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Learning Under Partial Cooperation and Uncertainty

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Abstract

It is now well known that in order to solve global environmental problems, such as global warming, a volunteer participation of sovereign countries to international environmental agreements is needed. However, the effects of greenhouse gases on global warming are not completely known; for instance, there is a lot of uncertainty about the impact of accumulated pollution on the global temperature. In this paper we consider a situation in which countries do not fully know the magnitude of the consequences caused by the accumulation of greenhouse gases. Countries are however able to increase their knowledge by using a Bayesian learning process, on the basis of their observation of the actual damages they incur. Moreover, we assume that some countries are engaged in an agreement aimed at reducing pollution emissions, while others are not. We study the consequences of uncertainty and learning in terms of pollution emissions and welfare, for both signatory and non-signatory countries.

Key Words: Uncertainty, Learning, International Environmental Agreements, Dynamic Games.

Résumé

On s'entend à l'heure actuelle sur le fait que, pour résoudre des problèmes environnementaux tels que le réchauffement global, il est nécessaire d'obtenir la participation volontaire de pays souverains à un accord environnemental international. Toutefois, l'impact des gaz à effet de serre sur le réchauffement global n'est pas parfaitement connu. Par exemple, l'impact de l'accumulation de pollution sur la température globale n'est pas connu avec certitude.

Dans cet article, nous considérons une situation où les pays ne connaissent pas parfaitement l'importance des conséquences de l'accumulation des gaz à effet de serre. Par ailleurs, les pays peuvent améliorer leur connaissance de cet impact par le biais d'un processus Bayésien, sur la base de leurs observations des dommages qu'ils subissent. De plus, nous supposons qu'une fraction des pays adhèrent à un accord pour la réduction de la pollution, alors que d'autres ne le font pas. Nous étudions les conséquences de l'incertitude et du processus d'apprentissage en termes d'émissions et de bien-être, pour les pays signataires comme pour les pays non signataires.

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1 Introduction

It is now well known that in order to solve global environmental problems, such as global warming and climate change, a voluntary participation of sovereign countries to international environmental agreements (IEAs) is needed. However, the participation to this kind of agreement may be affected by the uncertainty regarding the dimension of the environmental problem: more information is required on the magnitude of the environmental damage before making any commitment and sustaining the relative costs. For the impact of pollution emissions on global warming, this is true: there is a lot of uncertainty about the impact of accumulated greenhouse gases on the global temperature. Scientists do not know the precise value of what is called "climate sensitivity" as they do not have a full knowledge of the relationship between greenhouse gas levels and global mean temperature changes.[1]

In this paper we investigate how uncertainty and learning affect the emission decisions and the welfare of strategically interacting countries when an IEA is in place. We adopt a very simple environmental model that does not claim to be included among global warming models, which contain complex relations between greenhouse gas stocks and environmental damages, but it is still able to capture two key features of such models: namely, the dynamics of the pollution stock and of the related damage cost, and the negative externalities arising from the emissions. Using a simple model allows us to focus on the impact of the dynamics of the pollution stock and of the endogenous learning process on players' decisions and welfare.

The literature dealing with the size of stable IEAs has been mainly developed in a context where costs, benefits and damages are known by countries when they decide about their participation and emissions (see Barrett (2002) and Finus (2001) for an overview). Few papers study the same issue when decisions are made in a situation characterized by uncertainty and learning about some aspects of the problem. In general, in these papers, the learning process is exogenous, that is, the information is disclosed suddenly from outside the system. There is no modelling of how countries acquire knowledge. As a consequence, timing is the crucial problem, that is the moment when the information becomes available. Three scenarios are possible: after the membership and the emissions games (no learning occurs); before the membership and the emissions games, (complete learning); between the membership and the emissions games (partial learning). Moreover, these papers usually adopt static models, unable to account for the stock externality, and their main objective is to understand the effects of the different learning times on the size of the stable IEA. The general result they reach is pessimistic, as information usually has a negative impact on cooperation because of the strategic interactions between countries.

In this field a first contribution by Na and Shin (1998) examines the process of forming stable IEAs in a static model where uncertainty is about the distribution of net benefits coming from pollution abatement activities among three heterogeneous countries. Learning is exogenous and, by comparing ex-ante cooperation, e.g. before uncertainty is resolved, with ex-post cooperation, e.g. after uncertainty is resolved, they conclude that as learning brings to light the differences between countries, it makes cooperation more difficult.[2]

In a two-period model with a stock pollutant and signatory countries acting as Stackelberg leaders in the emissions game, Ulph (2004) introduces uncertainty about the level of benefits by assuming that the damage cost can be either high or low with equal probability. He then studies how exogenous learning affects the size of the stable IEA for the cases of fixed and variable membership, concluding that learning has a positive impact on cooperation and welfare if IEA membership is fixed.[3]

By using a static standard model of IEAs, Kolstad (2007) extends the model in Ulph (2004) by allowing the two states of the world to occur with variable probabilities and considering three different learning times. The results are that systematic uncertainty by itself decreases the size of an IEA. Moreover, learning can generate smaller coalitions when the environmental damage is high, mirroring the known result in the certain case.

[1] The estimated value for the climate sensitivity by the IPCC Fourth Assessment Report (AR4) is "likely to be in the range 2 to 4.5°C with a best estimate of about 3°C, and is very unlikely to be less than 1.5°C. Values substantially higher than 4.5°C cannot be excluded, but agreement of models with observations is not as good for those values."

[2] See Ulph (2004) and Kolstad and Ulph (2008) for criticisms on the emissions game adopted in this paper.

[3] He also looks at the impact of allowing or not allowing countries to revise their membership decision on welfare when learning does and does not occur.

Finally, Kolstad and Ulph (2008a) review prior knowledge on IEA membership under uncertainty and learning[4] and extend the analysis to include welfare implications of different learning times. They conclude that information has a negative value as the possibility of either complete or partial learning reduces the level of global welfare that can be achieved after forming an IEA, while partial learning reduces the size of stable IEAs.[5]

Our first contribution to the literature on uncertainty and learning in the context of IEAs is to introduce an endogenous learning process. Differently from the models above where countries are subject to passive learning, we assume that countries engage into a Bayesian learning process to discover the true value of the environmental impact of accumulated pollution emissions over time.[6] This is similar to the Bayesian learning process used by Kelly and Kolstad (1999) to resolve the uncertainty about climate sensitivity in a climate change model,[7] or by Leach (2007) who extends their model by introducing uncertainty over the value of the parameter governing the persistence of temperature changes, and by examining the effect of uncertainty on the optimal environmental policy. Karp and Zhang (2006) also adopt Bayesian learning dynamics in a model where the regulator has to decide about an optimal greenhouse gas policy when the relationship between the pollution stock and economic damages is not completely known. However, these endogenous learning papers do not consider the strategic interactions arising among countries when they act in an international context.

The second contribution of this paper is the adoption of a dynamic approach, that is, we solve a dynamic game. This is important as it allows us to account for the stock externality and for the fact that learning, by nature, is usually viewed as a dynamic process. This means that when countries decide on their emission strategies, they anticipate the future values of the pollution stock, of the environmental damage, and of their own belief about the damage parameters.

This paper differs from the literature on IEA stability because its main focus is not on the size of the stable IEA. Indeed, we assume that a stable IEA with a given number of members exists, and we study the impact of uncertainty and learning on the equilibrium emissions and welfare of signatory and non-signatory countries. As a motivation, we cite the Kyoto protocol signed in 1997 by 38 countries, and the formation of the IPCC in 1988.

In particular, we address two questions. First, what are the consequences, in terms of emissions and welfare for both signatory and non-signatory countries, of moving away from a context where everything is known to a context where the real environmental impact of accumulated pollution emissions is uncertain and subject to a Bayesian learning process? To answer this question we compare the equilibrium emissions and welfare of signatory and non-signatory countries in the case where the environmental impact is known to be either high or low versus the case where it is unknown.

The second question, given that countries have a limited knowledge, is about the environmental and economic convenience of adopting a sophisticated learning process. We address this question by comparing the equilibrium emissions and welfare of signatory and non-signatory countries when they account for the learning process dynamics versus the case where they simply use mixed strategies.

The paper is organized as follows: in Section 2, we present the basic model and in Section 3 the Bayesian learning process. Section 4 describes the dynamic emission game between signatory and non-signatory countries and it characterizes the equilibrium solution. Section 5 characterizes the equilibrium solution at the steady-state, when uncertainty is resolved. Section 6 describes the numerical approach adopted to solve for the equilibrium strategies and the value function. Section 7 reports on the results of numerical experiments and Section 8 concludes.

[4]This paper also contains a detailed review and comparison of the papers Na and Shin (1998), Ulph (2004) and Kolstad (2007).

[5]See also Kolstad and Ulph (2008b) and Finus and Pintassilgo (2009) for other discussions on how uncertainty about environmental issues affect the formation of stable IEAs.

[6]This is related to climate change model development: countries know how much they emit and they observe the environmental damage they cause, but they have an incomplete knowledge of the relationships linking these two elements.

[7]In Kelly and Kolstad (1999) the main question is about how long it will take to learn the true value of the climate sensitivity.

2 The model

We consider N identical countries, where s countries have agreed to participate in an IEA to reduce emissions with $2 \leq s \leq N-1$. For convenience, signatory countries are labeled from 1 to s while the remaining $(N-s)$ countries are called non-signatory countries and labeled from $s+1$ to N.

Denote q_{it} the production output of country i in period t. The production activity of country i in a given period t generates increasing and concave net revenues, which we assume to be quadratic and given by

$$R(q_{it}) = \left(b - \frac{1}{2}q_{it}\right) q_{it}.$$

However, the production activity of country i in period t also generates emissions x_{it}, assumed to be proportional to the output, such that

$$q_{it} = \gamma x_{it}$$

where γ is a parameter describing the degree of cleanliness of the production technology used by all countries.

In a given period, the welfare of a country i is thus given by

$$W_{it} = \left(b - \frac{1}{2}\gamma x_{it}\right) \gamma x_{it} - D(P_t)$$

where $D(P_t)$ is the environmental damage suffered in period t, arising from the global pollution stock. We analyze three different models for the environmental damage function, given by

$$\begin{aligned}
\text{(Model 1) } D(P_t) &= dP_t + \rho\varepsilon_t && (1)\\
\text{(Model 2) } D_t(P_t) &= (d + \rho\varepsilon_t)\, P_t && (2)\\
\text{(Model 3) } D_t(P_t) &= d\,(P_t)^2 + \rho\varepsilon_t && (3)
\end{aligned}$$

where P_t is the stock of pollution, d is an environmental damage parameter, assumed to be constant over time, and ε_t is a random variable having a normal distribution with mean 0 and variance 1. We assume the ε_t are independent, and that the value of d is not known with certainty. In this way, observing the level of the damage and the stock of pollution does not provide perfect information about the environmental damage parameter d. For these additive models, there exists the possibility of active learning (Karp and Zhang, 2006), meaning that the regulator is eventually able to influence the amount of learning by manipulating the level of stocks. Notice that in the first and third formulation, the amount of information obtained by monitoring the damage is more revealing when the level of pollution is high then when it is low (a larger value of P_t causes d to explain a greater proportion of the variation in damages).

The time evolution of the pollution stock is assumed to be governed by the linear discrete-time equation

$$P_t = P_{t-1}(1-\delta) + \sum_{i=1}^{N} x_{it} \qquad (4)$$

where $\delta \in (0,1)$ is the natural decay rate of the pollution stock.

3 The endogenous learning process

We assume that countries do not know the real impact of the accumulated pollution; however they form an estimate that is updated over time, and that is the same for all countries. This can be interpreted either by assuming that all countries adopt the same learning process, or that there is a research centre, like for example the IPCC, that periodically releases new data on the environmental damage.

In particular, we assume that the environmental parameter d is not known with certainty, but can take either one of two values: $d \in \{d_H, d_L\}$. In other words, there are only two possible states of the world, where the environmental impact can be either high or low respectively.[8] This kind of uncertainty is called systematic (Kolstad, 2007), as it concerns a variable which is common to all countries, that is, the uncertainty is about the level, and not the distribution, of the damages.

We denote by π the probability that d is worth d_H and by $(1-\pi)$ the probability that the value of d is d_L. Players are collectively engaged in an active learning process, where the probabilities assigned to the possible states of the world are common knowledge that update over time as countries emit and the stock of pollution evolves. In particular, we assume a Bayesian learning process: starting with an initial prior, countries observe (possibly via an environmental agency) the global environmental damage D and the level of pollution stock during a given time period. More information about d then becomes available, and the beliefs are updated, becoming the prior for next time period.

It is then possible to define a recursive formula describing the time evolution of π. Let π_t be the (prior) probability at time t of the environmental parameter being high. From the Bayesian rule, after observing the realization D_t, the updated π is the posterior

$$\begin{aligned}\pi_{t+1} &= \Pr(d_H \mid D_t) = \frac{f(D_t \mid d_H)\Pr(d_H)}{\sum_{k=L,H} f(D_t \mid d_k)\Pr(d_k)} \\ &= \frac{f_H \pi_t}{f_H \pi_t + f_L(1-\pi_t)},\end{aligned}$$

where $f_k \equiv f(D_t \mid d_k)$ is the conditional probability of observing the realization D_t when the environmental parameter value is d_k. By using the auxiliary variable $p_t = \frac{\pi_t}{1-\pi_t}$, we obtain the updating rule

$$p_{t+1} = \frac{f_H}{f_L} p_t.$$

Given that the environmental damage is a normally distributed random variable, this yields for the first model

$$\begin{aligned} D_t(P_t) \quad &\mid \quad {}_{d_k} \sim N(d_k P_t, \rho) \\ f_k &= \frac{1}{\rho\sqrt{2\pi}} \exp\left(-\frac{(D_t - d_k P_t)^2}{2\rho^2}\right),\end{aligned}$$

and it follows that

$$\begin{aligned} p_{t+1} &= \frac{\exp\left(-\frac{(D_t - d_H P_t)^2}{2\rho^2}\right)}{\exp\left(-\frac{(D_t - d_L P_t)^2}{2\rho^2}\right)} p_t \\ &= p_t \exp\left(-\frac{P_t}{2\rho^2}(d_H - d_L)(-2D_t + P_t(d_H + d_L))\right)\end{aligned}$$

or equivalently

$$\pi_{t+1} = \frac{1}{1 + \left(\frac{1-\pi_t}{\pi_t}\right)\exp\left(\frac{P_t}{2\rho^2}(d_H - d_L)(P_t(d_H + d_L) - 2D_t)\right)}. \tag{5}$$

Similar operations yield the following for Models 2 and 3:

$$\begin{aligned} \text{(Model 2)}\ D_t(P_t) \quad &\mid \quad {}_{d_k} \sim N(d_k P_t, P_t \rho) \\ p_{t+1} &= \frac{\exp\left(-\frac{(D_t - d_H P_t)^2}{2P_t^2\rho^2}\right)}{\exp\left(-\frac{(D_t - d_L P_t)^2}{2P_t^2\rho^2}\right)} p_t \\ \pi_{t+1} &= \frac{1}{1 + \left(\frac{1-\pi_t}{\pi_t}\right)\exp\left(\frac{1}{2P_t\rho^2}(d_H - d_L)(P_t(d_H + d_L) - 2D_t)\right)} \end{aligned} \tag{6}$$

[8]For example, according to the estimates for the equilibrium climate sensitivity by the IPCC Fourth Assessment Report (AR4), these two values could correspond to a climate sensitivity of 2°C or 4.5°C.

$$\begin{aligned}(\text{Model 3})\ D_t(P_t) \quad &| \quad _{d_k} \sim N\left(d_k P^2, \rho\right), \qquad (7)\\ p_{t+1} &= \frac{\exp\left(-\frac{(D_t - d_H P_t^2)^2}{2\rho^2}\right)}{\exp\left(-\frac{(D_t - d_L P_t^2)^2}{2\rho^2}\right)} p_t \\ \pi_{t+1} &= \frac{1}{1 + \left(\frac{1-\pi_t}{\pi_t}\right)\exp\left(\frac{1}{2}P_t^2\left(d_H - d_L\right)\frac{P_t^2(d_H + d_L) - 2D_t}{\rho^2}\right)}.\end{aligned}$$

4 The emission game

Given an IEA with membership size s, the emissions problem of a single non-signatory country consists in maximizing the expected value of its total discounted welfare over an infinite horizon, given the emission strategies of the other players:

$$\begin{aligned}&\max_{x_{it}}\left\{\sum_{t=0}^{\infty}\beta^t E\left[\gamma x_{it}\left(b - \frac{\gamma x_{it}}{2}\right) - D\left(P_t\right)\right]\right\}\\ \text{s.t.}\quad P_{t+1} &= P_t(1-\delta) + \sum_{j\neq i} x_{jt} + x_{it}.\end{aligned}$$

Note that we assume that the damage $D(P_t)$ incurred in a given period t depends on the stock of pollution at the beginning of period t, while it is observed at the end of period t.

4.1 Strategic learning

We first assume that players account for the dynamics of the learning process when determining the level of their emissions. We use a dynamic programming formulation, where the state variable P is the level of the stock of pollution and the state variable π is the common belief, or probability of having a high environmental damage coefficient, at the beginning of the period. Denote by $Y(P,\pi)$ the total emissions of the other players resulting from their feedback strategies. The total expected optimal discounted welfare over an infinite horizon of a non-signatory country i for a given $\pi \in (0,1)$ and $P \in \mathbb{R}^+$satisfies

$$V(P,\pi) = \max_x\left\{\left(b - \frac{\gamma x}{2}\right)\gamma x - E_\pi\left[D(P)\right] + \beta E_\pi\left[V\left(P(1-\delta) + Y(P,\pi) + x; T_k(\varepsilon,P,\pi)\right)\right]\right\} \qquad (8)$$

where the expectation $E_\pi[\cdot]$ of a function of the standard normal random variable ε depends on the player's belief π and where the transition state $T_k(\varepsilon,P,\pi)$ is defined by (5)–(7) for Models 1-3, that is,

$$T_k(\varepsilon,P,\pi) = \frac{1}{1 + \left(\frac{1-\pi}{\pi}\right)\exp F_k(\varepsilon,P)} \text{ if } d = d_k, k \in \{H,L\} \qquad (9)$$

where, denoting $\Delta \equiv (d_H - d_L)$,

Model 1:

$$\begin{aligned}F_H(\varepsilon,P) &= -\frac{P}{2\rho^2}\Delta\left(2\varepsilon\rho + P\Delta\right)\\ F_L(\varepsilon,P) &= -\frac{P}{2\rho^2}\Delta\left(2\varepsilon\rho - P\Delta\right)\end{aligned}$$

Model 2:

$$\begin{aligned}F_H(\varepsilon,P) &= -\frac{1}{2\rho^2}\Delta\left(2\rho\varepsilon + \Delta\right)\\ F_L(\varepsilon,P) &= -\frac{1}{2\rho^2}\Delta\left(2\varepsilon\rho - \Delta\right)\end{aligned}$$

Model 3:

$$\begin{aligned} F_H(\varepsilon, P) &= -\frac{P^2}{2\rho^2}\Delta\left(2\rho\varepsilon + \Delta P^2\right) \\ F_L(\varepsilon, P) &= -\frac{P^2}{2\rho^2}\Delta\left(2\rho\varepsilon - \Delta P^2\right). \end{aligned}$$

It follows that

$$\begin{aligned} E_\pi\left[V\left(\cdot; T_k(\varepsilon, P, \pi)\right)\right] &= \pi \int_{-\infty}^{\infty} V\left(\cdot; \frac{1}{1 + \left(\frac{1-\pi}{\pi}\right)\exp F_H(\varepsilon, P)}\right) d\varphi\left(\varepsilon\right) \\ &\quad + (1-\pi) \int_{-\infty}^{\infty} V\left(\cdot; \frac{1}{1 + \left(\frac{1-\pi}{\pi}\right)\exp F_L(\varepsilon, P)}\right) d\varphi\left(\varepsilon\right) \end{aligned} \tag{10}$$

where φ is the standard normal density function.

Assuming an interior solution, the first order condition yields

$$\gamma x^* = b + \frac{\beta}{\gamma} E_\pi\left[V_P\left(P(1-\delta) + Y\left(P, \pi\right) + x; T(\varepsilon, P, \pi)\right)\right]$$

where $V_P\left(\cdot\right)$ is the marginal impact of the pollution stock on the value function, evaluated at the transition state.

Signatory countries act as a single player and decide on their emissions in order to maximize the total welfare of the coalition. Denote by $B(P, \pi)$ the total emissions of the non-signatory countries resulting from their feedback strategies. Similarly to the optimization problem of the non-signatories, the value function representing the maximum expected total discounted welfare of the coalition members over an infinite horizon satisfies:

$$W(P, \pi) = \max_y \left\{ s\left(b - \frac{\gamma y}{2}\right)\gamma y - sE_\pi\left[D(P)\right] + \beta E_\pi\left[W\left(P(1-\delta) + X(P, \pi) + sy; T(\varepsilon, P, \pi)\right)\right]\right\}. \tag{11}$$

Assuming an interior solution, first order condition yields

$$\gamma y^* = b + \frac{\beta}{\gamma} E_\pi\left[W_P\left(P(1-\delta) + X(P, \pi) + sy; T(\varepsilon, P, \pi)\right)\right]$$

where $W_P\left(\cdot\right)$ is the marginal impact of the pollution stock on the signatories' total welfare value function, evaluated at the transition state.

Finally, at a given (P, π), the equilibrium emissions of signatory and non-signatory countries satisfy simultaneously

$$\gamma y^*(P, \pi) = b + \frac{\beta}{\gamma} E_\pi\left[W_P\left(P(1-\delta) + sy^*(P, \pi) + (N-s)\, x^*(P, \pi); T(\varepsilon, P, \pi)\right)\right] \tag{12}$$

$$\gamma x^*(P, \pi) = b + \frac{\beta}{\gamma} E_\pi\left[V_P\left(P(1-\delta) + sy^*(P, \pi) + (N-s)\, x^*(P, \pi); T(\varepsilon, P, \pi)\right)\right]. \tag{13}$$

4.2 Myopic players

In order to evaluate the impact of the endogenous learning, we now compute the welfare function of players when they do not account for the dynamics of the learning process. Feedback strategies are then only functions of the pollution stock level P. At a given π, the optimization problem solved by each type of player is then

$$\begin{aligned} V_\pi(P) &= \max_x \left\{ \left(b - \frac{\gamma x}{2}\right)\gamma x - E\left[D(P)\right] + \beta\left(V_\pi\left(P(1-\delta) + Y\left(P\right) + x\right)\right)\right\} \\ &= \max_x \left\{ \left(b - \frac{\gamma x}{2}\right)\gamma x - \pi D_H(P) - (1-\pi) D_L(P) + \beta\left(V_\pi\left(P(1-\delta) + Y\left(P\right) + x\right)\right)\right\} \end{aligned} \tag{14}$$

$$\begin{aligned} W_\pi(P) &= \max_y \left\{ s\left(b - \frac{\gamma y}{2}\right)\gamma y - sE\left[D(P)\right] + \beta\left(W_\pi\left(P(1-\delta) + X(P) + sy\right)\right)\right\} \\ &= \max_y \left\{ s\left(b - \frac{\gamma y}{2}\right)\gamma y - s\pi D_H(P) - s(1-\pi)D_L(P) \right. \\ &\quad \left. +\beta\left(W_\pi\left(P(1-\delta) + X(P) + sy\right)\right)\right\} \end{aligned} \tag{15}$$

where players do not anticipate the future changes in the value of their belief π. It is straightforward to see that these optimization problems are equivalent to the ones that the players would face in the full information case where the damage parameter is known to be equal to $d_\pi \equiv \pi d_H + (1-\pi)d_L$. The full information case is solved analytically in the next section.

However, even if players do not anticipate it, beliefs will change over time, and the welfare functions of players using myopic strategies $(x^\pi(P), y^\pi(P))$ satisfy

$$\begin{aligned} V_M(P,\pi) &= \left(b - \frac{\gamma x^\pi(P)}{2}\right)\gamma x^\pi(P) - \pi D_H(P) - (1-\pi)D_L(P) \\ &\quad +\beta E_\pi\left[V_M\left(P(1-\delta) + (N-s)x^\pi(P) + sy^\pi(P); T(\varepsilon,P,\pi)\right)\right] \\ W_M(P,\pi) &= s\left(\left(b - \frac{\gamma y^\pi(P)}{2}\right)\gamma y^\pi(P) - \pi D_H(P) - (1-\pi)D_L(P)\right) \\ &\quad +\beta E_\pi\left[W_M\left(P(1-\delta) + (N-s)x^\pi(P) + sy^\pi(P); T(\varepsilon,P,\pi)\right)\right]. \end{aligned}$$

These functions will be used to evaluate the welfare impact of accounting or not for the dynamics of the learning process when computing countries emission strategies.

5 Steady-state

As a result of the learning process, at some future time, the true value of the environmental impact d will become known with certainty and it will take one of two values: $d_k, k \in \{H, L\}$, corresponding respectively to the case $\pi = 1$ and $\pi = 0$. In this section, we solve for the equilibrium strategies and value functions for these limiting full information cases.

Countries' emissions problems consist in maximizing the expected value of their total discounted welfare over an infinite horizon, given the emission strategies of the other players. Denote $X_k(P)$ and $Y_k(P)$ the total emissions of the other countries resulting from feedback strategies when the damage parameter is known to take the value d_k in the optimization problem of the signatory and non-signatory players respectively. The total expected discounted welfare functions over an infinite horizon for non-signatory and signatory players satisfy:

$$V_k(P) = \max_x \left\{ \left(b - \frac{\gamma x}{2}\right)\gamma x - E\left[D(P)\right] + \beta\left(V_k\left(P(1-\delta) + Y(P) + x\right)\right)\right\} \tag{16}$$

$$W_k(P) = \max_y \left\{ s\left(b - \frac{\gamma y}{2}\right)\gamma y - sE\left[D(P)\right] + \beta\left(W_k\left(P(1-\delta) + X(P) + sy\right)\right)\right\}, \tag{17}$$

$$k \in \{H, L\}.$$

Since the damage function is linear in ε and the expected value of ε is 0, the expected damage $E\left[D(P)\right]$ in a given period, where the stock of pollution is P, is equal to d_kP in Models 1 and 2 and d_kP^2 in Model 3. In the following two subsections, we obtain analytical expressions for the equilibrium strategies and welfare functions for the linear (Models 1 and 2) and quadratic (Model 3) cases.

5.1 Linear damage function

Assume that $V_k(P) = \alpha_k - m_kP$. It follows that

$$V_k(P) = \max_x \left\{ \left(b - \frac{\gamma x}{2}\right)\gamma x - d_kP + \beta\left(\alpha_k - m_k\left(P(1-\delta) + Y_k\left(P\right) + x\right)\right)\right\},$$

where the function to be optimized is concave in x. Assuming an interior solution, first order condition yields

$$\begin{aligned}\gamma x^* &= b - m_k \frac{\beta}{\gamma} \\ V_k(P) &= \frac{1}{2\gamma^2}\left(b^2\gamma^2 - \beta^2 m_k^2\right) - d_k P + \beta\left(\alpha_k - m_k\left(P(1-\delta) + Y_k(P) + \frac{(b - m_k\beta)}{\gamma^2}\right)\right)\end{aligned}$$

which satisfies the assumption with

$$\begin{aligned}m_k &= \frac{d_k}{\beta(\delta-1)+1} \equiv \kappa d_k \\ \alpha_k &= \frac{1}{2(1-\beta)}\left(b^2 - \kappa\beta d_k\left(2Y_k(P) + \frac{2b - \kappa\beta d_k}{\gamma^2}\right)\right).\end{aligned}$$

In the same way, assume that $W_k(P) = \theta_k - n_k P$, first order condition yields

$$\begin{aligned}\gamma y^* &= \left(b - n_k \frac{\beta}{\gamma}\right) \\ W_k(P) &= \frac{1}{2\gamma^2} s\left(b^2\gamma^2 - \beta^2 n_k^2\right) - s d_k P + \beta\left(\theta_k - n_k\left(P(1-\delta) + X_k(P) + \frac{s(b - \beta n_k)}{\gamma^2}\right)\right)\end{aligned}$$

which satisfies the assumption with

$$\begin{aligned}n_k &= s\kappa d_k \\ \theta_k &= \frac{s}{2(1-\beta)}\left(b^2 - \kappa\beta d_k\left(2X_k(P) + s\frac{2b\gamma - s\kappa\beta d_k}{\gamma^2}\right)\right).\end{aligned}$$

Therefore, at a given P, the equilibrium emissions of signatory and non-signatory countries satisfy

$$\begin{aligned}\gamma y_k^* &= b - \frac{\beta}{\gamma} s\kappa d_k \\ \gamma x_k^* &= b - \frac{\beta}{\gamma}\kappa d_k,\end{aligned}$$

provided $b\gamma > \beta s\kappa d_k$. In the linear case (models 1 and 2), when uncertainty is resolved, players use strategies that are independent of the level of the stock and independent of each other. Production of both signatory and non-signatory countries are negatively affected by increases of the environmental impact, and positively affected by improvements in cleaner technologies, provided that $b\gamma > 2ds\kappa\beta$.

It is straightforward to compute the equilibrium welfare function parameters:

$$\begin{aligned}m_k &= \kappa d_k \\ \alpha_k &= \frac{1}{2(1-\beta)}\left(b^2 - \kappa\beta d_k \frac{2b\left((N-s)\gamma + 1\right) - \kappa\beta d_k\left(2(N-s)s + 1\right)}{\gamma^2}\right) \\ n_k &= s\kappa d_k \\ \theta_k &= \frac{s}{2(1-\beta)}\left(b^2 - \kappa\beta d_k s\frac{4b\gamma - \kappa\beta d_k(s+2)}{\gamma^2}\right).\end{aligned}$$

The welfare functions are linear and decreasing in P.

From these results, it is now easy to obtain the myopic equilibrium strategies (x^π, y^π) solving (14)–(15):

$$\begin{aligned}\gamma y^\pi &= b - \frac{\beta}{\gamma} s\kappa\pi\left(d_{kH} + (1-\pi)d_L\right) \\ \gamma x^\pi &= b - \frac{\beta}{\gamma}\kappa\left(d_{kH} + (1-\pi)d_L\right),\end{aligned}$$

which corresponds to "mixing" the strategies y_k^* and x_k^* according to the current belief of the players.

5.2 Quadratic damage function

Assume that $V_k(P) = \alpha_k - m_k P + \phi_k P^2$ and $Y_k(P) = A_k P + B_k$. It follows that the value function of non-signatory countries satisfies

$$\begin{aligned} V_k(P) &= \max_x \left\{ \left(b - \frac{\gamma x}{2}\right)\gamma x - d_k P^2 + \beta \left[V_k\left(P(1-\delta) + Y_k(P) + x\right)\right] \right\} \\ &= \max_x \left\{ \left(b - \frac{\gamma x}{2}\right)\gamma x - d_k P^2 + \beta \left(\begin{array}{c} \alpha_k - m_k\left(P(1-\delta) + A_k P + B_k + x\right) \\ +\phi_k \left(P(1-\delta) + A_k P + B_k + x\right)^2 \end{array} \right) \right\}, \end{aligned}$$

where the function to be optimized is concave in x if

$$2\beta\phi_k \leq \gamma^2.$$

In that case, assuming an interior solution, first order condition yields

$$\begin{aligned} x^* &= 2P\beta\phi_k \frac{A_k - \delta + 1}{\gamma^2 - 2\beta\phi_k} + \frac{b\gamma - \beta\left(m_k - 2\phi_k B_k\right)}{\gamma^2 - 2\beta\phi_k} \\ V_k(P) &= -P^2 \frac{\gamma^2 d_k - \beta\phi_k\left(2d_k + \gamma^2\left(A_k - \delta + 1\right)^2\right)}{\gamma^2 - 2\beta\phi_k} \\ &\quad + P\beta\gamma \frac{\left(2b\phi_k - \gamma\left(m_k - 2\phi_k B_k\right)\right)\left(A_k - \delta + 1\right)}{\gamma^2 - 2\beta\phi_k} \\ &\quad + \frac{1}{2} \frac{b^2\gamma^2 + \beta\left(2\alpha_k\left(\gamma^2 - 2\beta\phi_k\right) + \beta m_k^2 - 2\gamma^2\phi_k B_k^2 - 2\gamma\left(m_k - 2\phi_k B_k\right)\left(b + \gamma B_k\right)\right)}{\gamma^2 - 2\beta\phi_k} \end{aligned}$$

which satisfies the assumption, provided that the parameters satisfy

$$m_k = 2\beta\gamma\phi_k \frac{\left(b + \gamma B_k\right)\left(A_k - \delta + 1\right)}{\beta\gamma^2\left(A_k - \delta + 1\right) - \left(\gamma^2 - 2\beta\phi_k\right)} \tag{18}$$

$$\alpha_k = \frac{b^2\gamma^2 + \beta\left(\beta m_k^2 - 2\gamma m_k\left(b + \gamma B_k\right) + 2\gamma\phi_k B_k\left(2b + \gamma B_k\right)\right)}{2\left(1 - \beta\right)\left(\gamma^2 - 2\beta\phi_k\right)} \tag{19}$$

$$\gamma^2 d_k = \beta\phi_k\left(2d_k + \gamma^2\left(A_k - \delta + 1\right)^2\right) - \phi_k\left(\gamma^2 - 2\beta\phi_k\right). \tag{20}$$

In the same way, assuming that $W_k(P) = \theta_k - n_k P + \omega_k P^2$ and that $X_k(P) = C_k P + D_k$, the value function, representing the total discounted welfare of signatory players satisfies:

$$\begin{aligned} W_k(P) &= \max_y \left\{ s\left(b - \frac{\gamma y}{2}\right)\gamma y - s d_k P^2 + \beta\left(W_k\left(P(1-\delta) + B_k(P) + sy\right)\right) \right\} \\ &= \max_y \left\{ s\left(b - \frac{\gamma y}{2}\right)\gamma y - s d_k P^2 + \beta \left(\begin{array}{c} \theta_k - n_k\left(P(1-\delta) + C_k P + D_k + sy\right) \\ +\omega_k\left(P(1-\delta) + C_k P + D_k + sy\right)^2 \end{array} \right) \right\}, \end{aligned}$$

where the function to be maximized is concave in y if

$$2s\beta\omega_k \leq \gamma^2.$$

First order condition yields

$$\begin{aligned} y^* &= 2P\beta\omega_k \frac{C_k - \delta + 1}{\gamma^2 - 2s\beta\omega_k} + \frac{b\gamma - \beta\left(n_k - 2\omega_k D_k\right)}{\gamma^2 - 2s\beta\omega_k} \\ W_k(P) &= -P^2 \frac{s\gamma^2 d_k - \beta\omega_k\left(2s^2 d_k + \gamma^2\left(C_k - \delta + 1\right)^2\right)}{\gamma^2 - 2s\beta\omega_k} \\ &\quad + P\beta\gamma \frac{\left(2bs\omega_k - \gamma\left(n_k - 2\omega_k D_k\right)\right)\left(C_k - \delta + 1\right)}{\gamma^2 - 2s\beta\omega_k} \\ &\quad + \frac{1}{2} \frac{b^2 s\gamma^2 + \beta\left(2\theta_k\left(\gamma^2 - 2s\beta\omega_k\right) + s\beta n_k^2 - 2\gamma^2\omega_k D_k^2 - 2\gamma\left(n_k - 2\omega_k D_k\right)\left(\gamma D_k + bs\right)\right)}{\gamma^2 - 2s\beta\omega_k} \end{aligned}$$

which satisfies the assumption, provided that the parameters satisfy

$$n_k = 2\beta\gamma\omega_k \frac{(\gamma D_k + bs)(C_k - \delta + 1)}{\beta\gamma^2 (C_k - \delta + 1) - (\gamma^2 - 2s\beta\omega_k)} \tag{21}$$

$$\theta_k = \frac{b^2 s\gamma^2 + \beta \left(s\beta n_k^2 - 2\gamma n_k (\gamma D_k + bs) + 2\gamma\omega_k D_k (\gamma D_k + 2bs)\right)}{2(1-\beta)(\gamma^2 - 2s\beta\omega_k)} \tag{22}$$

$$s\gamma^2 d_k = \beta\omega_k \left(2s^2 d_k + \gamma^2 (C_k - \delta + 1)^2\right) - \omega_k \left(\gamma^2 - 2s\beta\omega_k\right). \tag{23}$$

Therefore, at a given P, the equilibrium emissions of signatory and non-signatory countries satisfy simultaneously

$$\begin{aligned} y^*(P) &= 2P\beta\omega_k \frac{C_k - \delta + 1}{\gamma^2 - 2s\beta\omega_k} + \frac{b\gamma - \beta(n_k - 2\omega_k D_k)}{\gamma^2 - 2s\beta\omega_k} \\ x^*(P) &= 2P\beta\phi_k \frac{A_k - \delta + 1}{\gamma^2 - 2\beta\phi_k} + \frac{b\gamma - \beta(m_k - 2\phi_k B_k)}{\gamma^2 - 2\beta\phi_k}, \end{aligned}$$

where

$$\begin{aligned} A_k P + B_k &= sy^* + (N - s - 1)\, x^* \\ C_k P + D_k &= (N - s) x^*, \end{aligned}$$

yielding by identification

$$\begin{aligned} A_k &= \frac{2(C_k - \delta + 1) s\beta\omega_k}{\gamma^2 - 2s\beta\omega_k} - \frac{2(A_k - \delta + 1)\beta\phi_k (s - N + 1)}{\gamma^2 - 2\beta\phi_k} \\ B_k &= \frac{s(b\gamma - \beta(n_k - 2\omega_k D_k))}{\gamma^2 - 2s\beta\omega_k} - \frac{(b\gamma - \beta(m_k - 2\phi_k B_k))(s - N + 1)}{\gamma^2 - 2\beta\phi_k} \\ C_k &= \frac{2(A_k - \delta + 1)\beta\phi_k (N - s)}{\gamma^2 - 2\beta\phi_k} \\ D_k &= \frac{(N - s)(b\gamma - \beta(m_k - 2\phi_k B_k))}{\gamma^2 - 2\beta\phi_k}. \end{aligned}$$

This reduces to

$$A_k = 2\beta(\delta - 1) \frac{\phi_k (N - s - 1) + s\omega_k}{2\beta\mu_k - \gamma^2} \tag{24}$$

$$C_k = 2\beta(\delta - 1) \frac{(N - s)\phi_k}{2\beta\mu_k - \gamma^2} \tag{25}$$

$$B_k = \frac{\gamma^2 (\beta m_k (N - s - 1) + s\beta n_k - b\gamma (N - 1)) + 2s\beta (\phi_k (b\gamma - \beta n_k) - \omega_k (b\gamma - \beta m_k))}{\gamma^2 (2\beta\mu_k - \gamma^2)} \tag{26}$$

$$D_k = (N - s)\left(\gamma^2 - 2\beta\phi_k\right) \frac{\gamma^2 (b\gamma - \beta m_k) + 2s\beta (\phi_k (b\gamma - \beta n_k) - \omega_k (b\gamma - \beta m_k))}{(2\beta\phi_k - \gamma^2)\gamma^2 (2\beta\mu_k - \gamma^2)}, \tag{27}$$

where $\mu_k \equiv (\phi_k (N - s) + s\omega_k)$ is the weighted average of parameters of the equilibrium value functions. Finally, replacing these values into (20)–(23), to find the equilibrium, one needs to solve for ϕ_k and ω_k :

$$\left(2\beta\mu_k - \gamma^2\right)^2 (\phi_k + d_k) + \beta\gamma^2\phi_k (\delta - 1)^2 \left(2\beta\phi_k - \gamma^2\right) = 0 \tag{28}$$

$$\left(2\beta\mu_k - \gamma^2\right)^2 (\omega_k + sd_k) + \beta\gamma^2\omega_k (\delta - 1)^2 \left(2s\beta\omega_k - \gamma^2\right) = 0. \tag{29}$$

Notice that, for a given μ_k, both equations are quadratic in ϕ_k and ω_k respectively, and one can show that roots of this system are always negative, and consequently satisfy the second order condition, so that both value functions are concave in P. Finally, if it exists, a solution to this non-linear system of equations yields parameter values that can readily be substituted into (24)–(27) to give the equilibrium strategies, and then into (18)–(19) and (21)–(22) to yield the equilibrium value functions of the players. In the quadratic case, the equilibrium strategies of the players are linear functions of the pollution stock.

Again, from the full information case result, one can obtain the myopic equilibrium strategies (x^π, y^π) solving (14)–(15) by replacing d_k by $d_\pi = (\pi d_H + (1 - \pi) d_L)$ in (24)–(29); however, by contrast to the linear case, these no longer correspond to simple mixed strategies.

6 Numerical approach

Except for the case where $\pi = 0$ or $\pi = 1$, the dynamic program (8)–(11) does not admit an analytical solution. We propose a value iteration algorithm to numerically compute the equilibrium stationary strategies and the value of the dynamic game for a given number of signatory countries. When initiated with the null function, the equilibrium value function provided by this algorithm at iteration t coincides with the solution of the same game played over a finite horizon of t periods. Provided unique equilibrium strategies exist in the successive finite horizon games, then this algorithm will converge to the value of the dynamic game over an infinite horizon.

We use a finite element approximation for the value functions by linear splines, a finite difference approximation for the derivatives of the value functions, a cobweb approach for the computation of the equilibrium strategies, and an analytic computation of the expected value in (10).

The algorithm is the following:

1. Initialize the parameter values and the number of signatory countries.
   Define a grid $\mathcal{G}_P \times \mathcal{G}_\pi$ on the state space $\left(0, \frac{Nb}{\delta\gamma}\right]$ and $(0,1)$, using equally spaced steps λ_P and λ_π.
   Define the tolerance ζ.
   Initialize the value functions $W^0(P_i,\pi_j)$ and $V^0(P_i,\pi_j)$ on all points of the grid. Set $W^0(P_i,0)$, $V^0(P_i,0)$, $W^0(P_i,1)$, $V^0(P_i,1)$ to their analytically known value in the certain case. Set $t = 0$.
2. Using a finite difference approximation, compute the derivatives $W_P^t(P_i,\pi_j)$ and $V_P^t(P_i,\pi_j)$ for all $(P_i,\pi_j) \in \mathcal{G}_P \times \mathcal{G}_\pi$.
   Set $t =: t+1$.
3. For all $(P_i,\pi_j) \in \mathcal{G}_P \times \mathcal{G}_\pi$
   (a) Initialize x and y.
   (b) Compute the transition pollution stock P_i' using (4)
   Compute the expectations $E_\pi\left[V_P^{t-1}(P';T(0,P_i,\pi_j))\right]$ and
   $E_\pi\left[W_P^{t-1}(P';T(0,P_i,\pi_j))\right]$.
   (c) Compute the optimal reactions x' and y' according to (13)–(12).
   (d) If $|y-y'| < \zeta$ and $|x-x'| < \zeta$, go to step 3e. Otherwise, set $y = y'$ and $x = x'$ and go to step 3b.
   (e) Compute $W^t(P_i,\pi_j)$ and $V^t(P_i,\pi_j)$ according to

$$\begin{aligned} V^t(P_i,\pi_j) &= \left(b - \frac{\gamma x}{2}\right)\gamma x - \pi D_H(P) - (1-\pi)D_L(P) \\ &\quad + \beta\left(E_\pi\left[V^{t-1}(P';T(0,P_i,\pi_j))\right]\right) \\ W^t(P,\pi) &= s\left(\left(b - \frac{\gamma y}{2}\right)\gamma y - \pi D_H(P) - (1-\pi)D_L(P)\right) \\ &\quad + \beta\left(E_\pi\left[W^{t-1}(P';T(0,P_i,\pi_j))\right]\right). \end{aligned}$$

4. If $\max_{ij}\left|W^t(P_i,\pi_j) - W^{t-1}(P_i,\pi_j)\right| < \zeta$ and $\max_{ij}\left|V^t(P_i,\pi_j) - W^{t-1}(P_i,\pi_j)\right| < \zeta$, go to step 5.
   Otherwise, set $t = t+1$ and go to step 3.
5. For all $(P_i,\pi_j) \in \mathcal{G}_P \times \mathcal{G}_\pi$, set the myopic equilibrium strategies $x^\pi(P)$ and $y^\pi(P)$ to their analytically known values. Initialize the welfare functions $W_M^0(P_i,\pi_j)$ and $V_M^0(P_i,\pi_j)$ on all points of the grid. Set $W_M^0(P_i,0), V_M^0(P_i,0)$, $W_M^0(P_i,1)$, $V_M^0(P_i,1)$ to their analytically known value.
6. For all $(P_i,\pi_j) \in \mathcal{G}_P \times \mathcal{G}_\pi$,
   (a) Compute the transition pollution stock P' using (4), $x^{\pi_j}(P_i)$ and $y^{\pi_j}(P_i)$
   (b) Compute $W_M^t(P_i,\pi_j)$ and $V_M^t(P_i,\pi_j)$ according to

$$\begin{aligned} V_M^t(P_i,\pi_j) &= \left(b - \frac{\gamma x^\pi(P)}{2}\right)\gamma x^\pi(P) - \pi D_H(P) - (1-\pi)D_L(P) \\ &\quad + \beta\left(E_\pi\left[V_M^{t-1}(P';T(0,P_i,\pi_j))\right]\right) \end{aligned}$$

$$W_M^t(P,\pi) = s\left(\left(b - \frac{\gamma y^\pi(P)}{2}\right)\gamma y^\pi(P) - \pi D_H(P) - (1-\pi)D_L(P)\right) + \beta\left(E_\pi\left[W_M^{t-1}\left(P'; T(0, P_i, \pi_j)\right)\right]\right)$$

7. If $\max_{ij}\left|W_M^t(P_i,\pi_j) - W_M^{t-1}(P_i,\pi_j)\right| < \zeta$ and $\max_{ij}\left|V_M^t(P_i,\pi_j) - W_M^{t-1}(P_i,\pi_j)\right| < \zeta$, stop. Otherwise, set $t = t+1$ and go to step 6.

7 Numerical illustration

7.1 Time evolution

To illustrate the dynamics of the model, we first show a sample result from a simulation of the time evolution of the variables of interest. This simulation is done under Model 1, using a relatively large value for the parameter ρ. In this case, the observation of the damage is not very informative because it contains a large amount of noise, and learning is slow. Figures 1 and 2 illustrate one possible time evolution of the state variables, emissions, damage and welfare.

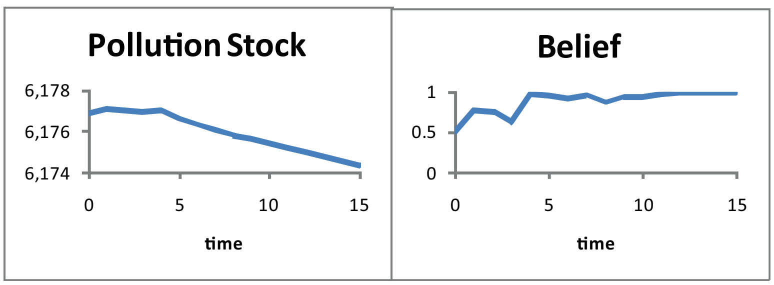

Figure 1: Evolution of the pollution stock and of the probability of a high marginal damage over time. $N = 10$, $s = 3$, $b = 500$, $\gamma = 10$, $\delta = 8\%$, $d_H = 10$, $d_L = 8$, $\beta = 0.85$, $\rho = 10000$. Belief is initialized at 0.5 and pollution stock at 6177.

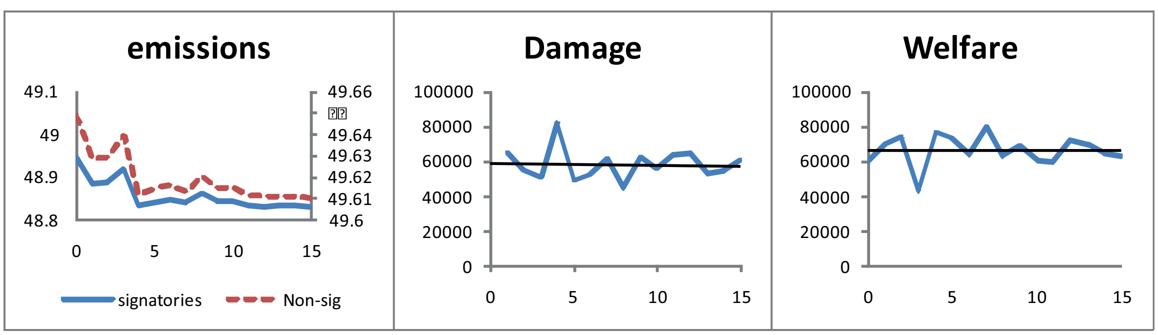

Figure 2: Evolution of emissions, damage and welfare over time. Parameters are as in Figure 1.

In this example, the true value of the damage parameter is d_H. Starting from an initial belief of 0.5, as time evolves, the value of π increases until uncertainty is revealed and the steady-state of $\pi = 1$ is attained. As players become more and more convinced that the value of the damage parameter is high, their emissions decrease over time, as well as the stock of pollution. In the left panel of Figure 2, notice that signatory countries (left vertical axis) always emit less than non-signatory countries (right vertical axis), because they internalize the stock externality. The right panel of Figure 2 represents the welfare of signatory countries. The welfare of non-signatories evolves in a similar way, because of the common damage, and it is always higher.

7.2 Equilibrium results

We now report on equilibrium results under uncertainty and learning for Model 1 (linear damage). Figure 3 shows the equilibrium emissions of signatory and non-signatory countries as a function of pollution stock and of the probability of high damage parameter. In this particular illustration, we use a relatively low value of ρ, so that learning is more valuable and has a greater impact on the players' strategies.

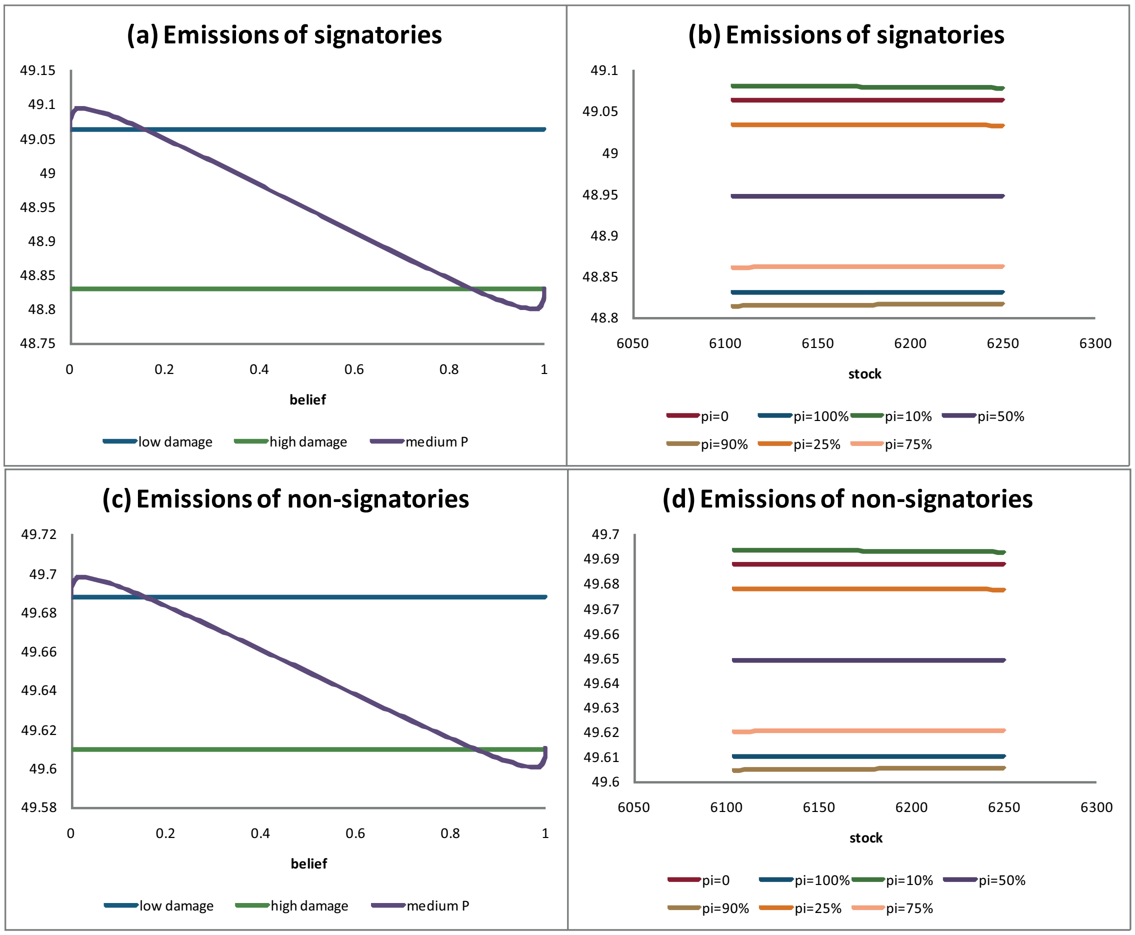

Figure 3: Equilibrium emission strategies. Parameter values are $N = 10$, $s = 3$, $b = 500$, $\gamma = 10$, $\delta = 8\%$, $d_H = 10$, $d_L = 8$, $\beta = 0.85$, $\rho = 3500$.

A first immediate result is that the strategies of both types of players have similar behaviors with respect to the pollution stock and belief (state variables), and that, as expected, signatory countries always emit less than non-signatories. Differently from the perfect information case, players' emissions are not constant in P, but are decreasing in P when π is small and increasing in P when π is large. Notice that equilibrium emissions can be higher than in the certain, low damage parameter case, or lower than in the certain, high damage case.

Figure 4 shows the equilibrium total discounted welfare of non-signatory countries. Welfare is almost linear decreasing in P and generally non-linear decreasing in π. Similar results hold for the signatories, except that they perform worst than non-signatories. Notice again that, for low values of π, the equilibrium welfare can be higher than what players achieve when the damage parameter is known to be low, and conversely, for high values of π, it can be lower than the equilibrium result in the certain case. The incentive for a signatory country to deviate from the cooperation strategy (computed as the difference in welfare) is almost constant in P and generally increasing with π (see Figure 5).

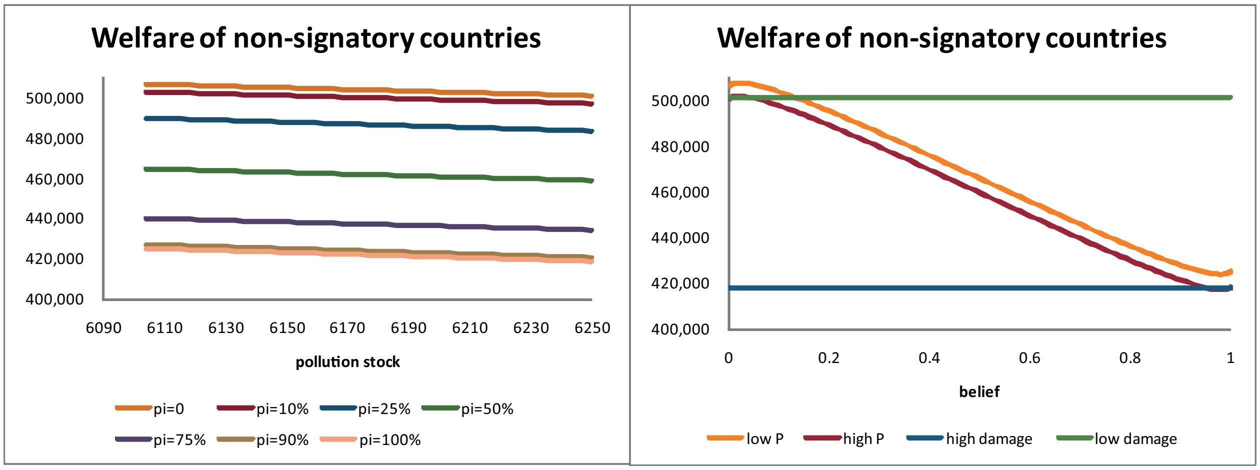

Figure 4: Welfare functions for non-signatory coutries. Parameters are as in Figure 3.

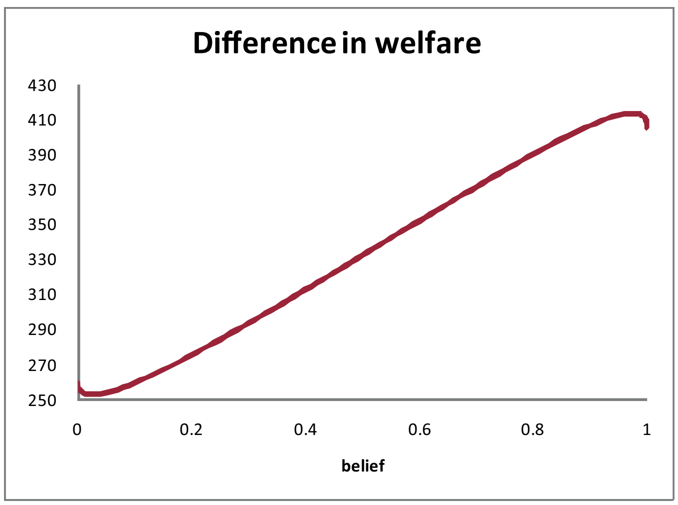

Figure 5: Incentive to deviate for signatory countries.

The shape of the equilibrium strategies and welfare functions are robust to the particular values of the parameters. Using these results, we now investigate the impact of uncertainty and learning on the environment and welfare.

Clearly, when the true value of the environmental damage parameter is low, emissions are in general lower under uncertainty; indeed, because countries are not sure about the true value of the parameter, they will be cautious and will emit less as long as the uncertainty persists. This is almost always the case, except when the probability of a high value of the damage parameter is very low, in which case players will in fact emit more than in the certain case (in the numerical illustration, this happens when $\pi \leq 0.15$). On the other hand, uncertainty has a negative impact on the environment when the true value of the environmental damage parameter is high, because players will emit more than in the certain case, except when the belief of a high value becomes very high (this happens when $\pi \geq 0.85$), in which case the players will emit less than in the certain case.

The impact of uncertainty and learning on welfare follows a similar pattern, that is, it can be positive or negative, depending on the true value of the uncertain parameter. Uncertainty is welfare worsening when the true environmental damage is low; instead it is welfare enhancing when the true environmental damage is high. Again, in the two regions where the belief is either very low or very high, the impact of uncertainty on welfare is not obvious. Countries are better-off when they believe that the probability of having a high damage parameter is low than when they know it is so.

A second issue of interest is the convenience in terms of emissions and welfare of accounting for the dynamics of the learning process. Figure 6 compares the equilibrium emission strategy of signatory countries using a myopic strategy. This consists in using a weighted average of the emissions corresponding to the two possible cases for the damage parameter. The two strategies coincide when the uncertainty is highest at $\pi = 0.5$, but otherwise they are different. We observe that the mixed strategy yields emissions that are lower than when players account for the learning process when $\pi < 0.5$, and higher when $\pi > 0.5$. The welfare implication is illustrated in Figure 7.

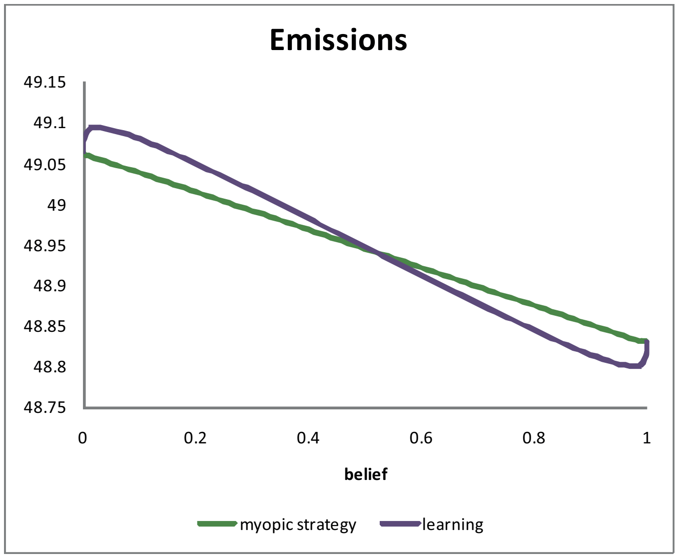

Figure 6: Comparison with myopic emission strategies (signatory countries).

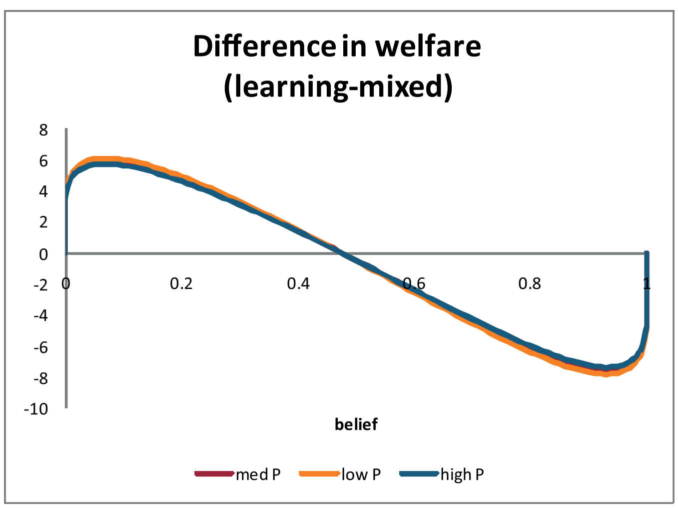

Figure 7: Welfare impact of accounting for the dynamics of learning.

The welfare difference is a non-linear function of the probability of a high environmental impact. Equilibrium welfares are higher when players account for the dynamics of the learning process only when $\pi < 0.5$. The difference in welfare is more pronounced for small values of the pollution stock.

8 Conclusion

In this paper we analyzed how uncertainty and learning about the true impact of the accumulated pollution on the environmental damage affect the emission decisions and welfare of strategically interacting countries when an IEA is in place. We assumed that both signatory and non-signatory countries are undergoing the same Bayesian learning process.

We investigated two main questions. The first one concerns the effects, in terms of emissions and welfare, of introducing uncertainty about the impact of the accumulated pollution on the environment. We found that under uncertainty, signatory and non-signatory countries show a similar behavior, with signatory countries always emitting less and performing worse in terms of welfare. We also found that acting under uncertainty can have both a positive or a negative impact on the environment and on countries' welfare, depending on the true state of nature. Our second objective was to evaluate whether or not there is any benefit in adopting a Bayesian learning process, both from an environmental and an economic point of view. This evaluation was made against the case where countries decide about their emissions by adopting a myopic strategy, which consists of a weighted average of the strategies from the two possible environmental outcomes. We found that accounting for the learning process can have either a positive or a negative impact on the environment and welfare, depending on the level of the belief in a high environmental impact.

This analysis was done by using numerical investigations under Model 1, that is, a linear damage function with a measurement noise that is independent of the level of the pollution stock. Qualitatively similar results are obtained under Model 3, that is, a quadratic damage function with independent measurement noise. The impact of learning is less important in Model 2, as the equilibrium solution is then very close to the mixed strategy.

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