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Scheduling Issues in Vehicle Routing

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Scheduling Issues in Vehicle Routing

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Abstract: Scheduling often plays an important role in vehicle routing. This paper describes several applications in which the author has been involved in recent years. These arise in the dial-a-ride problem, speed optimization in routing problems, pollution-routing problem, long-haul vehicle routing and scheduling with working hour rules, and synchronization in arc routing.

Key Words: vehicle routing, scheduling, arc routing, time windows, speed.

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1 Introduction

The aim of this paper is to describe several classes of routing problems in which time and scheduling play an important role. There exist numerous occurrences of such problems. Our intention is not to provide an exhaustive survey or classification of all conceivable cases, but rather to describe examples of interesting problems on which the author and some of his colleagues and students have worked in recent years.

Most research in vehicle routing is rooted in the study of the classical *Vehicle Routing Problem* (VRP) which consists of determining vehicle routes through a set of geographically scattered customers, subject to various side constraints. The standard objective is distance (or cost) minimization. The problem was introduced by Dantzig and Ramser (1959) and its study has since evolved into a rich research area (see, e.g., Toth and Vigo 2002; Golden, Raghavan and Wasil 2008; Laporte 2009). The most common objective in the VRP is distance minimization. When an upper limit is imposed on route duration, it is common to equate distance with travel times. There exist, however, several situations where distance and time must be treated separately. For example, in the *Vehicle Routing with Time Windows* (VRPTW), an interval is imposed on the time at which service must start at any given location. If the vehicle reaches a location before the opening of its time window, it must then wait. In such a context, traveled distance and route duration are clearly not equivalent.

The VRPTW can now be solved efficiently for relatively large instances. For example, Baldacci, Mingozzi and Roberti (2012) have developed an exact algorithm based on a set partitioning formulation of the problem, combined with the generation of valid inequalities, and capable of solving all but one of the Solomon (1987) instances involving 50 and 100 customers; Nagata, Bräysy and Dullaert (2010) have designed a highly efficient memetic algorithm which outperforms all known heuristics on the Solomon instances. It has improved 184 best-known solutions out of 356 instances comprising 56 Solomon instances and 300 instances introduced by Gehring and Homberger (1999).

The remainder of this paper is organized as follows. Sections 2 to 6 are respectively devoted to the following topics: the dial-a-ride problem, speed optimization, in routing problems, the pollution-routing problem, long-haul vehicle routing and scheduling with working hour rules, synchronization in arc routing. Conclusions follow in Section 7.

2 The dial-a-ride problem

With the aging of the population, an increasing number of cities are organizing and managing dial-a-ride services for elderly and handicapped people. In the dial-a-ride problem (DARP), n users specify transportation requests (v_i, v_{i+n}) between origins v_i and destinations v_{i+1}. These requests are fulfilled by a fleet of capacitated vehicles based at a depot v_0. A travel cost c_{ij} and a travel time t_{ij} are associated with each arc (v_i, v_j). Vehicle routes have maximal durations, and the time a user can spend in the vehicle is subject to a so-called ride time constraint. Users frequently make two trips during the same day: an outbound trip from home to a destination (e.g., a hospital), and an inbound trip back home. Users typically specify a time window on their desired arrival time at the destination and another time window on their desired pickup time from the destination. Depending on the context, the aim is either to satisfy all requests at least cost, using the required number of vehicles, or to maximize the number of satisfied requests with a given vehicle fleet. For surveys of DARP applications, models and algorithms, see Cordeau and Laporte (2007) and Parragh, Doerner and Hartl (2008a,b).

2.1 A tabu search algorithm for the DARP

The tabu search algorithm of Cordeau and Laporte (2003) exploits the scheduling issues encountered in the DARP. These are related to route duration, time windows and ride time.

As in the VRPTW, minimizing travel cost (or distance) or travel time may yield different results because waiting at the vertices is allowed. To illustrate this difference, consider the three-vertex example depicted in Figure 1. All arcs have unit costs and travel times, and there are no service times. Time windows are shown

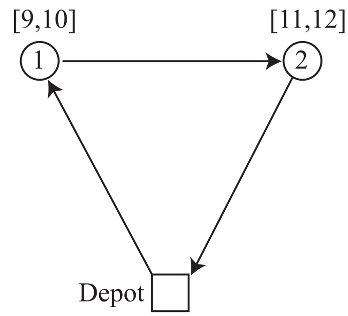


Figure 1: Single-request instance showing the effect of scheduling in the dial-a-ride problem

above the vertices. If the vehicle leaves the depot at 8h, it will arrive at vertex 1 at 9h, at vertex 2 at 10h and will be back at the depot at 12h. If it leaves the depot at 9h, it will arrive at vertex 1 at 10h but will still be back at the depot at 12h, which yields a one-hour trip reduction. This illustrates that arriving at a vertex as early as possible may lead to missing some feasible solutions (for example, if the maximum route duration is three hours, like here). The time F_i by which service at vertex v_i can be delayed without causing any time window violation is called the forward time slack. As shown by Savelsbergh (1992), given a route $(v_0, \ldots, v_q)$ it is computed as

$$F_i = \min_{i \leq j \leq q} \left\{ \sum_{i<p\leq j} \max\{0, a_p - A_p\} + (b_j - B_j) \right\}, \tag{1}$$

where $[a_i, b_i]$ is the time window at vertex v_i, A_i is the vehicle arrival time at v_i and $B_i \geq \max\{a_i, A_i\}$ is the time at which service starts.

Cordeau and Laporte (2003) have solved the DARP by means of a tabu search algorithm which iteratively removes a request from a vehicle route and inserts it into another one. Reinserting the request in its original route is tabu for a certain number of iterations. As is common in tabu search, the algorithm allows intermediate infeasible solutions. It does so by minimizing the function

$$f(s) = c(s) + \alpha q(s) + \beta d(s) + \gamma w(s) + \tau t(s), \tag{2}$$

where $c(s)$ is the routing cost of solution s, $q(s)$, $d(s)$, $w(s)$, and $t(s)$ denote the total violations of load, duration, time window and ride time constraints in solution s, and α, β, γ, and τ are self-adjusting positive coefficients Because of this feature, the authors modify formula (1) in order not to increase violations of time window and ride time constraints when seeking a neighbour solution. The computation of the forward time slack then becomes

$$F_i = \min_{i \leq j \leq q} \left\{ \sum_{i<p\leq j} W_p + (\min\{b_j - B_j, L - P_j\})^+ \right\}, \tag{3}$$

where $W_i = B_i - A_i$ is the waiting time at vertex v_i, L is the maximum ride time, $(x)^+ = \max\{0, x\}$, and P_i is the ride time of the user whose destination is v_i if $n+1 \leq i \leq 2n$, and $P_i = 0$ otherwise. It is also possible to show that the minimal route duration that does not increase constraints violations is given by $A_q - (a_0 + \min\{F_0, \sum_{0<p<q} w_p\})$.

Using these formulas, the authors have designed three versions of their algorithm, with increasingly complex neighbourhood exploration mechanisms:

V1: minimize time window violations;

V2: minimize route duration without increasing time window violations;

V3: minimize ride times by delaying the beginning of service at each origin vertex as much as possible without increasing route duration, time window or ride time violations.

The three versions of the algorithm were tested on 20 artificial instances and on six real instances from Denmark. Going from V1 to V3 gradually improves solution quality but also increases execution time. It was found that V2 offers the best compromise between computation time and solution quality.

2.2 Solution cost vs quality of service

A survey conducted by Paquette (2010) and also described in Paquette et al. (2012a) shows that users are highly sensitive to scheduling issues in DARP operations. Questionnaires were sent to 857 users of dial-a-ride services in Longueuil, a suburb of Montreal. The overall response rate was 38.6%. This survey helped identify three quality indicators reflecting some of the main concerns of the service users:

Q1: the inconvenience related to the arrival time at the destination of an outbound trip;

Q2: the waiting time at the beginning of an inbound trip;

Q3: the ratio between the actual time of a ride and the time corresponding to that of the most direct trip.

Using the survey responses, these indicators were modeled empirically as follows:

$$\mathbf{Q1} = ((b_i - a_i)/2 - B_i)^2$$
$$\mathbf{Q2} = (B_i - a_i)^2$$
$$\mathbf{Q3} = (L_i/t_{i,i+n})^2,$$

where a_i, b_i, B_i, t_{ij} have already been defined, and L_i is the ride time of a trip originating at v_i. The first indicator reflects the fact that users prefer to arrive around the middle of the time interval associated with the destination of an outbound trip. Variations around this mid-point are penalized quadratically. In contrast, for an inbound trip, users prefer to leave the origin as early as possible. Any delay is also penalized quadratically. Finally, the third penalty squares the ratio of the actual travel time between an origin and the destination to the travel time corresponding to a direct trip.

These three indicators were incorporated by Paquette (2010) and by Paquette et al. (2012b) within a multi-objective tabu search algorithm partly based on that of Cordeau and Laporte (2003). Solution quality was assessed by means of the hypervolume and the multiplicative unary epsilon indicators (Zitzler et al., 2003; Zitzler, Thiele and Bader, 2010). The algorithm was tested on 10 of the random instances of Cordeau and Laporte (2003) and on 12 real instances corresponding to data observed in Longueuil. Computational results showed that several non-dominated solutions could be generated within reasonable computing times. They also enabled an analysis of the possible trade-offs between solution cost and each of the three quality indicators Q1, Q2, and Q3. For example, it was found that if the three quality indicators were merged into a single one, an increase of one unit in user inconvenience yielded a cost saving of $0.31; similarly, a one dollar cost increase yielded a decrease of 2.28 units in user inconvenience.

2.3 The dynamic dial-a-ride problem

Two versions of the DARP can be defined according to the time at which requests are logged in. In the static version, which is the most common in practice, all requests are known at the time of planning; in the dynamic version, they are made in real-time, while vehicles operate. In practice, these two versions of the problem rarely exist in their pure form: static instances contain a certain degree of dynamism since last-minute cancellations often occur, and dynamic instances typically contain a certain number of known requests at the start of the day. An algorithm for the dynamic DARP must first decide whether or not to accept a new request. When a request is rejected, it cannot be reconsidered at a later time; when it is accepted it must be inserted in the best possible position in the current partial solution. The problem studied by Berbeglia, Pesant and Rousseau (2011, 2012) and by Berbeglia, Cordeau and Laporte (2012) concerns the real-time insertion of requests in partial routes. The latter study describes a hybrid heuristic combining tabu search (TS) and constraint programming (CP). Tabu search is rather good at optimally inserting a new request in a partial solution that is not too constrained. In contrast, constraint programming can quickly determine whether there exists a feasible solution capable of incorporating an incoming request. In the proposed implementation, TS and CP run in parallel from the moment the request is lodged. The request is accepted only when either algorithm identifies a feasible solution; it is rejected whenever CP proves the infeasibility of the request or after a preset computing time.

This study also defines and compares three strategies for inserting a new request in a partial solution, called basic scheduling, lazy scheduling and eager scheduling. The basic scheduling algorithm always serves a vertex as soon as possible. As shown by Cordeau and Laporte (2003), and as illustrated in Figure 1, this policy may cause the algorithm to miss some feasible solutions. The lazy algorithm transforms a solution into another one which minimizes the ride time violation of every request without increasing any time window violation. It does so by delaying the start of service at each vertex without increasing constraint violations. The eager scheduling algorithm transforms a solution into another one that minimizes the start of service at each vertex without increasing the time window violation of any vertex or the ride time violation of any request. In each of these three algorithms, it is preferable to delay as much as possible the departure of the vehicle from every vertex in order to create empty time periods that may be later used to feasibly insert future requests.

The algorithms just described were tested on a set of 28 static instances of Ropke, Cordeau and Laporte (2007) and on a set of 20 static instances of Cordeau and Laporte (2003). These were then transformed into dynamic instances. Results show the superiority of the eager algorithm over the lazy algorithm in that it usually accepts more requests. (The basic algorithm which is clearly dominated by the other two was not tested.) They also show that CP is sometimes able to accept or reject incoming requests, but TS tends to accept feasible requests faster. This shows that the hybrid algorithm is superior to either CP or TS executed alone. The capability of CP to prove infeasibility is highly related to the tightness of an instance.

3 Speed optimization in routing problems

Speed rarely appears as a decision variable in routing and scheduling problems. In problems like the DARP, driving at a higher speed can of course make it easier to identify feasible solutions. However, we are not aware of any vehicle routing study in which speed control is used to this end. The recent studies related to speed control have rather advocated speed reductions as a means of reducing costs and CO_2 emissions. This is particularly important in today's economic and environmental context.

Sea shipping is a transportation activity for which speed reduction can generate substantial savings. To illustrate this impact, it is estimated that with a fuel price of 450 USD/tonne, a one percent worldwide reduction in fuel consumption would yield a cost reduction of more than 1.2 billion USD, and a reduction of CO_2 emissions of 10.5 million tonnes (Hvattum et al., 2013). A computational study performed by Norstad, Fagerholt and Laporte (2011) has shown that under reasonable assumptions, optimizing speed on a ship route can reduce fuel consumption by as much as 14%.

The paper by Fagerholt, Laporte and Norstad (2010) proposes some speed control models and algorithms for sea transport. Put simply, CO_2 emissions are proportional to fuel consumption, which is a convex and increasing function of speed over some domain. Because of cost convexity, on a route with time windows it is suboptimal to sail at high speed if this means arriving at a vertex before the opening of its time window. It is always better to sail slower and not have to wait. The paper considers a fixed route $(v_0, \dots, v_n)$ where a time window $[a_i, b_i]$ is imposed on the start of service time at each vertex v_i. The total route duration is given, i.e., $a_0 = b_0$ and $a_n = b_n$. Using a discretization of time, this problem can readily be cast as a shortest path problem on a directed acyclic graph.

In a follow-up study, Hvattum et al. (2013) have proposed an exact $O(n^2)$ algorithm for this problem. It is valid as long as travel cost is a convex function of speed, and it applies to sea or ground transportation. Initially, the algorithm computes the speed necessary to reach v_n at time a_n. It then identifies the largest time window violation on the path $(v_0, \dots, v_n)$. For example, suppose this maximum violation corresponds to a late arrival at vertex v_i ($t_i > b_i$). Then the algorithm recomputes a higher speed on the subpath $(v_0, \dots, v_i)$ with an arrival time of b_i and a slower speed on the subpath $(v_i, \dots, v_n)$ with a departure time of b_i. It reiterates in a similar way on each subpath until no time window violations remain. To illustrate, consider the graph of Figure 2 depicting a ship route between vertex 0 and vertex 4. The distances (in

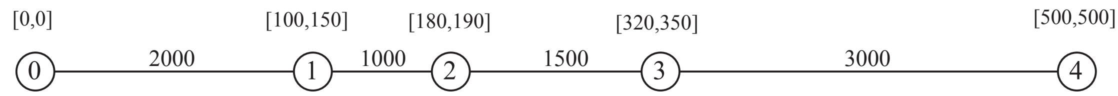


Figure 2: Ship route with time windows

nautical miles) are shown on the edges and the time windows (in hours) are shown on the vertices. To travel the total distance at constant speed, the ship must cruise at 15 knots (nautical miles/hour). At this speed, the arrival times at the vertices 1, 2, 3, and 4 are 133.33, 200, 300 and 500, respectively. The largest time window violation is at vertex 3. At the next iteration, the speed is decreased to 14.0625 knots on (0,1,2,3) and increased to 16.67 knots on (3,4). These speeds yield arrival times of 142.22, 213.33, 320 and 500 at the vertices. The only time window violation now occurs at vertex 2. The speed is then increased to 15.79 knots on (0,1,2) and decreased to 11.538 knots on (2,3). This yields a feasible and optimal solution with arrival times 126.66, 190, 320 and 500 at the four vertices.

In Section 4, we show how speed optimization can be used as a tool to reduce CO_2 emissions in freight transportation by truck.

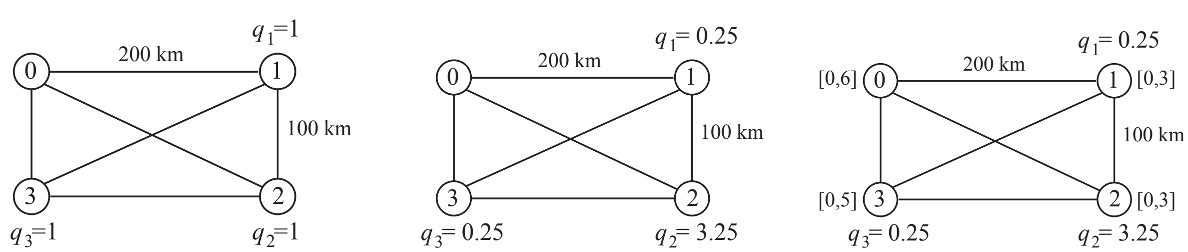
4 The pollution-routing problem

Freight transportation accounts for much of the CO_2 emissions in most countries. To illustrate, it is estimated that in the United Kingdom, freight transportation produces 21% of the emissions from the transportation sector, and 6% of the total emissions (McKinnon, 2007). The paper by Bektas and Laporte (2011) is one of the first to study CO_2 emissions in the context of vehicle routing. According to Barth, Younglove and Scora (2005) and Barth and Boriboonsomsin (2009), the energy consumed by a vehicle of a given type on a flat road segment can be approximated by the formula

$$\text{Energy } = A \text{ (load} \times \text{distance) } + B \text{ (speed}^2 \times \text{distance)} , \tag{4}$$

where A and B are positive constants. These two constants include technological factors which allow both terms to be measured in energy units (kWh). Speed is assumed to be above a certain threshold of about 40 km/h since it usually becomes less efficient to drive a commercial vehicle below this speed (see Bektas and Laporte 2011). The amount of CO_2 emissions produced by a vehicle in motion is more or less proportional to the amount of energy it consumes.

Through the development of a mathematical model and various examples, Bektas and Laporte (2011) illustrate various interactions between vehicle routing solutions. Consider first a three-ton vehicle traveling at a constant speed of 40 km/h on one of the graph depicted in Figures 3(a) or 3(b), showing edge lengths and delivery quantities in tons at the vertices. It is clear that traversing these graphs in the direction (0,1,2,3,0) or (0,3,2,1,0) yields the same traveled distance of 600 km. However, in graph 3(a), the first solution will yield a total weighted load of 2800 kmt, whereas the second solution will yield a smaller weighted load of 2600



(a) Equal delivery quantities (b) Unequal delivery quantities (c) Unequal delivery quantities and time windows

Figure 3: Graphs illustrating the effect of distance and weighted load on a solution

kmt. This is so because smaller loads are now carried on the longer edges. The latter solution is also optimal with respect to weighted load. On the graph of Figure 3(b), one can readily check that the first distance minimizing solution (0,1,2,3,0) yields a weighted load of 2950 kmt, whereas the longer solution (0,2,1,3,0) yields a smaller weighted load of 2886.07 kmt.

These examples were constructed with constant speed and without time windows. When time windows are imposed on arrival times at the vertices, it is easy to optimize speed on a given route by applying the Hvattum et al. (2013) algorithm described in Section 3. For example consider the time windows (in hours) shown in Figure 3(c). In Equation (4), we use the two realistic coefficient values $A = 0.05$ and $B = 0.00007$. On the optimal weighted load solution (0,2,1,3,0), the optimal speeds are 109.44 km/h on (0,2,1,3) and 100 km/h on (3,0). This yields an energy consumption of 673.08 kWh, which is not optimal since the distance minimizing solution (0,1,2,3,0) has a smaller energy consumption of 567.5 kWh, obtained by always traveling at 100 km/h.

The latter example clearly illustrates the effect of speed and scheduling on the minimization of CO_2 emissions in freight transportation. It is relatively straightforward to design mathematical models that will minimize energy consumption for one or several vehicles, with an objective function such as (4), and under the presence of time windows (Bektas and Laporte, 2011). However, such models cannot be solved exactly for medium size instances, even for a single vehicle. Demir, Bektas and Laporte (2012) have recently designed and tested and adaptive large neighbourhood search heuristic (Pisinger and Ropke, 2007) applicable to instances of realistic sizes.

In practice, it may be difficult to convince carriers to design routes under an energy minimization objective if this yields solutions more costly than those obtained under a cost minimization objective. As a rule, fuel consumption is minimized by driving slowly, which has an adverse effect on driving times and wages. Bektas and Laporte (2011) have shown through some computational experiments that this conclusion still holds when CO_2 costs and fuel costs are taken into account (and are even increased sharply) since drivers' costs tend to dominate the total solution cost. It is possible to minimize a global function combining operating costs and CO_2 emissions by means of an adaptive large neighbourhood search algorithm, for example (Demir, Bektas and Laporte, 2012).

5 Long-haul vehicle routing and scheduling with working hour rules

A complex routing and scheduling problem with multiple time windows arises in long-haul vehicle routing with work hour rules. The precise rules are different in settings like the European Union (e.g., Goel 2009; Kok et al. 2010; Kok, Hans and Schutten 2011; Prescott-Gagnon, Drexl and Rousseau 2010), Canada (Goel and Rousseau, 2011), Australia (Goel, Archetti and Savelsbergh, 2012), and the United States (Archetti and Savelsbergh 2009; Ceselli, Righini and Salani 2009; Goel and Kok 2012; Rancourt, Cordeau and Laporte 2013). The following description is based on the latter study.

The problem consists of optimally designing vehicle routes for a set of customers dispersed over a large territory. The routes may last several days and each customer must be visited once within one of several time windows. For example, each may have one time window per day. Standard vehicle routing constraints such as vehicle capacity and maximum route duration also apply, and the vehicle fleet is often heterogeneous. When visiting customers, drivers are subject to various rules on *driving time*, *on-duty time* (performing a task for the employer, including driving), *off-duty time* (free time) and *sleeper berth time* (time spent sleeping in the vehicle). Table 1, which combines Tables 2 and 3 of Rancourt, Cordeau and Laporte (2013), illustrates the intricacies of the working hour rules. It provides examples of five rules relative to driving time, on-duty time and sleeper berth time.

This problem was solved by Rancourt, Cordeau and Laporte (2013) by means of a tabu search heuristic in which the objective is the minimization of a linear combination of vehicle fixed costs and routing costs. At each iteration the heuristic moves a customer to a different route. It allows intermediate infeasible solutions with respect to vehicle capacity, the on-duty 70-hour limit, and time windows. As in Cordeau and Laporte (2003), constraint violations are multiplied by self-adjusting positive weights. Two versions of the algorithm

Table 1: Working hour rules in the United States

60/70-hour on-duty limit	A driver cannot drive after 70 hours on-duty in seven/eight consecutive days. He may restart an eight consecutive day period after 34 or more consecutive hours off-duty.
11-hour driving limit	A driver may only drive a maximum of 11 hours after 10 consecutive hours off-duty.
14-hour limit	A driver cannot drive beyond the 14$^{\text{th}}$ consecutive hour after coming on-duty, following 10 consecutive hours off-duty. Off-duty time does not expand the 14-hour period.
Sleeper berth partial rest periods	Drivers using the sleeper berth provision must spend: 1) at least eight consecutive hours (but less than 10 consecutive hours) in the sleeper berth; 2) a separate period of at least two consecutive hours (but less than 10 consecutive hours) either in the sleeper berth, off-duty, or in any combination of the two.
Driving and duty limits with partial rest periods	After the second required rest period is completed, a new calculation point for the 14-hour limit, starting at the end of the previous rest period, will have to be considered to determine the available on-duty and driving hours. In this calculation, only the period spent in the sleeper berth of at least eight consecutive hours will not be counted as part of the 14-hour limit; a period of less than eight hours will. The sleeper berth provision can be used continually until 10 consecutive hours off-duty are taken. After 10 consecutive hours off-duty, a driver has 11 hours of driving time and 14 hours of duty time available again.

were tested. In the first, only complete rest periods were considered whereas in the second, breaking rest periods was also allowed.

Both versions of the algorithm were tested on a set of modified Solomon instances (Solomon, 1987) and on a real instance from Groupe Robert, a Quebec-based company. On the Solomon instances, allowing split rest periods proved to be beneficial in terms of number of vehicles used, total distance traveled and total route duration. On the Groupe Robert instances, both versions of the algorithm yielded reductions in the number of vehicles and in the distance traveled with respect to the Groupe Robert solution. Allowing split rests also reduced the routing cost.

6 Synchronization in arc routing

In vehicle routing it is sometimes necessary to synchronize the schedules of several vehicles because these must operate at the same time or because their operations must follow a predetermined sequence. Drexl (2012) provides an exhaustive survey of applications of this class of problems. Here we describe two arc routing problems in which synchronization between vehicles is required.

The first arc routing synchronization problem studied by Salazar-Aguilar, Langevin and Laporte (2012) is related to snow plowing operations. The problem arises in contexts where some street or road segments have multiple lanes that must be plowed simultaneously by several vehicles, typically one per lane. Because not all segments have the same number of lanes, this requires some synchronization between the snow plowing vehicles and may generate waiting times in the solution. To illustrate, consider the graph depicted in Figure 4 showing travel times on the edges. In this example, two vehicles are used to plow five segments, only one of which, (3,0), requires two vehicles. The first vehicle will leave the depot at time 0 and return at time 8 after plowing arcs (0,1), (1,3) and (3,0). The second vehicle can leave the depot at time 2, and plow arcs (0,2), (2,3) and (3,0) before returning to the depot at time 8. The two vehicles are synchronized to meet at vertex 3 at time 4.

This problem is considerably more complicated to model and solve than standard arc routing problems such as the Rural Postman Problem or the Capacitated Arc Routing Problem. The authors have solved it by means of an adaptive large neighbourhood search heuristic which was applied to the street network of the

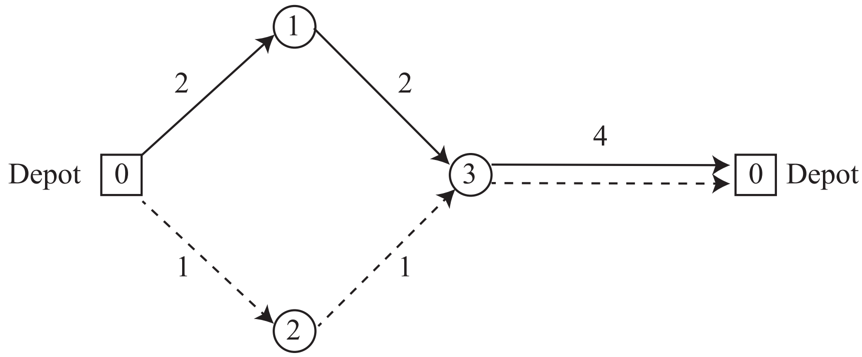


Figure 4: Arc routing problem with vehicle synchronization. ______: vehicle 1, _ _ _ _: vehicle 2

City of Dieppe in New Brunswick, Canada. This city has 144 km of streets to plow, all having one or two lanes, and representing a total of 363 lane-km.

A second problem considered by Salazar-Aguilar, Langevin and Laporte (2013) concerns the marking of rural roads by painting vehicles. Because of wear and erosion, white and yellow lines on road have to be repainted on a regular basis by means of capacitated specialized painting vehicles. This often means dispatching vehicles over a wide area, often far from the depot. When a painting vehicle runs out of paint, it can drive back to the depot to replenish before resuming its painting work, which can generate a fair amount of deadheading. Alternatively, one can make use of one or several replenishment vehicles which will supply the painting vehicles whenever they run out of paint. This gives rise to a combined arc and node routing problem with route synchronization: the arc routing problem is that of the painting vehicles whereas the node routing problem is that of the replenishment vehicles.

This problem is also very difficult to model and to solve. Salazar-Aguilar, Langevin and Laporte (2013) have devised a heuristic combining GRASP (greedy randomized adaptive search procedure – see Feo and Resende (1995)) and adaptive large neighbourhood search (Pisinger and Ropke, 2007) for this problem. They have applied it to the solution of instances involving between 60 and 400 vertices, between three and seven painting vehicles, and one replenishment vehicle. Three replenishment policies were implemented and compared over a set of randomly generated instances. Under the first policy (R), all replenishments are performed by the replenishment vehicle; under the second policy (D), the painting vehicle always returns to the depot to replenish. The third policy (RD) combines the first two: whenever a replenishment is needed, it implements policy R or D, depending on which one is the best. It was observed that D is usually preferable to R, except on the smaller instances of size 60, and the RD policy is always superior to the other two.

7 Conclusions

We have described several routing problems in which scheduling plays an important role. The need for scheduling often arises from the presence of time windows, like in the dial-a-ride problem and in the long-haul vehicle routing and scheduling problem with working hour rules. Scheduling is also associated with speed control which is often used as a means to reduce CO_2 emissions. We have provided two examples of speed control, one arising in ocean shipping and one in vehicle routing. Finally some problems require the synchronization of several vehicles, like in the two arc routing problems we have described.

The examples provided in this paper reflect the recent research interests of the author. They clearly do not represent the full range of problems in which scheduling plays an important role. The number of potential applications is very large and these will hopefully attract the interest of other researchers. There exist several promising avenues of research in the area of combined routing and scheduling. A first challenge is the inclusion of traffic congestion and speed variability in the computation of optimal vehicle routes and schedules. A second interesting research topic is the synchronization of vehicles operating at different levels of a supply chain, for example in the case of transshipment and cross-docking operations. Finally, the increased availability of real-time information, regarding congestion for example, calls for the development of algorithms capable of efficiently processing on-line information and of updating solutions dynamically.

References

Archetti, C., & Savelsbergh, M.W.P. (2009). The trip scheduling problem. *Transportation Science*, 43, 417–431.

Baldacci, R., Mingozzi, A., & Roberti, R. (2012). Recent exact algorithms for solving the vehicle routing problem under capacity and time window constraints. *European Journal of Operational Research*, 218, 1–6.

Barth, M., & Boriboonsomsin, K. (2009). Energy and emissions impacts of a freeway-based dynamic eco-driving system. *Transportation Research Part D*, 14, 400–410.

Barth, M., Younglove, T., & Scora, G. (2005). Development of a heavy-duty diesel modal emissions and fuel consumption model. Technical report UCP-ITS-PRR-2005-1, California PATH Program, Institute of Transportation Studies, University of California at Berkeley.

Bektas, T., & Laporte, G. (2011). The pollution-routing problem. *Transportation Research Part B*, 45, 1232–1250.

Berbeglia, G., Pesant, G., & Rousseau, L.-M. (2011). Checking the feasibility of dial-a-ride instances using constraint programming. *Transportation Science*, 45, 399–412.

Berbeglia, G., Pesant, G., & Rousseau, L.-M. (2012). Feasibility of the pickup and delivery problem with fixed partial routes: A complexity analysis. *Transportation Science*, 46, 359–373.

Berbeglia, G., Cordeau, J.-F., & Laporte, G. (2012). A hybrid tabu search and constraint programming algorithm for the dynamic dial-a-ride problem. *INFORMS Journal on Computing*, 24, 343–355.

Ceselli, A., Righini, R., & Salani, M. (2009). A column generation algorithm for a rich vehicle-routing problem. *Transportation Science*, 43, 56–69.

Cordeau, J.-F., & Laporte, G. (2003). A tabu search heuristic for the static multi-vehicle dial-a-ride problem. *Transportation Research Part B*, 37, 579–594.

Cordeau, J.-F., & Laporte, G. (2007). The dial-a-ride problem: Models and algorithms. *Annals of Operations Research*, 153, 29–46.

Dantzig, G.B., & Ramser, J.H. (1959). The truck dispatching problem. *Management Science*, 6, 80–91.

Demir, E., Bektas, T., & Laporte, G. (2012). An adaptive large neighborhood search heuristic for the pollution-routing problem. *European Journal of Operational Research*, 223, 346–359.

Drexl, M. (2012). Synchronization in vehicle routing – A survey of VRPs with multiple synchronization constraints. *Transportation Science*, 46, 297–316.

Fagerholt, K., Laporte, G., & Norstad, I. (2010). Reducing fuel emissions by optimizing speed on shipping routes. *Journal of the Operational Research Society*, 61, 523–529.

Feo, T.A., & Resende, M.G.C. (1995). Greedy randomized adaptive search procedures. *Journal of Global Optimization*, 6, 109–133.

Gehring, H., & Homberger, J.A. (1999). A parallel hybrid evolutionary metaheuristic for the vehicle routing with time windows. *Proceedings of EUROGEN99*, K. Miettinen, M.M. Makela and J. Toivanen (Eds.), Jyväskiä, Finland.

Goel, A. (2009). Vehicle scheduling and routing with drivers' working hours. *Transportation Science*, 43, 17–26.

Goel, A., Archetti, A., & Savelsbergh, M.W.P. (2012). Truck driver scheduling in Australia. *Computers & Operations Research*, 39, 1122–1132.

Goel, A., & Kok, A.L. (2012). Truck driver scheduling in the United States. *Transportation Science*, 46, 317–326.

Goel, A., & Rousseau, L.-M. (2011). Truck driver scheduling in Canada. *Journal of Scheduling*, forthcoming.

Golden, B.L., Raghavan, S., & Wasil, E.A. (2008). *The Vehicle Routing Problem: Latest Advances and New Challenges.* New York: Springer.

Hvattum, L.M., Norstad, I., Fagerholt, K., & Laporte, G. (2013). Analysis of an exact algorithm for the vessel speed optimization problem. *Networks*, forthcoming.

Kok, A.L., Hans, E.W., & Schutten, J.M.J. (2011). Optimizing departure times in vehicle routes. *European Journal of Operational Research*, 210, 579–587.

Kok, A.L., Meyer, C.M., Kopfer, H., & Schutten, J.M.J. (2010). A dynamic programming heuristic for the vehicle routing problem with time windows and European community social legislation. *Transportation Science*, 44, 442–454.

Laporte, G. (2009). Fifty years of vehicle routing. *Transportation Science*, 43, 408–416.

McKinnon, A. (2007). CO_2 emissions from freight transport in the UK. Technical report prepared for the Climate Change Working Group of the Commission for Integrated Transport, London, UK. http://www.isotrak.com/news/press/CO2_emissions_freight_transport.pdf. Accessed 11 February 2011.

Nagata, Y., Bräysy, O., & Dullaert, W. (2010). A penalty-based edge assembly memetic algorithm for the vehicle routing problem with time windows. *Computers & Operations Research*, 37, 724–737.

Norstad, I., Fagerholt, K., & Laporte, G. (2011). Tramp ship routing and scheduling with speed optimization. *Transportation Research C*, 19, 853–865.

Paquette, J. (2010). Mesure de la qualité de service et conception de tournées en transport adapté. Ph.D. Thesis, HEC Montréal, Canada.

Paquette, J., Bellavance F., Cordeau, J.-F., & Laporte, G. (2012a). Measuring quality of service in dial-a-ride operations: The case of a Canadian city. *Transportation*, 39, 539–564.

Paquette, J., Cordeau, J.-F., Laporte, G., & Pascoal, M.M.B. (2012b). Combining multicriteria analysis and tabu search for dial-a-rial problems. *Transportation Research B*, forthcoming.

Parragh, S.N., Doerner, K.F., & Hartl, R.L. (2008a). A survey of pickup and delivery models: Part I: Transportation between customers and depot. *Journal für Betriebswirtschaft*, 58, 21–51.

Parragh, S.N., Doerner, K.F., & Hartl, R.L. (2008b). A survey of pickup and delivery models: Part II: Transportation between pickup and delivery locations. *Journal für Betriebswirtschaft*, 58, 81–117.

Pisinger, D., & Ropke, S. (2007). A general heuristic for vehicle routing problems. *Computers & Operations Research*, 34, 2403–2435.

Prescott-Gagnon, E., Drexl, M., & Rousseau, L.-M. (2010). European driver rules in vehicle routing with time windows. *Transportation Science*, 44, 455–473.

Rancourt, M.-È., Cordeau, J.-F., & Laporte, G. (2013). Long-haul vehicle routing and scheduling with working hour rules. *Transportation Science*, 47, 81–107.

Ropke, S., Cordeau, J.-F., & Laporte, G. (2007). Models and branch-and-cut algorithms for pickup and delivery problems with time windows. *Networks*, 49(4), 258–272.

Salazar-Aguilar, M.A., Langevin, A., & Laporte, G. (2012). Synchronized arc routing for snow plowing operations. *Computers & Operations Research*, 39, 1432–1440.

Salazar-Aguilar, M.A., Langevin, A., & Laporte, G. (2013). The synchronized arc and node routing problem: Application to road marking. *Computers & Operations Research*, forthcoming.

Savelsbergh, M.W.P. (1992). The vehicle routing problem with time windows: Minimizing route duration. *ORSA Journal on Computing*, 4, 146–154.

Solomon, M.M. (1987). Algorithms for the vehicle-routing and scheduling problems with time window constraints. *Operations Research*, 35, 254–265.

Toth, P., & Vigo, D. (2002). *The Vehicle Routing Problem*. Philadelphia: SIAM.

Zitzler, E., Thiele, L., Laumanns, M., Fonseca, M., & da Fonseca, G.V. (2003). Performance assessment of multiobjective optimizers: An analysis and review. *IEEE Transactions on Evolutionary Computation*, 7, 117–132.

Zitzler, E., Thiele, L., & Bader, J. (2010). On set-based multiobjective optimization. *IEEE Transactions on Evolutionary Computation*, 14, 58–79.