Les Cahiers du GERAD ISSN: 0711–2440

Real Options in an Oligopoly Under Incomplete and Asymmetric Information

G. Janssens
P.M. Kort

G–2012–30

May 2012

Real Options in an Oligopoly Under Incomplete and Asymmetric Information

Gert Janssens

GERAD & HEC Montréal
Montréal (Québec) Canada, H3T 2A7
and Dawson College
Westmount (Quebec) Canada, H3Z 1A4
gert.janssens@gerad.ca

Peter M. Kort

Tilburg University
5037 AB Tilburg, The Netherlands
and University of Antwerp
2000 Antwerpen, Belgium
kort@uvt.nl

May 2012

Les Cahiers du GERAD

G–2012–30

Abstract

Standard real options theory states that there is a value of waiting and that irreversible investment should be postponed when revenue is uncertain. Past literature has shown that competition under complete information generates a preemption effect that reduces this value of waiting without completely eliminating it. This paper studies what happens to the value of waiting when a firm has incomplete information about its competitor in an oligopoly framework. We find that the value of waiting depends on the quality of the information, the size of the first-mover advantage and the number of competitors. We show that a unique and symmetric equilibrium exists under common and consistent priors. Investment is accelerated when one firm acquires complete information about the profitability of the other firm, while the other firm's information remains incomplete. We calculate the value of (acquiring) information under different assumptions on the priors.

Key Words: Game theory, real options, investment decisions, dynamic oligopoly.

1 Introduction

It is well-known that the widely used net present value criterion usually leads to suboptimal investment decisions. The real options paradigm exploits the analogy with financial options theory and enables decision makers to derive the optimal investment timing when a fixed-size investment with uncertain future returns is irreversible and can be postponed. A *value of waiting* is associated with the resolution of some market uncertainties over time and suggests that it is optimal to postpone investment beyond the net present value trigger.

Unlike on financial markets, the value of a real option to invest often depends on the decision of other agents to exercise their options. Interdependency and non-exclusivity affect optimal investment timing and warrant the recent development of *option games*. We can think of multiple firms competing to invest first in a prime location – as in Ghemawat (1986)'s Harvard Business School case study on Wal-Mart's strategy – buyers bidding on a property in a declining market, or researchers in a patent race. In an early application, Smets (1991) compares foreign direct investment with exports. Pawlina and Kort (2006) show that while the threat of preemption accelerates investment, the value of waiting remains relevant under imperfect competition when firms are asymmetric in investment costs only and when there is a first-mover advantage. The asymmetry endogenously determines which firm invests first.

Several authors (e.g. Kijima and Shibata (2005)) discuss whether this result continues to hold qualitatively under incomplete information about a competitor's profit. The present paper explores that question in detail. Following Pawlina and Kort (2006), our model involves incomplete information over investment costs only, but in general it is easy to find equivalent models where incomplete information affects some other private determinant of profitability.

We find that incomplete information about a competitor's investment cost can reduce or enhance the (perceived) option value of waiting, depending on the shape of uncertainty. Furthermore, we find that more competition or a greater first-mover advantage reduce the option value of waiting. This result is quite similar to the one in Kijima and Shibata (2005) for complete information. Moreover, we apply auction theory to show that a unique and symmetric equilibrium exists in a duopoly under the standard assumption of common and consistent priors. Finally, we solve a case where one firm has full information, while the other firm's information is incomplete. It turns out that firms rarely want to invest in information when this fact is common knowledge, as this always speeds up investment relative to incomplete information.

The remainder of this paper is structured as follows. First, we provide an overview of the literature in Section 2, after which we briefly review the classical case of the monopolist in Section 3. Section 4 expands on some aspects of the new market model of Pawlina and Kort (2006) that will serve as the basis for our generalization in Section 5. In Sections 5.1 to 5.3 and 5.7, we derive the optimal investment rule of a single firm in a non-strategic duopoly. We briefly consider the effect of the number of firms in Section 5.4 and root our generalized model firmly in the literature in Section 5.5. In Section 5.6 we study the effect of information quality by varying uncertainty around a constant mean. We show in Section 6 that there exists a unique symmetric equilibrium under the standard assumption of common and consistent priors.

In Section 7, we let go of the common and consistent prior assumption and study a case of asymmetric information. Finally, in Section 8, we summarize the main findings and we outline some directions for further research.

2 Literature

We distinguish literature on classical models, option games and models under incomplete information. Most of the classical references are reviewed in Dixit and Pindyck (1994), who also describe the symmetric duopoly of Smets (1991) that was later generalized and improved by Huisman (2000). The model that we develop in Sections 5 to 7 is based in turn on generalizations of this model (the case of negative externalities) by Huisman et al. (2004) (existing market model) and Pawlina and Kort (2006) (characterization of equilibria). In the model of Boyer et al. (2001) firms differ in initial capacity rather than investment costs. Their *tacit collusion* *equilibrium* is the non-cooperative *simultaneous equilibrium* of Pawlina and Kort (2006). Wu (2006) studies capacity investments in an industry that will start to shrink as it enters the last phase of its (product)lifecycle, at an exponentially distributed state. Huisman and Kort (2011) model strategic capacity investment and weigh flexibility versus scale. Kijima and Shibata (2005) extend Huisman (2000) model to an oligopoly and find, just as Grenadier (2002), that the value of waiting disappears in the limit of perfect competition. Shackleton et al. (2004) consider a two-dimensional *switching monopolist* model where operational profits are imperfectly correlated. Chu and Sing (2007) also allow for a firm-specific shock. Both Huisman et al. (2005) and Boyer et al. (2004) give an overview of the real option literature in a strategic environment.

This paper presents an option game under incomplete information. We generalize some related papers that essentially treat monopolies. Lambrecht and Perraudin (2003) describe two incompletely informed firms that fight over a natural monopoly. Pawlina and Kort (2005) describe the effect of a policy change on a monopolist, but their model can be given a dominant firm-competitive fringe interpretation. Hsu and Lambrecht (2007) consider asymmetric information in a game between an informed entrant and a non-informed incumbent. It is a *monopoly persistence* model. Graham (2011) finds that if firms are symmetric, but asymmetrically informed about their future profits, there may be no equilibrium. Nishihara and Fukushima (2008) study the scenario of a small pioneer that will someday be pushed out of the market by a more efficient player.

Usually, incomplete information applies to other parameters of the model. Bobtcheff and Mariotti (2010) study a monopoly with *potential competition*. Competitors are born (and become visible) *at random*. The firm that waits to invest (or the researcher that waits to publish), runs the risk that an unknown competitor surfaces and exercises his option first. Decamps et al. (2003) describe a firm whose profit flow fluctuates with the state variable only with some probability. In the traditional domain of contract theory, financing real options has attracted some attention. Lambrecht (2004) and Morellec and Zhdanov (2005) analyze a take-over battle where outsiders have incomplete information. Martzoukos and Zacharias (2008) consider a case where firms get access to better information about a potential monopolistic investment by pre-investments and cooperation. Grenadier and Malenko (2011) study a *signal-jamming* model, Morellec and Schrhoff (2011), Bustamante (2008), and Hennessy et al. (2010) treat true signaling, while Bouvard (2010) and Grenadier and Wang (2005) offer a screening approach.

3 The classical case: monopoly

A monopolist has a single, fixed-sized investment project with *Marshallian (or immediate investment) value function*

$$V^M(x(t) \mid m) = \frac{D_1 x(t)}{r-\mu} - k, \tag{3.1}$$

where D_1 indicates that there is only one active firm. The state variable $x(t)$ represents operational profit flow, m is the investment trigger, r is the discount rate, μ is the drift rate and the investment cost is k. A decision maker that refers to the classical net present value criterion will invest whenever its operational profits are equal to the *Marshallian trigger*

$$m = \frac{r-\mu}{D_1} k. \tag{3.2}$$

However, it is well-known that this criterion leads to suboptimally early investment for investments with uncertain future returns that are irreversible and can be postponed. This is because it neglects the value of waiting that arises under these circumstances. Modeling uncertainty of the profit flow over time t as a *geometric Brownian Motion*[1]

$$dx(t) = \mu x(t)dt + \sigma x(t)dB(t), \tag{3.3}$$

[1] This is standard in the real options literature but certainly not imperative (e.g. Kijima and Shibata, 2002). While under a Geometric Brownian Motion we can often derive analytical solutions, Davis (1998) actually fails to find broad empirical support. Returns on stocks for example have a fat tailed distribution over longer periods of time while the Geometric Brownian Motion implies a lognormal distribution. We will drop the time denotation for clarity henceforth.

with B a standard Wiener process and σ a measure for market volatility, we obtain the value function[2]

$$V^l(x \mid l) = \begin{cases} \left(\frac{l}{r-\mu} - k\right)\left(\frac{x}{l}\right)^\beta & \textit{before investment} \quad (x < l) \\ \frac{x}{r-\mu} - k & \textit{after investment} \quad (x \geq l) \end{cases}, \tag{3.4}$$

using the tools of dynamic programming. $\beta > 1$ is the positive root of the associated fundamental quadratic

$$\mathcal{Q} = \frac{1}{2}\sigma^2\beta(\beta-1) + (r-\mu)\beta - r = 0$$

and depends on σ, μ and r. It is often seen as an inverse measure of market volatility (with constant r and μ). We further note that $\left(\frac{x}{l}\right)^\beta$ equals the stochastic discount factor $E_0(e^{-rT})$, where T is the time that x hits l. The optimal investment trigger can be obtained from the so called value-matching and smooth-pasting conditions (see, e.g. Dixit and Pindyck (1994)):

$$l = \frac{\beta}{\beta-1}(r-\mu)k.$$

4 A duopoly under complete information

4.1 Introduction

When firms compete for an investment opportunity, the value of waiting has to be weighted against the threat of preemption. This section briefly describes the asymmetric new market[3] duopoly model of Pawlina and Kort (2006) and we derive the subgame perfect equilibria of this game under complete information. Models under complete information assume that all relevant information about one's competitor is public. In Section 4.2, we state the basic assumptions. Then, there are basically three steps to follow. First, in Section 4.3 we temporarily assume that the leader invests immediately at the start of the game, i.e. at $x(0)$, and we associate a leader and a follower role (with their associated value functions) to each firm. Second, we find the relevant equilibrium solution and endogeneously determine which firm leads and follows in Section 4.4. Finally, in Section 4.5, we let the leader's optimal investment timing influence the *authentic value functions*. This game has its earliest roots in Fudenberg and Tirole (1985). Simon and Stinchcombe (1989) described an asymmetric model, but Huisman (2000) was the first to provide a complete analysis for a stochastic and asymmetric model in continuous time.

4.2 Assumptions of the model

Two firms are considering a single investment in a new market to obtain an operational profit flow[4] of

$$\pi(x, D_N) = xD_N,$$

that depends multiplicatively on market uncertainty x -that follows a Geometric Brownian Motion- and a deterministic factor D_N -that depends on the number of active firms $N = \{0, 1, 2\}$. The assumption of negative externalities $D_1 > D_2$ implies that monopoly profits are higher than duopoly profits. The first-mover advantage can be measured by $\frac{D_1}{D_2}$. We normalize $D_0 = 0$ and assume that firms can make a profit in a duopoly, i.e. $D_2 > D_0$. Firms $i = 1, 2$ only differ in (sunk) investment costs k_1 resp k_2. We call $\frac{k_2}{k_1}$ the cost asymmetry. The stochastic process starts at $t = 0$ from $x(0)$ low enough so that immediate investment is not optimal for either firm. Investment cost asymmetry can be caused by differences in R&D, organizational flexibility (absorptive capacity), embedded real options, access to financial markets or other exogenous factors.

[2] See for example Dixit and Pindyck (1994).

[3] Pawlina and Kort (2006) also evaluate investments in existing markets.

[4] For simplicity we assume that prices and operational costs are the same for all firms.

4.3 Immediate investment value functions

We ascribe a hypothetical leader and a follower role to each firm. To simplify, assume for now that the leader invests at the start of the game at $x(0)$. Pawlina and Kort (2006) show that the value function for follower $i = \{1, 2\}$ after investment by leader $j = \{2, 1\}$ is strictly increasing, strictly convex, differentiable almost everywhere, and given by

$$V_i^F(x \mid f_i) = \begin{cases} \left(\frac{f_i D_2}{r-\mu} - k_i\right)\left(\frac{x}{f_i}\right)^\beta & \textit{if} \quad x \leq f_i \quad \textit{(wait)} \\ \frac{x D_2}{r-\mu} - k_i & \textit{if} \quad x > f_i \quad \textit{(invest)} \end{cases}. \tag{4.1}$$

Follower i's investment trigger is

$$f_i = \frac{\beta}{\beta - 1}\frac{(r-\mu)k_i}{D_2}. \tag{4.2}$$

The value function for leader i, when firm j is the follower, is strictly concave, differentiable almost everywhere, and given by

$$V_i^L(x \mid f_j) = \begin{cases} \frac{x D_1}{r-\mu} - k_i + \frac{f_j(D_2 - D_1)}{r-\mu}\left(\frac{x}{f_j}\right)^\beta & \textit{if} \quad x \leq f_j \\ \frac{x D_2}{r-\mu} - k_i & \textit{if} \quad x > f_j \end{cases}. \tag{4.3}$$

The term $\frac{xD_1}{r-\mu} - k_i$ is the operational profit for a perpetual monopolist i that invests immediately. The correction factor

$$C_i^L \equiv \frac{f_j(D_2 - D_1)}{r-\mu}\left(\frac{x}{f_j}\right)^\beta \tag{4.4}$$

reflects the decrease in the present value of the expected profit because of the eventual investment by follower j. From f_j onwards, firm i can only recuperate the duopoly profit $\frac{xD_2}{r-\mu} - k_i$. The leader value is usually not monotonic in x.

4.4 Strategic and non-strategic equilibrium

Firm i does not need to behave strategically if the investment cost k_j of its opponent is much higher than its own investment cost k_i. Firm i can act as a monopolist and invest at its *non-strategic trigger*

$$l_i = \frac{\beta}{\beta - 1}\frac{r-\mu}{D_1}k_i$$

in the *non-strategic domain*, i.e. as long as $k_j \geq k_j^S$. We want to find k_j^S such that for all $k_j \geq k_j^S$, firm j does not want to preempt firm i *if the latter invests at* l_i. This condition is satisfied if $V_j^L(x) < V_j^F(x)$ *for all* $x < l_i$. Then, firm i can behave as a monopolist and completely ignore the eventual investment of its competitor. This is so because that investment affects firm i's value before and after investment in the same way. Pawlina and Kort (2006) call this the *irrelevance of future alternatives*. The *non-strategic equilibrium* is given by the vector $\begin{bmatrix} l_i & f_j \end{bmatrix}$. We stress that our k_j^S is smaller than the k_j^* that Pawlina and Kort (2006) define as

$$k_j^* = \left(\frac{\left(\frac{D_1}{D_2}\right)^\beta - 1}{\beta\left(\frac{D_1}{D_2} - 1\right)}\right)^{\frac{1}{\beta-1}} k_i \tag{4.5}$$

and that separates the *preemptive region* $\left[k_i, k_j^*\right)$ from the *sequential region* $\left[k_j^*, \infty\right)$. Their k_j^* is the solution of the system

$$\begin{cases} \xi_j(p_i(k_j^*), k_j^*) = 0 \\ \frac{\partial \xi_j(p_i(k_j^*), k_j^*)}{\partial x} = 0 \end{cases}.$$

where the break-even function $\xi_j(x)$ is defined as

$$\xi_j(x) \equiv V_j^L(x) - V_j^F(x).$$

Hence, at k_j^*, firm j's leader value is tangent to its follower value. This implies that $V_j^L(x) < V_j^F(x)$ *for all* x where $k_j > k_j^*$. This condition is too strong. The inequality $V_j^L(x) < V_j^F(x)$ needs to hold only for $x < l_i$ because it does not matter if firm j wants to invest as a leader (or: the value functions cross) for $x > l_i$. In practice, it is relatively easy to compute k_j^S as the solution to $l_i = p_i(k_j^S)$. The function $p_i(\cdot)$ is the topic of Proposition 5.1.

In the *strategic domain, i.e.* $k_j \in \left[k_i, k_j^S\right)$, firm i can not disregard its opponent. It runs the risk of being preempted, because there exists an interval $x \in \left[b_j, \overline{b}_j\right]$ where $V_j^L(x) \geq V_j^F(x)$, and with $b_j < l_i$. The break-even point b_j is the lowest value of x where the high-cost firm j is indifferent between leading and following. It is thus the smallest real root of the breakeven function $\xi_j(x)$, i.e.

$$\xi_j(b_j) \equiv \frac{b_j D_1}{r-\mu} - k_j + \frac{f_i(D_2 - D_1)}{r-\mu}\left(\frac{b_j}{f_i}\right)^{\beta} - \left(\frac{f_j D_2}{r-\mu} - k_j\right)\left(\frac{b_j}{f_j}\right)^{\beta} = 0. \tag{4.6}$$

The *strategic equilibrium* is given by the vector $\begin{bmatrix} p_i & f_j \end{bmatrix}$, where $p_i \equiv b_j - \epsilon$ is the *strategic trigger.*

Proposition 4.1 *The strategic trigger p_i is optimal in the strategic domain, the non-strategic trigger l_i is optimal in the non-strategic domain.*

Proof. See appendix. □

In this game, the less efficient firm is a natural follower because the break-even trigger for a more efficient firm is always lower than that for a less efficient firm. Therefore, the lower bound of the strategic domain of firm i is k_i.

We illustrate the relationship between k_j and firm value for firm i with respect to k_j at $x = 4$ in Figure 4.1.

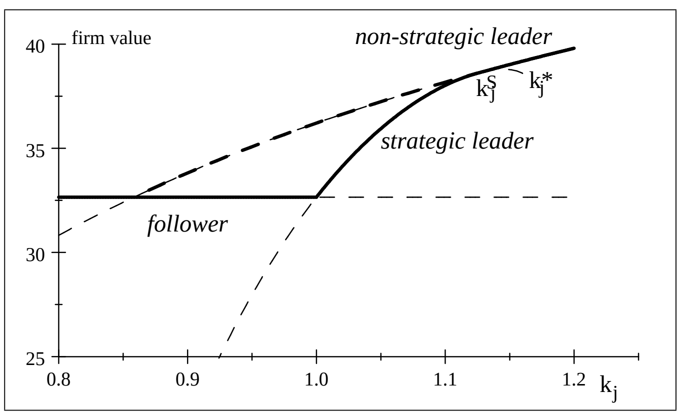

Figure 4.1: Composite value function for firm i in function of k_j for $x = 4$ (thick). The follower value holds for $k_j < k_i$. In $[1, k_j^S]$ firm i is the strategic leader. Beyond k_j^S firm i invests as a monopolist. Parametervalues are $r = 0.05$; $\mu = 0.015$; $\beta = 2$; $D_2 = 1$; $D_1 = 1.33$ and $k_i = 1$.

To construct this figure, we calculated three value functions, each corresponding to one trigger, respectively f_i, p_i, l_i over a broad range of k_j. The composite value function, indicated with a bold and solid line, selects the optimal feasible trigger at each k_j given the competitor's optimal strategy. Preemption precludes investment at the non-strategic trigger in the follower and strategic domains (bold and dashed line).

For symmetric firms, break-even and preemption points coincide. If one introduces the following tie-breaking rule that is common knowledge: each firm receives the leader value with equal probability,[5] then

[5] Under this rule, Huisman's (2001) *non-zero probability of mistake* vanishes at the preemption point (we have assumed that immediate investment is not optimal for either firm). Hence, firms are indifferent between leading and following and we obtain rent-equalization.

there exists a mixed strategy equilibrium. The latter can be motivated as the limit of a pure-strategy Bayesian equilibrium of a slightly perturbed game of incomplete information Harsanyi (1973). We derive such a game of incomplete information in Section 5.

Existence of a value of waiting is implied by $V_i^L(p_i) > 0$. This is so because, with leader i, $V_i^L(p_i) \geq V_j^L(p_i) \geq V_j^L(p_i) - V_j^F(p_i) = 0$, with equality in a symmetric natural monopoly. This implies that a value of waiting remains in a duopoly under complete information and suggests that the race to invest intensifies with weaker prospects for the follower and increasing symmetry.

To ensure that the strategic domain is indeed an interval, it is sufficient that the strategic trigger is increasing in k_j. We will show this in Proposition 5.1.

4.5 Authentic value functions

Now, instead of requiring immediate investment, we let the leader i choose its optimal investment timing.

Proposition 4.2 *As the optimal investment timing for the leader affects both leader and follower value functions in the same way, it does not affect the optimal investment triggers p_i and l_i.*

Proof. See appendix. □

We establish the following properties for future reference.

Proposition 4.3 *Both $V_i^{LL}(x \mid x_i, f_j)$ and $V_j^{FF}(x \mid x_i, f_j)$ are strictly increasing in x.*

Proof. See appendix. □

5 A duopoly under incomplete information: non-strategic competitor

Because firms usually have no more than a rough idea about the profitability of their competitors' projects, even if they are all public companies, complete information may not adequately reflect reality. In our model of incomplete information, firm-specific information is private while market information remains public. In particular, we assume that each firm has the same information about the state variable and knows its own investment cost with certainty, but is uncertain about the investment cost of its opponent -up to a prior $G_i(k_j)$. In this section, we assume that a firm i plays against a non-strategic competitor.[6] To solve this problem, we begin by characterizing the functional relationship between a firm's optimal strategic trigger p_i and the opponent's investment cost k_j in the strategic domain in Section 5.1. We will use this functional relationship to translate initial uncertainty over a firm's opponent investment costs $G_i(k_j)$ into uncertainty over a firm's own optimal strategic investment trigger $\psi_i(p_i)$ in Section 5.2. This also enables us to update our prior as more information becomes available. In Section 5.3, we derive the optimal Markovian investment rule of a single firm facing a non-strategic opponent. We briefly consider the effect of the number of firms in Section 5.4 and consider how our model generalizes previous models of incomplete information in Section 5.5. In Section 5.6 we study the effect of information quality using the mean-preserving spread. Then, in Section 5.7 we revert to the original problem with uncertainty over k_j instead of p_i. Finally, we allow for strategic interaction between firms. Section 6 shows that there exists a unique and symmetric Markovian Bayesian-Nash equilibrium under the standard assumption of common and consistent priors. In Section 7, we consider asymmetric information.

[6] Hence, we solve a single player decision problem. We do this primarly for pedagogical reasons, but we want to underline the relevance of this solution under the realistic assumption that priors are independent (see later).

5.1 Investment triggers and investment costs

To study the relationship between firm i's optimal investment trigger and its opponent's investment cost, we consider k_j as a variable. Neither the follower trigger f_i nor the non-strategic investment trigger l_i (irrelevance of future alternatives) depend on k_j. The strategic investment trigger $p_i = b_j - \epsilon$ does depend on k_j, directly and indirectly via $f_j(k_j)$. As before, we define b_j as the smallest positive real solution of the competitor's break-even function

$$\xi_j(b_j \mid k_j, f_j) = V_j^L(b_j \mid k_j) - V_j^F(b_j \mid f_j, k_j) = 0.$$

Firm j's follower trigger can readily be expressed as a function of its investment cost k_j

$$f_j(k_j) = \frac{\beta}{\beta-1}\frac{r-\mu}{D_2}k_j = \frac{f_i}{k_i}k_j = \frac{k_j}{k_i}f_i, \tag{5.1}$$

where k_i and f_i are known to firm i. We rewrite the break-even function as

$$\xi_j(b_j, k_j) = \frac{b_j D_1}{r-\mu} - k_j + \frac{f_i(D_2 - D_1)}{r-\mu}\left(\frac{b_j}{f_i}\right)^\beta - \left(\frac{\frac{k_j}{k_i}f_i D_2}{r-\mu} - k_j\right)\left(\frac{b_j}{\frac{k_j}{k_i}f_i}\right)^\beta = 0. \tag{5.2}$$

The implicit function theorem implies that there exists a unique optimal strategic investment trigger $p_i = b_j - \epsilon$ for firm i for each k_j in the strategic domain.

Proposition 5.1 *There exists a continuously differentiable, (strictly) increasing and strictly convex function* $p_i(k_j)$ *on* $\left[0, k_j^*\right]$ *that maps on codomain* $[0, l_i)$

Proof. See appendix. □

k_j^* is the upper bound of the domain[7] of $p_i(k_j)$ and $p_i'(k_j^*) \to +\infty$. The pre-image of the strategic domain $\left[k_i, k_j^S\right]$ maps on $[f_i, l_i]$. In the strategic domain, we can invoke the inverse function theorem.

Proposition 5.2 *There exists a continuously differentiable, (strictly) increasing and strictly concave function* $k_j(p_i)$ *on* $[f_i, l_i]$ *that maps on* $\left[k_i, k_j^S\right]$.

Proof. See appendix. □

Proposition 5.2 says that if firm i knows its optimal investment trigger p_i, then it also knows the (unique) investment cost of its opponent k_j. We will use this proposition in Section 5.2.

We illustrate Proposition 5.1 in Figure 5.1. The dashed lines form the contour of $\xi_j(x, k_j)$ at $\xi_j(p_i, k_j) = 0$. Limiting ourselves to the smallest real solution, the implicit function theorem describes a function $p_i(k_j)$, illustrated by the bold and dashed line. We use the bold and solid line to indicate the strategic trigger and the non-strategic triggers in their respective domains. We show that $k_j^S < k_j^*$ and that $l_i = p_i(k_j^S)$. Finally, the thin and solid line indicates the follower trigger f_i that applies on $k_j \in (0, k_i)$.

The implicit function theorem proves the existence of $p_i(k_j)$, but it may not yet be straigthforward to determine $p_i(k_j)$. When β is an integer, then $\xi_j(\cdot)$ is a polynomial and we can obtain $p_i(k_j)$ analytically. Otherwise, several strategies exist. First, we can apply the implicit function theorem to Equation (5.2) and numerically solve the resulting differential equation $p_i'(k_j)$ with boundary condition $\lim_{k_j \to 0} p_i(k_j) = 0$. This boundary condition expresses that immediate preemption is called for if your opponent has investment cost zero. Because the problem is convex, a numerical solution can probably be obtained more efficiently using the Newton-Raphson root-finding method on some grid of k_j. If the functional form of $p_i(k_j)$ is required, splines, spectral method interpolation or econometric techniques can be used.

[7] Pawlina and Kort's (2006) preemptive domain is the pre-image $\left[k_i, k_j^*\right]$ of this function. See footnote (5).

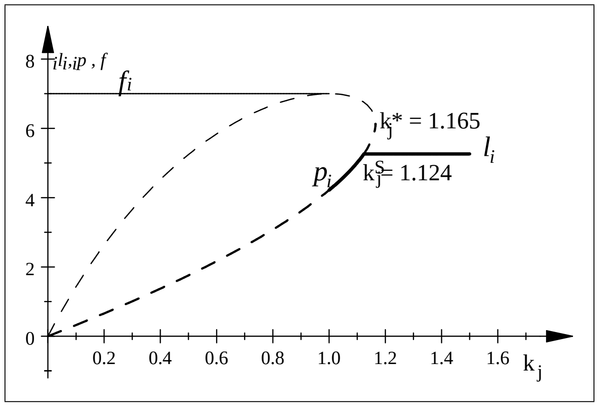

Figure 5.1: Functional relationship between k_j and investment triggers. Parameter values are $r = 0.05$; $\mu = 0.015$; $\beta = 2$; $D_2 = 1$; $D_1 = 1.33$ and $k_i = 1$.

5.2 Uncertainty and structure of learning

Assume that the uncertainty of firm i over its competitor's investment cost k_j is given by a prior $g_i(k_j)$ with support over an open interval $(k_{jL}, k_{jH}) \subset \mathbb{R}_0^+$ such that it is associated to a continuously differentiable cumulative density function $G_i(k_j)$. By Proposition 5.2, every p_i maps on a unique $k_j \in [k_i, k_j^S]$, so we can define a composition $\psi_i(p_i) \equiv G_i(k_j(p_i))$. Again by Proposition 5.2, this composition is an increasing function of p_i and thus qualifies as a cumulative probability distribution with support $(p_{iL}, p_{iH}) \equiv (p_i(k_{jL}), p_i(k_{jH})) \subset \mathbb{R}_0^+$. In this way, we have derived a powerful expression $\psi_i(p_i)$ of firm i's uncertainty over *its own optimal strategic investment trigger* in the strategic domain. In effect, $\psi_i(p_i)$ denotes the probability that firm j will invest first.[8]

We introduce a new state variable $\widehat{x}_t \equiv \max_{0\le\tau\le t}\{x_\tau\}$ to incorporate learning in our model. Each time x reaches a new high $\widehat{x}$ in (p_{iL}, p_{iH}) and firm j does not invest, firm i learns that no positive probability should have been atttributed to investment by its opponent at (or below) $\widehat{x}_t$. The truncated distribution

$$\psi_i(p_i \mid p_i > \widehat{x}_t) = \frac{\psi_i(p_i) - \psi_i(\widehat{x}_t)}{1 - \psi_i(\widehat{x}_t)} \tag{5.3}$$

denotes firm i's beliefs on the probability that firm j will invest first, conditional on the highest realization of the state, where the denominator inflates the conditional distribution so that the density still integrates to unity.

5.3 The expected value function and the optimal investment trigger for firm i

For an arbitrary trigger z_i, firm i invests first with probability $\frac{1-\psi_i(z_i)}{1-\psi_i(\widehat{x})}$ to capture the expected (authentic) leader value $\mathcal{E}_i\left[V_i^{LL}(x, z_i)\right]$. Otherwise, firm i will be the follower and its value will be $V_i^{FF}(x, z_i)$. Firm i chooses z_i such that its expected value before investment by either firm

$$\mathcal{E}_i\left[V_i^T(x, \widehat{x}, z_i)\right] = \frac{1-\psi_i(z_i)}{1-\psi_i(\widehat{x})}\mathcal{E}_i\left[V_i^{LL}(x, z_i)\right] + \left(1 - \frac{1-\psi_i(z_i)}{1-\psi_i(\widehat{x})}\right)V_i^{FF}(x, z_i) \tag{5.4}$$

is maximized. The expected leader value $\mathcal{E}_i\left[V_i^{LL}(x, z_i)\right]$ depends on firm i's expectations about the follower trigger for firm j. The latter depends on his expectation over firm j's investment cost k_j and can in turn be written as a function of the leader i's investment trigger z_i

$$\mathcal{E}_i\left[f_j(z_i)\right] = \frac{f_i}{k_i}\mathcal{E}_i\left[k_j(z_i)\right] = \frac{f_i}{k_i}\int_{p_{iL}}^{p_{iH}} k_j(z)\psi_i'(z)dz,$$

[8] To be precise, $\psi_i(p_i) = \psi_i(p_j \le p_i)$ denotes the probability that firm i attributes to firm j investing at a p_j smaller than p_i.

because $k_j(z_i)$ is non-negative and increasing, and where the first equality sign is implied by expression (5.1). We define

$$\kappa_j(z_i) \equiv \mathcal{E}_i\left[k_j(z_i)\right] = \int_{p_{iL}}^{p_{iH}} k_j(z)\psi_i'(z)dz$$

and write the expected leader value as

$$\mathcal{E}_i\left[V_i^{LL}(x,z_i)\right] = \left(\frac{z_i D_1}{r-\mu} - k_i + \frac{\frac{f_i}{k_i}\kappa_j(z_i)\left(D_2 - D_1\right)}{r-\mu}\left(\frac{z_i}{\frac{f_i}{k_i}\kappa_j(z_i)}\right)^\beta\right)\left(\frac{x}{z_i}\right)^\beta.$$

$\kappa_j(z_i)$ shares the basic properties of $k_j(z_i)$ because expectation is a linear and monotonic operator and the weighted (integral) sum of strictly concave functions is still strictly concave.

Corollary 5.1 *$\kappa_j(p_i)$ inherits the continuity, monotonicity and concavity properties from $k_j(p_i)$ on $[f_i, l_i]$.*

This yields the following result:

Proposition 5.3 *Firm i's expected value function before investment by either firm is*

$$\mathcal{E}_i\left[V_i^T(x,\widehat{x} \mid p_i)\right] = \frac{1-\psi_i(p_i)}{1-\psi_i(\widehat{x})}\mathcal{E}_i\left[V_i^{LL}(x \mid p_i)\right] + \left(1 - \frac{1-\psi_i(p_i)}{1-\psi_i(\widehat{x})}\right)V_i^{FF}(x \mid p_i), \tag{5.5}$$

where firm i's unique optimal strategic investment trigger p_i is implicitly defined by the first order condition

$$h_i(p_i) \equiv \frac{\psi_i'(p_i)}{1-\psi_i(p_i)} = \frac{\mathcal{E}_i\left[V_i^{LL\prime}(p_i)\right]}{\mathcal{E}_i\left[V_i^{LL}(p_i)\right] - V_i^{FF}(p_i)}. \tag{5.6}$$

That is, if the solution is interior, if the problem is compact valued, and if the second order condition is (strictly) satisfied, and where $h_i(p_i) = \frac{\psi_i'(p_i)}{1-\psi_i(p_i)}$ is the hazard rate.

Proof. See appendix. □

The hazard rate is the conditional probability that firm j invests at the very next instant, i.e. the risk of immediate preemption. The solution is not interior if either $\psi_i(p_i) = 1$ or $\psi_i(p_i) = 0$. In these cases, we have complete information and the optimal investment triggers are f_i and l_i, respectively. For $\psi_i(\widehat{x}) = 1$ the value of the problem would not be compact and a maximum would not necessarily exist. However, this is ruled out by the standard assumption that the true optimal trigger $p_i \in (p_{iL}, p_{iH})$; implying that $\lim_{p_i \to p_{iH}} \psi_i(p_i) = 1$ too.

Under complete information, we have $h_i(p_i) \to \infty$ at the preemption point and we obtain the familiar condition $\mathcal{E}_i\left[V_i^{LL}(p_i)\right] - V_i^{FF}(p_i) = 0$ from the first order condition (5.6). Rewriting it as

$$\left[\mathcal{E}_i\left[V_i^{LL}(p_i)\right] - V_i^{FF}(p_i)\right]\psi_i'(p_i) = \mathcal{E}_i\left[V_i^{LL\prime}(p_i)\right]\left(1-\psi_i(p_i)\right) \tag{5.7}$$

helps the interpretation. Firm i weighs the (cumulative) probability $1-\psi_i(p_i)$ that marginal delay increases profits with $\mathcal{E}_i\left[V_i^{LL\prime}(p_i)\right]$ against the (point) probability $\psi_i'(p_i)$ that it loses the expected incremental value of immediately investing first $\mathcal{E}_i\left[V_i^{LL}(p_i)\right] - V_i^{FF}(p_i)$. So, Equation (5.6) explains how firms weigh the expected marginal benefits of waiting against its expected marginal costs.

Before investment, both firms are valued at $\mathcal{E}\left[V_{i,j}^T(x,\widehat{x} \mid p_i)\right]$. A substantial part of uncertainty is resolved after investment by the leader i at p_i, so its value exhibits a positive jump to $\mathcal{E}\left[V_i^{LL}(x \mid p_i)\right]$ while the follower j is valued lower at $\mathcal{E}\left[V_j^{FF}(x \mid p_i)\right]$. The precise value of a firm is not known by incompletely informed investors[9] (nor by the leader) until all firms have invested, but the follower has complete information at this point and values the firms at the true $V_i^{LL}(x \mid p_i)$ and $V_j^{FF}(x \mid p_i)$ respectively. When the follower invests,

[9] We assume that investors and firms have consistent beliefs based on public data.

a second jump in the value occurs to $V_i^{LL}(x \mid p_i)$ and $V_j^{FF}(x \mid p_i)$. The direction of the second jump cannot be a priori determined. These jumps may explain volatility on financial markets.

If we expand the first order condition to

$$\begin{aligned} 0 = (1-\beta)\tfrac{D_1}{r-\mu} + \beta\tfrac{k_i}{p_i} + (1-\beta)\frac{\frac{\kappa_j'(p_i)}{k_i} f_i(D_2-D_1)}{r-\mu}\left(\tfrac{k_i}{\kappa_j(p_i)}\tfrac{p_i}{f_i}\right)^{\beta} \\ -\left(\tfrac{D_1 p_i}{r-\mu} - k_i + \frac{\frac{\kappa_j(p_i)}{k_i} f_i(D_2-D_1)}{r-\mu}\left(\tfrac{k_i}{\kappa_j(p_i) f_i}\right)^{\beta} - \left(\tfrac{f_i D_1}{r-\mu} - k_i\right)\left(\tfrac{p_i}{f_i}\right)^{\beta}\right) h_i(p_i), \end{aligned} \tag{5.8}$$

it becomes apparent that the optimal investment trigger does not explicity depend on the state variables.[10] This is because firms use threshold policies and, as such, firm i invests optimally when x reaches p_i for the first time, i.e. $p_i = x = \widehat{x}$. To see this, interpret $\mathcal{E}_i\left[V_i^T(x \mid \widehat{x}, p_i)\right]$ as a family of value functions in the (x, V) plane, where each function corresponds to a specific value of $\widehat{x}$ and is valid only in its domain $x \in [x(0), \widehat{x}]$. If firm j does not invest at a certain $x = \widehat{x}$, then firm i's value is given by a higher $\widehat{x}$-curve. The value function corresponds to an *envelope curve* $\mathcal{E}_i\left[V_i^T(x \mid \widehat{x}, p_i)\right]\big|_{x=\widehat{x}}$, composed of the end-points of all these $\widehat{x}$-curves precisely because $p_i = x = \widehat{x}$. At the optimal trigger, the envelope curve touches the Marshallian value function V^M from Section 3 (the value matching and smooth-pasting conditions).

The second order condition may be hard to verify. Proposition 5.4 provides a simplification.

Proposition 5.4 *A sufficient (but not necessary) condition that simplifies the second order condition is* $h_i'(p_i) \geq 0$.

Proof. See appendix. □

This condition applies to most common distributions and ensures that the value function is continuous and strictly concave in x.

5.4 Oligopoly and perfect competition

The generalization to an oligopoly with n firms is tedious yet rather straightforward. The terminology of leader and follower is no longer adequate. If firm i invests as the k^{th} (active) firm, its *investment profit* D_k will be corrected by all subsequent $n-k$ investments

$$\mathcal{E}_i\left[V_i^k(x \mid x_i, x_{k+1}...x_n)\right] = \left(\frac{x_i D_k}{r-\mu} - k_i + \sum_{j=k+1}^{n}\left(\frac{x_j\left(D_j - D_{j-1}\right)}{r-\mu}\right)\left(\frac{x_i}{x_j}\right)^{\beta}\right)\left(\frac{x}{x_i}\right)^{\beta}, \tag{5.9}$$

where x_i is the optimal trigger when firm i invests as the k^{th} firm and $x_{j\neq i}$ are the *expected* triggers of all firms that have not invested yet. To invest first when competing against $n-1$ firms, firm i has to invest before its most efficient competitor. If firm i has the same prior for each competitor, the probability that firm i invests as the k^{th} firm is given by

$$1 - \widetilde{\psi}_i^k(x_i) \equiv \prod_{j=1}^{n-k}\left[1 - \psi_i(x_j < x_i)\right] = \left(1 - \psi_i(x_i)\right)^{n-k}. \tag{5.10}$$

To find the optimal investment trigger, we construct the weighted value function.

Corollary 5.2 *The expected value function of firm i competing against $n-m-1$ competitors and after investment by $m \in \{0, ..., n-1\}$ firms is*

[10] $V_i^{FF\prime}(x) = 0$, so both x and $\psi_i(\widehat{x})$ apply to all terms of the first order condition (the derivative of the follower value is zero) and can be divided away.

$$\begin{aligned}&\mathcal{E}_i\left[V_{i,m}^T(x,\widehat{x}\mid m,x_i,x_1,...,x_{i-1},x_{i+1},...,x_{n-m})\right]\\ =&\sum_{k=m+1}^{n-m-1}\frac{1-\widetilde{\psi}_i^k(x_i)}{1-\psi_i(\widehat{x})}\mathcal{E}_i\left[V_i^k(x\mid x_i,x_{k+1}...x_{n-m})\right]+\left(1-\sum_{k=m+1}^{n-m-1}\frac{1-\widetilde{\psi}_i^k(x_i)}{1-\psi_i(\widehat{x})}\right)V_i^n(x\mid x_i),\end{aligned}$$

where the value maximizing trigger x_i can be found by applying the first and second order conditions if the solution is interior and if the problem is compact.

It follows immediately from Equation (5.10) that more competition leads to earlier investment.

Corollary 5.3 *Under perfect competition ($n \to \infty$) each firm invests at its break-even trigger.*

Information is no longer truly incomplete because $\psi_i(p_i)$ is no longer a probability distribution, but reflects the actual number of firms that will invest before firm i. As such, no firm can do better postponing investment. Thus, the value of waiting vanishes in the limit of perfect competition.

5.5 Natural monopoly and dominant leader versus competitive fringe

Lambrecht and Perraudin (2003) present a *natural monopoly* model under incomplete information. Firms race to invest first, but only one firm (the leader i) can actually make a profit. Their model is a special case of ours with $V_j^F(x) = 0$ for all x. This implies that $f_j \to \infty$ and thus that $C_j^L = 0$; see Equation (4.4). Firm i's expected value becomes

$$\mathcal{E}_i\left[V_i^T(x,\widehat{x}\mid p_i)\right]=\frac{1-\psi_i(p_i)}{1-\psi_i(\widehat{x})}\left(\frac{p_iD_1}{r-\mu}-k_i\right)\left(\frac{x}{p_i}\right)^{\beta}. \tag{5.11}$$

On the other hand, we can imagine a situation where the follower(s) capture some positive value, but do not affect the leader's profit. For example, a large part of the market is locked-in by the leader's technology (e.g. the Google search engine), but a fringe of smaller followers exist that share the remainder of the market with an alternative technology (e.g. Bing, Yahoo). The behavior of this fringe hardly affects the leader's profits, i.e. $C_j^L = 0$ even if $V_j^F(x) \neq 0$. We obtain the expected value function[11]

$$\mathcal{E}_i\left[V_i^T(x,\widehat{x}\mid p_i)\right]=\left(\frac{p_iD_1}{r-\mu}-k_i\right)\left(\frac{x}{p_i}\right)^{\beta}\frac{1-\psi_i(p_i)}{1-\psi_i(\widehat{x})}+\left(\frac{f_iD_2}{r-\mu}-k_i\right)\left(\frac{x}{f_i}\right)^{\beta}\left(1-\frac{1-\psi_i(p_i)}{1-\psi_i(\widehat{x})}\right). \tag{5.12}$$

In both these models, the leader gains a perpetual monopoly, so its value function does not depend on the investment trigger of the follower. Therefore, the value of a firm jumps only when it invests. The prospect of a perpetual monopoly profit accelerates investment compared to the situation of a duopoly in both these models. Moreover, waiting becomes riskier with decreasing follower payoff. Finally, asymmetry positively affects the value of waiting. In the limits of a natural monopoly (Lambrecht and Perraudin, 2003), the value of waiting vanishes completely. We conclude that real options theory is most relevant when there is a small first-mover advantage, the number of competitors is small, and where firms are (ex-post) more asymmetric.

5.6 Incomplete information and the choice of a prior

We want to find out how optimal investment timing depends on the quality of information, but unfortunately there is no simple answer. One traditional measure is the mean-preserving spread. In the mean-preserving spread, we start from the optimal investment trigger for firm i under complete information p_i^C and a given

[11] Pawlina and Kort (2005) consider a single firm that anticipates a policy change that will adversely affect its investment cost at an uncertain, but state-dependent moment in the future. Substitution of

$$k_{iH}\equiv\frac{f_i\,(D_1-D_2)+(r-\mu)k_i}{r-\mu}$$

in Equation (5.12), shows that we obtain an interpretation of their model in the context of industrial organization.

distribution and slowly increase the variance ω^2 around p_i^C. We measure how the optimal trigger changes. A change in variance affects the optimal trigger only indirectly via its effect on the hazard rate

$$\frac{\partial p_i}{\partial \omega^2} = \frac{\partial p_i}{\partial h_i(p_i \mid \omega^2)} \frac{\partial h_i(\omega^2 \mid p_i)}{\partial \omega^2}. \tag{5.13}$$

The sign of $\frac{\partial h_i(\omega^2|p_i)}{\partial \omega^2}$ does not have a straightforward practical connotation.[12] The effect of an increase of the variance on the hazard rate depends entirely on the distribution and on the initial p_i. We can show however that an increased risk of immediate preemption causes firms to invest sooner. This is very intuitive: a higher marginal probability that competitor j invests in the next moment, accelerates optimal investment by firm i.

Proposition 5.5 *The optimal investment trigger decreases monotonically in the hazard rate.*

$$\frac{\partial p_i}{\partial h(p_i \mid \omega^2)} < 0. \tag{5.14}$$

Proof. See appendix. □

The first factor in Equation (5.13) expresses how p_i must adapt to maintain equality in the first order condition (5.6) after a variance-induced change in the hazard rate. The change in the optimal investment trigger depends thus indirectly on the slope of the expected leader value function and the difference between expected leader and follower value at the current p_i.

Pawlina and Kort (2005) examine a normal distribution $N(p_i^C, \omega^2)$ and find that the optimal investment trigger first decreases but then starts to increase as the quality of information deteriorates in the mean-preserving spread. There is thus a finite standard deviation ω^e that corresponds to the lowest investment trigger. Beyond this critical value, greater uncertainty results in later investment:

$$\begin{cases} \frac{\partial p_i}{\partial \omega^2} < 0 & \textit{for} \quad \omega < \omega^e \\ \frac{\partial p_i}{\partial \omega^2} > 0 & \textit{for} \quad \omega > \omega^e \end{cases}.$$

This non-monotonicity is the result of repeated marginal analysis of Equation (5.6) and depends in a complicated and non-intuitive way on all the parameters of the model, and in particular on the shape of the distribution.[13] The hazard rate of uniform distributions is decreasing in ω^2 everywhere. This implies that additional uncertainty always delays investment. With sufficient uncertainty, investment may even be postponed relative to the complete information case. The hazard rate of a Pareto distributed variable decreases in ω^2 as well. However, due to its distinct asymmetry, it is not well-defined in the limit going to complete information, and the optimal investment trigger at the mean jumps down dramatically and then slowly increases when we introduce uncertainty. This can be easily checked, because these distributions have a closed form solution.

This warns us that careful determination of the prior is very important and can affect the optimal investment trigger in dramatic and non-trivial ways. In particular, it seems hard to justify the use of a Pareto distribution empirically[14] as the latter would imply a very high probability that an opponent has an investment cost just a little larger than a known and fixed minimum cost that carries zero probability itself. Apart from the (truncated) normal distribution, some other distributions with positive support could constitute more empirically plausible priors. The family of Weibull distributions with shape parameter $\lambda \geq 3.36$ approach a more and more leptokurtic normal distribution, while the t-distribution has fatter tails. For symmetric $G_i(\cdot)$ we obtain a left-skewed $\psi_i(\cdot)$ because $p_i(k_j)$ is convex. So, $\psi_i(\cdot)$ can be approximated directly by a loglogistic, lognormal, inverse-Gaussian, F or χ^2 distribution.

[12] Geometrically, the hazard rate is the point probability at p_i divided by the probability mass to the right of p_i (or the survival rate).

[13] Contrary to what Pawlina and Kort (2005) suggest, this monotonicity is not due to a trade-off between useful information and increasing noise. Applied to uniform or Pareto distributions, this argument would imply that these distributions are so noisy that they contain no useful information at all.

[14] The Pareto-distribution is nonetheless prominent in auction and preemption literature because it often allows analytical solutions.

5.7 A solution in terms of $G_i(k_j)$

We initially assumed uncertainty over the opponent's investment cost, but so far we have solved the problem only in terms of uncertainty over a firm's own optimal trigger. In this section we use the composition $G_i(k_j(p_i)) = \psi_i(p_i)$ to express the solution once again in terms of the prior over k_j. The problem then becomes a single non-linear first-order differential equation (5.15). To obtain a boundary condition (5.16), we determine p_{iH} from the fact that information about the opponent's investment trigger is complete at the upper boundary of the support of

$$\lim_{p_i \to p_{iH}} G_i(k_j(p_i)) = \lim_{p_i \to p_{iH}} \psi_i(p_i) = 1.$$

Proposition 5.6 *Firm i with prior $G_i(k_j)$ solves the non-linear differential equation*

$$k_j'(p_i) = \frac{1 - G_i(k_j(p_i))}{G_i'(k_j(p_i))} \frac{\mathcal{E}_i\left[V_i^{LL\prime}(p_i)\right]}{\mathcal{E}_i\left[V_i^{LL}(p_i)\right] - V_i^{FF}(p_i)}, \tag{5.15}$$

with implicit boundary condition

$$k_j\left(p_{iH} = \frac{(r-\mu)}{D_1}\widetilde{k}_i(p_{iH})\right) = k_{jH}, \tag{5.16}$$

where $\widetilde{k}_i(p_{iH}) \equiv k_i + V_i^F - \mathcal{E}_i\left[C_i^L(p_{iH})\right]$ is firm i's virtual investment cost at the upper boundary of the support of its prior.

Proof. See appendix. □

The inverse of the solution $k_j(p_i)$ for Equation (5.15) with boundary condition (5.16) provides the optimal investment trigger for firm i. This inverse exists if $k_j(p_i)$ is strictly increasing, which requires a *well-set* problem. A well-set problem has exactly one local solution through (k_j, p_i) and the solution curves of differential equation (5.15) form a *normal curve family* in the strategic domain. To show that a problem is well-set, we need to show existence, uniqueness and continuity of the solution on the initial conditions. Standard uniqueness proofs do not apply because $k_j'(p_i)$ in Equation (5.15) is not Lipschitz continuous at b_i (where $V_i^{LL}(b_i) = V_i^{FF}(b_i)$). The proof is somewhat more involved.

Proposition 5.7 *There exists a unique solution to differential equation (5.15) if $b_i < p_{iH}$*

Proof. See appendix. □

If $p_{iH} \leq b_i$ then firm i is the high cost firm ($G_i(k_j) = 1$) and invests as a follower. On the contrary, firm i invests at its non-strategic trigger l_i if $G_i(k_j) = 0$.

6 A duopoly under incomplete information: common and consistent priors

In an imperfectly competitive situation, players behave strategically and optimal strategies become interdependent. In order to avoid infinite regress (the need to specify beliefs and beliefs about beliefs and so on) under incomplete information and close our model, we require assumptions on the interdependence of the players' beliefs. The standard assumption is that each firm observes its own investment cost, while each firm's cost is drawn independently from a *common and consistent*[15] *prior* $G(k_j)$ with support (k_L, k_H). In a Bayesian-Nash equilibrium, no player can do better by unilaterally changing his strategy, given these beliefs.

[15] Common priors are *common knowledge* among the players. Consistent priors are priors that are the same for all players.

A Markovian version of such equilibrium $[(k_i, k_j), (G(k_i), G(k_j))]$ solves the following system of nonlinear first-order differential equations of the form (5.15) and boundary conditions of the form (5.16)

$$\begin{cases} k_j'(p) = \frac{1-G(k_j(p))}{G'(k_j(p))} \frac{\mathcal{E}_i\left[V_i^{LL\prime}(p)\right]}{\mathcal{E}_i\left[V_i^{LL}(p)\right]-V_i^{FF}(p)} \\ k_i'(p) = \frac{1-G(k_i(p))}{G'(k_i(p))} \frac{\mathcal{E}_j\left[V_j^{LL\prime}(p)\right]}{\mathcal{E}_j\left[V_j^{LL}(p)\right]-V_i^{FF}(p)} \\ k_j\left(p_H = \frac{(r-\mu)}{D_1}\widetilde{k}_i(p_H)\right) = k_H \\ k_i\left(p_H = \frac{(r-\mu)}{D_1}\widetilde{k}_j(p_H)\right) = k_H \end{cases}, \tag{6.1}$$

where $\widetilde{k}_i(p_{iH}) = \left(k_i + V_i^F - \mathcal{E}_i\left[C_i^L(p_{iH})\right]\right)$. This game possesses the characteristics of a *first-price (or sealed high-bid) auction with a continuum of types* Maskin and Riley (1986) and has a unique and symmetric equilibrium. In the language of auction theory, this is a private value auction because the value of investment differs between firms. We briefly check the conditions. First, $k_j(p)$ is a *continuous* function on the strategic domain. This is true because the (sufficient) condition for a maximum $h_i'(p) \geq 0$ ensures that the first order conditions yield a bijective mapping $p : k_j \to p(k_j)$ between costs and (optimal) investment triggers in the strategic domain. Second, $k_j(p)$ is *strictly increasing* on the strategic domain. We have established this in Proposition 5.7. Third, the optimal triggers coincide at the boundaries of the distribution over the investment costs. We show this in Proposition 6.1.

Proposition 6.1 *The endpoints of the prior map on the same values for both firms, i.e.* $p_i(k_L) = p_j(k_L)$ *and* $p_i(k_H) = p_j(k_H)$.

Proof. See appendix. □

Proposition 6.2 *There exists a unique, symmetric equilibrium for the game of system (6.1) if* $b_i < p_H$ *and* $b_j < p_H$

Proof. See appendix. □

Once again, the optimal investment trigger is the inverse of the solution of system (6.1). In Section 5.6, we investigated the impact of information quality on the optimal investment trigger of a single firm. Under common and consistent priors, these results should remain valid as both firms react in the same fashion.

While philosophically captivating,[16] common and consistent priors are a safe assumption only when nature's moves represent public events that are common knowledge (e.g. Binmore (2007); Fudenberg and Tirole (1991)). This assumption, that nature attributes a cost from the same distribution $G(\cdot)$ to each firm, implies here that both firms have a priori identical information about each other's investment costs.

7 A duopoly under incomplete information: pure asymmetric information

Now we study what happens if information is not identically distributed over the firms.[17] Firms may be able to achieve a competitive advantage if they possess better information than their competitor(s). Competition between a public (uninformed) firm, that has to divulge a wealth of sensitive information to its stakeholders, and a private (informed) firm is an example. In this case, we can assume that priors are common but not consistent. Competition between a firm that carefully studies its competitor by an elaborate *marketing information system* or resorts to outright industrial espionage, and a firm that does not pursue such endeavours

[16] The so-called *Harsanyi doctrine* has strong philosophical foundations going back to Rawls (1971) veil of ignorance.

[17] Surprisingly little has been published on alternative assumptions. Maskin and Riley (2000a,b) and Lebrun (1999) show when an equilibrium exists in *independent private value auctions* (common but inconsistent priors) and identify the interesting case of *first order stochastic dominance*, where $\forall k_i, k_j : |G(k_j) - G(k_i)| \geq 0$. Lebrun (2006) and Maskin and Riley (2003) provide sufficient conditions for uniqueness. Lizzeri and Persico (2000) and Maskin and Riley (2000a,b, 2003) also establish uniqueness and existence under affiliation (pairwise correlated distributions).

provides another example. In the latter example, priors become common when espionage is detected and it is known what strategic data have been stolen.[18]

Since a full analysis would be quite untractable,[19] we investigate – in neo-classical economic tradition – only an extreme case of *pure asymmetric information*[20] where the informed firm i has complete information about the investment cost of the uninformed firm u, but the latter is only certain about the investment cost of firm i up to a prior $G_u(k_i)$. We maintain the common prior assumption.

We evaluate the decision of the uninformed firm first. The uninformed firm choses its optimal investment trigger p_u, in the knowledge that it will be preempted if the *break-even trigger* of the informed firm is lower. This happens with probability[21]

$$\psi_u(b_i < p_u) = G_u(k_i(b_i)) = G_u(k_i).$$

This probability cannot be larger than $\psi_i(p_i < p_u)$ from Section 5.2 because $b_i \leq p_i$. It follows immediately that the asymmetric information forces the uninformed firm to give up more value of waiting than in the case of incomplete information. The value function of the uninformed firm is

$$\mathcal{E}_u\left[V_u^T(x,\widehat{x} \mid p_u)\right] = \frac{1-\psi_u(p_u)}{1-\psi_u(\widehat{x})}\mathcal{E}_u\left[V_u^{LL}(x,p_u)\right] + \left(1 - \frac{1-\psi_u(p_u)}{1-\psi_u(\widehat{x})}\right)V_u^{FF}(x,p_u), \tag{7.3}$$

with the optimal p_u given by

$$h_u(p_u) = \frac{\mathcal{E}_u\left[V_u^{LL\prime}(p_u)\right]}{\mathcal{E}_u\left[V_u^{LL}(p_u)\right] - V_u^{FF}(p_u)}. \tag{7.4}$$

Because of common knowledge, the informed firm can make the same calculations and will invest at $p_u - \epsilon$ if its true cost is such that $b_i < p_u$. The equilibrium thus has a distinct Stackelberg flavor with the uninformed firm being the leader, because the informed firm can take p_u as given before making its decision to invest.

Contrary to what one would expect, there is only one situation where the (informed) firm can benefit from asymmetric information. The inefficient and informed firm can overcome its natural follower role and capture the leader value if the efficient but uninformed firm underestimates the latter's profitability so much that it postpones investment beyond the break-even point of the inefficient firm. Public firms can be at such strategic disadvantage if they seriously underestimate their privately owned competitors. Remember that under incomplete information (and common and consistent priors), the efficient firm postpones investment to maximize its expected value based on $\psi_i(p_j < p_i)$. If it is known that an inefficient firm acquires complete information, then its efficient but uninformed opponent wants to accelerate investment to $\max\{b_{efficient}, p_u\}$ with $p_u \equiv p_{efficient}^{asymmetric} < p_{efficient}^{incomplete}$ to maximize its expected value, taking $\psi_u(b_j < p_i)$ as a reference. The inefficient firm will invest first if $p_u \geq b_{inefficient}$.

An efficient firm can never benefit from complete information about its uninformed competitor if this is common knowledge. The uninformed and inefficient firm will accelerate planned investment to $\max\{b_{inefficient}, p_u\}$ with $p_u \equiv p_{inefficient}^{asymmetric} < p_{inefficient}^{incomplete}$ and the efficient firm will have to ϵ-preempt sooner, thereby losing some value of waiting.

[18] Strategic information was allegedly stolen at Renault in 2011 and this fact received lots of publicity. See The economist (2011) *The Renault "spying" affair. A new twist.* (March 10th, 2011) [online] Available at `http://www.economist.com/node/18332938/print` Accessed on 07/09/2011.

[19] We study a private value auction, while Graham (2011) studies a common value auction under asymmetric information.

[20] This case is not covered by Maskin and Riley (2003a,b).

[21] It is worth mentioning that Hsu and Lambrecht (2007) extend the model of Lambrecht and Perraudin (2003) to pure asymmetric information and find that the efficient firm always invests at its break-even trigger. This simply cannot be true as firms still use marginal analysis. With a follower value of zero, the value of the uninformed firm u

$$V_u^T(x,\widehat{x} \mid p_u) = \frac{1-\psi_u(p_u)}{1-\psi_u(\widehat{x})}\left(\frac{p_u D_1}{r-\mu} - k_u\right)\left(\frac{x}{p_u}\right)^{\beta} \tag{7.1}$$

is maximized for

$$p_u = -\frac{\beta + p_u h_u(p_u)}{1-\beta-p_u h_u(p_u)}\frac{(r-\mu)}{D_1}k_u \tag{7.2}$$

and we conclude that $p_u \to b_u = \frac{(r-\mu)}{D_1}k_u$ only if $-\frac{\beta+p_u h_u(p_u)}{1-\beta-p_u h_u(p_u)} \to 1$, i.e. if $h_u(p_u) \to \infty$.

Whether the efficient or inefficient firm engages in such activities, we conclude that espionage always works counterproductively when discovered (because this always accelerates investment). Since no firm knows a priori whether it is the more efficient firm or not, a firm i should only invest an amount $C_i(\Im)$ to obtain complete information where priors are common, if

$$\begin{aligned} 0 \quad \leq \quad & -C_i(\Im) + \frac{1-\psi_i(p_y)}{1-\psi_i(\widehat{x})}\mathcal{E}_i\left[V_i^{LL}(p_u) - V_i^{LL}(p_y)\right] \\ & + \left(1 - \frac{1-\psi_i(p_y)}{1-\psi_i(\widehat{x})}\right)\left(\frac{1-\psi_u(p_u)}{1-\psi_u(\widehat{x})}\mathcal{E}_i\left[V_i^{LL}(p_u)\right] + \left(1-\frac{1-\psi_u(p_u)}{1-\psi_u(\widehat{x})}\right)V_i^{FF}(p_u) - V_i^{FF}(p_y)\right), \end{aligned}$$

where p_y is the optimal trigger under incomplete information and p_u is the optimal trigger under pure asymmetric information. We conjecture that this is almost never true and that the net value of information (given that it is common knowledge that the firm possesses this information) is almost always non-positive.

This obviously does not hold for espionage that remains covert. In that case, we conjecture that the uninformed firm will keep investing *as if* information were incomplete for both firms and priors were common and consistent; i.e. at the $p_y \equiv p^{incomplete}_{inefficient}$ or the $p_y \equiv p^{incomplete}_{efficient}$ from Section 6. The spying firm can invest right before p_y as long as $b_i < p_y$. It is optimal for a firm to invest in information if

$$0 \leq -C_i(\Im) + \left(\frac{\psi_i(p_y) - \psi_u(p_y)}{1-\psi_i(\widehat{x})}\right)\mathcal{E}_i\left[V_i^{LL}(p_y)\right] + \left(\frac{\psi_u(p_y) - \psi_i(p_y)}{1-\psi_i(\widehat{x})}\right)V_i^{FF}(p_y).$$

We conclude that espionage pays particularly where the first-mover advantage and the increase in probability of investing first are large. Furthermore, we conclude that covert espionage can be more rewarding than open espionage, because both the probability of becoming the leader and the leader value obtained are lower under common priors.

8 Conclusion

We describe competition between firms that are racing to invest first in a new and uncertain market where there is a first-mover advantage. Firms are not certain about each others' investment cost either. Our model generalizes the complete information model of Pawlina and Kort (2006), a reinterpreted Pawlina and Kort (2005) and the natural monopoly models of Lambrecht and Perraudin (2003) and Hsu and Lambrecht (2007).

First, we derive the optimal investment trigger in a duopoly under the important assumption that one's opponent does not behave strategically. We find that investment is accelerated in markets with more competitors, markets with an important first-mover advantage and markets where technological lock-in forces competitors into the competitive fringe.

We evaluate how investment timing is affected by information quality as the *mean-preserving spread* under a given distribution. We find that firms do not always invest sooner as the quality of their information deteriorates and may even invest later than they would have under complete information. Firms postpone investment as long as the increased expected marginal loss due to preemption is outweighed by the marginal increase in the option value of waiting.

When we allow for strategic interaction, assumptions about the interdependence of beliefs characterize the equilibrium solution. Under the standard assumption of common and consistent priors, we find that a unique and symmetric *Bayesian-Markov equilibrium* exists.

We relax the consistent prior assumption to investigate what happens when it is known that one firm has better information than its competitor. In a game between a public and a private firm, such a common prior assumption is plausible. We find that firms invest sooner than under incomplete information in a game of *pure* asymmetric information with common priors, where one firm has complete information but the other firm does not. We conclude that firms will not usually benefit if it is known that they have acquired better information and that industrial espionage is usually counterproductive when discovered.

The assumption that both firms are perfectly knowledgeable about what their opponents believe but not about their investment costs seems to be quite a stretch, especially in a situation of industrial espionage (where these beliefs could be false). A more general model of asymmetric information could be the subject of further research. However, without some assumption on the interdependence of the priors, an equilibrium solution must be precluded. It is not clear what assumption would be both tractable and plausible and perhaps a case could be made for the assumption that beliefs are often not interdependent in reality. In that case, we conjecture that each firm might solve a single player decision problem of Section 5. In our opinion, these rather philosophical questions have not attracted adequate attention.

Finally, our model does not easily generalize to more complicated *existing market models* (see Pawlina & Kort, 2006). Such models have an additional continuum of equilibria, called *simultaneous equilibria.* We think that the resulting equilibrium selection problem could be challenging under incomplete information.

A Appendix: Proofs of propositions

Proof of Proposition 4.1. First, because the strategic trigger is not defined for $k_j > k_j^*$ (see Proposition 5.1), l_i is optimal there by default. Second, we show that there exists an interval $\left[k_j^S, k_j^*\right)$. We observe that $l_i = p_i(k_j^S)$ on the lower bound of the non-strategic domain and solve $\xi_j(l_i, k_j^S) = 0$ for k_j^S. We obtain

$$\xi_j(l_i, k_j^S) \equiv \frac{l_i D_1}{r-\mu} - k_j^S + \frac{f_i(D_2 - D_1)}{r-\mu}\left(\frac{l_i}{f_i}\right)^{\beta} - \left(\frac{f_j D_2}{r-\mu} - k_j^S\right)\left(\frac{l_i}{f_j}\right)^{\beta} = 0. \tag{A.1}$$

$V_j^L(x)$ and $V_j^F(x)$ intersect at $b_j = l_i$ for investment cost k_j^S. The former is increasing and strictly concave, the latter is increasing and strictly convex. Therefore, k_j^S cannot lie to the right of k_j^* (where $V_j^L(x)$ is tangent to $V_j^F(x)$). This proves that there exists an interval $\left[k_j^S, k_j^*\right)$. Third, we show that firm i prefers its non-strategic trigger l_i over its strategic trigger p_i in this interval. First, we compute the authentic leader value (see Section 4.5) for each trigger. We obtain

$$V_i^{LL}(k_j \mid l_i) = \left(\frac{l_i D_1}{r-\mu} - k_i + \frac{f_i \frac{k_j}{k_i}(D_2 - D_1)}{r-\mu}\left(\frac{l_i}{f_i \frac{k_j}{k_i}}\right)^{\beta}\right)\left(\frac{x}{l_i}\right)^{\beta}$$

for the non-strategic trigger, and (using Proposition 5.1), we find that the strategic trigger is:

$$V_i^{LL}(k_j, p_i(k_j)) = \left(\frac{p_i\left(k_j\right) D_1}{r-\mu} - k_i + \frac{f_i \frac{k_j}{k_i}(D_2 - D_1)}{r-\mu}\left(\frac{p_i(k_j)}{f_i \frac{k_j}{k_i}}\right)^{\beta}\right)\left(\frac{x}{p_i(k_j)}\right)^{\beta}.$$

The value functions coincide per definition at k_j^S. Now, we prove that their slopes coincide as well (see also Figure 4.1). The authentic value for a non-strategic firm has positive derivative:

$$\frac{\partial V_i^{LL}(k_j \mid l_i)}{\partial k_j} = (1-\beta)\frac{\left(\frac{f_i}{k_i}\right)^{1-\beta}(D_2 - D_1)}{r-\mu} x^{\beta} k_j^{-\beta}.$$

For a strategic firm we find derivative

$$\frac{\partial V_i^{LL}(k_j \mid p_i\left(k_j\right))}{\partial k_j} = (1-\beta)\frac{p_i\left(k_j\right)^{-\beta} p_i'\left(k_j\right) D_1}{r-\mu} x^{\beta} + \beta k_i p_i(k_j)^{-\beta-1} p_i'(k_j) x^{\beta} + (1-\beta)\frac{\left(\frac{f_i}{k_i}\right)^{1-\beta}(D_2 - D_1)}{r-\mu} x^{\beta} k_j^{-\beta}.$$

The difference between these derivatives is

$$\begin{aligned} \Delta &\equiv \frac{\partial V_i^{LL}(k_j \mid p_i\left(k_j\right))}{\partial k_j} - \frac{\partial V_i^{LL}(k_j \mid l_i)}{\partial k_j} \\ &= \left((1-\beta)\frac{p_i\left(k_j\right) D_1}{r-\mu} + \beta k_i\right) p_i(k_j)^{-\beta-1} p_i'(k_j) x^{\beta}. \end{aligned}$$

The sign of Δ depends on the the first factor, which we can multiply by $\frac{1}{\beta-1}\frac{r-\mu}{D_1} > 0$ to obtain

$$sgn\left\{\Delta\right\} = sgn\left\{\left(l_i - p_i\left(k_j\right)\right)\right\}.$$

It follows immediately that

$$\begin{cases} \frac{\partial V_i^{LL}(k_j|p_i(k_j))}{\partial k_j} < \frac{\partial V_i^{LL}(k_j|l_i)}{\partial k_j} & for \quad k_j > k_j^S \\ \frac{\partial V_i^{LL}(k_j|p_i(k_j))}{\partial k_j} > \frac{\partial V_i^{LL}(k_j|l_i)}{\partial k_j} & for \quad k_j < k_j^S \\ \frac{\partial V_i^{LL}(k_j|p_i(k_j))}{\partial k_j} = \frac{\partial V_i^{LL}(k_j|l_i)}{\partial k_j} & for \quad k_j = k_j^S \end{cases} .$$

Finally, because $V_i^{LL}(k_j \mid l_i)$ increases faster to the right of common point k_j^S and decreases slower to the left than $V_i^{LL}(k_j \mid p_i(k_j))$, we obtain that

$$\begin{cases} V_i^{LL}(k_j \mid p_i(k_j)) < V_i^{LL}(k_j \mid l_i) & for \quad k_j > k_j^S \\ V_i^{LL}(k_j \mid p_i(k_j)) > V_i^{LL}(k_j \mid l_i) & for \quad k_j < k_j^S \\ V_i^{LL}(k_j \mid p_i(k_j)) = V_i^{LL}(k_j \mid l_i) & for \quad k_j = k_j^S \end{cases} .$$

Hence, firm i uses the strategic trigger when $k_j < k_j^S$ and the non-strategic trigger when $k_j \geq k_j^S$. □

Proof of Proposition 4.2. Application of the Feynman-Kac theorem immediately yields the *authentic value function* of leader i

$$V_i^{LL}(x \mid x_i, f_j) = \left(\frac{x_i D_1}{r-\mu} - k_i + \frac{f_j (D_2 - D_1)}{r-\mu} \left(\frac{x_i}{f_j} \right)^\beta \right) \left(\frac{x}{x_i} \right)^\beta, \tag{A.2}$$

where $x_i \in \{p_i, l_i\}$ is the optimal investment trigger for the leader. Single firm optimization with respect to x_i now yields indeed the non-strategic trigger l_i. However, usually we have to take follower j into account. The authentic value function for follower j is

$$V_j^{FF}(x \mid x_i, f_j) = \left(\frac{f_j D_2}{r-\mu} - k_j \right) \left(\frac{x_i}{f_j} \right)^\beta \left(\frac{x}{x_i} \right)^\beta . \tag{A.3}$$

It then follows immediately that

$$V_j^{LL}(x_i) - V_j^{FF}(x_i) = V_j^L(x_i) - V_j^F(x_i). \tag{A.4}$$

□

Proof of Proposition 4.3. We rewrite $V_i^{FF}(x \mid x_j, f_i)$ as $V_i^F(x_j) \left(\frac{x}{x_j} \right)^\beta$. $V_i^F(x)$ is strictly increasing from $V_i^F(0) = 0$. So $V_i^F(x_j)$ is a positive number. $\left(\frac{x}{x_j} \right)^\beta$ is non-negative, strictly increasing and strictly convex, and $V_i^{FF}(x)$ inherits these properties. Similarly, $V_i^L(x_i) \geq 0$ because $V_i^L(x_i) - V_i^F(x_i) = 0$ and $V_i^F(x_i) > 0$. So $V_i^{LL}(x)$ is also non-negative, strictly increasing and strictly convex. □

Proof of Proposition 5.1. First, since $\xi_j(p, k_j)$ is continuously differentiable in an open set around each (p_i^0, k_j^0) in $[0, k_j{}^*)$ with $\xi_j(p_i^0, k_j^0) = 0$ and $\frac{\partial \xi_j(p_i^0, k_j^0)}{\partial p_i} \neq 0$, we know by the implicit function theorem that a function $p_i = p_i(k_j)$ exists locally[22] and is continuously differentiable with derivative

$$\frac{dp_i(k_j)}{dk_j} = - \frac{\frac{\partial \xi_j(p_i, k_j)}{\partial k_j}}{\frac{\partial \xi_j(p_i, k_j)}{\partial p_i}}. \tag{A.5}$$

Second, to show that $p_i'(k_j) > 0$ for $k_j \in [0, k_j^*)$, consider the numerator first. The derivative of Equation 5.2 with respect to k_j is

$$\frac{\partial \xi_j(p_i, k_j)}{\partial k_j} = -1 + (\beta - 1) \left(\frac{\frac{f_i}{k_i} D_2}{r-\mu} - 1 \right) \left(\frac{k_i}{f_i} \frac{p_i}{k_j} \right)^\beta .$$

[22] Since we consider only the smallest real solution p_i of $\xi_j(x, k_j)$, the implicit function theorem is also globally true. Every p_i of the domain has a unique image in $\mathbb{R}^+$.

Using Equations (4.2) and (5.1) this simplifies to

$$\frac{\partial \xi_j(p_i,k_j)}{\partial k_j} = -1 + \left(\frac{p_i}{f_j}\right)^{\beta}, \tag{A.6}$$

and (because $p_i < f_j$) we conclude that

$$\frac{\partial \xi_j(p_i,k_j)}{\partial k_j} < 0.$$

The denominator

$$\frac{\partial \xi_j(p_i,k_j)}{\partial p_i} = \frac{D_{10}}{r-\mu} + \beta\left(\frac{f_i(D_2-D_1)}{r-\mu}\left(\frac{1}{f_i}\right)^{\beta} - \left(\frac{f_jD_2}{r-\mu} - k_j\right)\left(\frac{1}{f_j}\right)^{\beta}\right)p_i^{\beta-1} > 0$$

is zero at $k_j^* > k_j^S$, where k_j^* was defined in Equation (4.5). It is monotonically decreasing. To wit, its derivative

$$\frac{\partial^2 \xi_j(p_i,k_j)}{\partial p_i^2} = \beta(\beta-1)\left(\frac{f_i(D_2-D_1)}{r-\mu}\left(\frac{1}{f_i}\right)^{\beta} - \left(\frac{f_jD_2}{r-\mu} - k_j\right)\left(\frac{1}{f_j}\right)^{\beta}\right)p_i^{\beta-2} \tag{A.7}$$

is negative because $D_2 < D_1$ (for the second term use Equation (5.1) again). We conclude that the denominator must be strictly positive for all $k_j \in [0, k_j^*)$ and it follows immediately that $p_i'(k_j)$ is strictly increasing. Finally, $\xi_j(p_i,k_j)$ is strictly concave in p_i and in k_j because the Hessian

$$H = \begin{bmatrix} \frac{\partial^2 \xi_j(p_i,k_j)}{\partial k_j^2} & \frac{\partial^2 \xi_j(p_i,k_j)}{\partial k_j \partial p_i} \\ \frac{\partial^2 \xi_j(p_i,k_j)}{\partial k_j \partial p_i} & \frac{\partial^2 \xi_j(p_i,k_j)}{\partial p_i^2} \end{bmatrix}$$

is negative definite. (The leading principal minors of the negative of the Hessian are positive.) This in turn implies strict convexity of $p_i(k_j)$, which coincides with the upper level set of $\xi_j(p_i,k_j)$ at $\xi_j = 0$. □

Proof of Proposition 5.2. The strategic codomain $[f_i, l_i]$ is a subset of the image of $[0, k_j^*)$, hence $p_i(k_j)$ is onto (surjective). Since $p_i(k_j)$ is continuous and strictly increasing, it is a one-to-one mapping (injective). By the inverse function theorem, k_j can be mapped one-to-one onto p_i for all k_j in the strategic domain. $k_j(p_i)$ is well-defined because $p_i(k_j)$ is a strictly increasing function. The inverse of a strictly increasing and strictly convex function is strictly increasing and strictly concave. We conclude that $k_j(p_i)$ is continuously differentiable, strictly increasing and strictly concave in $p_i[k_i, k^S)$. □

Proof of Proposition 5.3. The value function can be obtained from Equation (5.4) as

$$\max_{z_i \in [f_i, l_i]} V_i^T(x, \widehat{x}, z_i) = V_i^T(x, \widehat{x} \mid p_i). \tag{A.8}$$

Assuming interior solutions and compactness, the strategic trigger p_i optimizes $V_i^T(x, \widehat{x}, z_i)$ in the strategic domain. The first order condition is

$$\max_{p_i \in (p_{iL}, p_{iH})} V_i^T(x, \widehat{x} \mid p_i) = 0, \tag{A.9}$$

or equivalently

$$0 = \frac{-\psi_i'(p_i)}{1-\psi_i(\widehat{x})}\mathcal{E}_i\left[V_i^{LL}(x \mid p_i)\right] + \frac{1-\psi_i(p_i)}{1-\psi_i(\widehat{x})}\mathcal{E}_i\left[V_i^{LL\prime}(x \mid p_i)\right] + \frac{\psi_i'(p_i)}{1-\psi_i(\widehat{x})}V_i^{FF}(x \mid p_i). \tag{A.10}$$

We can rewrite this as

$$h_i(p_i) \equiv \frac{\psi_i'(p_i)}{1-\psi_i(p_i)} = \frac{\mathcal{E}_i\left[V_i^{LL\prime}(x \mid p_i)\right]}{\mathcal{E}_i\left[V_i^{LL}(x \mid p_i)\right] - V_i^{FF}(p_i)}. \tag{A.11}$$

□

Proof of Proposition 5.4. We expand the expected value function by substitution of Equations (A.2) and (A.3), and apply the results from Section 5.1 to obtain

$$V_i^T(x,\widehat{x}\mid p_i)=\left(\frac{1-\psi_i(p_i)}{1-\psi_i(\widehat{x})}\right)\left[\left(\frac{D_1p_i}{r-\mu}-k_i+\left(\frac{\frac{\kappa_j(p_i)}{k_i}f_i\left(D_2-D_1\right)}{r-\mu}\right)\left(\frac{p_i}{\frac{\kappa_j(p_i)}{k_i}f_i}\right)^{\beta}\right)\left(\frac{x}{p_i}\right)^{\beta}\right] \\ +\left(1-\frac{1-\psi_i(p_i)}{1-\psi_i(\widehat{x})}\right)\left[\left(\frac{f_iD_2}{r-\mu}-k_i\right)\left(\frac{p_j}{f_i}\right)^{\beta}\left(\frac{x}{p_j}\right)^{\beta}\right]. \tag{A.12}$$

The expanded first-order condition is

$$\begin{aligned}(1-\beta)\tfrac{D_1p_i^{-\beta}}{r-\mu}+\beta k_ip_i^{-\beta-1}+(1-\beta)\tfrac{\kappa_j'\kappa_j^{-\beta}k_i^{\beta-1}f_i^{1-\beta}(D_2-D_1)}{r-\mu}\\ +\left(-\tfrac{D_1p_i^{1-\beta}}{r-\mu}+k_ip_i^{-\beta}-\tfrac{\kappa_j(p_i)^{1-\beta}k_i^{\beta-1}f_i^{1-\beta}(D_2-D_1)}{r-\mu}+\tfrac{f_i^{1-\beta}D_2}{r-\mu}-k_if_i^{-\beta}\right)h_i(p_i)=0\end{aligned}. \tag{A.13}$$

We differentiate to find the second order condition for a (unique) maximum

$$\begin{aligned}-\beta(1-\beta)\tfrac{D_1p_i^{-\beta-1}}{r-\mu}-\beta(\beta+1)k_ip_i^{-\beta-2}+(1-\beta)\left(\kappa_j''\kappa_j-\beta\left(\kappa_j'\right)^2\right)\tfrac{\kappa_j^{-\beta}k_i^{\beta-1}f_i^{1-\beta}(D_2-D_1)}{r-\mu}\\ +\left(-(1-\beta)\tfrac{D_1p_i^{-\beta}}{r-\mu}-\beta k_ip_i^{-\beta-1}-(1-\beta)\tfrac{\kappa_j'\kappa_j^{-\beta}k_i^{\beta-1}f_i^{1-\beta}(D_2-D_1)}{r-\mu}\right)h_i(p_i)\\ +\left(-\tfrac{D_1p_i^{1-\beta}}{r-\mu}+k_ip_i^{-\beta}-\tfrac{\kappa_j{}^{1-\beta}k_i^{\beta-1}f_i^{1-\beta}(D_2-D_1)}{r-\mu}+\tfrac{f_i^{1-\beta}D_{11}}{r-\mu}-k_if_i^{-\beta}\right)h_i'(p_i)\end{aligned}<0. \tag{A.14}$$

We rewrite the second order condition as

$$\begin{aligned}\beta\left[(\beta-1)D_1p_i-(\beta+1)(r-\mu)k_i\right]p_i^{-\beta-2}+(1-\beta)\left(\kappa_j''\kappa_j-\beta\left(\kappa_j'\right)^2\right)\kappa_j^{-\beta}k_i^{\beta-1}f_i^{1-\beta}\left(D_2-D_1\right)\\ +\left(\left[(\beta-1)D_1p_i-\beta\left(r-\mu\right)k_i\right]p_i^{-\beta-1}-(1-\beta)\kappa_j'\kappa_j{}^{-\beta}k_i^{\beta-1}f_i^{1-\beta}\left(D_2-D_1\right)\right)h_i(p_i)\\ +\left(\frac{1}{\beta-1}k_if_i^{-\beta}-\left(D_1p_i-(r-\mu)k_i\right)p_i^{-\beta}-k_j(p_i)^{1-\beta}k_i^{\beta-1}f_i^{1-\beta}\left(D_2-D_1\right)\right)h_i'(p_i)\end{aligned}<0. \tag{A.15}$$

and evaluate the sign of each line. We know that p_i lies between the Marshallian and non-strategic trigger, so $\frac{r-\mu}{D_1}k_i<p_i<\frac{\beta}{\beta-1}\frac{r-\mu}{D_1}k_i$ and we know that the hazard rate is positive (per definition). Finally,

$$\kappa_j''\kappa_j-\beta\left(\kappa_j'\right)^2\leq 0 \tag{A.16}$$

because κ_j is positive, increasing and strictly concave. If we impose the sufficient (but not necessary) condition

$$h_i'(p_i)\geq 0, \tag{A.17}$$

then each line in Equation (A.15) is negative and the second order condition is strictly satisfied. □

Proof of Proposition 5.5. We take the derivative of Equation (5.6) with respect to p_i:

$$\frac{\partial h_i(p_i)}{\partial p_i}=\frac{\partial}{\partial p_i}\frac{V_i^{LL'}(p_i)}{V_i^{LL}(p_i)-V_i^{FF}(p_i)}=\frac{V_i^{LL''}(p_i)\left[V_i^{LL}(p_i)-V_i^{FF}(p_i)\right]-\left[V_i^{LL'}(p_i)\right]^2}{\left[V_i^{LL}(p_i)-V_i^{FF}(p_i)\right]^2}<0. \tag{A.18}$$

The numerator is negative because $V_i^{LL\prime}(p_i)$ is decreasing in p_i and $V_i^{LL}(p_i)-V_i^{FF}(p_i)$ is positive (see Proposition 4.3). Hence $\frac{\partial h_i(p_i)}{\partial p_i}$ and its inverse are negative as well. Pawlina and Kort (2005) provide an alternative proof. □

Proof of Proposition 5.6. We express the hazard rate in terms of $G_i(k_j(p_i))$ using the chain rule

$$h_i(p_i)=\frac{\psi_i'(p_i)}{1-\psi_i(p_i)}=\frac{G_i'(k_j(p_i))}{1-G_i(k_j(p_i))}k_j'(p_i) \tag{A.19}$$

and substitute in first order condition (5.6) to obtain Equation (5.15). The boundary condition reflects the fact that information becomes complete if competitor j has not invested yet as x approaches $p_{iH} = p_i(k_{jH})$. In the limit $\lim_{p_i \to p_{iH}} \psi_i(p_j \le p_i) = 1$, firm i is certain to be preempted in the next instant. Thus, its hazard rate goes to infinity in the limit. Define $h_i^*(p_i) \equiv p_i h_i(p_i)$ and rewrite the expanded first order condition (5.8) as

$$(1-\beta - h_i^*(p_i))D_1 p_i + (\beta + h_i^*(p_i))(r-\mu)k_i +$$
$$\left((1-\beta)\left(\tfrac{\kappa_j}{k_i}\right)^{-\beta\prime}\tfrac{\kappa_j'}{k_i} f_i(D_2 - D_1)p_i - \left(\tfrac{\kappa_j}{k_i}\right)^{1-\beta} f_i(D_2-D_1)h_i^*(p_i) + f_i D_2 h_i^*(p_i)\right)\left(\tfrac{p_i}{f_i}\right)^{\beta} = 0.$$

Now, divide by $h_i^*(p_i)$ and let $p_i \to p_{iH}$ (and thus $\lim_{p_i \to p_{iH}} h_i^*(p_i) = +\infty$). We obtain

$$\begin{aligned} p_{iH} &= \frac{(r-\mu)}{D_1}k_i + \frac{(r-\mu)}{D_1}\left(\frac{f_i D_2}{r-\mu} - k_i - \left(\frac{\kappa_j}{k_i}\right)^{1-\beta} f_i\left(\frac{D_2-D_1}{r-\mu}\right)\right)\left(\frac{p_{iH}}{f_i}\right)^{\beta} \\ &= \frac{(r-\mu)}{D_1}\left(k_i + V_i^F - \mathcal{E}_i\left[C_i^L(p_{iH})\right]\right) = \frac{(r-\mu)}{D_1}\widetilde{k}_i(p_{iH}), \end{aligned}$$

with

$$\widetilde{k}_i(p_i) \equiv k_i + V_i^F - \mathcal{E}_i\left[C_i^L(p_i)\right]).$$

So, the boundary condition is

$$k_i\left(p_{iH} = \frac{(r-\mu)}{D_1}\widetilde{k}_i(p_{iH})\right) = k_{iH}. \tag{A.20}$$

We call $\widetilde{k}_i(p_i)$ the *virtual investment cost* because it refers to an adjusted net present value criterion

$$\frac{D_1}{r-\mu} - \widetilde{k}_i(p_i) = 0.$$

that takes into account market uncertainty and duopolistic competition under complete information (or here with preemption nearly certain at p_{iH}). Firm i behaves as a monopolist with investment cost $\widetilde{k}_i(p_i)$ in a deterministic market. With $\widetilde{k}_i(p_i) \ge k_i$ under negative externalities, we find again that firm i postpones investment relative to its Marshallian trigger $p_i \ge \frac{(r-\mu)}{D_1}k_i$. □

Proof of Proposition 5.7. Let b_i be the break-even threshold for firm i. Then,

$$\lim_{p_i \to b_i} V_i^{LL}(p_i) - V_i^{FF}(p_i) = 0. \tag{A.21}$$

We integrate Equation (5.15). If $b_i < p_{jH}$ then the integral

$$\ln\left[1 - G_i(k_j(p_i))\right] = -\int_{p_{iL}}^{p_i} \frac{V_i^{LL\prime}(z)}{V_i^{LL}(z) - V_i^{FF}(z)}dz \tag{A.22}$$

has to converge because the lefthand side is negative for all $k_j < k_{jH} = k_j(p_{iH})$. Hence, there exists a p_{iL} such that $V_i^{LL}(b_i) = V_i^{FF}(b_i)$. We show that there exists a unique p_{iL} such that boundary condition $k_j(p_{iH}) = k_{jH}$ is respected. Suppose without loss of generality and to obtain a contradiction that there exist two constants $p_{iL}^* > p_{iL}^{**}$ such that there exists two solutions $k_j^*(p_i)$ and $k_j^{**}(p_i)$ to Equation (5.7). Since $k_j^*(p_i^*) = k_j^{**}(p_i^{**}) = k_{jL}$ we have $k_j^*(p_{iL}^{**}) < k_j^{**}(p_{iL}^{**})$. Substitution in Equation (A.22) and substraction gives us

$$\ln\frac{1 - G(k_j^*(p))}{1 - G(k_j^{**}(p))} = \int_{p_{iL}^*}^{p} \frac{V_i^{LL\prime}(z)}{V_i^{LL}(z) - V_i^{FF}(z)}dz - \int_{p_{iL}^{**}}^{p} \frac{V_i^{LL\prime}(z)}{V_i^{LL}(z) - V_i^{FF}(z)}dz. \tag{A.23}$$

For any $p_i \ge p_{iL}^*$ we have that $\frac{k_j^*(p_i)}{k_j^{**}(p_i)} < 1$ and decreasing in p_i and cannot converge to unity when $p_i \to b_i$ (the righthand side of Equation (A.23) goes to zero). It follows immediately that there exists a unique solution to the differential equation and boundary condition $k_j(p_{iH}) = k_{jH}$ whenever $b_i < p_i(k_{jH})$. □

Proof of Proposition 6.1. First, suppose -to obtain a contradiction- that $p_j(k_L) < p_i(k_L)$. Thus, firm i does not invest in the interval $[p_j(k_L), p_i(k_L)]$ and the preemption hazard for firm j is zero. Substitute $h_j(p_j) = 0$ in firm j's expanded first order conditions (cfr. 5.8) to obtain

$$p_j = \frac{\beta}{\beta - 1} \frac{1}{D_1 + \frac{k_L'}{k_L}\left(\frac{p_j}{f_j}\right)^\beta f_j\,(D_2 - D_1)}(r-\mu)k_L. \tag{A.24}$$

Firm j postpones investment more than firm i, whose hazard is not zero at k_L. Since the threat of preemption accelerates investment $\frac{\partial p_i}{\partial h_i} < 0$ (see Proposition 5.5) we have $p_i(k_L) < p_j(k_L)$. Alternatively, one can work out the first order condition. We obtain a similar contradiction if we assume $p_i(k_L) < p_j(k_L)$ and conclude that $p_i(k_L) = p_j(k_L)$. Second, suppose -again to obtain a contradiction- that $p_i(k_H) < p_j(k_H)$. As x approaches $p_i(k_H)$ firm i's hazard of preemption goes to infinity. Firm i invests at

$$p_i(k_H) = \frac{(r-\mu)}{D_1}\left(k_H + V_i^F - C_i^L(k_H)\right),$$

as in Proposition 5.6. However, firm j's hazard rate is less than infinity at this point and because $\frac{\partial p_i}{\partial h_i} < 0$, we find that $p_j(k_H) > p_i(k_H)$, another contradiction. A similar contradiction can be obtained for $p_i(k_H) > p_j(k_H)$, so we conclude that $p_j(k_H) = p_i(k_H)$. □

Proof of Proposition 6.2. Since $k_j'(b_i)$ is not Lipschitz continuous at $V_i^L(b_i) - V_i^F(b_i)$, we cannot show uniqueness easily. We show first that there cannot exist asymmetric equilibria. By integration of the system of differential equations and application of the chain rule,

$$\begin{cases} k_j'(p) = \frac{1-G(k_j(p))}{G'(k_j(p))} \frac{\mathcal{E}_i\left[V_i^{LL\prime}(p)\right]}{\mathcal{E}_i\left[V_i^{LL}(p)\right] - V_i^{FF}(p)} \\ k_i'(p) = \frac{1-G(k_i(p))}{G'(k_i(p))} \frac{\mathcal{E}_j\left[V_j^{LL\prime}(p)\right]}{\mathcal{E}_j\left[V_j^{LL}(p)\right] - V_j^{FF}(p)} \end{cases}, \tag{A.25}$$

we obtain

$$\begin{cases} \ln\left[1 - G(k_j(p))\right] = -\int_{p_L}^p \frac{\mathcal{E}_i\left[V_i^{LL\prime}(z)\right]}{\mathcal{E}_i\left[V_i^{LL}(z)\right] - V_i^{FF}(z)} dz \\ \ln\left[1 - G(k_i(p))\right] = -\int_{p_L}^p \frac{\mathcal{E}_j\left[V_j^{LL\prime}(z)\right]}{\mathcal{E}_j\left[V_j^{LL}(z)\right] - V_j^{FF}(z)} dz \end{cases}. \tag{A.26}$$

(Note that $G(k_L) = 0$.) Substraction of the second equation from the first (with $p < p_H$) leads to

$$\ln\frac{1 - G(k_j(p))}{1 - G(k_i(p))} = \int_{p_L}^p \left(\frac{\mathcal{E}_j\left[V_j^{LL\prime}(z)\right]}{\mathcal{E}_j\left[V_j^{LL}(z)\right] - V_j^{FF}(z)} - \frac{\mathcal{E}_i\left[V_i^{LL\prime}(z)\right]}{\mathcal{E}_i\left[V_i^{LL}(z)\right] - V_i^{FF}(z)}\right) dz. \tag{A.27}$$

Suppose first that $k_i < k_j$. The lefthand side of Equation (A.27) is then strictly negative. To obtain a contradiction we must show that the integrand is nonnegative for all $p_L \le z \le p$. Or, given that all quantities are positive, we can show the stronger conditions that

$$\mathcal{E}_j\left[V_j^{LL}(z)\right] - V_j^{FF}(z) \le \mathcal{E}_i\left[V_i^{LL}(z)\right] - V_i^{FF}(z) \tag{A.28}$$

and

$$\mathcal{E}_j\left[V_j^{LL\prime}(z)\right] \ge \mathcal{E}_i\left[V_i^{LL\prime}(z)\right] \tag{A.29}$$

hold. Condition (A.28) should be intuitively clear. Low cost firm i -the natural leader- will finally invest at $p = b_j - \epsilon$. There, $\xi_j(b_j) = 0$, but firm i still has an incentive to become the leader, i.e. $\xi_i(b_j) > 0$. A formal proof for $p_L \le z \le p$ simply works out Condition (A.28) using Definitions (A.2) and (A.3). Now we consider Condition (A.29), that says that the marginal benefit of waiting is higher for a high cost firm than for a low cost firm for any value of the state. Substract the first positive derivative ($z_i < l_i$)

$$\mathcal{E}_i\left[V_i^{LL'}(z)\right] = \left((1-\beta)\frac{zD_1}{r-\mu} + \beta k_i + (1-\beta)\frac{\left(\frac{f_i}{k_i}\right)^{1-\beta}\kappa_j(z)^{-\beta}\kappa_j'(z)\,(D_2 - D_1)}{r-\mu}\right)\left(\frac{x}{z}\right)^\beta \frac{1}{z} \tag{A.30}$$

from the second

$$\mathcal{E}_j\left[V_j^{LL'}(z)\right] = \left((1-\beta)\frac{zD_1}{r-\mu} + \beta k_j + (1-\beta)\frac{\left(\frac{f_j}{k_j}\right)^{1-\beta}\kappa_i(z)^{-\beta}\kappa_i'(z)\left(D_2 - D_1\right)}{r-\mu}\right)\left(\frac{x}{z}\right)^{\beta}\frac{1}{z} \tag{A.31}$$

and simplify to complete the proof using the rational conjecture that $\kappa_j(z) = \kappa_i(z)$ under common and consistent priors. Conditions (A.28) and (A.29) are enough for a contradiction and a similar contradiction can be obtained for $k_i > k_j$. We conclude that $k_i(p) = k_j(p)$. There are no asymmetric equilibria. Since there are no asymmetric equilibria, the game in (6.1) reduces to

$$\begin{cases} k'(p) = \frac{1-G(k(p))}{G'(k(p))}\frac{\mathcal{E}\left[V^{LL'}(p)\right]}{\mathcal{E}[V^{LL}(p)]-V^{FF}(p)} \\ k\left(p = \frac{(r-\mu)}{D_1}\widetilde{k}(p)\right) = k_H \end{cases}.$$

Existence and uniqueness then follow immediately from Proposition 5.7. Note that both the lefthand and the righthand side of Equation (A.27) are zero for symmetric firms. □

References

Binmore, K., 2007. Playing for real. Oxford University Press.

Bobtcheff, C., Mariotti, T., 2010. Potential competition in preemption games. IDEI Working Papers 594, Institut d'Économie Industrielle (IDEI), Toulouse. URL http://ideas.repec.org/p/ide/wpaper/21999.html

Bouvard, M., 2010. Real option financing under asymmetric information. SSRN eLibrary.

Boyer, M., Gravel, E., Lasserre, P., 2004. Real options and strategic competition: a survey. Tech. rep., CIRANO.

Boyer, M., Lasserre, P., Mariotti, T., Moreaux, M., 2001. Real options, preemption, and the dynamics of industry investments. Cahiers de recherche du Département des sciences économiques, UQAM 20-10, Université du Québec à Montréal, Département des sciences économiques. URL http://ideas.repec.org/p/cre/uqamwp/20-10.html

Bustamante, M. C., 2008. The dynamics of going public. SSRN eLibrary.

Chu, Y., Sing, T. F., 2007. Optimal Timing of Real Estate Investment under an Asymmetric Duopoly. Journal of Real Estate Finance and Economics 34(3).

Decamps, J.-P., Mariotti, T., Villeneuve, S., 2003. Investment Timing Under Incomplete Information. SSRN eLibrary.

Dixit, A. K., Pindyck, R. S., 1994. Investment under Uncertainty. Princeton University Press.

Fudenberg, D., Tirole, J., 1985. Preemption and rent equilization in the adoption of new technology. Review of Economic Studies 52 (3), 383–401. URL http://ideas.repec.org/a/bla/restud/v52y1985i3p383-401.html

Fudenberg, D., Tirole, J., 1991. Game Theory. MIT Press.

Ghemawat, P., 1986. Wal-mart store's discount operations. Tech. rep., Case Study 9.387-018, Harvard Business School, Boston.

Graham, J., 2011. Strategic real options under asymmetric information. Journal of Economic Dynamics and Control 35 (6), 922–934. URL http://EconPapers.repec.org/RePEc:eee:dyncon:v:35:y:2011:i:6:p:922-934

Grenadier, S. R., 2002. Option exercise games: An application to the equilibrium investment strategies of firms. Review of Financial Studies 15 (3), 691–721. URL http://EconPapers.repec.org/RePEc:oup:rfinst:v:15:y:2002:i:3:p:691-721

Grenadier, S. R., Malenko, A., 2011. Real options signaling games with applications to corporate finance. Review of Financial Studies 24 (12), 3993–4036. URL http://rfs.oxfordjournals.org/content/24/12/3993.abstract

Grenadier, S. R., Wang, N., 2005. Investment timing, agency, and information. Journal of Financial Economics 75 (3), 493–533. URL http://ideas.repec.org/a/eee/jfinec/v75y2005i3p493-533.html

Harsanyi, J. C., 1973. Games with randomly disturbed payoffs: A new rationale for mixed-strategy equilibrium points. International Journal of Game Theory 2, 1–23, 10.1007/BF01737554. URL http://dx.doi.org/10.1007/BF01737554

Hennessy, C. A., Livdan, D., Miranda, B., 2010. Repeated signaling and firm dynamics. Review of Financial Studies 23 (5), 1981–2023. URL http://rfs.oxfordjournals.org/content/23/5/1981.abstract

Hsu, Y.-W., Lambrecht, B. M., 2007. Preemptive patenting under uncertainty and asymmetric information. Annals OR 151, 5–28.

Huisman, K., 2000. Technology investment: A game theoretic real options approach. Open Access publications from Tilburg University urn:nbn:nl:ui:12-84087, Tilburg University. URL http://ideas.repec.org/p/ner/tilbur/urnnbnnlui12-84087.html

Huisman, K., Kort, P., 2011. Strategic capacity investment under uncertainty. Tech. rep., Tilburg University Working Paper.

Huisman, K., Kort, P., Pawlina, G., Thijssen, J., 2004. Strategic investment under uncertainty: Merging real options with game theory. Open Access publications from Tilburg University urn:nbn:nl:ui:12-142400, Tilburg University. URL http://ideas.repec.org/p/ner/tilbur/urnnbnnlui12-142400.html

Huisman, K., Kort, P., Pawlina, G., Thijssen, J., 2005. Strategic investment under uncertainty: a survey of game theoretic real option models. Journal of Financial Transformation 13, 111–118.

Kijima, M., Shibata, T., 2005. Real options in an oligopoly market. The Kyoto Economic Review 74 (1), 47–64.

Lambrecht, B., Perraudin, W., 2003. Real options and preemption under incomplete information. Journal of Economic Dynamics and Control 27 (4), 619–643. URL http://www.sciencedirect.com/science/article/pii/S0165188901000641

Lambrecht, B. M., 2004. The timing and terms of mergers motivated by economies of scale. Journal of Financial Economics 72 (1), 41–62. URL http://EconPapers.repec.org/RePEc:eee:jfinec:v:72:y:2004:i:1:p:41-62

Lebrun, B., 1999. First price auctions in the asymmetric n bidder case. International Economic Review 40 (1), 125–142. URL http://dx.doi.org/10.1111/1468-2354.00008

Lebrun, B., 2006. Uniqueness of the equilibrium in first-price auctions. Games and Economic Behavior 55 (1), 131–151. URL http://EconPapers.repec.org/RePEc:eee:gamebe:v:55:y:2006:i:1:p:131-151

Lizzeri, A., Persico, N., 2000. Uniqueness and existence of equilibrium in auctions with a reserve price. Games and Economic Behavior 30 (1), 83–114. URL http://www.sciencedirect.com/science/article/pii/S0899825698907047

Martzoukos, S. H., Zacharias, E., 2008. Real option games with r&d and learning spillovers. MPRA Paper 12686, University Library of Munich, Germany. URL http://ideas.repec.org/p/pra/mprapa/12686.html

Maskin, E., Riley, J., 2000a. Asymmetric auctions. Review of Economic Studies 67 (3), 413–38. URL http://ideas.repec.org/a/bla/restud/v67y2000i3p413-38.html

Maskin, E., Riley, J., 2000b. Equilibrium in sealed high bid auctions. Review of Economic Studies 67 (3), 439–54. URL http://ideas.repec.org/a/bla/restud/v67y2000i3p439-54.html

Maskin, E., Riley, J., 2003. Uniqueness of equilibrium in sealed high-bid auctions. Games and Economic Behavior 45 (2), 395–409. URL http://ideas.repec.org/a/eee/gamebe/v45y2003i2p395-409.html

Maskin, E., Riley, J. G., 1986. Existence and uniqueness of equilibrium in sealed high bid auctions. UCLA Economics Working Papers 407, UCLA Department of Economics. URL http://ideas.repec.org/p/cla/uclawp/407.html

Morellec, E., Schrhoff, N., 2011. Corporate investment and financing under asymmetric information. Journal of Financial Economics 99 (2), 262–288. URL http://www.sciencedirect.com/science/article/pii/S0304405X10002096

Morellec, E., Zhdanov, A., 2005. The dynamics of mergers and acquisitions. Journal of Financial Economics 77 (3), 649–672. URL http://ideas.repec.org/a/eee/jfinec/v77y2005i3p649-672.html

Nishihara, M., Fukushima, M., 2008. Evaluation of firms loss due to incomplete information in real investment decision. European Journal of Operational Research 188 (2), 569–585. URL http://www.sciencedirect.com/science/article/pii/S0377221707004249

Pawlina, G., Kort, P. M., 2005. Investment under uncertainty and policy change. Journal of Economic Dynamics and Control 29 (7), 1193–1209. URL http://ideas.repec.org/a/eee/dyncon/v29y2005i7p1193-1209.html

Pawlina, G., Kort, P. M., 2006. Real options in an asymmetric duopoly: Who benefits from your competitive disadvantage? Journal of Economics & Management Strategy 15 (1), 1–35. URL http://ideas.repec.org/a/bla/jemstr/v15y2006i1p1-35.html

Rawls, J., 1971. A theory of justice. Belknap.

Shackleton, M. B., Tsekrekos, A. E., Wojakowski, R., 2004. Strategic entry and market leadership in a two-player real options game. Journal of Banking & Finance 28 (1), 179–201. URL http://ideas.repec.org/a/eee/jbfina/v28y2004i1p179-201.html

Simon, L. K., Stinchcombe, M. B., 1989. Extensive form games in continuous time: Pure strategies. Econometrica 57 (5), 1171–1214. URL http://www.jstor.org/stable/1913627

Smets, F., 1991. Exporting versus fdi: the effect of uncertainty, irreversibility and strategic interactions. Working paper, Yale University.

Wu, J., 2006. Capacity preemption in a duopoly market under uncertainty. Tech. rep., Working Paper, University of Arizona.