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Abstract: This paper proposes a stochastic integer programming (SIP) formulation to address the optimization of long-term mine production schedules, whereby the supply of metal from a mineral deposit is considered uncertain and can be described by a set of equally probable orebody representations. The proposed SIP formulation maximizes the expected net present value of the production over the life of the mine while reducing the risk of deviating from production targets. The proposed modelling methodology considers a set of limiting surfaces to facilitate the scheduling of larger size deposits, where a surface defines a limit that separates mining blocks assigned to two consecutive production periods, and is represented by a discrete set of elevations. A key characteristic of the formulation is that it facilitates a divide-and-conquer approach to scheduling, where the scheduling can be performed sequentially, thus controlling the number of binary variables in multi-period production scheduling, which in turn facilitates production scheduling for large mineral deposits

The proposed SIP production scheduling formulation is tested at a copper deposit. First, scheduling with the proposed SIP shows the intricacies and performance of the formulation. Subsequently, the method is applied sequentially to show the equivalence of solutions while assessing computational requirements and demonstrates empirically that the proposed SIP formulation can reduce the processing time from days to minutes.

Key Words: Mine planning, production scheduling, geological uncertainty, surface, stochastic optimization, stochastic orebody models, risk analysis.

1 Introduction

Mineral projects aim to produce metal available in the Earth's subsurface to meet the needs of society and its development through the sustainable utilization of mineral resources while generating revenue. The longstanding conventional approach of using exploration drilling data to estimate the attributes of interest (grade, material types, density, etc.) in a mineral deposit has been found limiting and does not capture the intrinsic geological variability and uncertainty. As a result, conventional optimization formulations used to optimize mine production schedules and assess discounted cashflows over production years have been shown to be both misleading in production forecasts (Dimitrakopoulos et al, 2002; Dowd, 1997) and undervalues potential metal production and project value (Godoy, 2003). A review of these issues as well as overview of recent developments in dealing with uncertainty (stochasticity) in optimizing mine design and production scheduling can be found in Dimitrakopoulos (2011).

Stochastic optimization for long-term mine production scheduling using simulated annealing was first introduced by Godoy and Dimitrakopoulos (2004). Although the simulated annealing framework remains relevant and is being expanded to control production deviations in mining complexes with multiple mines and processing destinations (Goodfellow and Dimitrakopoulos, 2012), it may be limited in its ability to guarantee an optimal solution. Stochastic integer programming with recourse was introduced as an alternative framework (Ramazan and Dimitrakopoulos, 2005) to address the limitations of simulated annealing. Ramazan and Dimitrakopoulos (2012) detail an SIP formulation with recourse that maximizes total discounted cash flows and minimizes deviations from production targets (ore tonnage, grade and metal).The key drawback of the SIP approach is computational: as the number of binary variables increases with the number of mining blocks being scheduled, the amount of time required to generate an optimal solution often becomes impractical, particularly as mineral deposits may be represented by Notable variations of the SIP framework includes long- and short- term mine production scheduling based on simulated future grade control data (Jewbali and Dimitrakopoulos, 2009) and using the SIP formulation for pushback design and demonstrating that stochastically generated pit limits are larger than the corresponding conventional ones (Albor and Dimitrakopoulos, 2009). Menabde et al. (2007) proposes an alternate formulation that uses a variable cut-off grade and relies on aggregations of blocks to ensure the problem is computationally tractable. Boland et al. (2008) propose a multi-stage stochastic programming approach that considers both processing and mining decisions, however, their formulation has practical limitations, in terms of usability for mining engineering problems and also computational limitations.

To address the computational and size limits of the above SIP mine scheduling formulations, Lamghari and Dimitrakopoulos (2012) introduce Tabu and variable neighborhood search, bypassing the need to solve SIP formulations with conventional solvers such as CPLEX. Results reported are very close to optimality, while executing orders of magnitude faster. Despite the improvements in terms of solving faster than before, further developments are needed if very large size deposits are to be scheduled.

Goodwin et al. (2005) propose a new "mine state" concept for open pit production scheduling. A mine state is defined as a set of elevations, one for each (x, y) coordinate of a three-dimensional orebody model, which are used to represent the pit depth (the distance from a fixed higher elevation) at a given mining period. The evolution from one period to the next is given by downward vertical increments, updating the mine state each time. Slope constraints are controlled over each mine state, comparing only adjacent elevations. Each state is always below previous ones, as only positive increments are allowed. Unfortunately, in their formulation, the objective function is non-convex, which limits the use of conventional integer programming techniques. In addition, uncertainty is not accounted for. The mine state concept is the same as the surface concept used herein and is presented in the subsequent sections, with the difference that surfaces carry exact elevations in space, instead of measures of depth from a fixed elevation.

The present paper expands on previous work by proposing a SIP formulation to address the optimization of mine production scheduling, defining constraints over surfaces to reduce the number of constraints in the mathematical formulation. Uncertainty and risk are controlled through additional hard constraints, instead of resource actions over the objective function as formulated in prior work. It is important to note that the management of slope constraints is simpler and more general, given the linear aspect of the formulation proposed. The concepts and stochastic scheduling formulation based on surfaces is detailed will first be discussed. Following this, a sequential implementation of the proposed method is revisited in an effort to use its properties to yield more computationally efficient implementations that have solutions close to optimality. A case study using a copper deposit demonstrates the full implementation of the proposed SIP formulation, the sequential implementation and the performance aspects. Conclusions follow.

2 A Stochastic Programming Formulation Based on Surfaces

2.1 Surfaces and related concepts

Surface relations used herein are based on the fact that blocks are not independently distributed in space and can be grouped into vertical columns. More specifically, surfaces are defined as sets of elevations in which the periods are divided. Each column of blocks (fixed x and y coordinates) can be partitioned by T surfaces into $T+1$ groups of blocks, which then becomes the T mining periods in addition to the blocks that are not extracted. For each surface (or period) T, cell c is defined by a fixed pair of coordinates (x, y), and each cell c has an elevation e_{ct} associated with period t. Variables e_{ct} are linear and assume values from the origin up to the highest elevation allowed in the orebody model. Using the concept of surfaces, maximum slope angle constraints can be controlled over surfaces, rather than blocks. This is a tradeoff, where new variables e_{ct} are included but fewer constraints are required, as shown in subsequent sections.

Past approaches for modelling the production scheduling problem are typically based on binary variables x_{it}, where $x_{it} = 1$, if block i is mined in period t, and zero otherwise. Here, the notation of x_{it} is modified to x^z_{ct}, where each i corresponds to a pair (c, z), for correlating e_{ct} (elevation; linear) with x^z_{ct} (block; binary). The x^z_{ct} variable assumes the value 1 only if block (c, z) is the deepest block being mined in period t and in column c; otherwise, the x^z_{ct} variable is zero.

Block attributes such as total/ore/metal tonnages are accumulated starting from the topography down to the last block over each column, with cumulative values being stored at each level; attributes for single blocks are discarded for the optimization process, which allows quick operations between surfaces by taking differences. A key aspect of this approach is the need to associate blocks with surface cells and is performed by comparing their elevations in space (Figure 1): a block (c, z) will be considered the last block mined over c in a given period t if its centroid elevation E^z_c lies between e_{ct} and $e_{ct} + \triangle z$, where $\triangle z$ is the block dimension in z. Figure 2 shows, in a sectional view, cells representing the end of first and second periods, with blocks valued and coloured accordingly. Note that for each column (given x and y) and each period, $\sum x^z_{ct} = 1$, which is a strong constraint given by the approach proposed herein that reduces processing time.

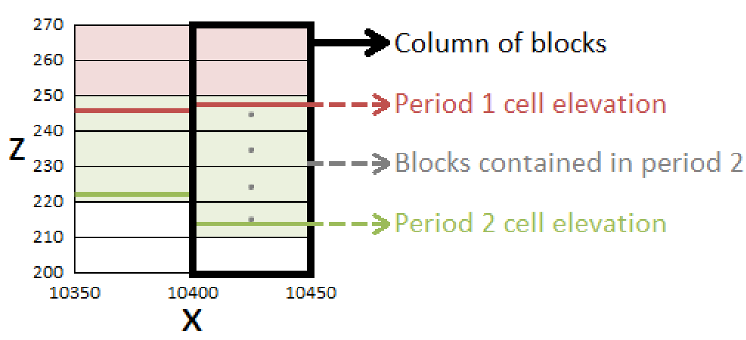

Figure 1: Example showing a cross-section of blocks being represented by cells.

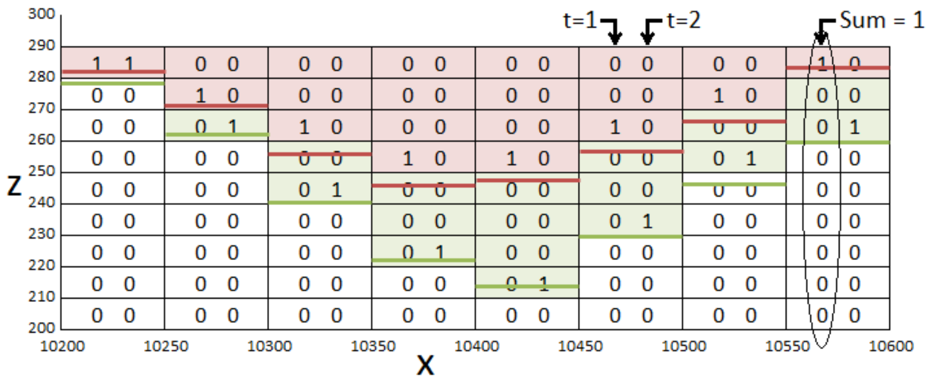

Figure 2: Assigning a period to a block according to surface elevations.

2.2 Notation

The following notation is used:

Indices

1. M: number of cells in each surface; where $M = x \times y$ represents the number of mining blocks in x and y dimensions.
2. c: cell index corresponding to each (x, y) block/cell location, $c = 1, \ldots, M$.
3. Z: number of levels in the orebody model.
4. z: level index, $z = 1, \ldots, Z$.
5. T: number of periods over which the orebody is being scheduled and also defines the number of surfaces considered.
6. t : period index, $t = 1, \ldots, T$.
7. S: number of simulated orebody models considered.
8. s: simulation index, $s = 1, \ldots, S$.

Constants and sets

9. E_c^z: elevation of the centroid for a given block (c, z).
10. H_x: maximum difference in elevation for adjacent cells in contact laterally in the x direction, calculated by $H_x = \triangle x \times tan(\theta)$, where $\triangle x$ is the block size in x and θ is the maximum slope angle.
11. H_y: maximum difference in elevation for adjacent cells in contact laterally in the y direction, calculated by $H_y = \triangle y \times tan(\theta)$, where $\triangle y$ is the block size in y.
12. H_d: maximum difference in elevation for adjacent cells in contact diagonally, calculated by $H_d = \sqrt{\triangle x^2 + \triangle y^2} \times tan(\theta)$.
13. X_c, Y_c and D_c: equivalent to H_x, H_y and H_d concept, the sets of adjacent cells, laterally in x, in y and diagonally, for a given cell c, respectively.
14. T_c^z: cumulative tonnage of block (c, z) and all blocks above it (scenario independent).
15. O_{cs}^z: cumulative ore tonnage of block (c, z) and all blocks above it in scenario s.
16. M_{cs}^z: cumulative amount of metal contained in block (c, z) and all blocks above it in scenario s.
17. EV_{cs}^z: cumulative economic value of block (c, z) and all blocks above it in scenario s.

Parameters

18. $\underline{T_t}$ and $\overline{T_t}$: lower and upper limits, respectively, in total tonnage to be extracted during period t.
19. $\underline{O_t^R}$ and $\overline{O_t^R}$: lower and upper limits, respectively, on ore tonnage to be processed over period t, where R is used to denote "risk".
20. $\underline{O_t^A}$ and $\overline{O_t^A}$: lower and upper limits, respectively, on expected ore tonnage to be processed over period t, where A is used to denote "average".
21. α: economic discount rate.
22. V_{cts}^z: discounted economic value to be generated for all blocks extracted up to period t in scenario s.

Variables

23. e_{ct}: scenario-independent linear variables associated with each cell c for each period t, representing cell elevations.
24. x_{ct}^z: binary variable that assumes 1 if block (c, z) is the last block being mined in period t over c, and 0 otherwise.

2.3 Formulation

2.3.1 Objective function

The proposed objective function in Eq. (1) maximizes the expected net present value from mining and processing selected blocks over all considered periods. Recall that all values are cumulative and $x_{ct}^z - x_{c,t-1}^z$ is associated with the difference between the surfaces. $(x_{ct}^z - x_{c,t-1}^z)$ results in 0, 1 or -1, depending on location (c, z) and period (t).

$$\max \sum_{s=1}^{S} \sum_{t=1}^{T} \sum_{c=1}^{M} \sum_{z=1}^{Z} V_{cts}^z (x_{ct}^z - x_{c,t-1}^z) \tag{1}$$

2.3.2 Constraints

The constraints presented in Equations (2)–(7) and (10) are scenario-independent, while constraints (8)–(9) are scenario-dependent stochastic constraints.

Surface constraints

The following set of constraints (Eq. 2) guarantee that each surface t has, at maximum, the same elevation as surface $t-1$, which is used to avoid crossing surfaces and blocks being mined more than once.

$$e_{c,t-1} - e_{ct} \geq 0 \qquad c = 1, \ldots, M; \quad t = 2, \ldots, T \tag{2}$$

Slope constraints

The maximum surface slope angle is guaranteed herein by Equations (3)–(5). Each cell elevation is compared to the elevation of the 8 adjacent cells, which therefore represents a set of $8 \times nx \times ny \times T$ linear constraints. Slope constraints controlled by surface relations do not depend on the slope angle, which is efficient in the sense that it requires fewer constraints.

$$e_{ct} - e_{xt} \leq H_x \qquad c = 1, \ldots, M; \quad t = 1, \ldots, T; \quad x \in X_c \tag{3}$$

$$e_{ct} - e_{yt} \leq H_y \qquad c = 1, \ldots, M; \quad t = 1, \ldots, T; \quad y \in Y_c \tag{4}$$

$$e_{ct} - e_{dt} \leq H_d \qquad c = 1, \ldots, M; \quad t = 1, \ldots, T; \quad d \in D_c \tag{5}$$

Link constraints

Blocks and surfaces are linked together in the formulation by comparing the elevation of each block centroid with the elevation of each surface. Variables x_{ct}^z will assume value 1 only for block centroids that are the closest above the correspondent surface (index t). Constraints (6) guarantee this link and, consequently, guarantee that there is only one block defining the end of each period t over each column c of blocks z.

$$0 \leq \sum_{z=1}^{Z} (E_c^z x_{ct}^z) - e_{ct} \leq \triangle z \qquad c = 1, \dots, M; \quad t = 1, \dots, T \tag{6}$$

Mining constraints

Constraints (7) ensure that the requirements on the mining production (ore and waste) are respected during each period:

$$\underline{T_t} \leq \sum_{c=1}^{M} \sum_{z=1}^{Z} T_c^z (x_{ct}^z - x_{c,t-1}^z) \leq \overline{T}_t \qquad t = 1, \dots, T \tag{7}$$

Variability constraints

Constraints (8) guarantee that processed ore tonnages are not outside lower and upper bounds, $\underline{O_t^R}$ and $\overline{O_t^R}$. These hard constraints guarantee, when there is a solution, that acceptable deviations from production targets are met.

$$\underline{O_t^R} \leq \sum_{c=1}^{M} \sum_{z=1}^{Z} O_{cs}^z (x_{ct}^z - x_{c,t-1}^z) \leq \overline{O_t^R} \qquad s = 1, \dots, S; \quad t = 1, \dots, T \tag{8}$$

Target constraints

Constraints (9) guarantee that expected processed ore tonnages are not outside lower and upper bounds, $\underline{O_t^A}$ and $\overline{O_t^A}$, keeping results close to defined targets. Other constraints over metal and grade, for multiple variables, can be modeled using the same approach.

$$\underline{O_t^A} \leq \frac{1}{S} \sum_{s=1}^{S} \sum_{c=1}^{M} \sum_{z=1}^{Z} O_{cs}^z (x_{ct}^z - x_{c,t-1}^z) \leq \overline{O_t^A} \qquad t = 1, \dots, T \tag{9}$$

Tightening constraints

Constraints (10) are optional and are related to the concept that there is only one block representing the end of a given period for a given column (Figure 2). Link constraints already guarantee this property, but it has been shown empirically that the following constraints may help solving the optimization faster.

$$\sum_{z=1}^{Z} x_{ct}^z = 1 \qquad c = 1, \dots, M; \quad t = 1, \dots, T \tag{10}$$

Variables definition

$$e_{ct} \in \mathbb{R} \qquad c = 1, \dots, M; \quad t = 1, \dots, T \tag{11}$$

$$x_{ct}^z \in \{0, 1\} \qquad c = 1, \dots, M; \quad t = 1, \dots, T; \quad z = 1, \dots, Z \tag{12}$$

3 Alternative Implementation

3.1 Intricacies

The formulation proposed in Section 2 has a specific characteristic that is can be exploited; the binary variables in the formulation are related to surface elevations by constraints (6). When solving the linear relaxation of the formulation previously presented, situations like the ones presented in Figure 3 become feasible.

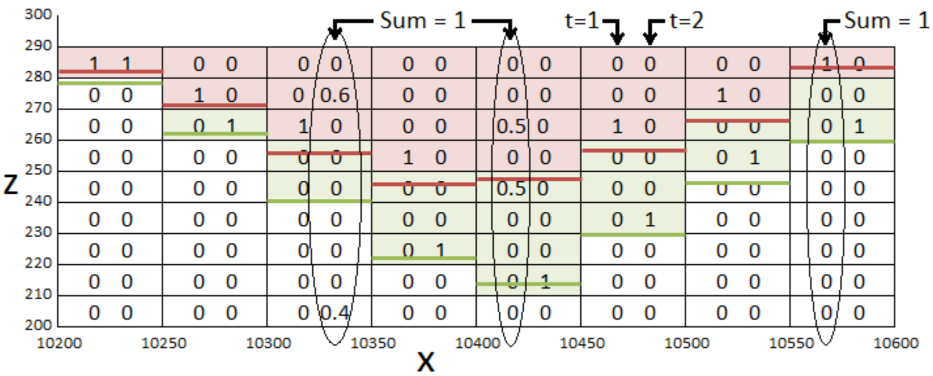

Figure 3: Linear relaxation of binary variables.

Given a cell elevation e_{ct}, any combination of fractional values for variables x^z_{ct} for each c and t that respect (6) and (10) will be part of a valid solution. However, to comply with constraints (6), fractional values must exist both above and below the cell elevation. Establishing strong superior or inferior limits for surface elevations, where blocks are fixed to zero outside, reduces considerably the possible combinations of fractional solutions, which consequently reduces the processing time. Past SIP formulations do not have this characteristic, although they are more efficient for unconstrained cases, as waste blocks can be defined as linear variables (Ramazan and Dimitrakopoulos, 2004). As a result, the formulation presented in this section is only efficient when considered as part of an approach with limiting surfaces.

The number of variables and constraints in the formulation in Section 2 is proportional to the number of periods in the schedule; increasing the number of mining periods results in an increased number of possible solutions, and the solution time using integer programming techniques becomes even slower. The opposite happens when the scheduling process can be first broken into single period optimizations and the subsequent periods are sequentially included in the process, while the previous period(s) is also adjusted. A new period t is added to the optimization process only after the method returns its best solution for $t-1$.The scheduling process starts from the first period and the results from each step are used as reasonable limiting assumptions for the next steps, until the last period is included and the algorithm terminates.

3.2 Sequential Implementation

The sequential implementation of the proposed method considers some engineering aspects of a mine's production schedule. The first objective of the sequential implementation is to find the best available limits when mining a single period. The original topography serves as an upper limit for this single period. For defining reasonable lower limits, realistic constraints should be considered. More specifically, the implementation considers the following.

Bench limits

Mining operations have limits on minimum/maximum benches over a production year, productivity and operational constraints related to long-term plans and mining capacities. For this reason, bench limits are established for each period to define the maximum depth for a surface. For the illustrative example depicted in Figures 4 to 8, periods 1 and 2 are not permitted to mine below the 6^{th} and 3^{rd} benches, respectively, while the third period is left unconstrained.

Fractional periods

Given that it is possible mine many benches over each long-term production period, bench limits are not enough to reduce the complexity of the optimization process. Thus, instead of running the optimization for the entire first period at once, the algorithm runs fractional periods sequentially, rescaling the full period targets and limits. For example, consider all mining targets and limits for quarters of production; the algorithm runs four consecutive smaller schedules, always using the previous period result as an upper limit for the next; the bottom of these four quarters is considered a surface for the entire mining period. The number of fractional periods (four in the given example) is user-defined and unlimited. Fractional periods guide the developments of the mine to regions where short-term periods will also tend to respect defined constraints (exception is the case where the optimization process may need to spend the entire production period stripping). Bench limits can also be defined for each fractional period.

Maximum mining depth

While the bench limit do not allow mining below a fixed elevation, the maximum mining depth parameter defines the maximum distance that each cell of the surface can move, creating a bottom limit parallel to the top. This parameter is valid only for fractional periods, considering that it is operationally impossible to mine deeply in the same location and in a short amount of time. The combination of these parameters (bench limits, number of fractional periods and maximum depth) affects the efficiency of the optimization process, as decisions are taken locally and sequentially. Figure 4 shows an example of two fractional periods considering maximum depth of 20 meters for each. The maximum depth is represented by arrows for each column until reaching the blue limit, 20 meters below the actual surface. Note that, for clarification purposes, only mining blocks and limits are presented in the figure, however all operations are related to cell elevations, which are linear variables and are not required to be at the bottom elevation of the blocks. The bottom of the last fractional period is considered the surface representing the first period of the schedule.

Tolerance range

Until this point, all decisions are taken optimally within the use of local constraints. To assess potential further improvements, a distance range is introduced above and below the first period surface found as the new top and bottom limits, and the optimization is executed again. The algorithm loops over this process until no better solution is found. In this step, the surface is allowed to move, blending blocks from different parts of the mine that are located inside given ranges. Bench limits are still respected in this step. Figure 5 illustrates one loop over this step with a range of 10 meters (dashed lines) over the surface of period 1 (continuous line), where elevations are changed, including two blocks and excluding the other two.

Having the best result allowed by the method for period 1, period 2 is then considered. Instead of creating an optimization of two periods, thereby doubling the number of variables, the optimization is first run for a single period, step by step, until completing the last fractional period, which contains enough material for periods 1 and 2 together. The surface defined for period 1 is considered as a top limit. Results obtained for period 1 are summed with targets for period 2 and fractional periods consider cumulative targets. The result will be a feasible initial solution for the surface of period 2, as illustrated in Figure 6.

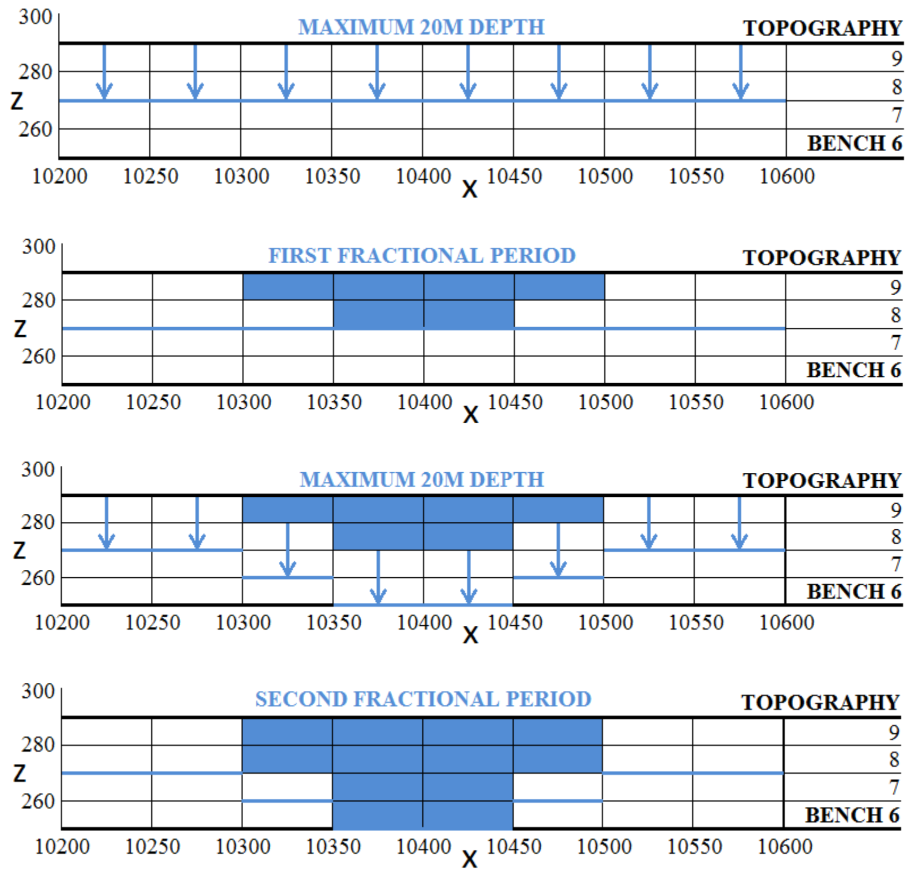

Figure 4: Example of two sequential fractional period optimizations within limits.

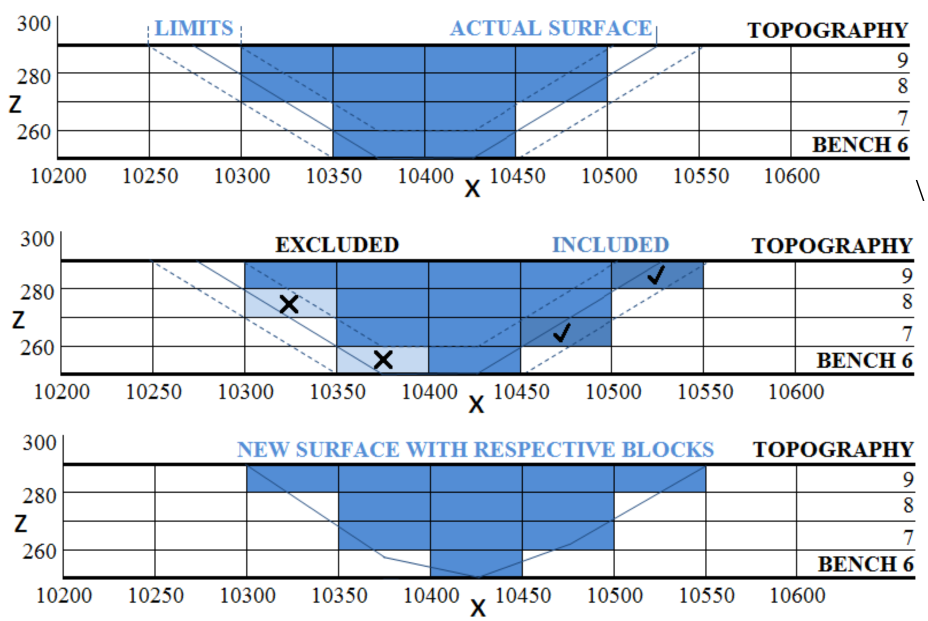

Figure 5: Example of tolerance range (one loop) applied to results of period 1.

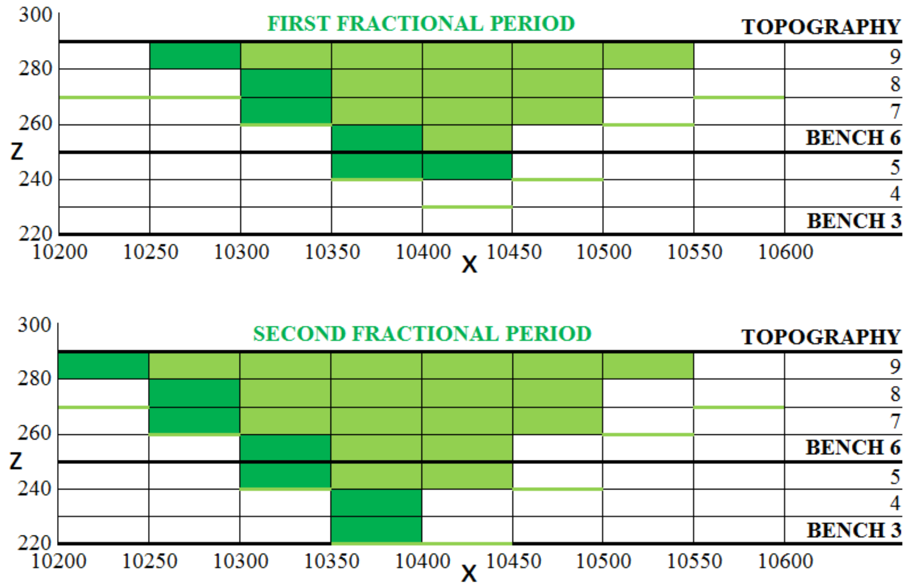

Figure 6: Single optimization process achieving targets for the two first periods.

Simultaneous optimization

Although feasible, single period schedules do not take into account combined economical discounting and possible combinations that could improve expected NPV. The schedule found separately for periods 1 and 2 in previous steps are then combined into a joint optimization for two periods. The tolerance range step is now considered simultaneously to both periods, allowing combined adjustments to increase the expected NPV of the project, as presented in Figure 7. Looking for improvements periodically, while less periods are involved in the optimization, reduces the computational effort to improve results in later steps. Figure 7 presents two blocks swapping periods, allowing the inclusion of a new block in the schedule. The steps presented are repeated until the end of the schedule.

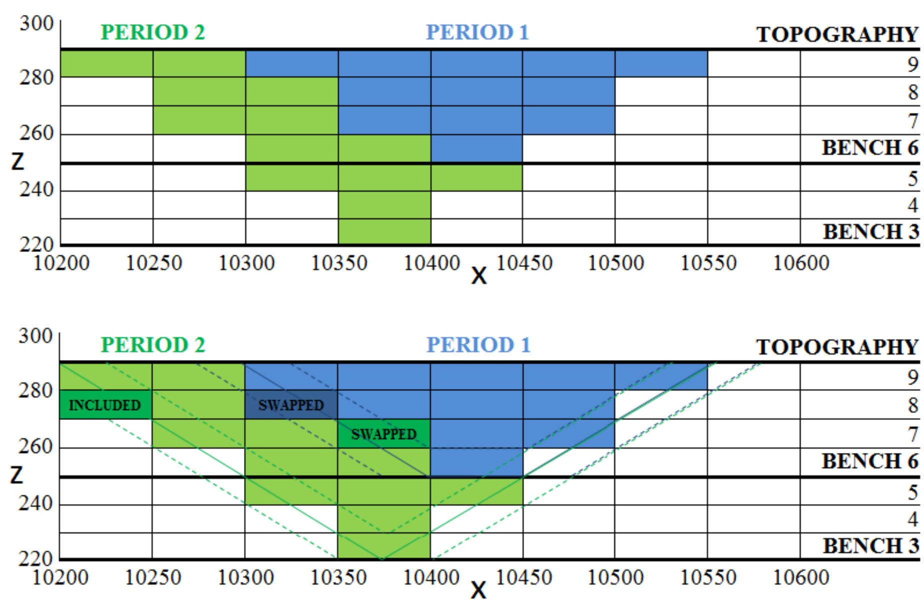

Figure 7: Two-period joint optimization, applying tolerance ranges (one loop) to both.

Larger pit limits

An additional search is performed beyond the limits of the last period to allow additional mining blocks to be included in the schedule. This step is performed by fixing all previous periods and setting as the top limit for running again the last joint optimization, that is, new blocks can be included but not excluded. Figure 8 shows the resulting 3-period schedule, including one extra block.

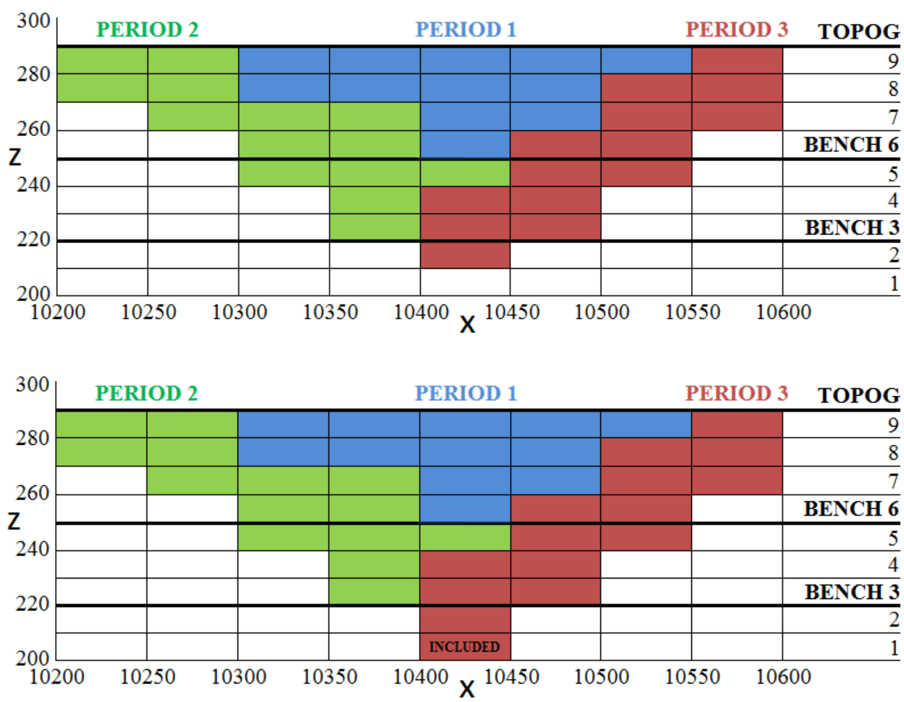

Figure 8: Final schedule after including the third period and looking deeper.

The above implementation considerations focus on generating solutions during the first periods, before evaluating later ones. A summary of the main steps of the sequential implementation is as follows:

1. Define the top limit as the actual topography.
2. Define the bottom limit by eliminating external volumes containing only mining blocks with low probability of being ore. No surface in any period can go beyond these limits.
3. Rescale mining and processing targets and limits according to the number of fractional periods defined.
4. Run fractional periods sequentially, using bench and maximum depth limits, until achieving targets for period 1.
5. Define a tolerance range and rerun for period 1 (not fractional). Loop over this step until no better solution is found.
6. Run fractional periods for period 2, using targets of periods 1 and 2 together and the final surface found in step 5 as top limit.
7. Define tolerance ranges and re-optimize all periods jointly. Loop until there are no further improvements in the solution.
8. Loop over steps 7 and 8 until the last period of the schedule.
9. Freeze all periods except by the last and look for extra blocks to be included.

The sequential implementation of the SIP formulation in Section 2, as discussed above, raises the questions of optimality of the related production sequence. This topic is addressed and explored next through an application.

4 Tests at a Copper Deposit

A copper deposit is used in this case study and is the one described by and simulated in Leite (2006). The orebody model consists of 9953 blocks, each measuring 20 × 20 × 10 m^3, simulated using the direct block simulation method (Godoy, 2003). The parameters considered for scheduling are given in Table 1.

Table 1: General parameters.

Copper price, $/lb	1.9
Selling cost, $/lb	0.4
Mining cost, $/t	1.0
Processing cost, $/t	9.0
Processing recovery	0.9
Discounting rate per period, %	10
Block tonnage, t	10800
Maximum mining production, Mt/year	28.0
Minimum expected ore production, Mt/year	7.45
Maximum expected ore production, Mt/year	7.55
Cutoff grade, %	0.3

The potential annual ore production variability is controlled with a set of hard constraints defined experimentally and assuming a life of mine of 8 years (as known from previous studies (Albor and Dimitrakopoulos, 2010). Table 2 presents the predefined levels of acceptable deviations for yearly production targets used in this study.

Table 2: Accepted deviations from ore production targets.

PERIOD	MINIMUM (Mt)	MAXIMUM (Mt)
1	7.4	7.6
2	7.3	7.7
3	7.2	7.8
4	7.1	7.9
5	7.0	8.0
6	6.9	8.1
7	6.8	8.2
8	*not restricted*	*not restricted*

In the following subsections, 7 variants in scheduling the copper deposit are presented. The application of the "Full SIP" presented in Section 2 is first detailed. Then, Case 1 introduces fractional periods, maximum depth limits and weak bench constraints; Case 2 establishes more reasonable bench limits; Case 3 tests the sensitivity of the maximum depth parameter; and, lastly, Case 4 explores variability constraints.

4.1 Scheduling with the Full SIP

In generating the life-of-mine production schedule for the above copper deposit and information provided in Tables 1 and 2, the results from the optimization of the formulation presented in Section 2, or the "Full SIP", is presented here. Results are summarized in Figures 9 and 10, which report the average forecasts of ore, waste, metal and cumulative cashflows along with the corresponding P10, P50 and P90. The Full SIP results are based on 15 simulated realizations of the deposit (Albor and Dimitrakopoulos, 2009), while the production forecasts reported and their probabilistic assessment is based on 20 simulations. The expected NPV is $277 million, which is 23% higher than the results generated from the conventional best industry practices, as reported in Leite and Dimitrakopoulos (2007) and using the same information and parameters. On average, ore production is close to the specified ore production target and is associated with lower variability over the first two periods. Although not controlled directly in the model, the waste risk profile increases gradually over time. Metal production starts at higher levels at the beginning of the life of mine and decreases in the end, which is expected given that the optimizer looks for more profitable blocks when there is less of an impact from the time value of money (discounting). Figure 10 presents the cumulative NPV for this project showing a 90% probability of achieving at least 254 million dollars.

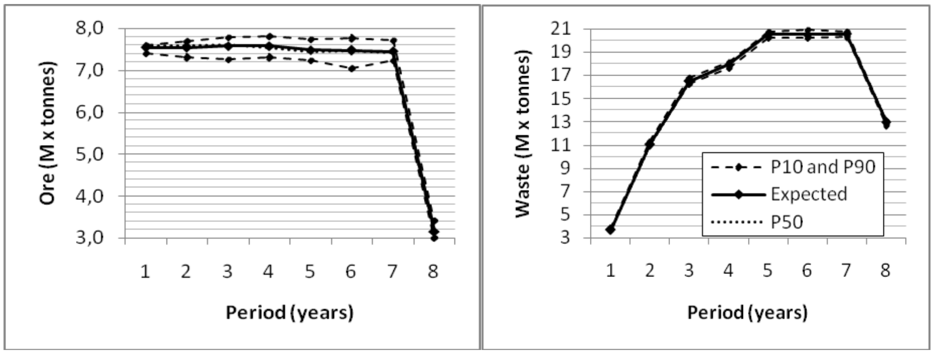

Figure 9: Risk profile for ore and waste tonnages for Full SIP solution.

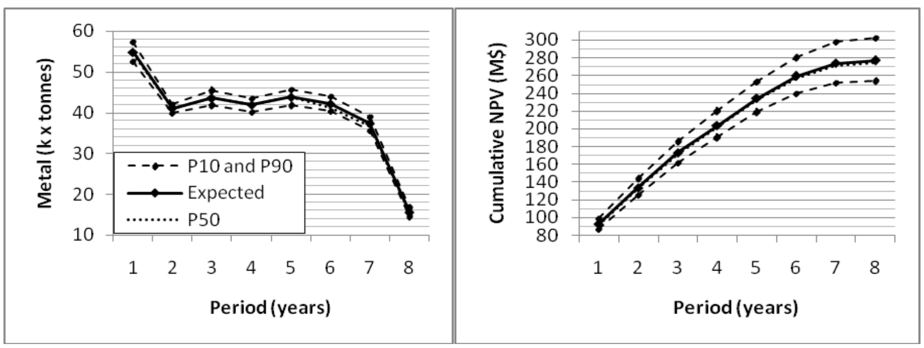

Figure 10: Risk profile for metal tonnages and NPV for Full SIP solution.

Figures 11 and 12 present the coefficient of variation (CV) for ore, waste, metal tonnages, and NPV over mining periods. The presented CV's are very low in general and make a very eloquent point: the spaces of uncertainty being mapped by ore, metal, waste production and cashflows are less than 0.1 and with two exceptions (Figure 12) that are below 0.3. This is distinctly low a CV and, thus demonstrates that the use of larger numbers of simulated realizations of the deposit would not add any additional information to the results for the project indicators of interest. This is distinctly different from the CV of datasets of metal grades from exploration drilling used to model the deposits considered in scheduling studies, which usually have CVs of over 2. Figure 13 shows, in a vertical section, the physical schedule obtained.

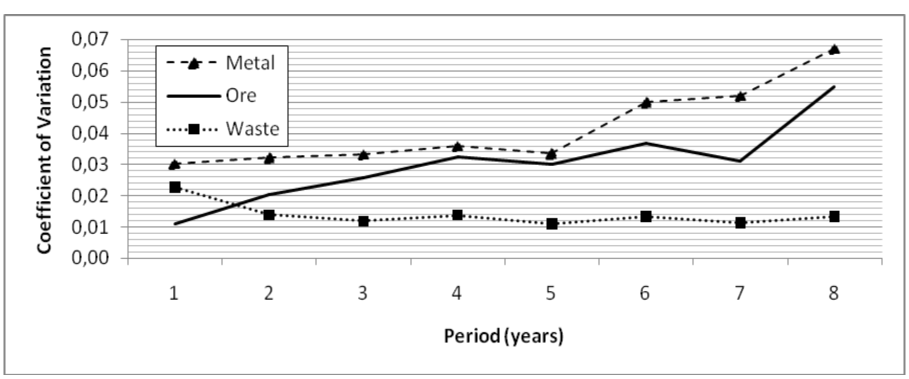

Figure 11: Coefficient of variation of ore, waste and metal over mining periods.

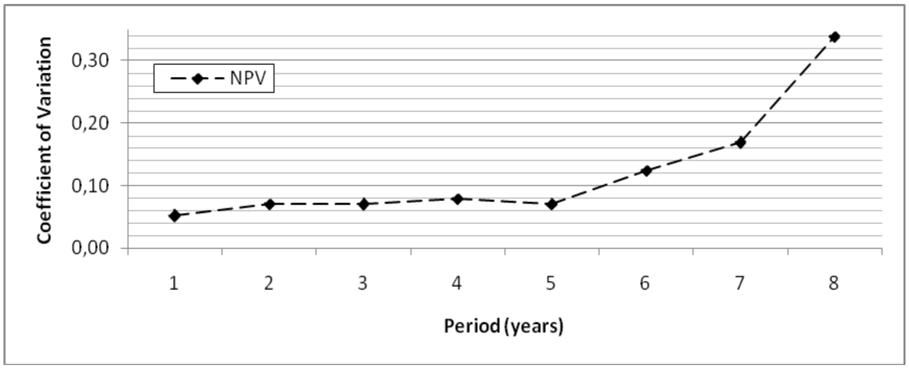

Figure 12: Coefficient of variation of NPV over mining periods.

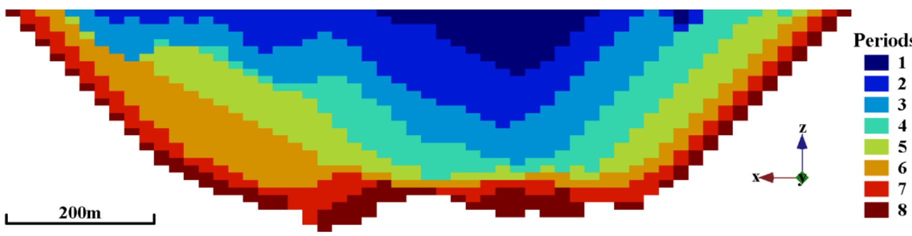

Figure 13: East-West deepest vertical section (N10290) for Full SIP schedule.

4.2 Sequential implementation of the Full SIP

Case 1

Case 1 uses the same parameters from the Full SIP (Tables 1 and 2). Additionally, mining depth is limited to 12 benches per period with 4 fractional periods, with a dynamic maximum mining depth, starting from 20m and increasing to 30m if no solution is found. The resulting schedule appears identical to the schedule generated for the Full SIP, with the exception of two blocks. As a result, the risk profiles and the chosen vertical section are the same as in Figures 9, 10 and 13.

Case 2

In Case 2, bench limits are set to the highest possible level that contains less than 10 mining blocks below it. As a result, the number of benches that are allowed to be mined from periods 1 to 8 have the following bench limit configuration: 9, 6, 5, 3, 2, 1, 1, *free*.

All other parameters are kept the same as in Case 1. All results are quite similar to Case 1, as presented in Figures 14 to 16. The solution indicates that an additional average of 0.4Mt of ore and 3Mt of waste being mined in this case with the same average NPV of 277 million dollars, with 90% chance of being higher than 252 million dollars. Comparing these numbers, vertical sections and risk profiles, the schedules in Cases 1 and 2 are identical for all practical purposes.

Case 3

Regarding the maximum depth parameter, defining deeper limits allows finding better solutions at each step given that more options are available; however, it incurs additional processing time. Case 3 is executed for a fixed maximum depth of 30 meters and uses the same parameters as Case 2. Figures 17 to 19 indicate that there are negligible differences in the risk profiles and vertical sections of the two cases.

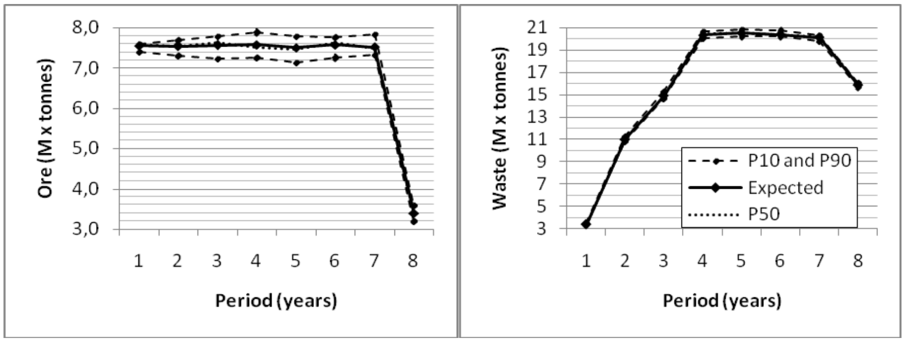

Figure 14: Risk profile for ore and waste tonnages for Case 2.

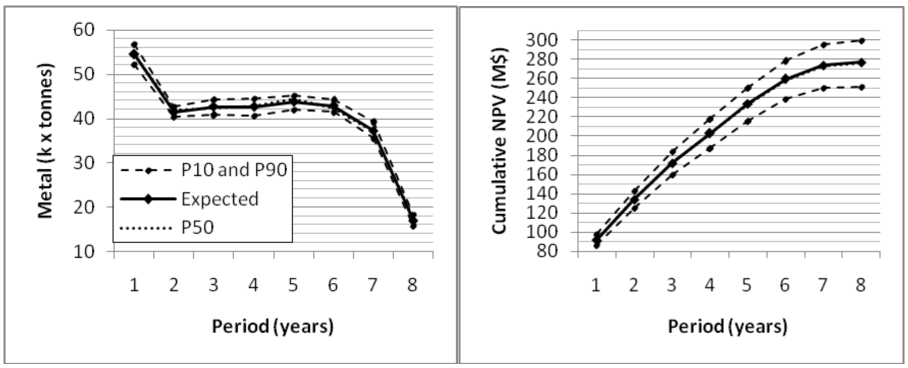

Figure 15: Risk profile for metal tonnages and NPV for Case 2.

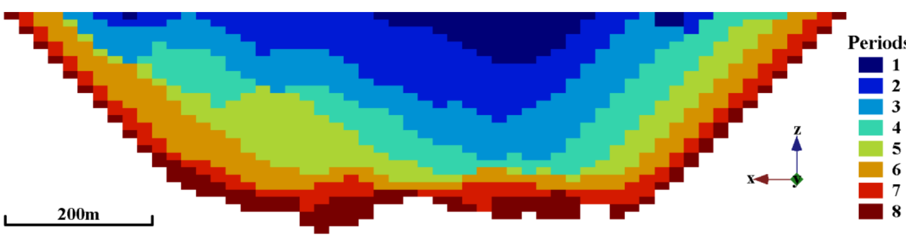

Figure 16: East-West vertical section (N10290) for Case 2 schedule.

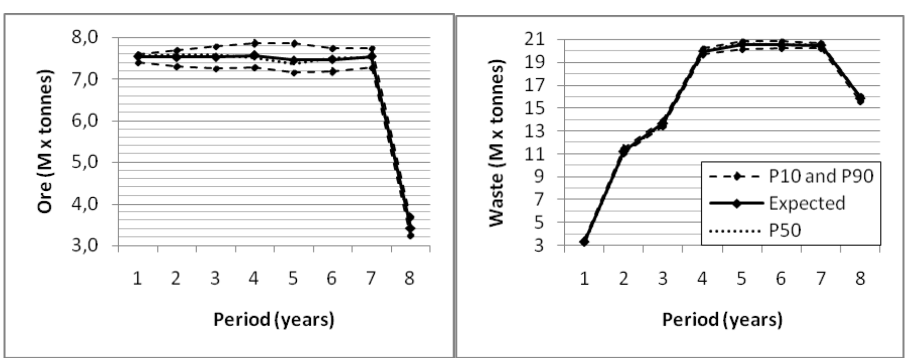

Figure 17: Risk profile for ore and waste tonnages for Case 3.

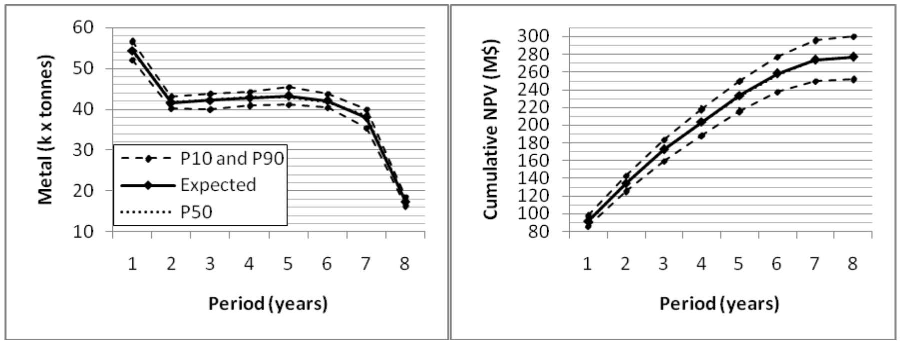

Figure 18: Risk profile for metal tonnages and NPV for Case 3.

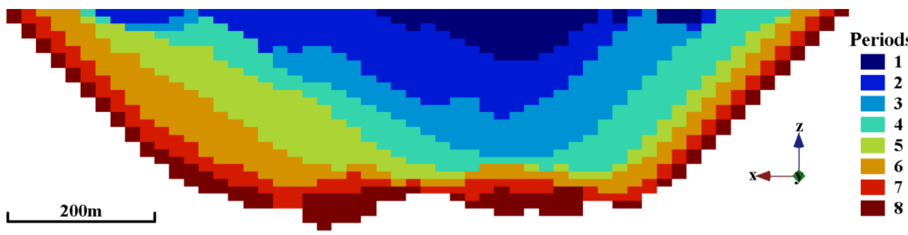

Figure 19: East-West vertical section (N10290) for Case 3 schedule.

Case 4

Case 4 presents results when experimenting with constraining the amount of acceptable risk (ore production variability) through the constraints presented in Eq. (8). The parameters used are shown in Table 3. Results obtained show similar physical schedule and risk profiles when compared to previous cases, however, there is minimally improved control in the ore production variability, which increases over time, as presented in Figures 20 to 22.

Table 3: Maximum and minimum ore production accepted for Case 4.

PERIOD	MINIMUM (Mt)	MAXIMUM (Mt)
1	7.45	7.55
2	7.40	7.60
3	7.35	7.65
4	7.30	7.70
5	7.25	7.75
6	7.20	7.80
7	*not restricted*	*not restricted*
8	*not restricted*	*not restricted*

4.3 Comparisons

All cases present similar physical schedules for practical purposes. This shows that, in this case study, the proposed approach converges to a similar solution, regardless of when different parameters are used. There are, however, differences in computing times for the various cases. The following reporting is based on using a 64-bit computer with Intel® Core™ i7-2600S CPU @ 2.80GHz, 8Gb RAM, 8 processors and using CPLEX 12.4. The proposed alternative sequential implementation reduces substantially the processing times. Results for Cases 1 to 4 are given in Table 4.

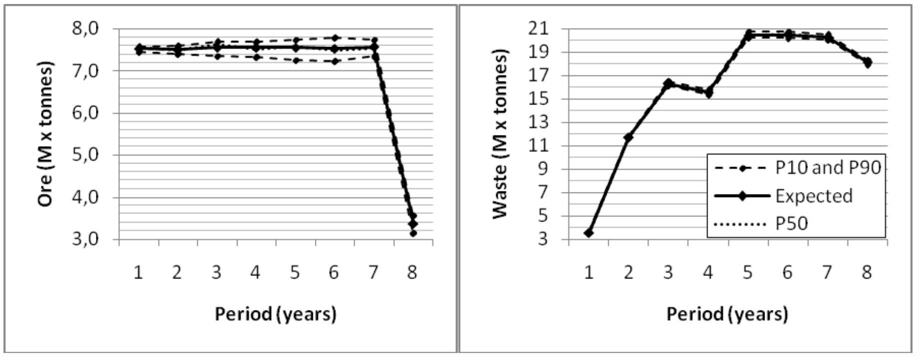

Figure 20: Risk profile for ore and waste tonnages for Case 4.

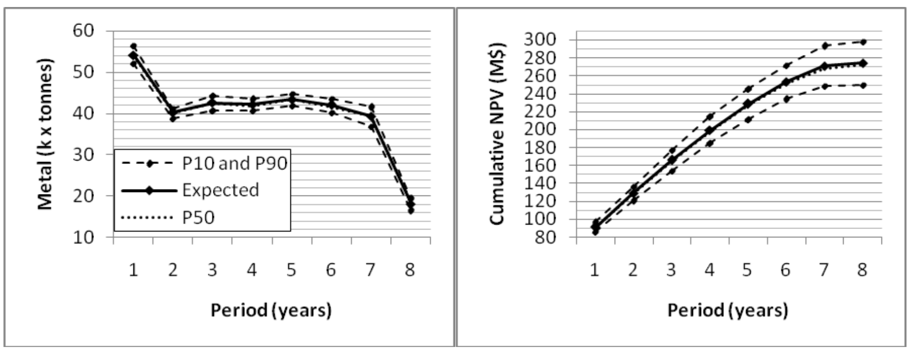

Figure 21: Risk profile for metal tonnages and NPV for Case 4.

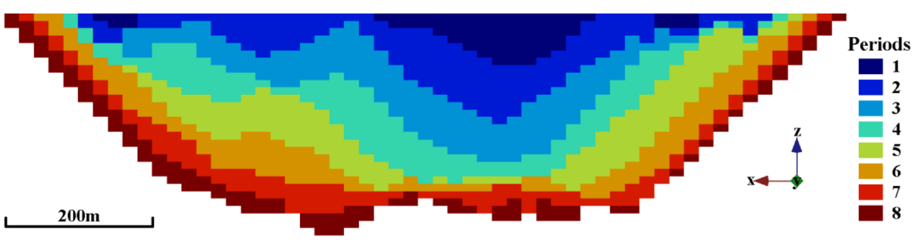

Figure 22: East-West vertical section (N10290) for Case 4 schedule.

Table 4: Binary variables and processing times for Cases 1 to 4, and Full SIP.

CASE	MAX NUMBER OF BINARIES	PROCESSING TIME
Full SIP	163276	˜3 days
1	11265	31 minutes
2	11813	25 minutes
3	9939	29 minutes
4	9030	24 minutes

5 Conclusions

The formulation proposed in this study introduces the use of limiting surfaces in the context of SIP for mine production scheduling optimization, adding the benefits of better slope angle management. The proposed sequential implementation divides the scheduling problem to smaller sub-problems using surfaces found in single optimization processes, based on the engineering aspects of multi-period scheduling. Fractional periods are defined to increase efficiency and guide the developments of the mine to regions where short-term periods also tend to respect constraints. Each multi-period solution is further optimized by allowing iterative changes in surface elevations of all periods simultaneously. The case study at a copper deposit shows the generation of equivalent physical schedules for different parameters. The expected NPV is similar to previous stochastic scheduling approaches and more than 23% higher than the schedule reported obtained in a related conventional (deterministic) production scheduling study. The proposed method can be used to generate results in approximately 12 minutes of processing time, against days for the full SIP case.

References

Albor, F., Dimitrakopoulos R. (2009) Stochastic mine design optimization based on simulated annealing: Pit limits, production schedules, multiple orebody scenarios and sensitivity analysis. IMM Transactions, Mining Technology, 118(2), 80–91.

Boland, N., Dumitrescu, I., Froyland, G. (2008) A multistage stochastic programming approach to open pitmine production scheduling with uncertain geology. Optimization Online www.optimizationonline.org/DBHTML/2008/10/2123.html.

Dimitrakopoulos, R., Farrelly, C., Godoy, M.C. (2002) Moving forward from traditional optimisation: Grade uncertainty and risk effects in open pit mine design. Transactions of the IMM, Section A Mining Industry, 111, A82–A89.

Dimitrakopoulos, R., Martinez L., Ramazan, S. (2007) A maximum upside / minimum downside approach to the traditional optimization of open pit mine design. Journal of Mining Science, 43(1), 73–82.

Dimitrakopoulos, R., Ramazan, S. (2004) Uncertainty based production scheduling in open pit mining. SME Transactions, 316, 106–112.

Dowd, P.A. (1997) Risk assessment in reserve estimation and open-pit planning. Transactions of the Institution of Mining and Metallurgy, 103, A148–A154.

Godoy, M. (2003) The efficient management of geological risk inlong-term production scheduling of open pit mines, PhD thesis, University of Queensland, Brisbane.

Godoy, M., Dimitrakopoulos, R. (2004) Managing risk and waste mining in long-term production scheduling. SME Transactions, 316, 43–50.

Goodfellow, R., Dimitrakopoulos, R. (2012) Mining supply chain optimization under geological uncertainty. COSMO Research Report No. 6, McGill University, Montreal.

Goodwin, G.C., Seron, M.M., Middleton, R.H., Zhang, M., Hennessy, B.F., Stone, M.S., Menabde, M. (2005) Receding horizon control applied to optimal mine planning. Automatica, 42(8), 1337–1342.

Jewbali, A., Dimitrakopoulos, R. (2009) Stochastic mine planning: Example and value from integrating long- and short-term mine planning through simulated grade control. In proceedings, Orebody Modelling and Strategic Mine Planning 2009, AusIMM, 327–334.

Lamghari, A., Dimitrakopoulos, R. (2010) Metaheuristics for the open pit mine production scheduling problem with uncertain supply. COSMO Report No. 4, McGill University, Montreal, pp. 110–149.

Leite, A. (2008). Application of stochastic mine design and optimization methods. MEng Thesis, McGill University, Montreal.

Leite, A., Dimitrakopoulos, R. (2007) Stochastic optimization model for open pit mine planning: Application and risk analysis at a copper deposit. Transactions of the Institution of Mining and Metallurgy: Mining Technology, 116(3), 109–118.

Leite, A., Dimitrakopoulos, R. (2010) Production scheduling under metal uncertainty – Application of stochastic mathematical programming at an open pit copper mine and comparison to conventional scheduling. The Australasian Institute of Mining and Metallurgy, Spectrum Series, 17.

Menabde, M., Froyland, G., Stone, P., Yeates, G. (2007) Mining schedule optimisation for conditionally simulated orebodies. The Australasian Institute of Mining and Metallurgy, Spectrum Series, 14.

Padberg, M., Rinaldi, G. (1991) A Branch-and-Cut algorithm for the resolution of large-scale symmetric travelling salesman problems. Society for Industrial and Applied Mathematics, SIAM Review, 33(1), 60–100.

Ramazan, S., Dimitrakopoulos, R. (2004) Recent applications of operations research in open pit mining. SME Transactions 316, 73–78.

Ramazan, S., Dimitrakopoulos, R. (2007) Stochastic optimization of long-term production scheduling for open pit mines with a new integer programming formulation. In proceedings, Orebody modelling and strategic mine planning: Uncertainty and risk management models, AusIMM Spectrum Series 14, 2nd Edition, pp. 385–392.

Ramazan, S., Dimitrakopoulos, R. (2012) Production scheduling with uncertain supply: a new solution to the open pit mining problem. Optimization and Engineering, 13, 1–20.