of bid on an energy market, we do not consider bid production, since the topic has been well documented in a number of recent studies [20–25]. Of course, pool operations should intervene at some level of the broker's operations, but the challenge raised by the management of the two-sided portfolio needs to be addressed separately, prior to considering pool activities.

The aim of this paper is to propose a model for an energy broker, and to devise algorithmic schemes for its solution. The broker operations are set within a stochastic and dynamic environment, over a time horizon that spans several months, for typical bilateral contracts. Moreover, discrete decisions are involved, and a multi stage stochastic mixed integer formulation yields an optimization problem that state of the art computational methods could only tackle for a toy portfolio [26, 27]. Alternatively, addressing the curse of dimensionality directly though statistical learning schemes [28, 29] is problematic on account of the involved combinatorics. In this respect, our contribution is twofold. First, we position an energy broker attached to the smart grid which manages a two-sided portfolio of forward contracts, respectively with its clients and the pool. Two achieve this, we devised a two-frame model that involves a robust mixed integer short term formulation, covering the broker against any weekly demand scenarios, and a long term formulation that captures the full extent of all contracts, based on the notion of availability configurations. On the algorithmic side, this naturally leads to an optimization framework where a long term model passes information to a short term model.

This paper is organized as follows. The problem's formulation is presented in Section 2, the algorithmic framework is developed in Section 3, and a numerical experiment is documented in Section 4. The conclusion is then followed by the list of notation. Proofs of the various theoretical results is deferred to the appendix.

2 Formulation

Let us consider the activities of an energy broker managing a two-sided portfolio of bilateral contracts with distributed generators and the grid. A grid contract is set for an amount of power to be delivered to the grid, upon request by the grid, within broker specific time frames. Let D be the set of grid contracts, referred to as *demands*. Similarly, a generator contract is set for an amount of power to be produced by the generator, upon request by the broker, within generator specific time frames. Let R be the set of generator contracts, referred to as *resources* in the sequel. Each contract is associated with a fixed number of requests (*tokens*) r_i^{res} and r_j^{res}, where $(i, j) \in R \times D$. Generator contracts also allow for a predefined number of maintenance periods $(m_i)_{i \in R}$, during which the generator is not required to respond to requests.

Note that the heterogenous nature of the broker aggregated resources, whose underlying processes can involve hard operational constraints (ramping delay, successive requests delay) restricts the compatibility of resources and demands. The subset of resources eligible to participate in responding to a request from demand j is expressed $R_j \subset R$. Similarly, the subset of demands for which Resource i is compatible is expressed $D_i \subset D$.

In the formulation, the time horizon is partitioned into a finite set of weeks T. In turn, each week $t \in T$ is partitioned into a finite set of time slots $S(t)$ of possibly uneven durations. Let $R^{ts} \subset R$ and $D^{ts} \subset D$ denote the subsets of contracts available and eligible for receiving or sending requests, respectively, during time slot $s \in S(t)$. A contract with no remaining tokens is unavailable for the rest of the time horizon. Contract eligibility is encapsulated in notation $DR^{ts} \subset R^{ts} \times D^{ts}$, representing the sets of compatible resource-demand couples that are available on time slot s. We denote by R_j^{ts} (resp. D_i^{ts}) the set of available resources (resp. demands) compatible with demand j (resp. resource i) on time slot s. The notations r_{tj}^{dem} and r_{ti}^{res} refer to the number of request tokens for demand j and resource i (respectively) at the beginning of week t.

Energy delivery must be monitored. At the beginning of each week, before any demand request has been received, the broker broadcasts to all parties involved a mobilization schedule that assigns available and compatible resources to available demands, for each weekly time slot. Mobilized resources can then be used to respond to requests at the current week. We assume the mobilization schedule is fixed for the entire week. Let $(x_{ij}^{ts})_{(i,j) \in DR^{ts}, s \in S(t)}$ be the mobilization policy for week t, where x_{ij}^{ts} is set to 1 if resources i is assigned to demand j on time slot $s \in S(t)$. In the short term, the mobilization policy must ensure that sufficient power is gathered for each demand, that is, $x^{ts} \in X_{\text{pow}}(R^{ts}, D^{ts})$ where

$$X_{\text{pow}}(R, D) = \left\{ x : \begin{array}{lr} \sum_{j\,:\,(i,j) \in DR} x_{ij} \leq 1 & i \in R \\ \sum_{i\,:\,(i,j) \in DR} \text{power}_i \, x_{ij} \approx \text{power}_j & j \in D \end{array} \right\}. \tag{1}$$

These constraints are separable by time slot. The first constraint ensures that each resource is assigned to at most one demand on any given time slot. In the second constraint, parameter $\text{power}_j \in \mathbb{R} \times \mathbb{R}$ is the interval of valid power levels (for demand j). The symbol $\approx$ specifies that demand should be approximately satisfied, i.e., up to a prespecified tolerance.

Resource contracts include weekly availability throughout the contract validity period, as well as mobilization costs. Let c_i^{mob} be the marginal cost of mobilizing resource i, independently of the fact that a request is actually received. This cost may vary among resources having similar parameters but distinct numbers of maintenance weeks. We do also assume that all contracts involve some fixed cost paid (resp. received) in advance, for the total amount of power to be delivered (resp. received). But since these costs are constant, we are only concerned by the variable costs incurred by the broker in the covering of the time horizon:

$$\text{cost}(x) \;=\; \sum_{t \in T} \sum_{s \in S(t)} \sum_{(i,j) \in DR^{ts}} \lambda^{ts} c_i^{\text{mob}} \, x_{ij}^{ts}, \tag{2}$$

where λ^{ts} is the duration of the time slot $s \in S(t)$.

Stochasticity impacts the broker's operations at two levels: demand requests and resource maintenance. We assume that maintenances are announced with at least a week's notice, allowing us to account for them directly when specifying the sets R^{ts} and DR^{ts}, which are themselves random. Let y be a demand scenario, such that y_j^{ts} is set to 1 if a request from demand j is received on time slot s of week t. We assume that the width of time slots is sufficiently small, so that the probability of more than one request within a single time slot is negligible. Independent of any particular assumption on the demands distribution, we must have $y \in Y$, where

$$Y \subset \left\{ y_j^{ts} \in \{0,1\}, t \in T, s \in S(t), j \in D^{ts} : \sum_{t \in T} \sum_{s \in S(t)} y_{tsj} \leq r_j^{\text{dem}} \right\},$$

and a sequence of mobilizations $(x^t)_{t \in T}$ that covers a scenario $y \in Y$ is such that

$$\sum_{t \in T} \sum_{s \in S(t)} \sum_{(i,j) \in DRts} x_{ij}^{ts} y_j^{ts} \leq r_i^{\text{res}}, \quad i \in R.$$

The mobilization x^t chosen at the beginning of week t is a function of all past information $(y^{t'})_{t'<t}$ and $(R^{t'})_{t' \leq t}$, and we assume that it covers the broker against any demand scenario at week t:

$$x^t \left((y^{t'})_{t'<t}, (R^{t'})_{t' \leq t} \right) \in X_{\text{week}}^t \left((y^{t'})_{t'<t}, (R^{t'})_{t' \leq t} \right)$$

where

$$X_{\text{week}}^t \left((y^{t'})_{t'<t}, (R^{t'})_{t' \leq t} \right) = \{ x^{ts} \in X_{\text{pow}}(R^{ts}, D^{ts}), s \in S(t') : \sum_{t' \leq t} \sum_{s \in S(t')} \sum_{(i,j) \in DR_{t's}} x_{ij}^{t's} y_j^{t's} \leq r_i^{\text{res}}, \quad y \in Y, \, i \in R \}. \tag{3}$$

In the sequel, parameters $(y^{t'})_{t'<t}$ and $(R^{t'})_{t' \leq t}$ will be dropped whenever the context is clear, and we simply write: $x^t \in X_{\text{week}}^t$.

At a high level, the broker problem is expressed as the multi stage mixed integer stochastic program:

Program 1

$$\begin{aligned} \min_x \quad & \mathbb{E}\left[\text{cost}(x)\right] \\ \text{s.t.} \quad & x^t \in X_{\text{week}}^t \quad t \in T. \end{aligned}$$

Implicit in the above formulation is the assumption that the broker resources are *a priori* sufficient to cover any demand scenario. The issue is how to achieve this at minimal cost, and using a minimal amount or resources. Throughout this process, we also assume that the broker is honest, in the sense that he will not mobilize resources that have insufficient request tokens to go through the week, along the chosen mobilization schedule. Accordingly, unsatisfied requests, and more generally uncovered time slots, never occur. Obviously, in practice, failure to cover a time slot will yield severe penalties to the broker. Next, considering the risk averse environment within which the broker operates, together with the fact that failures not only impact the broker's welfare in the short term, but perhaps more importantly impact negatively his ability to secure future contracts with the grid. Uncovered time slots are to be avoided at all cost, and are simply not modeled here.

As an illustration, consider the small Portfolio 1 described in Table 1, where time slots are set to days for simplicity. Any one of the two resources $R1$ and $R2$ can alone cover requests from the only demand $D1$.

Table 1: Portfolio 1: Contracts parameters.

Dem/Res	req	pow	cost	t_1	t_N	Mon	Tue	Wed	Thu	Fri	Sat	Sun
D1	12	1	n/a	1	15	x	x	x	x	x	x	
R1	10	1	2	1	15	x	x	x	x	x	x	
R2	5	1	5	1	15	x	x					

Resource 1 is only available 2 days a week, while Resource 2 is available 6 days a week. Observe that while the combined resources allow for a total of 15 requests, only 10 requests can be satisfied between Thursday and Saturday. Table 2 provides the data of a demand scenario history and two associated sequences of mobilization policies leading to either the failure to cover all time slots (top), or a coverage of the entire time horizon (bottom). Entries in each column correspond to a (weekly) mobilization policy (set at the beginning of the week) where either resource $R1$ or $R2$ is assigned to cover demand $D1$. Circles correspond to requests, and uncovered time slots are labeled with symbol '*'. Mobilization costs are displayed at the bottom.

Table 2: Portfolio 1: Two simulation runs.

$s \backslash t$	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Mon	1	1	1	①	1	1	1	1	2	2	2	2	2	2	2
Tue	1	1	1	1	1	①	2	2	2	②	2	2	2	2	2
Wed	1	1	①	1	1	1	1	①	①	1	1	*	*	*	*
Thu	1	1	1	1	①	1	1	1	1	1	①	*	(*)	*	*
Fri	1	1	1	1	1	1	1	1	1	1	①	*	*	*	*
Sat	1	1	1	1	1	①	1	1	1	1	①	*	*	*	*
Sun															
Cost	12	12	12	12	12	12	15	15	18	18	18	∞	∞	∞	∞

s$\backslash t$	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Mon	2	2	2	②	2	2	2	2	2	2	2	2	2	2	2
Tue	2	2	2	2	2	②	2	2	2	②	2	2	2	2	2
Wed	1	1	①	1	1	1	1	①	①	1	1	1	1	1	1
Thu	1	1	1	1	①	1	1	1	1	1	①	1	①	1	1
Fri	1	1	1	1	1	1	1	1	1	1	①	1	1	1	1
Sat	1	1	1	1	1	①	1	1	1	1	①	1	1	1	1
Sun															
Cost	18	18	18	18	18	18	18	18	18	18	18	18	18	18	18

Straightforward combinatorics yields that at least 4 percent of all possible request scenario cannot be covered:

$$\frac{\binom{60}{11}\binom{30}{1}+\binom{60}{12}}{\binom{90}{12}} \approx 0.04. \tag{4}$$

However, this optimistic estimate does not account for the weekly robustness of Program 1. Inspection of Portfolio 1 suggests to favor the use of $R2$ over $R1$ on Monday and Tuesday, as $R1$ must also cover the rest of the week. To simplify the exposition, let us refer to Monday and Tuesday as *day type 1* and Wednesday to Saturday as *day type 2*. Consider the following policy: resource $R2$ is mobilized on days of type 1 until it has a single request token left, and then it is mobilized each week on a single day of type 1, while resource $R1$ covers the rest of the week. If, on a given week, 7 requests have been received on days of type 2, and less then 2 requests on days of type 1, then $R2$ has 3 tokens left to cover 4 time slots (Wednesday to Saturday), on which demand $D1$ can still make more than 3 requests. It follows that the broker can no longer guaranty the feasibility of the portfolio. It is readily seen that this is the only failure under this mobilization policy, and the resulting probability of failure, given equiprobable demand scenarios, is

$$p' + p'' \approx 0.20, \tag{5}$$

where

$$\begin{aligned} p' &= \mathbb{P}\{6 \text{ requests on day type 2 and 0 request on day type 1}\} \approx (2/3)^7 \\ p'' &= \mathbb{P}\{6 \text{ requests on day type 2 and 1 request on day type 1}\} \\ &\approx 7(2/3)^7(1/3). \end{aligned}$$

3 Numerical resolution

Multistage mixed integer stochastic programs are challenging. Due to the scale of the instances we wish to tackle (typical forward contracts span periods between 3 months to a year), exact resolution scheme such as [26,27] must be ruled out, while scenario based heuristics proposed in [30,31] involve but a small number of scenarios, which in our case are difficult to sample. Moreover, taking into account the huge number of states, the set of scenarios required to properly implement non-anticipativity would by far exceed our computational capabilities. It is otherwise difficult to see how approximate dynamic programming schemes such as [28, 29] can properly account for the problem combinatorial features.

The resolution scheme we propose involves a two time scale model in which a simpler long term model passes long term information to a short term model, where a single week of operations is considered. The two formulations are solved in sequence, over a rolling time horizon. The short term formulation covers the broker against any possible weekly demand scenario. The long term formulation is deterministic and covers the broker against expected amounts of requests arriving on so called *time slot types*, independently of the order of their arrival. The long term solution is labelled as *semi online*, and is expressed in terms of the number of times a resource can safely used to respond to a given demand, on a given time slot type, without putting the feasibility of the portfolio at risk.

3.1 Robust short term feasibility

Mobilization schedules must account for every possible weekly demand scenario. This condition is implied by our positioning of an *honest* broker, who will leave no time slot uncovered. For example, a resource assigned to a single demand over a full week should either (i) be assigned to no more time slots than it has request tokens, or (ii) be assigned to a demand that has at most as many request tokens as the resource does. The following result provides necessary and sufficient conditions for the covering of any weekly demand scenario in the general case, where one resource can cover multiple demands on different time slots.

Theorem 1 *A mobilization policy $x^t \in X^t_{\text{week}}$ if and only if $x^{ts} \in X_{\text{pow}}(R^{ts}, D^{ts})$ for each $s \in S(t)$ and*

$$r^{\text{res}}_{ti} \geq \sum_{j\in D_i} \min\left\{ \sum_{s:(i,j)\in DR^{ts}} x^{ts}_{ij}, r^{\text{dem}}_{tj} \right\}, \qquad i \in R. \tag{6}$$

Proof. See Appendix. □

The above result allows the characterization of weekly feasibility through a system of linear inequalities.

Theorem 2 *A mobilization policy $x^t \in X^t_{\text{week}}$ if and only if $x^{ts} \in X_{\text{pow}}(R^{ts}, D^{ts})$ for each $s \in S(t)$ and there exist binary vectors (z_{ij}) and $(\tilde{x}^s_{ij})$ such that*

$$r^{\text{res}}_{i} \geq \sum_{j\in D_i} \left(r^{\text{dem}}_{tj} z_{ij} + \sum_{s:(ij)\in DR^{ts}} x^{ts}_{ij} - \tilde{x}^{ts}_{ij} \right) \qquad i \in R \tag{7}$$

$$\sum_{s:(i,j)\in DR^{ts}} x^{ts}_{ij} - r^{\text{dem}}_{tj} - \mu \leq M z_{ij} \qquad (i,j) \in DR \tag{8}$$

$$\sum_{s:(i,j)\in DR^{ts}} x^{ts}_{ij} - r^{\text{dem}}_{tj} \geq -M(1 - z_{ij}) \qquad (i,j) \in DR \tag{9}$$

$$\tilde{x}^{ts}_{ij} \leq x^{ts}_{ij} \qquad (i,j) \in DR^{ts} \quad s \in S(t) \tag{10}$$

$$\tilde{x}^{ts}_{ij} \leq z_{ij} \qquad (i,j) \in DR^{ts} \quad s \in S(t) \tag{11}$$

$$\tilde{x}^{ts}_{ij} \geq x^{ts}_{ij} + z_{ij} - 1 \qquad (i,j) \in DR^{ts} \quad s \in S(t) \tag{12}$$

from any $0 < \mu < 1$.

Proof. See Appendix. □

Note that, in practice, it is unnecessary to ensure robust feasibility when the remaining number of resources exceeds the number of weekly time slots.

3.2 Rolling Time Horizon heuristic

Let q_{ij}^{ts} be the cost associated with the mobilization of resource i for demand j on time slot type $s \in S(t)$, and consider the single stage optimization problem:

Program 2

$$\begin{aligned} \min_{x^t} \; & \sum_{s\in S(t)} \sum_{(i,j)\in DR(t,s)} q_{ij}^{ts}\, x_{ij}^{ts} \\ \text{s.t} \quad & x^t \in X^t_{\text{week}}. \end{aligned}$$

This *short term model* is solved from one week to the next, over a rolling time horizon, on the basis of simulated demand requests and resource maintenances.

Before introducing the long term model, let us consider the following simple scheme. The *Contract Level* (CL) information is defined as

$$q_{ij}^{ts} = r_{tj}^{\text{dem}} - r_{ti}^{\text{res}} \qquad t \in T,\ s \in S(t),\ (i,j) \in DR^{ts}, \tag{13}$$

and favors the use of resources having an amount of request tokens that exceeds the demand for which they are mobilized, the rational being that an uncovered time slot can only arise if at least one resource runs out of tokens before a demand.

The CL information is evaluated at week $t = 1$ for Portfolio 1 (Tables 1–2) in Table 3, and was actually used in the simulation at the top of Table 2. Over a total of 100 simulations based on uniform demand distribution, sequentially solving Program 2 using the CL information evaluated at the beginning of each week allowed to fully cover 19 of the scenarios, out of the 80 expected to be feasible (see (4)).

Table 3: Portfolio 1: CL information at week $t = 1$.

Resource	Mon-Tue	Wed-Sat
R1	2	2
R2	7	n/a

3.3 Aggregated demand scenario based on availability configurations

Our next approach to obtaining long term information requires to first cast the broker problem into a static framework defined in terms of the total number of requests received on time slot *types*, characterized by available demand-resource configuration pairs $(\hat{D}^k, \hat{R}^k) \subset D \times R$. Similar to the CL scheme, resource maintenances are ignored, but now mobilization costs are factored in.

Let $(\pi_k)_{k\in K}$ be a partition of the time horizon, where class π_k (time slots of type k), are characterized by the set of online and compatible demand-resource pairs $(\hat{D}^k, \hat{R}^k) \subset D \times R$. A uniform distribution of demand yields the expected number of requests $\overline{y}_{kj}$ from contract j on time slots of type k:

$$\overline{y}_{kj} = \begin{cases} r_j^{\text{dem}} |\pi_k| / n_j^{\text{res}} & \text{if } j \in \hat{D}^k \\ 0 & \text{otherwise.} \end{cases} \tag{14}$$

Let $k(t,s) \in K$ be the type of time slot $s \in S(t)$, and define $Y_{\text{uni}} \subset Y$ as the set of *compatible* demand scenarios, i.e., $y \in Y_{\text{uni}}$ if and only if $y \in Y$ and

$$\lfloor \overline{y}_{kj} \rfloor \leq \sum_{t \in T} \sum_{s \in S(t)} \mathbf{1}(k(s,t) = \pi_k) y_j^{ts} \leq \lceil \overline{y}_{kj} \rceil.$$

Note that the uniformity assumption is readily generalized to forecast data, and is otherwise consistent with a Bayesian approach given that no predictive information is available.

The key to the long term model formulation is the expression of the aggregate forecast in terms of the maximum number of requests issued simultaneously from demand subsets. The left-hand side of Figure 1 displays a request scenario's history. It corresponds to requests from demands j_1, j_2 and j_3 occurring on time slots of a given type, say Monday mornings, for weeks $t = 1, \ldots, 15$. The corresponding histogram is shown in the middle. The data on the right-hand side corresponds to the maximum number of simultaneous requests from demand subsets $\{j_1, j_2, j_3\}, \{j_2, j_3\}$ and $\{j_2\}$.

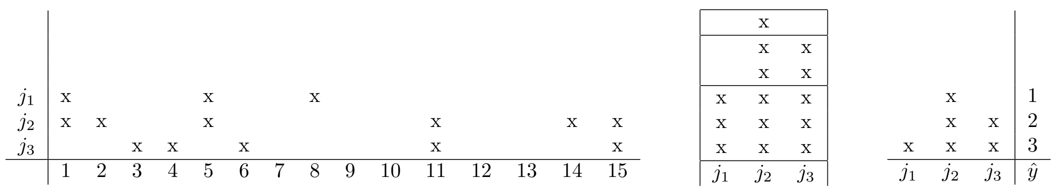

Figure 1: Request history (left hand side) with the associated histogram (center). On the right-hand side is given the maximum number of requests $\hat{y}$ associated to each subset of demands $\{j_1, j_2, j_3\}$.

The following procedure is instrumental in computing quantities relevant to the analysis. Consider the sequence of nested sets

$$\hat{D}^{kn_k} \subset \ldots \subset \hat{D}^{k1} \subset D^k \tag{15}$$

and the vector $\hat{y}_k \in \mathcal{R}^{n_k}$, recursively constructed as follows: first, set

$$\hat{D}^{k1} = \{j \in \hat{D}^k : \overline{y}_{kj} > 0\} \tag{16}$$

and let $\hat{y}^{k1}$ be the smallest quantity such that

$$\hat{D}^{k2} = \{j \in \hat{D}^{k1} : \overline{y}_{kj} - \hat{y}_{k1} > 0\} \neq \hat{D}^{k,1}, \tag{17}$$

then update $\overline{y}_{kj} \leftarrow \overline{y}_{kj} - \hat{y}_{k1}$ for each $j \in \hat{D}^{k1}$ and repeat the operation until no positive components remains in vector $\overline{y}_k$, for each $k \in K$.

Theorem 3 *Let $\hat{D}^{k\ell}$ and $\hat{y}_{k\ell}$ obtained from procedure (14–17). Then $\hat{y}_{k\ell}$ is an upper bound on the number of requests received simultaneously from the demands $\hat{D}^{k\ell}$, on any one time slot of type k, in any compatible scenario $y \in Y_{\text{uni}}$.*

Proof. See Appendix. □

3.4 Min-Cost Flow Model

Assuming that the set of mobilizations $X_{\text{pow}}(\hat{D}^{k\ell}, R^k)$ are provided explicitly for each time slot type, let us consider the problem of responding to requests, independently of the order of their arrival, within the static framework described in the previous section. We now show that, if fractional request are allowed, the broker's problem reduces to a min-cost flow problem.

To this aim, let $M^{k\ell}$ be an ordering of $X_{\text{pow}}(\hat{D}^{k\ell}, R^k)$ and let $\delta^{k\ell}$ be the corresponding mobilization-resource incidence matrix:

$$\delta^{k\ell}_{mi} = \begin{cases} 1 & \text{if the } m\text{-th mobilization of } X_{\text{pow}}(\hat{D}^{k\ell}, R^k) \text{ uses resource } i \text{ for} \\ & \text{one of the demands in } \hat{D}^{k\ell}, \\ 0 & \text{otherwise.} \end{cases}$$

Let $C^{k\ell}_m$ be the cost associated with mobilization $m \in M^{k\ell}$ such that

$$C^{k\ell}_m = \sum_{i,j \in DR^{k\ell}} \delta^{k\ell}_{mi} \mathbf{1}\left\{ j \in \hat{D}^{k\ell} \right\} c_i, \tag{18}$$

and let variable $w_{k\ell m} > 0$ denote the number of time slots on which joint requests from demands $\hat{D}^{k\ell}$ are satisfied by mobilization m. In other words, variable $w_{k\ell m}$ represents the number of tokens from the resources that are used in mobilization m for responding to joint requests from demands $\hat{D}^{k\ell}$ on time slots of type k. Let

$$\overline{w}_{ki} = \sum_{\ell \le n_k} \sum_{m \in M^{k\ell}} \delta^{k\ell}_{mi} w_{k\ell m},$$

be the total amount of tokens from resource i used to respond to requests on time slots of type k, and let $\hat{y}_{t\cdot}$ be the aggregate forecast constructed following procedure (15–17), at the beginning of week t. Now, consider the set of flow constraints $W(\hat{y}, r^{\text{res}})$ defined by the inequalities

$$\sum_{m \in M^{k,\ell}} w_{k\ell m} \ge \hat{y}_{k\ell} \quad k \in K \quad \ell \in L_k \tag{19}$$

$$\sum_{k \in K} \overline{w}_{ki} \ge r^{\text{res}}_i \qquad i \in R, \tag{20}$$

$$w_{k\ell m} \ge 0 \qquad k \in K \quad \ell \in L_k,$$

together with the mathematical program:

Program 3

$$\min_{w \in W(\hat{y}_t, r^{\text{res}}_t)} \quad \sum_{k \in K} \sum_{\ell \in L_k} \sum_{m \in M^{k\ell}} C^{k\ell}_m w_{k\ell m}, \tag{21}$$

whose optimal solution ensures that a maximum number of demand requests are satisfied at minimum cost. Constraint (19) models demand satisfaction and ensure that, given sufficient resources are present, all demands are satisfied, and then all components of Constraint (20) are tight. Otherwise, the slack variable associated with the resource constraints (20) provide information on missing resources. More formally:

Theorem 4 *Given that resources are sufficient to satisfy all demands, that is, there exists a* $w \in W(\hat{y}_t, r^{\text{res}}_{t\cdot})$ *such that*

$$\sum_{m \in M^{k\ell}} w^*_{k\ell m} = \hat{y}_{tk\ell}, \qquad k \in K, \ell \in L_k \tag{22}$$

$$\sum_{k \in K} \sum_{\ell \in L_k} \sum_{m \in M^{k\ell}} \delta^{k\ell}_{mi} w^*_{k\ell m} \le r^{\text{res}}_{ti}, \quad \forall i, \tag{23}$$

then all components of Constraint (20) are tight in Program 3.

Proof. See Appendix. □

3.5 Long term information

With respect to an optimal solution of Program 3, two situations may arise. First, there exists an index $i \in R$ such that (20) is not tight, i.e., resource i is lacking request tokens, and then at least one of the component of (19) is tight, say component (k, ℓ), where additional resources are required to satisfy demands $\hat{D}^{k\ell}$. The opposite situation entails that all demands are covered. This is consistent with our general positioning of the broker problem, where resources are assumed to be sufficient. In this context, the mobilization policy chosen at the current week should be such that the long term model remains feasible in the subsequent weeks.

To satisfy the above requirement, we factor in bounds on the use of each resource (provided by the long term solution) and restrain the current week's mobilization accordingly. The stochastic nature of the aggregated forecast $\overline{w}$ (on the basis of which the long term model is build) suggests not to implement this as hard constraints. In the proposed scheme, the coefficients of the short term objective are corrected in the following manner. Consider the restriction $\hat{x}(w)$ of the long term solution w to the time slots at current week t, that is, the expected amount of resource i used to satisfy requests from demand j on time slot $s \in S(t)$, for each $(i,j) \in DR^{ts}$:

$$\hat{x}^{ts}_{ij}(w) = p_{t,k(ts)} \sum_{\ell \in L_{k(t,s)}: j \in \hat{D}^{k\ell}} \sum_{m \in M^{k(t,s)\ell}} \delta^{k(t,s)\ell}_{mi} w_{k(t,s)\ell m},$$

where p_{tk}, is the conditional probability that a request on a time slot of type k arrive at current week t under the uniform demand assumption, is expressed as

$$p_{tk} = |\{s \in S(t) : k(ts) = k\}| / |\pi_k|.$$

Since incorporating the constraints

$$p_{t,k(ts)}\, x^{ts}_{ij} \leq \hat{x}^{ts}_{ij}(w) \quad (ij) \in DR^{ts},\ s \in S(t)$$

may make the short term model infeasible, we look for a feasible mobilization policy x^* such that

$$p_{t,\cdot}\, x^* \in \arg \min_{x^t \in X^{\text{week}}_t} \|\hat{x}^t(w), x^t\|.$$

Equivalently, the square of the norm can be minimized:

$$\begin{aligned}
\|\hat{x}^t(w), x^t\|^2 &= \sum_{s \in S(t)} \sum_{(i,j) \in DR(t,s)} (\hat{x}^{ts}_{ij}(w) - x^{ts}_{ij})^2 \\
&= \sum_{s \in S(t)} \sum_{(i,j) \in DR(t,s)} (\hat{x}^{ts}_{ij}(w))^2 - 2\hat{x}^{ts}_{ij}(w) x^{ts}_{ij} + (x^{ts}_{ij})^2 \\
&= \sum_{s \in S(t)} \sum_{(i,j) \in DR(t,s)} [1 - 2\hat{x}^{ts}_{ij}(w)] x^{ts}_{ij} + \text{constant} \qquad (24)
\end{aligned}$$

where we used $(x^{ts}_{ij})^2 = x^{ts}_{ij}$, as $x^{ts}_{ij} \in \{0, 1\}$. We now have:

$$p_{t,\cdot}\, x^* \in \arg \min_{x^t \in X^{\text{week}}_t} \sum_{s \in S(t)} \sum_{(i,j) \in DR(t,s)} [1 - 2\hat{x}^{ts}_{ij}(w)]\, x^{ts}_{ij}.$$

We then set the long term information in the short term model (Program 2) to

$$q^{ts}_{ij} = (p_{t,k(ts)})^{-1} (1 - 2\hat{x}^{ts}_{ij}(w)), \qquad (25)$$

which is referred to as the projected flow (PF) information in the sequel.

Consider again Portfolio 1 (Tables 1 and 2), which involves two time slot types (referred to as day types above): Mondays and Tuesdays for $k = 1$, and the remaining week days for $k = 2$, and thus $K = \{1, 2\}$. As a single resource suffices to cover a request, mobilizations can be expressed directly in terms of the mobilized resource: We have $M^1 = \{R1\}$ and $M^2 = \{R1, R2\}$, and we also have $\overline{w}_{ki} = w_{k1i}$, that is, the amount of resource i assigned to day type k matches the flow of the corresponding mobilization on the unique slice 1. Variable $w_{k\ell m}$ represents the amount of resource i mobilized on time slot of type k to cover the demands $\hat{D}^{k\ell}$, where $k \in \{1,2\}$, $\ell = 1$ and $m \in 1, 2$. Mobilization 1 (resp. mobilization 2) corresponds to the use of resource $R1$ (resp. resource $R2$) and $\hat{D}^{11} = \hat{D}^{12} = \{1\}$, that is, a single demand is involved. Recalling that $c_1 = 2$ and $c_2 = 5$, Program 3 takes the form:

$$\begin{aligned}
\min_{w \geq 0} \quad & 2(w_{111} + w_{211}) + 5w_{112} \\
\text{s.t.} \quad & w_{111} + w_{112} \geq \overline{y}_1^1 = 4 \\
& w_{211} \geq \overline{y}_1^2 = 8 \\
& \overline{w}_{11} + \overline{w}_{21} \geq r_1^{\text{res}} = 10 \\
& \overline{w}_{12} \geq r_2^{\text{res}} = 5.
\end{aligned}$$

Its optimal solution is illustrated on the right-hand side of Figure 2. For example, at week $t = 1$ and for time slot $s = 1$, resource $i = 1$ ($D1$) and demand $i = 2$ ($R2$), we have $p_{11} = 1/15$ and $\hat{x}_{21}^{11} = 1/3$, and the corrected cost

$$q_{21}^{11} = (1/15)^{-1}[1 - 2(1/3)] = 5.$$

The PF information at week $t = 1$ is summarized in Table 4. The left-hand side of Figure 2 provides a graphical comparison between the CL and PF information.

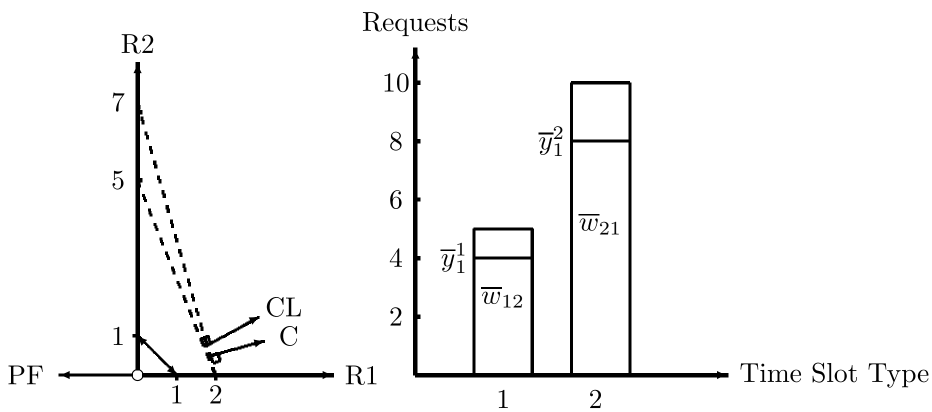

Figure 2: Portfolio 1: On the left-hand side – CL and PF information comparison at week 1 for time slots of type 1 (Monday and Tuesday); On the right-hand side – Solution of the long term model.

Table 4: Portfolio 1: PF information at week $t = 1$.

Resource	Mon-Tue	Wed-Sat
R1	0	−5
R2	5	n/a

A simulation run using FB information on Portfolio 1 is presented at the bottom of Table 2, and compared with one based on CL information. Note that all time slots are now covered. CL and FP information are compared on Portfolio 1 in Table 5, where 100 simulations have been performed. The PF information provides a coverage of 83 percent of all scenarios, compared to 19 percent for the CL information.

While the FB information performs better, both schemes “agree” that Portfolio 1 cannot be covered with probability 1, which confirms our previous observations.

Table 5: Portfolio 1: CL vs TP information.

information type	mobilization cost	proportion of covered scenarios
CL	176.88	19%
FP	223.50	83%

4 Example

We now compare CL and FP on the larger Portfolio 2 described in Table 6. For each time slot type, the first and the last week within which the type is represented (Lines 1-2), the total power available from the demands and the resources (lines 3-4), upper bounds on the power that can be requested and rolled out (Lines 5-6), the number of mobilization candidates (Line 7) and the available demands (Lines 8-10) and resources (Lines 11-20), from Monday to Sunday. Associated data is available in Figures 3, 4 and Table 7.

The results of 50 simulations are summarized in Table 8 and Table 9, respectively. The statistics are gathered for each time slot type: average mobilization costs, average number of uncovered time slots (failures)

Table 6: Portfolio 2: Contract parameters.

dem/ res	pow	req	main	t_0	t_N	Mon	Tue	Wed	Thu	Fri	Sat	Sun
R1	2	10	1	1	16		x	x		x		
R2	2	10	1	1	24	x		x		x		
R3	2	20	0	1	24			x	x	x		
R4	1	10	4	1	16		x	x		x		
R5	1	10	4	1	16		x	x				
R6	1	10	4	1	16		x	x		x		
R7	1	10	1	1	24	x	x	x	x	x		
R8	1	10	1	1	16	x		x	x	x		
R9	1	20	0	1	24	x	x	x	x	x	x	
R10	1	10	1	1	24					x	x	
D1	1	15	n/a	1	16	x	x	x		x	x	
D2	2	20	n/a	1	24	x		x	x	x		
D3	3	25	n/a	1	16		x	x		x		

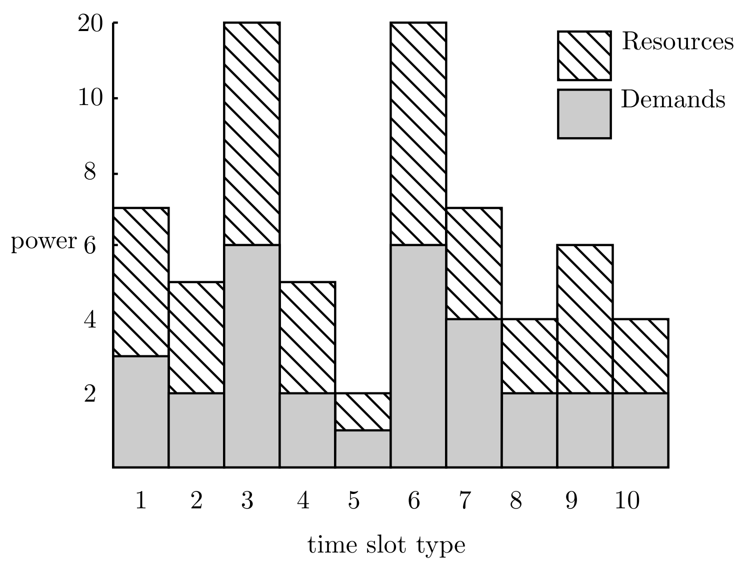

Figure 3: Portfolio 2: Demand vs resource power (power constraints).

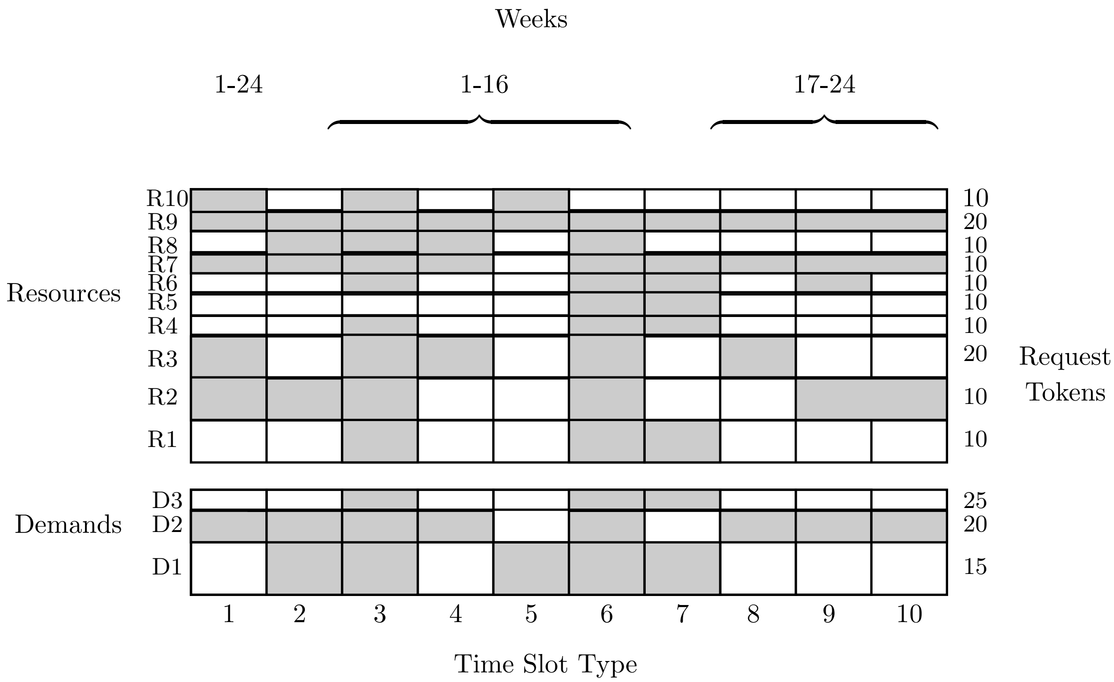

Figure 4: Portfolio 2: Demand vs resource request tokens (request constraints).

Table 7: Portfolio 2: Day type information – number of candidate mobilizations.

time slot type	1	2	3	4	5	6	7	8	9	10
nb. mobilizations	5	6	960	4	2	960	40	2	3	2

and the proportion of simulations where all demands are fully covered. At the bottom, we provide the average number of unused resource tokens, for each resource, as well as the total proportion of lost power.

Considering the distribution of uncovered time slots along their different types, both algorithms (information type) broadly agree that resources are lacking on time slots of type 1 and time slot type 7 (week 1 to week 16) and time slot type 10 (week 17 to week 24). The CL information otherwise manages a lower average mobilization cost of 887.62, compared to 1037.64 when using the FP information. But the extra mobilization cost allows the FP scheme to fully cover 76 percent of scenarios, compared to none for the CL scheme, and this is achieved using only 2 percent of additional resources.

Table 8: Portfolio 2 – CLI: Simulation summary (averages over 50 runs).

time slot type	1	2	3	4	5	6	7	8	9	10	total
cost	84.80	34.28	188.46	65.80	30.58	224.44	192.84	25.44	30.18	10.80	887.62
nb. failures	0.60	0.06	0.14	0	0.28	0.20	0.58	0	0.40	4.52	6.78
prop. covering	0.62	0.94	0.88	1	0.90	0.80	0.64	1	0.82	0.02	0
resource	1	2	3	4	5	6	7	8	9	10	lost
unused tokens	0.12	0	2.42	5.52	3.82	1.60	0.16	1.12	11.38	5.72	0.21

Table 9: Portfolio 2 – PF: Simulation summary (averages over 50 runs).

time slot type	1	2	3	4	5	6	7	8	9	10	total
cost	105.42	31.82	213.18	88.78	32.60	277.22	204.92	41.64	25.46	16.60	1037.64
nb. failures	0.06	0	0	0	0	0	0.04	0	0	0.22	0.32
prop. covering	0.94	1	1	1	1	1	0.96	1	1	0.84	0.76
resources	1	2	3	4	5	6	7	8	9	10	lost
unused tokens	1.32	1.72	7.98	1.28	1.34	0.94	0.46	0.84	1.48	1.72	0.19

5 Conclusion

In the present work, we have introduced a model for an energy broker attached to the smart grid which operates in a novel way, using bilateral contracts with both his clients and the grid. While the management of this novel contractual framework yields a hard combinatorial problem, we could propose for its solution original and efficient optimizing tools aimed at maximizing the broker's profitability. Our deterministic two time frame semi-online mathematical model actually covers the broker against a family of demand scenarios in the long run, while insuring feasibility. The next challenge consists in addressing a probabilistic generalization of the model, which will be done in a companion paper.

Notation

In the following, a contract is said to be valid at week t if week t is within the contract validity period and if it as a positive number of request tokens left at the end of week $t-1$. Additionally, a resource contract that has announced a maintenance for week t is not valid at week t.

CL	Contract Level information.
PF	Projected Flow information.
T	Set of weeks.
$S(t)$	Set of time slots in week t.
λ^{ts}	Duration of time slot $s \in S(t)$.
K	Set of time slots types.
$k(t,s)$	Type of time slot $s \in S(t)$.
D	Set of demands (grid contracts).
R	Set of resources (generator contracts).
D^{ts}	Set of valid demand contracts on time slot $s \in S(t)$.
R^{ts}	Set of valid resource contracts on time slot $s \in S(t)$.
DR^{ts}	Set of valid and compatible demand/resource contract pairs on time slot $s \in S(t)$.
R_j^{ts}	Set of valid resources on time slot $s \in S(t)$ that are compatible with demand i.
D_j^{ts}	Set of valid demands on time slot $s \in S(t)$ that are compatible with resource i.
$\hat{D}^k$	Set of demand contracts on time slots of type k.
$\hat{D}^{k\ell}$	Subset of valid demand contracts in $\hat{D}^k$ forming the ℓ-th horizontal slice of the corresponding demand histogram.
n_k	Number of slices in demands $\hat{D}^k$ histogram.
L_k	Slide indices on time slot type k: $L_k = [1, \ldots, n_k]$.
$X_{\text{pow}}(R, D)$	Static feasibility set associated to the resources R and the demands D.
X_{week}^t	Robust feasible at week t.
Y	Set of feasible demand request scenarios.

Y_{uni} Subset of feasible demand request scenarios compatible with the uniform forecast.

$w_m^{k\ell}$ Positive decision variable modeling to the quantity of mobilization m that is used on time slots of type k along the ℓ-th slice (to respond to simultaneous requests from demands $\hat{D}^{k\ell}$).

x_{ij}^{st} Binary decision variable set to 1 if resource i is assigned to demand j on time slot $s \in S(t)$.

y_j^{st} Binary parameters set to 1 if demand j makes a request on time slot $s \in S(t)$.

$\hat{y}^{k\ell}$ Upper bound on the expected number of simultaneous requests from demands $\hat{D}^{k\ell}$ on time slots of type k.

$\overline{y}_{kj}$ Expected number of requests from demand j on time slots of type k.

r_{ti}^{res} Number of request tokens in resource i at the beginning of week t.

r_{tj}^{dem} Number of requests tokens in demand j at the beginning of week t.

q_{ij}^{ts} Corrected cost of resource i on demand j at time slot $s \in S(t)$.

c_i Cost for the mobilization of resource i for one unit of time.

$M^{k\ell}$ Set of mobilizations for time slots of type k for demands in $\hat{D}^{k\ell}$.

$C_m^{k\ell}$ Total cost for the mobilization of the resources in mobilization $m \in M^{k\ell}$.

$\delta^{k\ell}$ Mobilization incidence matrix associated with slice ℓ of time slot type k.

λ^{ts} Duration of time slot $s \in S(t)$.

power_ℓ Power delivered (resp. received) in resource (resp. demand) ℓ.

6 Appendix

Proof of Theorem 1

($\Rightarrow$) Let $x \in X_{\text{week}}^t$ and suppose that $\exists t \in T, i \in R$:

$$\sum_{j\in D_i} \min\{\sum_{s:(i,j)\in D^{ts}} x_{ij}^{ts}, r_{tj}^{\text{dem}}\} > r_{ti}^{\text{res}}.$$

We can choose $y \in Y$ such that

$$\sum_{j\in D_i}\sum_{s(i,j)\in D^{ts}} x_{ij}^{ts} y_j^{ts} = \sum_{j\in D_i} \min\{\sum_{s:(i,j)\in D^{ts}} x_{ij}^{ts}, r_{tj}^{\text{dem}}\}$$

But then

$$\sum_{j\in D_i}\sum_{s:(i,j)\in D^{ts}} x_{ij}^{ts} y_j^{ts} > r_{ti}^{\text{res}}$$

and thus $x \notin X_{\text{week}}^t$, contradicting the hypothesis.

($\Leftarrow$) Let $x \in X_{\text{week}}^t$ and suppose that there exists a resource i such that

$$\sum_{s\in S(t)}\sum_{(i,j)\in DR_{ts}} x_{ij}^{ts} y_j^{ts} > r_{ti}^{\text{res}}.$$

According to (2) we have

$$\begin{aligned}\sum_{j\in D}\sum_{s\in S(t)} y_j^{ts} x_{ij}^{ts} &\leq \sum_{j\in D} \min\{\sum_{s\in S(t)} x_{ij}^{ts}, r_{tj}^{\text{dem}}\} \\ &\leq r_{ti}^{\text{res}},\end{aligned}$$

contradicting the hypothesis.

Proof of Theorem 2

Constraints (8–9) require that

$$z_{ij} = \begin{cases} 1 & \text{if } \sum_{s\in S(t)} x_{ij}^s > r_i^R \\ 0 & \text{if } \sum_{s\in S(t)} x_{ij}^s \le r_i^R. \end{cases}$$

Constraints (10–12) require that

$$w_{ij}^s = x_{ij}^s \, z_{ij},$$

and thus Constraint (7) requires that

$$\begin{aligned} r_i^{\text{res}} &\ge \sum_{j\in D^t} \left(r_j^D z_{ij} + \sum_{s\in S(t)} x_{ij}^s (1 - z_{ij}) \right) \\ &= \sum_{j\in D^t} \min\{r_j^D, \sum_{s\in S(t)} x_{ij}^s\}. \end{aligned}$$

Proof of Theorem 3

The argument is geometric. Consider the histogram associated with the quantities $\overline{y}_{kj}, j \in \hat{D}^k$. Given an appropriate ordering of $\hat{D}^k$, the histogram can be partitioned into a finite number of horizontal slices, the ℓ-th slice having height $\hat{y}^{k,\ell}$, and horizontally covering the columns associated with demands in $\hat{D}^{k,\ell}$. The conclusion follows.

Proof of Theorem 4

By contradiction, let w' satisfy Constraints (22–23) and w^* be an optimal solution of Program 2 such that there exists $i \in R$ such that

$$\sum_{k\in K} \sum_{\ell\le n_k} \sum_{m\in M^{k\ell}} \delta_{mi}^{k\ell} w_m^{*k\ell} > r_{ti}^{\text{res}}.$$

Now consider the following equivalent expression of Program 2:

$$\min_{\psi,w} \hat{f}(w) = \sum_{i\in I} c_i \psi_i \tag{26}$$

$$\text{s.t.} \quad \sum_{m\in M^{k\ell}} w_m^{k\ell} \ge \hat{y}_t^{k\ell} \qquad \forall k \quad \ell \le n_k \tag{27}$$

$$\psi_i \ge r_{ti}^{\text{res}} \qquad \forall i \tag{28}$$

$$\psi_i = \sum_{k\in K} \sum_{\ell\le n_k} \sum_{m\in M^{k\ell}} \delta_{mi}^{k\ell} w_m^{k\ell} \qquad \forall i \tag{29}$$

$$w_m^{k\ell} \ge 0 \qquad \forall k \quad \ell \in L_k \quad \forall m.$$

Expressing w' and w^* in terms of the new variable yields

$$\psi' = \sum_{k\in K} \sum_{\ell\le n_k} \sum_{m\in M^{k\ell}} \delta_{mi}^{k,\ell} w_m^{'k\ell}$$

$$\psi^* = \sum_{k\in K} \sum_{\ell\le n_k} \sum_{m\in M^{k\ell}} \delta_{mi}^{k,\ell} w_m^{*k\ell}$$

From the optimality of w^* we deduce: $\sum_i c_i \psi_i^* \le c_i \sum_i \psi_i'$, while the feasibility of w' implies that $\psi_i' \le \psi_i^*, \forall i$. The strict positivity of c implies that w' is optimal, which in turn implies that $\sum_i c_i \psi_i^* = c_i \sum_i \psi_i',$. The contradiction follows from the inequality $\psi_i^* > \psi_i'$.

References

[1] F. Rahimi, A. Ipakchi, Demand response as a market resource under the smart grid paradigm, IEEE Transactions on Smart Grid 1 (1) (2010) 82–88.

[2] N. Hatziargyriou, H. Asano, R. Iravani, C. Marnay, Microgrids, Power and Energy Magazine, IEEE 5 (4) (2007) 78–94.

[3] S. You, C. Træholt, B. Poulsen, A market-based virtual power plant, in: Clean Electrical Power, 2009 International Conference on, IEEE, 2009, pp. 460–465.

[4] A. Mondal, S. Misra, Dynamic coalition formation in a smart grid: a game theoretic approach, in: Communications Workshops (ICC), 2013 IEEE International Conference on, pp. 1067–1071.

[5] P. Li, X. Guan, J. Wu, D. Wang, An integrated energy exchange scheduling and pricing strategy for multi-microgrid system, in: TENCON 2013 IEEE Region 10 Conference (31194), pp. 1–5.

[6] H. Nunna, S. Doolla, Demand response in smart distribution system with multiple microgrids, IEEE Transactions on Smart Grid 3 (4) (2012) 1641–1649.

[7] M. Rahimiyan, L. Baringo, A. J. Conejo, Energy management of a cluster of interconnected price-responsive demands, Power Systems 29 (2014) 645–655.

[8] M. Negnevitsky, T. Nguyen, M. de Groot, Novel business models for demand response exchange, in: Power and Energy Society General Meeting, 2010 IEEE, IEEE, pp. 1–7.

[9] M. G. Bosman, V. Bakker, A. Molderink, J. Hurink, G. J. Smit, Planning the production of a fleet of domestic combined heat and power generators, European journal of operational research 216 (1) (2012) 140–151.

[10] D. Wu, R. Wang, Combined cooling, heating and power: a review, Progress in Energy and Combustion Science 32 (5) (2006) 459–495.

[11] G. Chicco, P. Mancarella, Distributed multi-generation: a comprehensive view, Renewable and Sustainable Energy Reviews 13 (3) (2009) 535–551.

[12] R. Angelino, A. Bracale, G. Carpinelli, M. Mangoni, D. Proto, A fuel cell-based dispersed generation system providing system ancillary services through power electronic interfaces, Renewable Energy 36 (9) (2011) 2312–2323.

[13] B. Roossien, A. van den Noort, R. Kamphuis, F. Bliek, M. Eijgelaar, J. de Wit, Balancing wind power fluctuations with a domestic virtual power plant in europe's first smart grid, in: PowerTech, 2011 IEEE Trondheim, pp. 1–5.

[14] D. Pudjianto, C. Ramsay, G. Strbac, Virtual power plant and system integration of distributed energy resources, Renewable Power Generation, IET 1 (1) (2007) 10–16.

[15] C. Cecati, C. Citro, P. Siano, Combined operations of renewable energy systems and responsive demand in a smart grid, IEEE Transactions on Sustainable Energy 2 (4) (2011) 468–476.

[16] P. Koponen, J. Ikaheimo, A. Vicino, A. Agnetis, G. De Pascale, N. Ruiz Carames, J. Jimeno, E. Sanchez-Ubeda, P. Garcia-Gonzalez, R. Cossent, Toolbox for aggregator of flexible demand, in: Energy Conference and Exhibition (ENERGYCON), 2012 IEEE International, IEEE, 2012, pp. 623–628.

[17] Y. G. Rebours, D. S. Kirschen, M. Trotignon, S. Rossignol, A survey of frequency and voltage control ancillary services–Part i: Technical features, IEEE Transactions on Power Systems 22 (1) (2007) 350–357.

[18] Y. G. Rebours, D. S. Kirschen, M. Trotignon, S. Rossignol, A survey of frequency and voltage control ancillary services–Part ii: economic features, IEEE Transactions on Power Systems 22 (1) (2007) 358–366.

[19] M. Albadi, E. El-Saadany, Demand response in electricity markets: An overview, in: IEEE Power Engineering Society General Meeting, Vol. 2007, pp. 1–5.

[20] J. Gordijn, H. Akkermans, Business models for distributed generation in a liberalized market environment, Electric Power Systems Research 77 (9) (2007) 1178–1188.

[21] C. Corchero, F. Heredia, et al., A stochastic programming model for the thermal optimal day-ahead bid problem with physical futures contracts, Computers & Operations Research 38 (11) (2011) 1501–1512.

[22] M. Zdrilic, H. Pandzic, I. Kuzle, The mixed-integer linear optimization model of virtual power plant operation, in: Energy Market (EEM), 2011 8th International Conference on the European, IEEE, pp. 467–471.

[23] M. Peik-Herfeh, H. Seifi, M. Sheikh-El-Eslami, Decision making of a virtual power plant under uncertainties for bidding in a day-ahead market using point estimate method, International Journal of Electrical Power & Energy Systems 44 (1) (2013) 88–98.

[24] H. Pandžić, J. M. Morales, A. J. Conejo, I. Kuzle, Offering model for a virtual power plant based on stochastic programming, Applied Energy 105 (2013) 282–292.

[25] E. Mashhour, S. M. Moghaddas-Tafreshi, Bidding strategy of virtual power plant for participating in energy and spinning reserve markets–Part i: Problem formulation, IEEE Transactions on Power Systems 26 (2) (2011) 949–956.

[26] S. Ahmed, A. J. King, G. Parija, A multi-stage stochastic integer programming approach for capacity expansion under uncertainty, Journal of Global Optimization 26 (1) (2003) 3–24.

[27] G. Lulli, S. Sen, A branch-and-price algorithm for multistage stochastic integer programming with application to stochastic batch-sizing problems, Management Science 50 (6) (2004) 786–796.

[28] W. B. Powell, Approximate Dynamic Programming: Solving the curses of dimensionality, Vol. 703, John Wiley & Sons, 2007.

[29] L. Buşoniu, B. De Schutter, R. Babuška, Approximate dynamic programming and reinforcement learning, in: Interactive collaborative information systems, Springer, 2010, pp. 3–44.

[30] A. Løkketangen, D. L. Woodruff, Progressive hedging and tabu search applied to mixed integer (0, 1) multistage stochastic programming, Journal of Heuristics 2 (2) (1996) 111–128.

[31] C. C. Carøe, R. Schultz, Dual decomposition in stochastic integer programming, Operations Research Letters 24 (1) (1999) 37–45.