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Abstract

Cognitive radio networks (CRNs) benefit from several features, such as decision-making, spectrum-awareness and reconfigurability, which enable them to perform channel recovery after operating channel failures due to the appearance of primary users or if the channel quality becomes unacceptable. As a result of their frequency and bandwidth agility, the cognitive radios can use heterogeneous channels with different parameters, such as availability, recovery time, and transmission rate. We thus propose a general hierarchical recovery model where the channels are classified based on their parameters into distinct sets. Instead of performing a flat channel search over all channels, the CRN first selects a channel set and then performs a restoration over the selected set. The decision process to select, as a function of the CRN current state and its knowledge about the different channel parameters, the best channel set to perform the restoration mechanism is the focus of this research work. We first target a two-set case and propose different heuristic algorithms to solve this selection problem. Numerical and simulation results are provided illustrating the benefits of the different decision algorithms for channel recovery over heterogeneous channels.

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1 Introduction

To solve the problem of spectrum scarcity and usage inefficiency, the idea of using vacant portions of the spectrum licensed to a primary network to deploy a secondary network was proposed [1] and because of its spectrum-awareness and reconfigurability abilities the cognitive radio (CR) [2] technology was selected as the best candidate to implement this concept. The opportunistic use of the spectrum by CR networks (CRNs) implies that a channel should be vacated by the CRs when the primary licensees return to the channel or if the channel quality becomes unacceptable. The event of CRs vacating a channel can be modeled as a failure [3] and, when it occurs, the CRs must spend an amount of time, called the *recovery time*, to find a new vacant channel and re-establish their communication on this channel. Similar to any failure recovery approach, the recovery objective is to switch to the best possible channel in the shortest time.

In the literature, the focus has been more on the flat recovery models where the CR user decides among a set of mostly homogeneous channels. Thus, the objective of the recovery has been generally to find the first available channel which is free of primary users. For instance in [4], different search schemes over a flat structured channel set have been investigated. In [5, 6, 7, 8, 9], the authors propose algorithms to determine the recovery sensing orders for CRNs. They also discuss some trade-offs between stopping the recovery and using the found channel, and continuing the recovery hoping that a better channel may be found. Several recovery schemes embedded in medium access (MAC) protocols have also been also proposed for homogeneous channels [10, 11].

However, due to their frequency and transmission bandwidth agility, the CRs can be reconfigured to use heterogeneous channels with different parameters such as availability, recovery time, and transmission rate. To fully take advantage of their abilities, the CRs must perform, following a failure event, the recovery mechanism over the heterogeneous channels and not limit themselves to homogeneous channels, similar to the one that was in use. But the CRs are faced with a trade-off in performing a recovery mechanism over heterogeneous channels, as some channels may provide higher quality but are less available and require a longer restoration period, while some channels may remain available for a longer duration but their transmission rate can be lower. The objective of this paper is thus to propose a decision process for the CRs facing this heterogeneous recovery trade-off.

To tackle this problem, we introduce a general hierarchical model for channel recovery in CRNs where available channels are classified into multiple sets based on their characteristics, such as availability and service capacity. To perform the recovery, the CRN first selects a channel set (type) and then performs a restoration over the selected set. We then focus on a two-set case and model the problem as an optimization and decision-making problem where the objective is to minimize the average total transmission time of a flow of packets. Heuristic algorithms are proposed to solve this problem and numerical results are provided to illustrate their benefits. The main contribution of this paper is therefore to propose a recovery procedure for CR networks with heterogeneous channels and framing it as an optimization and decision-making problem within a novel hierarchical channel recovery model. To the best of our knowledge, this approach to recovery in CRNs has never been discussed before.

The rest of the paper is organized as follows. In Section 2, the hierarchical channel recovery model is presented. In Section 3, some heuristics for the decision-making problem between two different channel sets are proposed. We discuss the numerical and simulation results in Section 4. Finally, Section 5 concludes the paper with some remarks on future research directions.

2 Hierarchical Recovery Model

In this section, we propose a novel hierarchical recovery model for cognitive radios which, due to their frequency and bandwidth agility, can perform the recovery procedure over channels with dissimilar characteristics. As illustrated in Figure 1, we assume that I sets (types) of channels are available for recovery. For instance, each set could belong to a different licensee or network such as TV channels, ISM bands, or Telephony networks. Each set of channels has, from the perspective of the CRN, different distributions for the availability and unavailability periods, and different service capacity. In each set i, N_i channels can be

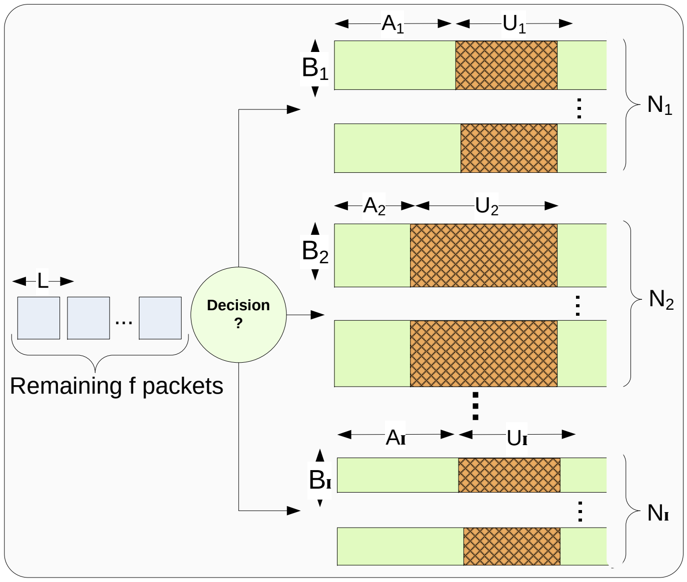

Figure 1: General decision model for a channel hierarchy in cognitive radio networks.

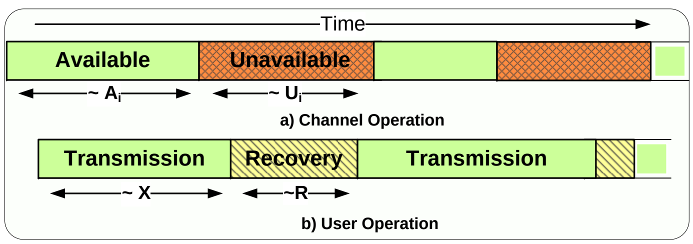

Figure 2: **a)** Alternating renewal processes of channel type i operation: availability and unavailability. **b)** Alternating processes of a CR user: transmission and recovery.

used by the CRN and the average service capacity of the channels of type i is denoted by B_i. The transmission time of a packet of length L over a link with a service capacity B_i is equal to $\frac{L}{B_i}$ units of time. The unit of time can be assumed any appropriate value based on other parameters. All timing parameters from now on are thus represented in unit of time. We also assume that the CR users are equipped with only one CR transceiver which can be used either for data communication or channel sensing purposes, besides a traditional radio for signaling and controlling purposes.

From the perspective of the CRN, the main cause of channel unavailability is the appearance of primary users. However, it could also include other reasons such as excessive interference, shadowing, deep fading, and traffic congestion [3]. As illustrated in Figure 2.a, let A_i and U_i be the random variables, with probability density function (PDF) f_{A_i} and f_{U_i} (we denote by F_Z and f_Z, respectively, the cumulative distribution functions (CDF) and PDF of any random variable Z), respectively, which represent the length of the available and unavailable periods of a type i channel. Those periods form an alternating renewal processes [12] for the channels.

Assume that there are f^0 packets in the source buffer that needs to be transmitted (a flow size equal to f^0 packets). The two endpoints of the link decide to select a common available channel to serve those f^0 packets. In the proposed hierarchical recovery model, instead of using a flat recovery procedure over all channels, the CR users first select the channel set and then perform a restoration over the selected set. For the restoration procedure, the endpoints prepare a list of channels in the selected set, called *channel sensing order* (CSO), to be sensed one by one and follow this order until they find an available channel with an acceptable quality. The time spent to sense each channel can be different for different channels which is represented by τ^n for channel C^n, as illustrated in Figure 3. If the user makes its decision after sensing the N' first channels of the

CSO, the restoration time can be given by:

$$R = \sum_{n=1}^{N'} \tau^n + O(1) \tag{1}$$

where $O(1)$ represents some constant timing overhead such as negotiation time which are independent of the channel sensing order. The channel sensing order depends on the policy that the CR nodes employ which is called a *channel search/sensing scheme*. The restoration time thus depends on several factors such as the channel availability, the channel bandwidth, the restoration policy, the number of users contending for those channels and their traffic level, etc. Furthermore, the restoration time is a random variable. In each set, the total recovery time spent to find a new channel is thus modeled with a random variable R_i with PDF f_{R_i}. As illustrated in Figure 2.b, after a recovery time of length R_i, the user transmits for a random length X_i over the selected channel of type i until, as determined by the sensing result of the current operating channel, it experiences a failure. The endpoints then select a new channel using the procedure described previously to transmit the remaining packets. This procedure is repeated until all f^0 packets are transmitted. The operation of the user is thus modeled with alternating processes where the user consecutively alternates between recovery and transmission.

It should be noted that as illustrated in Figure 4, X_i is a random variable which represents the length of the remaining available period of the selected channel of type i. X_i is also known as the operating time over a channel of type i, or recurrence time in the context of alternating renewal theory [12]. That is, when the user senses a channel available and starts using it, it does not know for how long this channel has been available, so a uniform access to the channel during an available period is assumed. With this assumption, one can find the distribution of X_i based on A_i as follows [12]:

$$f_{X_i}(t) = \frac{1 - F_{A_i}(t)}{\bar{A}_i}, \tag{2}$$

where $\bar{A}_i$ represents the expected value of A_i. Let us also find the Laplace-Stieltjes transform (LST) of f_{X_i} which can be written as:

$$\mathcal{L}[(f_{X_i}(t)] = X_i(S) = \frac{1 - A_i(S)}{S\bar{A}_i} \tag{3}$$

where $A_i(S)$ stands for the LST of the random variable A_i.

The objective of any recovery scheme may differ depending on the application requirements. For example, a delay sensitive application might require the shortest down time while a high data rate application would need a channel with the highest available data rate. In this paper, the recovery scheme objective is to

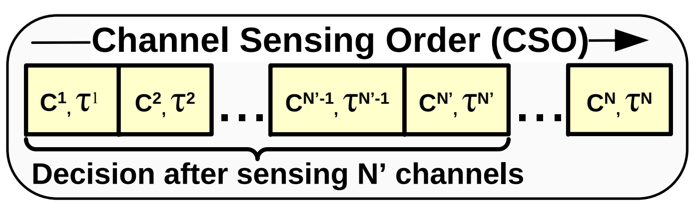

Figure 3: Channel sensing order (CSO) for a CR link.

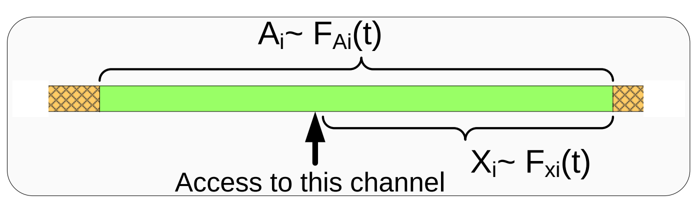

Figure 4: Operating period of a user over a found channel is the remaining part of the channel availability period.

minimize the total transmission time of the f_0 packets. Furthermore, we focus on the problem of selecting, among these I sets of channels, the set for which the restoration procedure will take place and we do not study the exact restoration procedure for this set. In the following, we focus on the two-set model, although most of the results can be extended to multiple sets. Finally, it should be noted that, as will become apparent later, although we consider in the model the complete transmission of the initial f_0 packets, the decision procedure could be applied to a system with a packet arrival process and where the recovery scheme objective would be to make the set selection which will minimize the total transmission time of all packets that are currently in the buffer. This will be equivalent to minimizing the average transmission time per packet in the long-term.

3 Decision-making and Analysis

The objective of the set selection optimization problem for the two-set case is to select, after each failure, the appropriate channel set to perform the recovery mechanism in order to minimize the total transmission time of a flow with f^0 packets initially. At each decision instant, let $D(f)$ be the total transmission time of the remaining f packets (at the process start we have $f = f^0$). $D(f)$ can be written as a recursive function equal to:

$$D(f) = \begin{cases} R_1 + X_1 + D(f - f_1) & \text{Selects type 1 channels} \\ R_2 + X_2 + D(f - f_2) & \text{Selects type 2 channels} \end{cases} \tag{4}$$

where f_i represents the random number of packets transmitted over a type i channel during a period X_i, which is given by $\lfloor \frac{X_i B_i}{L} \rfloor$. As $\frac{L}{B_i}$ can be assumed small, we neglect the impact of the interrupted packet transmission and do not use $\lfloor \cdot \rfloor$ in the remainder of the paper. We also define $\bar{f}_i$ as the expected value of f_i:

$$\bar{f}_i = \frac{\bar{X}_i B_i}{L}, \ i = 1, 2 \tag{5}$$

In the following, we propose some heuristics for the users' decision-making. Each heuristic could be more appropriate for a specific scenario: for instance, when the channels' availability or recovery distribution is not known or when an analytical approximation is required. It is assumed that the number of channels in both sets is large enough so that during a channel recovery, the user eventually finds an available channel. Moreover, the recovery time (channel search time) can also be a function of the distribution of the channels' availability and unavailability periods which implies that the two random variables R_i and X_i are not independent. For simplicity, we assume for now that they are independent and the impact of their dependence will be discussed in the conclusion.

3.1 Average-based Decision-making

In this heuristic, the user decides based only on average values. This scheme is thus particularly suitable for the cases where the exact distributions are not known. The number of remaining packets to be transmitted, f, is compared with the average number of packets that can be transmitted over an operating instance of each channel type.

As illustrated in Figure 5, when the remaining number of packets to be transmitted, f, is decreasing over time (from left to right), three zones are defined: when f is larger than both $\bar{f}_1$ and $\bar{f}_2$, when f is between $\bar{f}_1$ and $\bar{f}_2$ and when it is smaller than both $\bar{f}_1$ and $\bar{f}_2$. The pseudo-code of the decision-making process is

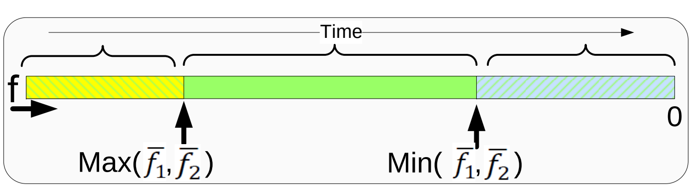

Figure 5: Boundary values for the remaining number of packets in the Average-based decision making scheme.

Algorithm 1 Pseudo-code for the Average-based decision-making scheme.

```
if f >= Max(f̄_1, f̄_2) then
  if R̄_1 + X̄_1 > R̄_2 + X̄_2 then
    if (X̄_2 + R̄_2 − R̄_1)B_1 < X̄_2 B_2 then
      Decision=2;
    else
      Decision=1;
    end if
  else
    if (X̄_1 + R̄_1 − R̄_2)B_2 < X̄_1 B_1 then
      Decision=1;
    else
      Decision=2;
    end if
  end if
else if Min(f̄_1, f̄_2) < f < Max(f̄_1, f̄_2) then
  i=Index of Max ; j=Index of Min
  if R̄_i + fL/B_i < R̄_j + X̄_j then
    Decision=i;
  else
    if (X̄_j + R̄_j − R̄_i)B_i < X̄_j B_j then
      Decision=j;
    else
      Decision=i;
    end if
  end if
else
  %f <= Min(f̄_1, f̄_2)%
  if R̄_1 + fL/B_1 > R̄_2 + fL/B_2 then
    Decision=2;
  else
    Decision=1;
  end if
end if
```

given in Alg. 1. For clarity of presentation, each section of the algorithm refers to one of the three zones in Figure 5.

When f is larger than both $\bar{f}_1$ and $\bar{f}_2$, the user in average can not transmit all remaining packets in just one operating instance. We thus compare the time to the next failure, $R_i + X_i$, of both channel types. Assume that this time is larger for channels of type 1; this implies that if a type 2 channel is selected, the user experiences a failure sooner on the type 2 channel. At the next failure point, the number of packets transmitted so far is compared with the number of packets that would have been transmitted during the same time if the user had selected a channel of type 1. If it is larger, this means that the user has made a correct decision by selecting a type 2 channel otherwise selecting a channel of type 1 would have been the correct decision.

When f is smaller than $\bar{f}_i$ (i=1,2), we can assume that before the next failure, all remaining packets can be transmitted over a channel of type i. Therefore, the transmission time of the remaining packets are used in the decision-making. Without loss of generality, assume that $\bar{f}_1 < f < \bar{f}_2$. Then, if the total transmission time of all remaining packets over the type 2 channel is shorter than the time to the next failure over the channel of type 1, a channel of type 2 should be selected. Otherwise, we again compare the number of packets transmitted up to the next failure over the type 1 channel with the packets which would have been transmitted in the same time over a type 2 channel. The one which is higher represents the correct decision.

Finally, in the third zone when f is smaller than both $\bar{f}_1$ and $\bar{f}_2$, the transmission time of all remaining packets is compared. The one which is shorter corresponds the correct decision.

3.2 Utility-based and Random Decision-making

In this method, the approach is similar, but the distribution function of the remaining channel availability period, $F_{X_i}(t)$, is used. We define two utility functions as follows for the incoming operating period which is the average number of packets that can be transmitted over the time spent for transmission:

$$Z_i = \big(1 - P_i^F(f)\big)\frac{B_i \bar{X}_i(\frac{fL}{B_i})}{L\left(\bar{R}_i + \bar{X}_i(\frac{fL}{B_i})\right)} + P_i^F(f)\frac{f}{\bar{R}_i + \frac{fL}{B_i}}. \tag{6}$$

where $P_i^F(f)$ stands for the probability that all remaining f packets can be transmitted in the incoming operating period over a channel of type i, before the next failure, which can be given by:

$$P_i^F(f) = Pr(X_i > \frac{fL}{B_i}) = 1 - F_{X_i}(\frac{fL}{B_i}). \tag{7}$$

$\bar{X}_i(\frac{fL}{B_i})$ represents the conditional average of X_i when X_i is smaller than $\frac{fL}{B_i}$. It can be written as:

$$\bar{X}_i(\frac{fL}{B_i}) = E(X_i|X_i <= \frac{fL}{B_i}) = \frac{1}{F_{X_i}(\frac{fL}{B_i})}\int_{0^+}^{\frac{fL}{B_i}} t f_{X_i}(t)dt. \tag{8}$$

In this equation, we make the same assumption that if all the remaining packets can be transmitted before a new failure, the time spent for transmission is the real transmission time in addition to the restoration time. Otherwise, it is the average conditional operating time in addition to the restoration time. The decision function can be written as:

IF $Z_1 > Z_2$

Decision=1

Else

Decision=2

This utility function can also be employed in mathematical programming models.

The random decision-making algorithm is similar to this approach but it uses a random selection procedure based on the utility functions in Eq. (6) to decide between the two possible options. Let us define the probability θ^* as follows:

$$\theta^* = \frac{Z_1}{Z_1 + Z_2}. \tag{9}$$

Using this probability, the decision function at each decision point can be written as:

Generate a random number γ.

IF $\gamma < \theta^*$

Decision=1

Else

Decision=2

3.3 Threshold-based Decision

In this section, we propose an approach which enables us to find analytically the average total transmission time. For this end, we use renewal theory results to find a threshold value for the remaining number of packets so that before this threshold value, the same channel type is used repeatedly and after this value, the other channel type is repeatedly used.

Considering Eq. (4), suppose that the recursive function always recalls the same channel type, say i (i=1,2). We can then write the total transmission time of the *remaining* f packets as

$$\begin{aligned} D_i(f) &\leq R_{i,1} + X_{i,1} + R_{i,2} + X_{i,2} + ... + R_{i,K_i} + X_{i,K_i} \\ &= \frac{fL}{B_i} + R_{i,1} + R_{i,2} + ... + R_{i,K_i} \end{aligned} \tag{10}$$

where $R_{i,j}$ and $X_{i,j}$ respectively represent the j-th independent and identical instances of R_i and X_i (i.e., the j-th recovery period and operating period). The unknown here is K_i which is the number of times that the function recalls itself until transmitting all f packets. This is equivalent to the number of failures where for each failure, the same decision has been made. It is worth noting that the reason of the first inequality in the equation above is that in the last operating period, the transmission of the remaining packets may finish before the end of the channel availability period.

We define a new random variable $S_{i,g}$ as the sum of the g instances of X_i,

$$S_{i,g} = \sum_{n=1}^{g} X_{i,n}, \tag{11}$$

then K_i is the smallest value of k such that

$$S_{i,k} \geq \frac{fL}{B_i}. \tag{12}$$

That is, it should be greater than the total time required to transmit all packets over a type i channel. From [12, Chap. 3], we can find the average of K_i as being:

$$\bar{K}_i - 1 = m_i(\frac{fL}{B_i}) = \sum_{g=1}^{\infty} F_{S_{i,g}}(\frac{fL}{B_i}) \tag{13}$$

where $F_{S_{i,g}}$ is the CDF of the random variable $S_{i,g}$ and $m_i(t)$ is the mean renewal function for the operating period over a channel of type i [12]. Note that, the reason to have '-1' in the left side is that the mean renewal function does not take into account the first operating period before the first renewal. To find $m_i(\frac{fL}{B_i})$, we apply differentiation and the Laplace Transform on both sides of the equality to have:

$$\mathcal{L}[m_i'(t)] = X_i(S) + [X_i(S)]^2 + ... = \sum_{g=1}^{\infty} [X_i(S)]^g. \tag{14}$$

Assuming $|X_i(S)| < 1$ and using Eq. (3), we can write:

$$\mathcal{L}[m'(t)] = \frac{X_i(S)}{1 - X_i(S)} = \frac{1 - A_i(S)}{A_i(S) + S\bar{A}_i - 1}. \tag{15}$$

One can find $m'(t)$ and consequently $m(t)$ from the equation above and thereby derive $\bar{K}_i$. The average total transmission time can then be given by:

$$E[D_i(f)] = \frac{fL}{B_i} + \bar{K}_i\bar{R}_i, \tag{16}$$

where it is assumed that all recovery times are identical and independent and that they are also independent of the number of renewals. For our problem, the latter is equivalent to assuming that when the user decides and starts the recovery over a channel set, it keeps searching and does not leave the selected set until it finds a channel in this set.

To make a decision between two types of channel, we can compare the average transmission time of the remaining packets where we assume that all these packets will be transmitted over the same type of channel. That is,

IF $E[D_1(f)] < E[D_2(f)]$

Decision=1

Else

Decision=2

For our objective function, this usually results in finding a threshold value f^* for the number of packets such that $E[D_1(f^*)] = E[D_2(f^*)]$. When $f > f^*$, we select one type and otherwise the other type of channel will be selected. It is worth mentioning that in this approach the user may always operate over only one channel type when the threshold value is negative or larger than f^0. For the applications where the delay variation is more important, the decision function can be a weighted sum of the mean and the higher moments of the total transmission time; for instance, $w_1 E[D_i(f)] + w_2 Var[D_i(f)]$, where the variance can be found again from the renewal theory results.

4 Numerical and Simulation Results

In this section, we numerically compare the transmission time of a flow of packet for the four proposed schemes. We also included the transmission time of an unrealistic optimal decision obtained through an enumerative decision tree. Note that to obtain such an optimal minimum transmission time, it is assumed that the future is known in advance; for this reason the optimal decision cannot be practically implemented and it is solely used for comparison purposes as it provides a lower bound on the performance of any realizable scheme. The value of the different parameters for the three scenarios studied in this section are given in Table 1. The parameter values are selected such that the average values of the availability and recovery periods are the same for all scenarios. Furthermore, the parameter values are such that the existence of a threshold value between the two type of channels is guaranteed. The results of the proposed heuristics are also compared with the cases that only one channel type is used repeatedly for the entire flow, which would be the case for an homogeneous recovery scheme. "Only type 1" thus represents the case that only type-1 channels are used, so no decision is made during the flow transmission, and inversely for the "Only type 2" case.

Numerical and simulation experiments have been done using MATLAB where each scenario has been repeated for different initial number of packets (f^0) and the same experiment has been repeated 8000 times. For visual clarity and since the confidence intervals are tight, they are not shown in the figures. Note that we have repeated the experiments for several other parameter values and distributions, which are not included here due to space constraints, and obtained similar results.

4.1 First Scenario: Exponential Distribution

As a very common scenario, we assume that all timing random variables (X_i and R_i) are exponentially distributed. Exponentially distributed available and unavailable periods (a.k.a, idle and busy periods in the literature) have been assumed in several papers, such as [10, 13]. The assumption of an Exponential distribution for the recovery time is also realistic when the channel searching scheme is random such that the user senses homogeneous channels one by one until finding the first available channel. From the memoryless property, we know that when A_i is exponentially distributed with parameter λ_i (i.e., $F_{A_i} = 1 - e^{-\lambda_i t}$), X_i is also exponentially distributed with the same parameter value. Using Eq. (14), $\mathcal{L}[m_i'(t)]$ can be found equal to $\frac{\lambda_i}{S}$ and from that $m(t)$ is equal to $\lambda_i t$. This verifies our relations since the mean renewal function for an

Table 1: Numerical analysis and simulation parameters.

		Numerical			**Simulation**
Par.		1st	2nd	3rd	1st Scen.
X_1	Dist.	Exp	2-Erl	Uniform	Exp
	Avg.	45	45	45 [30 60]	$\bar{A_1} = \bar{U_1} = 15$
X_2	Dist.	Exp	Exp	Exp	Exp
	Avg.	65	65	65	$\bar{A_2} = 25, \bar{U_2} = 6.25$
R_1	Dist.	Exp	Exp	Exp	Geom
	Avg.	15	15	15	2 ($P = \frac{15}{30}$)
R_2	Dist.	Exp	Det	Det	Geom
	Avg.	4	4	4	1.25 ($P = \frac{25}{31.25}$)
L		1	1	1	1
B_1		5 (4,8)	5	5	5
B_2		3.5 (4-8)	3.5	3.5	3.5

exponential distribution is a Poisson process with average λt. $\bar{K_i}$ can be then given by:

$$\bar{K_i} = \lambda_i(\frac{fL}{B_i}) + 1 \tag{17}$$

and from that and using Eq. (16), the threshold number of packets, f^*, is the value that we have:

$$\frac{f^*L}{B_1} + \left(\lambda_1(\frac{f^*L}{B_1}) + 1\right)\bar{R_1} = \frac{f^*L}{B_2} + \left(\lambda_2(\frac{f^*L}{B_2}) + 1\right)\bar{R_2} \tag{18}$$

The results given in Figure 6 show that the average total transmission times are very close for all proposed schemes except for the random utility-based decision-making approach. For a flow size much larger than the crossing point of 290 packets between the type 1 and type 2 transmission times, the threshold-based, utility-based and average-based schemes provide similar results. However, for smaller values there is a jump when the threshold-based scheme switches from one type of channels to the other type (here from type 1 to type 2) which makes the threshold-based scheme worse for smaller flow sizes. This jump can be clearly seen in the magnified Figure 7. The random scheme is farther than the optimal result when the flow size is large while it approaches other schemes for small flow sizes, although its performance is always slightly worst. Note that all experiments have been done for a wide range of packets, but from now on, the figures are magnified around the threshold values to clearly show the difference between the schemes.

In Figure 8, the initial number of packets is fixed to 700 and the service capacity of type 2 channels, B_2, is variable. It can be seen again that all proposed schemes except the random one provide similar results in average. When $B_1 = 4$, there is no switching for the threshold-based scheme while for $B_1 = 8$, a breaking (turning) point can be observed at $B_2^* \approx 5.95$ (Eq. (18)) where a switching from one channel type to the other one occurs.

4.2 Second Scenario: 2-Erlang Distribution

In the second scenario, it is assumed that the recovery time of type-2 channels is constant. This is a realistic assumption when the state of these channels is known in advance. Furthermore, X_1 is distributed with a 2-Erlang distribution with the same average as the previous scenario. From Eq. (13) and putting

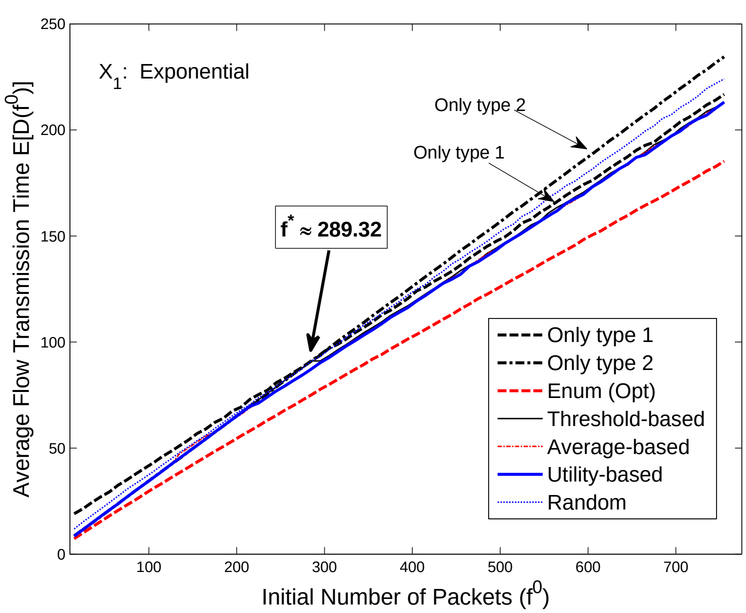

Figure 6: Numerical comparison of the average total transmission time (1st scenario, whole range).

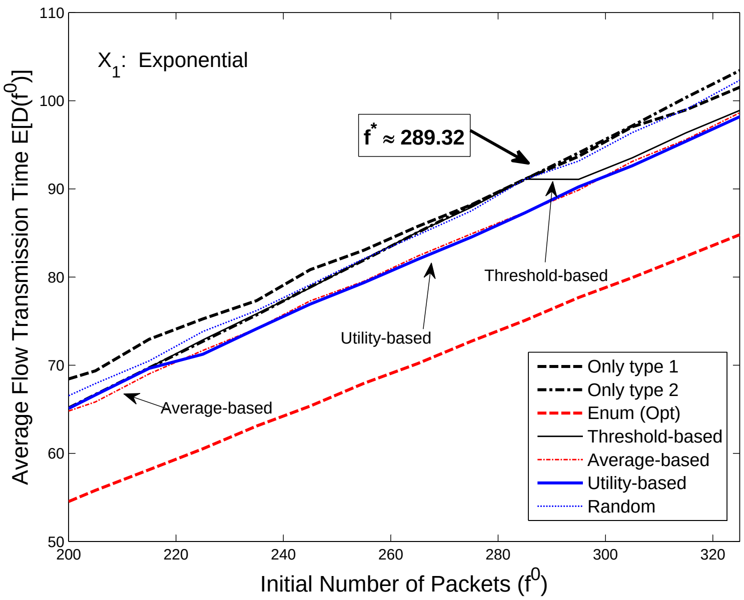

Figure 7: Numerical comparison of the average total transmission time (1st scenario, magnified).

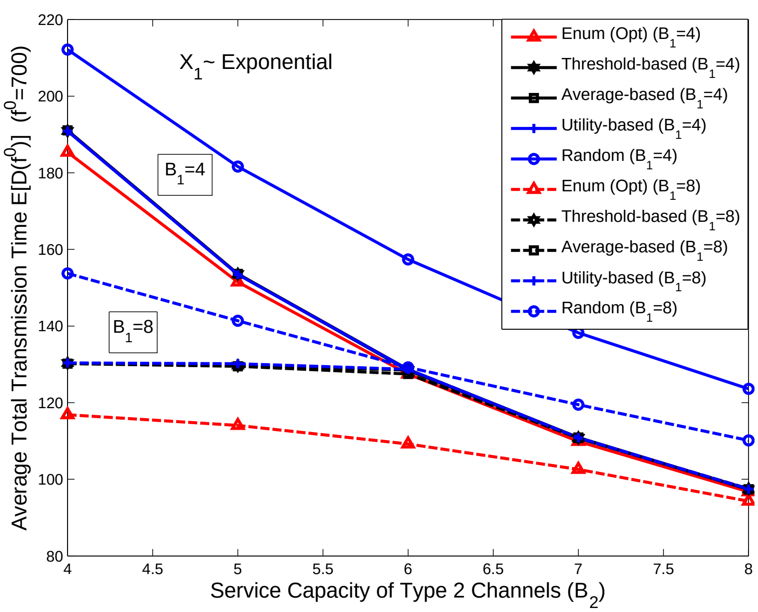

Figure 8: Numerical comparison of the average total transmission time for a fixed size flow and different service capacities (1st scenario).

$E[D_1(f^*)] = E[D_2(f^*)]$, f^* is such that

$$\frac{f^*L}{B_1} + \left((\frac{f^*L}{B_1\bar{X}_1}) + \frac{1}{4}e^{\frac{-4f^*L}{B_1\bar{X}_1}} + \frac{3}{4} \right) \bar{R}_1 = \frac{f^*L}{B_2} + \left(\lambda_2(\frac{f^*L}{B_2}) + 1 \right) \bar{R}_2 \quad (19)$$

which results in $f^* \approx 193$ packets. As illustrated in Figure 9, the trend of the results is almost similar to the previous scenario and the average-based and utility-based schemes still provide the best results.

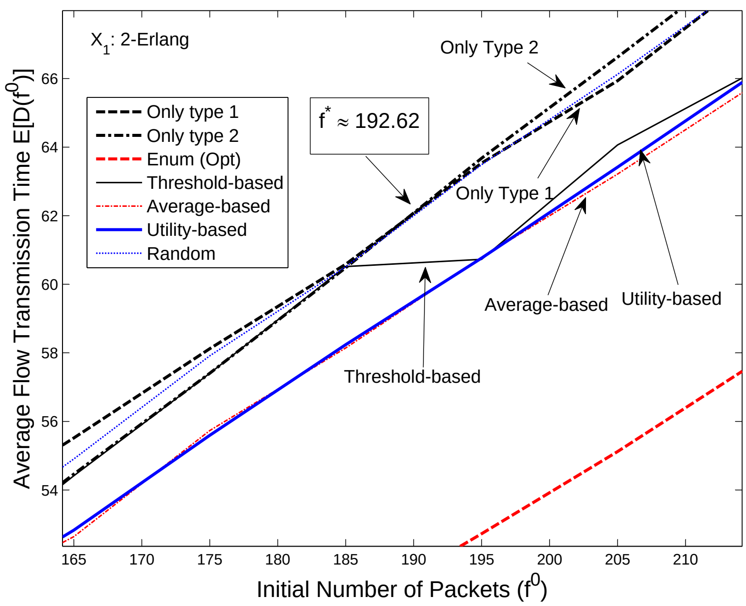

Figure 9: Numerical comparison of the average total transmission time (2nd scenario).

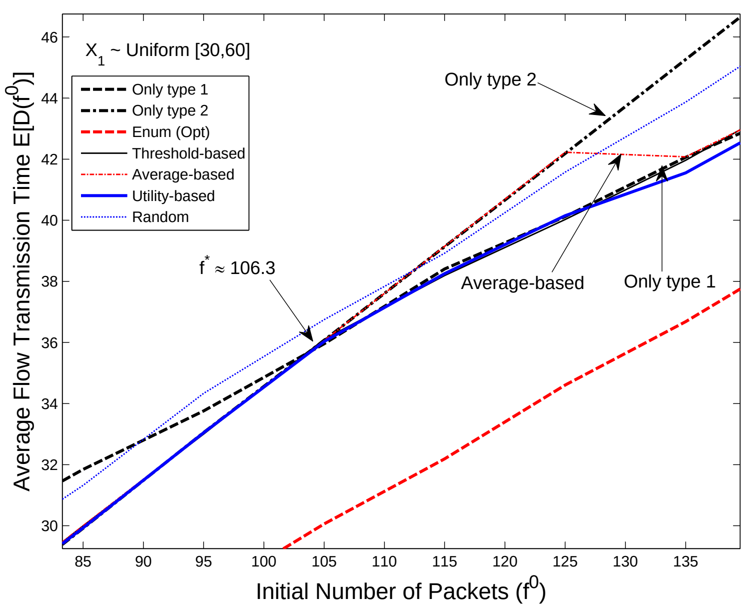

Figure 10: Third scenario results around the threshold value of the number of packets (f^*).

4.3 Third Scenario: Uniform Distribution

The only change compared to the previous scenario is that now X_1 is distributed with a uniform distribution between 30 and 60. We can still see that the results of all schemes except the random one are close to each other. The threshold value, obtained experimentally, equals 106. As shown in Figure 10, if we focus around the threshold value we can see that the three other schemes outperform the average-based scheme. This can be explained considering the fact that for a uniform distribution, contrary to previous distributions, any value of X_i may occur with the same probability as $\bar{X}_i$. Therefore, as the range of the uniform distribution increases, the performance of the average-based scheme decreases and other schemes which take the distribution into account outperform the average-based scheme.

4.4 Simulation Results for a Two-set Model

In the previous sections, we provided numerical results obtained using simulation of the system based on the availability and recovery period distributions. In this section, we actually simulate a real cognitive network with a large number of channels and the simulation parameters listed in Table 1. Due to space limit, we provide here the results of only one scenario where all distributions are Exponential. In this scenario, a new available channel should be found by sensing. The unit of time is the time required to sense one channel (τ in Eq. (1)) for both types of channel (their bandwidths are close to each other). For both sets of channel, the channel sensing order (CSO) is created randomly which implies that the recovery times (R_1 and R_2) follow a geometric distribution. The parameter of these geometric distributions, P_i^a, can be approximated as in [13] and [14] by:

$$P_i^a = \frac{\bar{A}_i}{\bar{A}_i + \bar{U}_i}, \tag{20}$$

calculated in Table 1. The service capacity of type-2 channels is lower but they have longer availability periods.

As illustrated in Figure 11, the performance of the first three schemes in simulation results is close to each other and is approximately the same as the numerical results based on the distributions. Due to the discrete nature of the simulation which results in a Geometric distribution instead of an exponential one, there is a small gap between the simulation and numerical results. Note that in the shown range, all three schemes except the random one always use the same set of channels so no jump and threshold value can be observed.

4.5 Discussions and Analysis of Results

All results show that when the remaining number of packets to be transmitted is high such that the probability to transmit all of them in the incoming operating period is low, no channel type switching in consecutive decision-making points are necessary. In other words, our assumption in the proposed threshold-based scheme which repeatedly uses the same type of channel before a threshold is realistic. The reason can be the independence of the channels and for a fixed channel, the independence of the consecutive channel availability periods. When such an assumption is applied, this implies that in a new decision point compared to the previous one, the only change is the remaining number of packets. Therefore, for the large number of remaining packets where the decision horizon is long, the same decision is valid in several consecutive decision points. Thus all scheme will decide to use the same type of channels, except for the random based scheme where

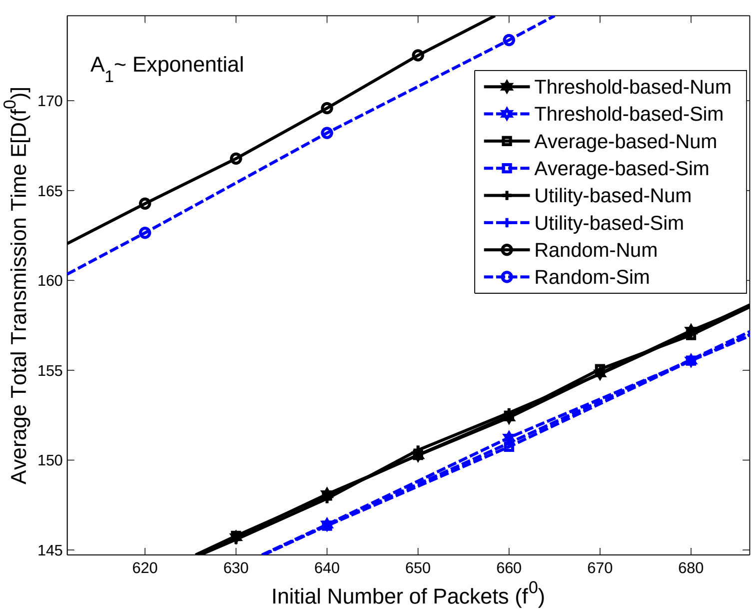

Figure 11: Simulation and numerical results for a single-link CR network.

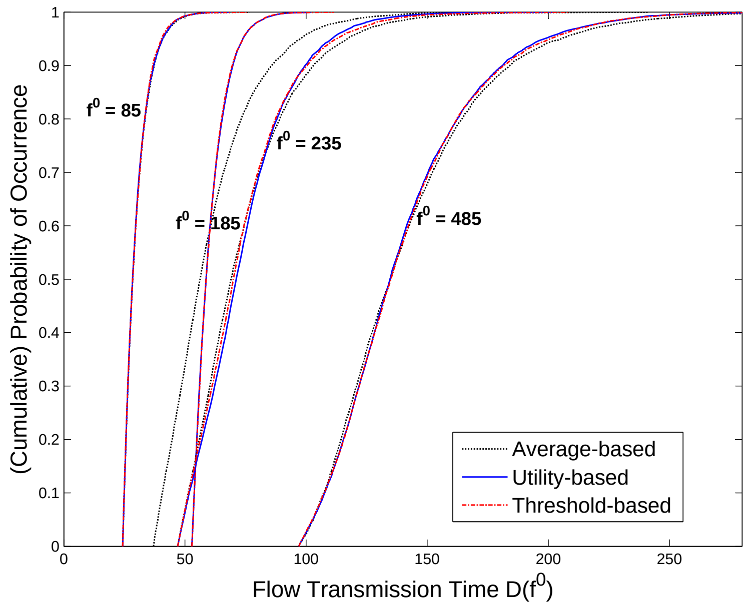

Figure 12: Distribution of the total transmission time for different flow sizes.

there is non-zero probability that the other channel type is selected, which explains its worst performance. However, close to the threshold values for different schemes, a channel type switching could be necessary which differentiates the different proposed schemes since they switch at different points. For a low number of packets after the switching, they all gradually make again the same decisions for the small remaining number of packets. This implies that for large flow sizes, it is expected that all non-random schemes provide almost the same results since they all select the same type of channel in most decision points. The small differences between different schemes come from the different decisions around the threshold value(s). As these few different decisions do not contribute substantially to the total transmission time, the final results are almost similar for all schemes except the random one. However for a flow size around the switching points, this contribution is more significant thus more performance differences between different schemes can be observed.

It is worth noting that even though all schemes, except the random one, provide similar results in average, the higher moments of the transmission time can be substantially different. For instance, Figure 12 illustrates the cumulative distribution of the flow transmission time for the first scenario (Exponential X_1) for four different flow sizes. For a small flow size equal to 85, the results are almost the same as all schemes only use type-2 channels. For a larger flow size equal to 185, we can see a considerable difference between the average-based and utility-based schemes which is due to the fact that they have different thresholds and boundary values to decide. We can see that although both scheme have similar average total transmission time, the utility-based scheme exploits its knowledge about the distribution to decrease the tail of the transmission time distribution. When the flow size increases, all schemes select the same type of channel at the beginning, but the decision is different when a few packets remain to be transmitted. As discussed, the results will be again similar for large flow sizes because the contribution of the few packets which experience different decisions is low. Similar results have been obtained for other scenarios and distributions.

Moreover, Table 2 compares the minimum and maximum performance difference between the three main proposed schemes and the result found by enumeration for the first numerical scenario. It represents the minimum and maximum difference between the total transmission time of the flow with four different size, observed among 8000 runs. It can be seen that for small flow sizes, the utility-based scheme is closer to the enumerative result while for the larger flow sizes, the threshold-based scheme is closer. However as already discussed, the result obtained by enumeration is not achievable and only provide a lower performance bound on realizable schemes. This table again highlights that while they have almost similar average results, the various schemes have different behaviour.

Table 2: Minimum (left) and maximum (right) flow transmission time difference between the proposed schemes and the result found by enumeration for the first numerical scenario.

	$f^0 = 105$	305	505	705
Threshold-based	(9, 21.6)	(0, 161.9)	(0, 137)	(0, 179)
Utility-based	(9, 13.4)	(0, 123.2)	(0, 181.3)	(0. 249.4)
Average-based	(9, 17.5)	(0, 158.6)	(0, 161.7)	(0, 196)
Only type-1	(0, 141.6)	(0, 171)	(0, 214)	(0, 243.6)
Only type-2	(9, 14.2)	(26.1, 18.4)	(43.2, 58.8)	(60.3, 61)

Table 3: Average and standard deviation of the remaining number of packets when type-2 channels is selected for the first time (first scenario).

	Threshold-based	Utility-based	Average-based
Average	174	128	71
Std	80	62	37

To better highlight the difference of the proposed schemes, and since all three schemes first use channel type 1 and then switch to type-2 channels, we have also compared the remaining number of packets when for the first time type-2 channels are used. In Table 3, the average and the standard deviation of the results for different schemes in the first scenario are provided. The average and standard deviation have been found conditionally, assuming that a switching to type-2 channel has occurred. This implies that the experiments in which there was no switching have not been considered. As expected, the average value is between zero and the threshold values. It can be observed that although the average performance is similar for the three schemes, their switching thresholds are quite different.

The switching value can be validated as follows for the threshold-based scheme. For this scheme and only when the channel availability periods are exponentially distributed, the distribution of the number of remaining packets called n', when for the first time type-2 channel is used, can be found easily as follows:

$$\begin{aligned} Pr(n' > n_0)|(n' < f^*) = Pr(n < f^* - n_0)|(n' < f^*) = \\ Pr(X_1 < \frac{(f^* - n_0)B_1}{L})|(X_1 < \frac{f^* B_1}{L}) = \frac{F_{X_1}(\frac{(f^*-n_0)B_1}{L})}{F_{X_1}(\frac{f^* B_1}{L})} \end{aligned} \quad (21)$$

where n is the number of packets transmitted over type-1 channel after the threshold value and before a new failure (before switching to type-2 channels). Conditional average of n can be found equal to $\frac{B_1 \bar{X}_i(\frac{f^* L}{B_1})}{L}$ where $\bar{X}_i(.)$ is given in Eq. (8). The memoryless property of the exponential distribution implies that we can focus only on the threshold point. At this point, the number of remaining packets is known and the distribution of the availability period of the current operating type-1 channel is also known. So, the type-2 channel is employed for the first time when a failure occurs on the current type-1 channel. Subtracting the number of packets which are transmitted during this time from the threshold number of packets gives the number of packets when, for the first time, type-2 channels are employed, as obtained in Eq. 21. For the first numerical scenario and assuming the threshold equal to 290 packets, $E[n']$ can be analytically found equal to 175.32 which is close to 174 in Table 3. For other distributions which are not memoryless, we should take into account the number of transmitted packets before the threshold since the last failure until the current point (threshold value), which is not practical. For other schemes, the threshold value is not known analytically, so no analytical discussion is provided.

5 Conclusion and Future Work

Using the cognitive radio networks capabilities of reconfigurability, decision-making, and channel sensing and switching, we showed that there is a decision trade-off between performing a restoration over channel sets with different features. To analyze such a decision, a hierarchical multi-set channel model in CRNs was introduced and some heuristics and analytical results for a two-set model were provided. The results demonstrate that when the remaining number of packets of an interrupted flow is high, such that the probability to transmit all of them before a new failure is low, using repeatedly the same type of channels is optimal since no other dependency exists between two consecutive decision points. Thus, all proposed schemes except the random one make the same decision and their results are almost similar except for smaller flow sizes. However, the higher moments of the results are generally different for different schemes.

In this paper we focused on solving the hierarchical recovery problem for two-set channels. The extension to the general multi-set model represents an interesting area of future work. A simple solution is to use the second heuristic (utility-based) with multiple utility functions. That is, for each set, a similar utility function as in Eq. (6) is defined and then the largest one will represent the best set to be selected. The random scheme is also applicable when the probability to select a channel of type i can be given by $\theta_i^* = \frac{Z_i}{\sum_{n=1}^{I} Z_n}$. For a threshold-based scheme, we can also compare the objective function (e.g., the average total transmission time) in each decision point when it is assumed that the same channel set will be used repeatedly. This results in multiple threshold values where for each threshold the selected channel set will change. A dynamic programming (DP) model [15] could also be used for a general decision procedure for the multi-set recovery problem.

For simplicity, we also assumed in this paper that the recovery time is independent of the channel availability. In reality, they can be dependent because when channels are less available, the probability to find an available channel decreases and consequently the recovery time increases. As we saw in Eq. (20), for a random recovery scheme and assuming independent channels, the recovery time over channels of type i can be defined as a Geometric distribution with a parameter P_i^a equal to $P_i^a = \frac{\bar{A}_i}{\bar{A}_i + \bar{U}_i}$. When the number of channels is large and the distribution of A_i and U_i are not heavy-tailed, it can be seen that the recovery time is only a function of the expected value of A_i and U_i. Thus, for stationary processes, the recovery time can be assumed independent of the channels' availability distribution. In CRNs, thanks to the features such as external spectrum servers which provide spectrum information [1], or learning-based schemes which provide a smaller decision space for recovery [8], the dependence of the recovery time and the channel availability could be even less. Therefore, our assumption in this paper are still valid in several real scenarios. However, deriving a decision procedure when the dependency between the recovery and availability can not be neglected represents a challenging area for future research.

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