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The Trade-Off of Quality Improvement in a Marketing Channel with Advertising- and Quality-Based Goodwill

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Abstract

We consider a marketing channel with a single manufacturer and a single retailer, where both advertising and quality improvement contribute to the build-up of goodwill. In a non-coop scenario, the retailer controls the price and the advertising while the manufacturer controls the quality. Although improving quality contributes positively to goodwill, it also increases the production cost, reducing the manufacturer's profit. In a coop scenario, the manufacturer supports the retailer's advertising while abandoning the quality improvement strategy. We investigate the conditions under which a coop program is beneficial when such a trade-off occurs. Our results show that a coop program is always successful if operational inefficiency is high. When the operational inefficiency and the quality effectiveness are both low, a coop program only benefits the retailer. In the ideal situation of high quality effectiveness and low operational inefficiency, both players prefer to play a non-coop game.

Key Words: Marketing channel, Differential game, Advertising, Quality improvement, Support program, Feedback equilibrium.

Résumé

On considère un canal Marketing avec un seul producteur et un seul distributeur et où la publicité et l'amélioration de la qualité contribuent au *Goodwill* ou à la survaleur. Dans un scénario non-coopératif, le distributeur contrôle le prix et la publicité alors que le producteur contrôle la qualité. Bien que l'amélioration de la qualité contribue positivement au *Goodwill*, elle augmente aussi les coûts de production et donc diminue le profit du fabricant. Dans un scénario coopératif, le producteur appuie la publicité du distributeur et abandonne sa stratégie d'amélioration de la qualité. Nous examinons les conditions sous lesquelles un programme coopératif est bénéfique quand un tel compromis aura lieu. Nos résultats montrent qu'un programme coopératif sera toujours prospère si l'inefficience opérationnelle est élevée. Quand l'inefficience opérationnelle et l'efficacité de la qualité sont faibles, un programme coopératif sera seulement à l'avantage du distributeur. Idéalement, dans une situation où l'efficacité de la qualité est élevée et l'inefficience opérationnelle est faible, les deux joueurs favoriseront un jeu non-coopératif.

Mots clés : Canal de distribution, jeu différentiel, publicité, qualité, programme coop, équilibres en rétroaction.

1 Introduction

The integration of marketing and operations management represents one of the main and recent research developments. Numerous contributions characterize the interfaces between different research domains, such as, quality and advertising (Nair and Narasimhan, 2006), pricing and operational knowledge (De Giovanni, 2009a), pricing and inventory (Jørgensen, 1986) and sales, production, pricing and inventory (Jørgensen et al., 1999). Developing strategic decisions across functions enhances the players' understanding of how to optimize their own payoffs, choose the best strategies and gain competitive advantage. The managers of firms require specific knowledge, skills and responsibilities in order to successfully manage the integration of business functions that are generally characterized by conflicting relationships and divergent objectives (De Giovanni, 2009b).

We contribute to this research domain by presenting a differential game of marketing and operations management. We characterize advertising, pricing and quality improvement strategies in a dynamic setting, where the demand depends on both price and goodwill. We confine our interest to a simple marketing channel with one single manufacturer, player M, and one single retailer, player R. The manufacturer controls the operational tool, that is, the rate of quality improvement. The retailer controls the marketing tools, that is, pricing and advertising. Both advertising and quality positively influence the goodwill dynamics. Nair and Narasimhan (2006) develop a similar state variable where both advertising and quality contribute to goodwill. One of the main assumptions of Nair and Narasimhan (2006) is that the production cost does not depend on quality. Nevertheless, numerous contributions in marketing and operations management have shown direct and increasing production costs due to quality improvement (Fine, 1986, 1988; Tapiero, 1989; Chand et al., 1995; Vörös, 2006). Investing in quality involves a set of operational challenges: new controls and standards, training, setup and trials, which negatively impact the unit profit margin due to increasing production costs (De Giovanni, 2009c).

Our model follows this last assumption. Quality improvement increases the demand through goodwill, but also increases the production cost, thus reducing the unit profit margin. We address the question of whether the manufacturer should abandon an operational tool – quality improvement – to embrace a marketing tool – support of advertising. Although an increase in quality increases the goodwill, it may also reduce the manufacturer's profit, depending on the operational inefficiency. We investigate the conditions in which a coop program is an attractive, alternative solution to face that trade-off. Such trade-offs between strategies are not new in marketing and operations management. For instance, Jørgensen et al. (2003) characterize a similar trade-off for (sales) promotion, which has a positive impact on demand but a negative impact on the goodwill dynamics. Similarly, Jørgensen and Zaccour (2003) introduce a channel with multiple-retailer promotions that have a positive effect on sales and a negative effect on brand image.

Two scenarios are analyzed. The first is played à la Nash, then the two firms choose their strategies non-cooperatively and simultaneously. In this case, operations management is only an upstream issue, while marketing remains a set of downstream decisions. Both research areas lack a coop interface. The second scenario is modelled à la Stackelberg and characterizes a coop program where the manufacturer supports the retailer's advertising effort and acts as the leader of the chain. Quality improvement and advertising support become substitutable strategies for the manufacturer, who adequately face the trade-off. We compare the strategies and the outcomes, taking the non-coop scenario as a benchmark and investigating the effectiveness of a coop program.

Firms develop coop programs to attain specific targets. For instance, Jørgensen et al. (2003) investigate the conditions under which a manufacturer is willing to support the retailer's advertising expenditures. Similarly, Karray and Zaccour (2006), Jørgensen et al. (2000, 2001, 2003) develop numerous coop programs in marketing to evaluate the benefits obtainable through collaboration. If it is well known that coop programs

may be Pareto-improving, Jørgensen et al. (2003) demonstrate that this collaboration is only beneficial under specific conditions. The authors characterize a coop program that is beneficial only when the brand image is low or the negative effect of promotion is not too damaging. In this sense, this paper addresses the following research questions:

- 1. How does the manufacturer face the trade-off of quality improvement in the non-coop scenario?
- 2. Under which conditions is a coop program beneficial for both players?
- 3. How does a coop program influence the players' strategies?

In order to answer to these research questions, the paper is organized as the follows. In Section 2 we describe the differential game model and in Section 3 we characterize the Nash equilibria in a coop program, as well as the Stackelberg equilibria in a non-coop game. Section 4 compares both strategies and their outcomes and Section 5 is comprised of the concluding remarks.

2 The model

Let a conventional marketing channel be formed of one manufacturer player M and one retailer player R. Suppose that the manufacturer controls the quality improvement rate, d(t), while the retailer controls the price p(t) as well as the advertising rate, A(t).

Efforts in both advertising and the quality improvement enhance the goodwill. This is the main objective of the players who wish to increase the demand and then the profits by adopting both marketing and operational strategies. In a dynamic framework, the goodwill may be investigated by mean of the following dynamic equation:

$$\dot{G} = A(t) + \gamma d(t) - \delta G(t), \quad G(0) = G_0 \ge 0 \tag{1}$$

where $\delta > 0$ is the decay rate or forgetting effects of goodwill. $\gamma > 0$ represents the marginal contribution of quality improvement on goodwill. The players' strategies increase the stock of goodwill. Nevertheless, the manufacturer faces the negative effect of quality improvement on his production cost, whose function assumes the following form:

$$Q(d(t)) = cd(t) \tag{2}$$

The production cost is an increasing function of quality improvement, so any increase in quality implies a higher production cost. This function was developed by Vörös (2006) as well as by Fine (1988) who modelled a production cost increasing in the quality improvement. We disregard the fixed unit cost of production that was used by Fine (1988) and Tapiero (1989), since it would only represent a parameter that would not provide any additional knowledge. The manufacturer sells the products to the retailer at the wholesale price, $\omega > 0$. The unitary profit margin, $\pi(d(t))$, decreases in the quality improvement and it is given by $\pi(d(t)) = \omega - cd(t) > 0$. When the manufacturer does not invest in quality improvement, the unit profit margin coincides with the wholesale price. In this sense, the manufacturer faces a trade-off when choosing the quality improvement rate. High (low) quality improvement efforts increase (decrease) the goodwill (and consequently also both demand and profits) in (1) as well as the production cost in (2). Quality improvement plays a positive role in goodwill and a negative role in the unit profit margin, thus the manufacturer maximizes his pay-off facing this trade-off.

Customer demand depends on price and goodwill and it is given by:

$$D(p(t), G(t)) = \alpha - \beta p(t) + \theta G(t)$$
(3)

where $\alpha > 0$ represents the market potential and $\beta > 0$ and $\theta > 0$ represent the effects on current sales of pricing and goodwill, respectively. According to (3), the retailer controls the demand directly by means

of the price while both players influence it indirectly through quality improvement and advertising. In the non-coop scenario, the manufacturer is concerned only with the operational issues of the channel, while the retailer controls the marketing tools exclusively. In coop scenarios, the manufacturer supports the retailer's advertising effort and evaluates the shift from an operational to a marketing strategy.

Advertising and quality improvement costs may be represented by means of a convex function taking the following quadratic form:

$$C(A(t)) = \frac{u_A A(t)^2}{2}$$
 $C(d(t)) = \frac{u_d d(t)^2}{2}$ (4)

where u_A and u_d are positive cost parameters. To save notations, we suppose that both parameters take value 1. Denote by B(t) the manufacturer's support rate. It represents the amount that the manufacturer contributes to the retailer's advertising efforts and its value exists in the interval [0,1]. Assuming an infinite time horizon and a positive discount rate ρ in a non-coop program, the manufacturer's objective functional is:

$$J_M = \int_0^\infty e^{-\rho t} \left\{ \left(\alpha - \beta p(t) + \theta G(t)\right) \left(\omega - cd(t)\right) - \frac{1}{2}d(t)^2 \right\} dt \tag{5}$$

and the retailer's objective functional is:

$$J_R = \int_0^\infty e^{-\rho t} \left\{ \left(\alpha - \beta p(t) + \theta G(t)\right) \left(p(t) - \omega\right) - \frac{1}{2} A(t)^2 \right\} dt \tag{6}$$

where $\omega > 0$ is the wholesale price and it is assumed to be constant. Through (1), (5), and (6) we have defined a two-player differential game with three controls $A(t) \geq 0$, $d(t) \geq 0$, and $p(t) \geq 0$, and one state $G(t) \geq 0$. In the coop scenario, there is one additional control variable, $B(t) \geq 0$, related to the support. From now on the time argument is omitted.

3 Equilibria

We start by analyzing the first scenario in which the players implement a non-coop program. We use the subscripts N to signify "Non-coop scenario" that is played à la Nash. The players do not coordinate their strategies; thus B(t)=0. The second scenario characterizes a coop game. We use the subscript S to signify "Coop scenario", where the game is played à la Stackelberg; the manufacturer supports the retailer's advertising efforts and acts as the leader of the marketing channel. In this case, $1 \ge B(t) > 0$.

3.1 A non-coop scenario

In this scenario, the manufacturer decides the quality improvement rate and the retailer controls both advertising and price. Both players act simultaneously and independently, therefore the game is played à la Nash. The main research question to be addressed is how the manufacturer manages the trade-off of quality improvement in absence of a coop program.

Let V_M^N and V_R^N denote the players' value functions, the Hamiltonian-Jacobi-Bellman (HJB) equations are:

$$\rho V_M^N(G) = \max_{d \ge 0} \left\{ \left(\alpha - \beta p^N + \theta G^N \right) \left(\omega - c d^N \right) - \frac{1}{2} d^{N^2} + V_M^{N'} \left(A^N + \gamma d^N - \delta G^N \right) \right\}$$
 (7)

$$\rho V_R^N(G) = \max_{p \ge 0, A \ge 0} \left\{ \left(\alpha - \beta p^N + \theta G^N \right) \left(p^N - \omega \right) - \frac{1}{2} A^{N^2} + V_R^{N'} \left(A^N + \gamma d^N - \delta G^N \right) \right\}$$
(8)

Proposition 1 characterizes the equilibrium strategies.

Proposition 1 The equilibrium price, advertising and quality improvement are given by:

$$p^{N} = \frac{\alpha + \theta G^{N} + \beta \omega}{2\beta} \tag{9}$$

$$A^{N} = \begin{cases} \varsigma_{1}G^{N} + \varsigma_{2} & if \ G^{N} > -\frac{\varsigma_{2}}{\varsigma_{1}} \\ 0 & otherwise \end{cases}$$
 (10)

$$A^{N} = \begin{cases} \varsigma_{1}G^{N} + \varsigma_{2} & \text{if } G^{N} > -\frac{\varsigma_{2}}{\varsigma_{1}} \\ 0 & \text{otherwise} \end{cases}$$

$$d^{N} = \begin{cases} \frac{(2\varphi_{1}\gamma - c\theta)G^{N} + 2\varphi_{2}\gamma - c(\alpha - \beta\omega)}{2} & \text{if } G > \frac{c(\alpha - \beta\omega) - 2\varphi_{2}\gamma}{2\varphi_{1}\gamma - c\theta} \\ 0 & \text{otherwise} \end{cases}$$

$$(10)$$

where the parameters φ_1 , φ_2 , ς_1 and ς_2 satisfy $V_M^N(G)$ and $V_R^N(G)$.

Proof. See Appendix A.

The interpretation of the pricing strategy is straightforward. Indeed, optimal pricing is determined by the market potential, α , by both wholesale price, ω , and goodwill, as well as by both sensitivity parameters of price and goodwill, β and θ , respectively. Increasing the level of goodwill implies an increasing price, which is a result that is already known from the literature (e.g., Zaccour, 2008; Taboubi and Zaccour, 2002).

However, we need a thorough understanding of the signs of the identified parameters for an exhaustive interpretation of the other strategies. Unfortunately, this information cannot be obtained analytically since the Ricatti's system of equations is non-linear. Therefore, we use a numerical illustration in Section 4 to check for (10)–(11), to test the robustness of the results, as well as to compare the strategies.

(10) requires that ζ_1 and ζ_2 have opposite signs. This guarantees that (1) is satisfied and advertising is positive. Moreover, the stability requires that quality improvement is positive, therefore we need to satisfy the following condition:

$$G^{N} > \frac{c(\alpha - \beta\omega) - 2\varphi_{2}\gamma}{2\varphi_{1}\gamma - c\theta}.$$
(12)

Substituting for advertising and quality improvement into the goodwill dynamics leads to:

$$\dot{G}^{N} = 2\left(\varphi_{2}\gamma^{2} + \varsigma_{2}\right) - c\gamma\left(\alpha - \beta\omega\right) + \left[2\left(\varphi_{1}\gamma^{2} + \varsigma_{1} - \delta\right) - c\theta\gamma\right]G^{N}$$
(13)

Where the steady state, G_{SS}^N , is given by:

$$G_{SS}^{N} = \frac{c\gamma (\alpha - \beta \omega) - 2(\varphi_{2}\gamma^{2} + \varsigma_{2})}{2(\varphi_{1}\gamma^{2} + \varsigma_{1} - \delta) - c\theta\gamma}$$

$$(14)$$

We numerically verify all assumptions for $G_{SS}^N>0$ in Section 4.

3.2A coop program with the manufacturer as leader

In this game, the manufacturer behaves as the leader of the channel and supports the retailer's advertising effort. The manufacturer wishes to increase the profitability of the channel by increasing the stock of goodwill. Supporting the retailer's advertising represents a suitable, alternative strategy to increase the demand and the profits avoiding any kind of trade-off. Supporting advertising and improving quality become substitutable strategies for the manufacturer. He may prefer the substitution of an operational tool with a marketing tool to increase the goodwill. From an economic perspective, the manufacturer avoids an increasing production cost. If this strategy appears suitable for the manufacturer, the retailer has to evaluate the convenience of a coop program, which will affect the price as well as the advertising strategies. Moreover, the retailer's profits are not influenced negatively when the manufacturer increases the quality, but decreasing quality improvement decreases the stock of goodwill. Therefore, the research questions to be addressed in this game are:

- Is this coop program beneficial for the retailer?
- How does a coop program affect the strategies?
- Under which condition are both players better off, compared with the non-coop scenario?

Let V_M^S and V_R^S denote the players' value functions when the manufacturer is the leader; the HJB equations

$$\rho V_{M}^{S}(G) = \max_{d \ge 0, B \ge 0} \left\{ \left(\alpha - \beta p^{S} + \theta G^{S} \right) \left(\omega - c d^{S} \right) - \frac{1}{2} d^{S^{2}} - \frac{B^{S}}{2} A^{S^{2}} + V_{M}^{S'} \left(A^{S} + \gamma d^{S} - \delta G^{S} \right) \right\}$$
(15)

$$\rho V_{R}^{S}(G) = \max_{p \ge 0, A \ge 0} \left\{ \left(\alpha - \beta p^{S} + \theta G^{S} \right) \left(p^{S} - \omega \right) - \frac{(1 - B^{S})}{2} A^{S^{2}} + V_{R}^{S'} \left(A^{S} + \gamma d^{S} - \delta G^{S} \right) \right\}$$
(16)

where $1 \ge B^S > 0$ is the manufacturer's support rate. Proposition 2 characterizes the equilibrium strategies.

Proposition 2 The equilibrium price, advertising, quality improvement and the manufacturer's support rate are given by:

$$p^{S} = \frac{\alpha + \theta G^{S} + \omega \beta}{2\beta} \tag{17}$$

$$A^{S} = \begin{cases} \frac{(2\sigma_{1} + \mu_{1})G^{S} + 2\sigma_{2} + \mu_{2}}{2} & \text{if } G^{S} > -\frac{2\sigma_{2} + \mu_{2}}{2\sigma_{1} + \mu_{1}} \\ 0 & \text{otherwise} \end{cases}$$
 (18)

$$d^{S} = \begin{cases} \frac{(2\sigma_{1}\gamma - c\theta)G^{S} + 2\sigma_{2}\gamma - c(\alpha - \omega\beta)}{2} & \text{if } G^{S} > \frac{c(\alpha - \omega\beta) - 2\sigma_{2}\gamma}{2\sigma_{1}\gamma - c\theta} \\ 0 & \text{otherwise} \end{cases}$$
(19)

$$d^{S} = \begin{cases} \frac{(2\sigma_{1}\gamma - c\theta)G^{S} + 2\sigma_{2}\gamma - c(\alpha - \omega\beta)}{2} & \text{if } G^{S} > \frac{c(\alpha - \omega\beta) - 2\sigma_{2}\gamma}{2\sigma_{1}\gamma - c\theta} \\ 0 & \text{otherwise} \end{cases}$$

$$B^{S} = \begin{cases} \frac{(2\sigma_{1} - \mu_{1})G^{S} + 2\sigma_{2} - \mu_{2}}{(2\sigma_{1} + \mu_{1})G^{S} + 2\sigma_{2} + \mu_{2}}} & \forall G^{S} > 0 : 1 \ge B^{S} > 0 \\ 0 & \text{otherwise} \end{cases}$$

$$(19)$$

Just as in the non-coop scenario, the price depends on the standard assumptions linking the price to demand in terms of customer sensitivity, wholesale price and market potential. The level of goodwill influences the price. An increasing dynamic also implies an increasing price in the Stackelberg formulation. Investigating the dynamic of goodwill as well as the quality improvement and the support rate requires a full understanding of the sign of the identified parameters. Opposite signs for the numerator and the denominator of (18) assure that the results of advertising are positive. Goodwill plays a central role in all of the strategies and its positive value assures that (19)–(20) are satisfied. Substituting for advertising and quality improvement into the goodwill dynamics leads to:

$$\dot{G}^{S} = \frac{\left[2\sigma_{1}\left(1+\gamma^{2}\right)-2\delta+\mu_{1}-c\theta\gamma\right]G^{S}+2\sigma_{2}\left(1+\gamma^{2}\right)-c\gamma\left(\alpha-\omega\beta\right)+\mu_{2}}{2}$$
(21)

where the steady state, G_{SS}^{S} , is given by:

$$G_{SS}^{S} = \frac{c\gamma \left(\alpha - \omega\beta\right) - 2\sigma_2 \left(1 + \gamma^2\right) - \mu_2}{2\sigma_1 \left(1 + \gamma^2\right) - 2\delta + \mu_1 - c\theta\gamma}$$
(22)

The system of equations of the identified parameters has a non-linear form, therefore we check all assumptions numerically for $G_{SS}^S>0$ and stability in Section 4.

4 Numerical analysis

Variations in the steady-state values

Unfortunately, the system of parameters characterizing each game is non-linear. The resulting identifications reported in the Appendix show recursive parameters therefore each of the Ricatti's systems need to be solved

numerically. Using Maple 10, we are able to compare the strategies and the outcomes of the games, while confining our interest to the long-term behaviour of the control and the state variables, the demand and the players' profits, that is, the values of the different variables are at the steady state.

Our main research question relates to the conditions in which a coop program helps a manufacturer to face the trade-off of quality improvement and whether it is beneficial for both players. The marginal production cost (c) and the marginal contributions of quality improvement to goodwill (γ) represent the key parameters to be investigated when comparing the scenarios. The players' decisions as well as the outcomes of the game, in fact, are influenced by the amplitude of both parameters. We run a simulation, setting the following parameters for the benchmark case:

Demand parameters: $\alpha = 1, \beta = 0.8, \theta = 0.5$

Operational parameters: $\omega = 1, c = 0.7$ Goodwill parameters: $\gamma = 0.7, \delta = 0.2$

Dynamic parameters: $\rho = 0.2$.

These parameters have been selected according to previous studies in marketing and operations management in feedback form, such as Amrouche et al. (2008), Nain and Narasimhan (2006), De Giovanni (2009a). They allow for an exhaustive illustration and comparison of strategies and outcomes. Nevertheless, the solution obtained is only one of the possible solutions of the Ricatti's systems. As the equations are not linear, several solutions of these systems can exist. In order to provide the most robust possible result, we carry out sensitivity analysis on each parameter, which is reported in Table 1. The purpose of this analysis is twofold. On one hand, we check that all the assumptions reported in (10)–(11) as well as in (18)–(20) are satisfied. This represents a required condition for running any further comparisons and illustrations. On the other hand, this analysis does not serve to compare the strategies and the payoffs (which are done later), but rather to show the variations in the steady state of each variable when varying a given parameter. The invariant sign of each variable at the steady state provides an indication of a robust analysis, thus reinforcing our research findings. A similar analysis has recently been developed by Amrouche et al. (2008). We investigate the steady-state values of the control and state variables as well as profits and demand.

For each associated parameter-variable, two signs are reported in Table 1. The first sign (the one outside the brackets) should be interpreted as a variation in the steady-state value of one variable when changing a given parameter. For instance, a positive variation in the values of α from 1 to 1.3 implies a positive variation of the steady state of the goodwill. When a parameter is varied, all the others remain at the benchmark values. The second sign (the one inside of the brackets), indicates the sign of the variable when varying a given parameter. Positive values of control and state variables satisfy the conditions of stability in (10)–(11) and (18)–(20).

Table 1: Variation in the steady state values

	G'	SS	p^{S}	SS	d^{S}	SS	A'	SS	B^{SS}	D'	SS	V_{Λ}^{S}	8 S 1	V_{I}	SS R
	N	S	N	S	N	S	N	S	S	N	S	N	S	N	S
$\beta = \beta(1 - 1.3)$	-(+)	-(+)	-(+)	-(+)	+(+)	+(+)	+(+)	+(+)	+(+)	-(+)	-(+)	-(+)	-(+)	-(+)	-(+)
c = c(.58)	-(+)	-(+)	-(+)	-(+)	-(+)	-(+)	-(+)	-(+)	+(+)	-(+)	-(+)	-(+)	-(+)	-(+)	-(+)
$\gamma = \gamma(1 - 1.3)$	+(+)	+(+)	+(+)	+(+)	+(+)	+(+)	+(+)	+(+)	-(+)	+(+)	+(+)	+(+)	+(+)	+(+)	+(+)
$\theta = \theta(.46)$	+(+)	+(+)	+(+)	+(+)	-(+)	-(+)	-(+)	-(+)	-(+)	+(+)	+(+)	+(+)	+(+)	+(+)	+(+)

In the same manner as Amrouche et al. (2008), we screen the parameters to be used in the sensitivity analysis. Precisely, we analyze only the parameters that imply a change in control and state variables (Amrouche et al., 2008). The following comments relate to Table 1:

• An increasing sensitivity to price, β , lowers both the steady-state values of demand and players' profits. This is quite conceivable. From there, the chain undergoes the effect of a lower price and goodwill and a higher advertising and quality improvement. When implementing a coop program, the manufacturer gives more support to reduce the negative effect on demand of a high customers' price-sensitivity.

- An increasing production cost, c, has a negative effect for the entire chain. Advertising and quality improvement decreases, and hence, a lower goodwill and demand imply lower profits. The manufacturer decreases his investments in quality improvement. As the retailer knows that with a high production cost, the manufacturer will decreases his efforts in improving quality, she lowers her advertising investment. The retailer does not want to contribute alone to goodwill. However, she tries to increase demand by reducing the price. In a coop scenario, the manufacturer increases his support, hoping that the retailer will increase her advertising.
- An increasing marginal contribution of goodwill, γ , has a positive impact on the steady-state values of both demand and profits. The manufacturer invests more in quality improvements as the benefits obtainable by increasing goodwill are higher than the losses due to his increasing production costs. When γ assumes a high value, the retailer knows that the manufacturer will increase his quality effort. Therefore, she also increases advertising. Consequently to an increasing goodwill, the retailer is also able to increase the price without damaging neither the demand nor the players' profits. In a coop scenario, the manufacturer's willingness to support decreases as he is able to succeed also without implementing any coop program.
- An increasing marginal contribution of goodwill to demand, θ , leads to increasing demand and profits at the steady-state. Both players reduce their investments since small investments in quality and advertising increase the steady-state value of demand. The manufacturer can invest less in quality improvements to save costs from an operational point of view, while the retailer can decrease his advertising and increase the price without damaging the demand and the profits. Increasing values of θ decrease the manufacturer's support rate. The manufacturer does not wish to cooperate when the impact of the goodwill is high because this may benefit the retailer too much.

4.2 Comparison between scenarios

In this section, we compare the strategies and the outcomes of a coop and a non-coop scenario to provide exhaustive answers to our research questions. We compare the control and the state variables by varying both c and γ . Precisely, both parameters assume values 0.3 and 0.9, which represent the cases of low and high operational inefficiency, and low and high quality effectiveness. All of the illustrations are reported in Figure 1 and represent four possible cases.

Case 1. Low production cost, low contribution of quality. Goodwill is equal in both the coop and the non-coop scenarios, therefore it does not depend on the program. In terms of strategies, pricing is similar in both coop and non-coop scenarios while advertising and quality strategies vary. A coop program leads to a higher level of advertising. The retailer increases the advertising and the manufacturer abandons the quality improvement strategy to support more. Although the production cost is low, the quality effectiveness is only marginal. Thus, the manufacturer shifts from a quality to an advertising strategy in a coop scenario, lowering his quality investment and offering intermediate support. However, the interface between marketing and operational strategies benefits only the retailer. The manufacturer's profits, in fact, are equal in both scenarios. The results of coop and non-coop programs are the same for the manufacturer. This result may be due to the high support that he provides and to the low contribution to goodwill of quality. Although the advertising increases, the demand remains constant as both pricing and goodwill are similar in both coop and non-coop scenarios. The implementation of a coop program appears to be an operational tool that only increases the internal efficiency of the channel and only benefits the retailer.

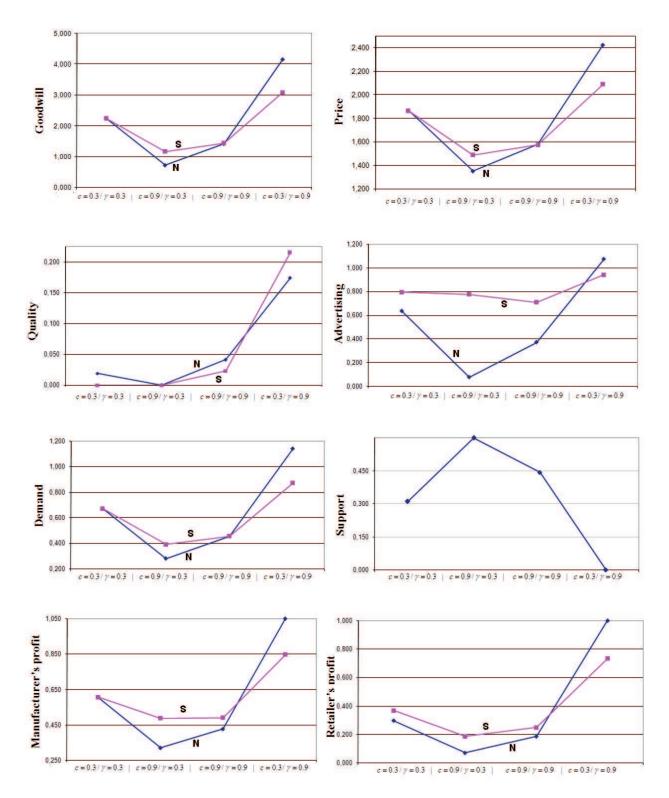


Figure 1: Numerical illustrations

Case 2. High production cost, low contribution of quality. Under these conditions, both players prefer a coop program. In the non-coop scenario, in fact, the manufacturer's quality effort is null. The retailer knows that the manufacturer is no longer willing to invest when those conditions occur. As a consequence, she decreases her advertising efforts and combines the marketing strategies (pricing and advertising) consequently. Since the production cost is high, while the quality effectiveness is low, the manufacturer is willing to implement a coop program. He does not invest at all in quality also under coop scenario while providing the highest possible support. The retailer increases her advertising while the manufacturer supports her strategy. The retailer increases the price without hurting the demand since her contribution to goodwill overcomes the negative effect of an increasing price. Marketing and operations interface to improve the total profits of the channel. The marketing tools enhance the demand, while abandoning the quality strategy is the operational challenge.

Case 3. High production cost, high contribution of quality. Coop and non-coop scenarios lead to the same level of goodwill. In terms of strategies, the retailer is indifferent when deciding the price in both scenarios. Consequently, the demand remains unchanged. However, the other strategies and the profits vary. In this case, the interface between marketing and operations represents only an internal issue in the channel. Both players' profits increase under collaboration although the demand does not depend on the coop program. The manufacturer saves production costs when supporting the advertising, while the retailer benefits from the other player's contribution. Thus, a coop program represents an operational issue rather than a marketing issue.

Case 4. Low production cost, high contribution of quality. This is the ideal case for the manufacturer. Under these conditions, in fact, the strategy of quality improvement is more successful than the support of advertising. Goodwill in a non-coop scenario is higher than in a coop scenario, mainly due to a higher level of advertising and to increasing quality. The latter is higher than it was in all previous cases. In a coop scenario, the manufacturer prefers to invest in quality rather than support. Finally, both players are better-off in a non-coop scenario. This is due to the high internal efficiency of the channel. A coop program represents neither a marketing nor an operational tool. That is, it increases neither the demand nor the players' profits.

We now use the results of the previous cases to modify the subscripts and provide some claims. For instance, we use S2 to signify "Coop scenario, case 2".

Claim 1 The manufacturer only prefers the implementation of a coop program for a high level of operational inefficiency. This results in:

$$V_M^{N4} > V_M^{S1} = V_M^{N1} > V_M^{S3} > V_M^{S2}$$

The manufacturer is willing to cooperate only when his production cost, due to quality improvement, is high. In this case, he prefers to shift from an operational to a marketing strategy. His ideal situation is high γ and low c where he prefers a non-coop program. However, when both of these two parameters are low, he prefers to play a non-coop game. A coop program, in fact, does not provide him with any benefits in terms of demand, profits and goodwill, but increases only the retailer's profits.

Claim 2 The retailer always prefers the implementation of a coop program except the case of high quality effectiveness and low operational inefficiency.

$$V_R^{N4} > V_R^{S1} > V_R^{S3} > V_R^{S2}$$

In the ideal situation of high γ and low c, the retailer prefers a non-coop program. Since the manufacturer's support is null, increasing advertising results in lower profits. In all other cases, a coop program translates into increasing efficiency of the channel and improved outcomes for the players.

Claim 3 A coop program is always Pareto-improving for high levels of operational inefficiency and independent of quality effectiveness.

$$V_i^{S2} > V_i^{N2}$$

$$V_i^{S3} > V_i^{N3}$$

with i = M, R.

When implementing a coop program under high level of operational inefficiency, both players are better-off, independent of quality effectiveness. Creating efficiency along the channel represents their main challenge.

Claim 4 The implementation of a coop program is driven only by operational rather than by operational and marketing motivations for equal level of operational efficiency and quality effectiveness.

$$G_i^{S1} = G_i^{N1}; D_i^{S1} = D_i^{N1}; p_i^{S1} = p_i^{N1}$$

$$G_i^{S3} = G_i^{N3}; D_i^{S3} = D_i^{N3}; p_i^{S3} = p_i^{N3}$$

with i = M, R.

When both γ and c are equal, a coop program creates efficiency along the chain by saving costs. Demand, goodwill, and price are equal in both coop and non-coop scenarios. Under those conditions, a coop program is an operational rather than a marketing tool.

5 Conclusion

This paper identifies the conditions under which a coop program is beneficial in a marketing channel in which demand depends on goodwill and price; both quality improvement and advertising contribute to goodwill, while the quality efforts enhance a critical decisional trade-off. Although managers have many valuable reasons to implement a quality improvement strategy – for instance, increasing goodwill – this strategy negatively impacts the production costs and reduces the manufacturer's profit. Increasing quality enhances goodwill and then demand, but also increases the production cost at the same time. We addressed this trade-off in a marketing channel with one manufacturer and one retailer, where the manufacturer controls the quality improvement and the retailer controls both price and advertising. In a coop scenario, the manufacturer may face that trade-off by supporting the retailer's adverting rather than by investing in quality improvements. In practical terms, we investigated the conditions under which a manufacturer is willing to shift from an operational (quality improvement) strategy to a marketing strategy (advertising support).

Our results – summarized in Table 2 – show that a coop program makes both players better-off, depending on production efficiency and quality effectiveness.

Table 2: Players' payoffs

		Operational	Operational inefficiency			
		Low	High			
	Low	$V_M^N=V_M^S$	$V_M^S > V_M^N$			
Quality	Low	$V_R^S > V_R^N$	$V_R^S > V_R^N$			
effectiveness	High	$V_M^N > V_M^S$	$V_M^S > V_M^N$			
	mgn	$V_R^N > V_R^S$	$V_R^S > V_R^N$			

Our findings demonstrate that independently from the contribution of quality improvement to goodwill, whenever the production is characterized by high operational inefficiency – high production cost due to quality improvement – both players prefer the implementation of a coop program. Operational tools are abandoned and marketing tools become successful alternative strategies. Shifting from a non-coop to a coop scenario is always Pareto-optimal for high levels of operational inefficiency, and independently from the quality effectiveness. In the ideal situation of low operational inefficiency and high quality effectiveness, both players are no longer willing to implement a coop program. In this sense, the use of both marketing and operational tools improves the outcomes of the channel. Finally, when the efficiency is high and the contribution of quality is low, the retailer still prefers a coop program while the manufacturer is indifferent. He does not have any incentive for a coop program that only increases the other player's profit. When shifting from an operational to a marketing support strategy, the implementation of a coop program benefits only the downstream player.

Despite the importance of the findings and the managerial implications, our results indeed present some limitations related to the simplifying assumptions in the model. Furthermore, the solution of the Ricatti's system is only one of the possible solutions available. It would be interesting in future research to integrate other strategies beyond pricing, advertising, and quality improvement, such as product development, service, green investments, or some other quality features, such as conformance quality and durability. Further, several contract schemes, such as reverse revenue sharing contract, could be considered when coordinating players' strategies. Including competition or considering other players in the up- and down-stream of the channel could represent a further avenue to explore. Finally, it would be interesting to empirically test our results with a case study and qualitative research.

Appendix

Proof of Proposition 1. We need to establish the existence of bounded and continuously differentiable value functions $V_M^N(G)$, $V_R^N(G)$ such that there exists a unique solution G(t) to (1) and the HJB equations. To obtain an equilibrium à la Nash we first determine the players' necessary conditions from the HJBs:

$$\rho V_M^N(G) = \max_{d \ge 0} \left\{ \left(\alpha - \beta p^N + \theta G^N \right) \left(\omega - c d^N \right) - \frac{1}{2} d^{N^2} + V_M^{N'} \left(A^N + \gamma d^N - \delta G^N \right) \right\}$$
 (23)

$$\rho V_R^N(G) = \max_{p \ge 0, A \ge 0} \left\{ \left(\alpha - \beta p^N + \theta G^N \right) \left(p^N - \omega \right) - \frac{1}{2} A^{N^2} + V_R^{N'} \left(A^N + \gamma d^N - \delta G^N \right) \right\}$$
(24)

where the necessary conditions are:

$$p = \frac{\alpha + \theta G + \beta \omega}{2\beta} \tag{25}$$

$$d^{N} = \frac{2\gamma V_{M}^{N'} - c\left(\alpha + \theta G - \beta \omega\right)}{2} \tag{26}$$

$$A = V_R^{N'} \tag{27}$$

Inserting (25)–(27) inside the HJB we obtain

$$\left(\alpha + \theta G^N - \beta \omega\right) \left[4\omega - 4c\gamma V_M^{N'} + c^2 \left(\alpha + \theta G^N - \beta \omega\right)\right] + 4V_M^{N'} \left(2V_R^{N'} + \gamma^2 V_M^{N'} - 2\delta G^N\right) - 8\rho V_M^N(G) = 0 \quad (28)$$

$$\left(\alpha + \theta G^N - \beta \omega\right)^2 + 2\beta V_R^{N'} \left(V_R^{N'} + 2\gamma^2 V_M^{N'} - c\gamma \left(\alpha + \theta G^N - \beta \omega\right) - 2\delta G^N\right) - 4\beta \rho V_R^N(G) = 0$$
 (29)

We may satisfy (28) and (29) by conjecturing quadratic value functions. We define $V_M^N(G) = \frac{\varphi_1}{2}G^2 + \varphi_2G + \varphi_3$ and $V_R^N(G) = \frac{\varsigma_1}{2}G^2 + \varsigma_2G + \varsigma_3$ where φ_1 , φ_2 , φ_3 , ς_1 , ς_2 and ς_3 are constant parameters. By inserting V_M^N and V_R^N and their derivatives into (28) and (29), we obtain the following six algebraic Ricatti equations,

$$4\varphi_1 \left(2\varsigma_1 + \gamma^2 \varphi_1\right) - c\theta \left(4\gamma \varphi_1 - c\theta\right) - 4a_1 \varphi_1 = 0 \tag{30}$$

$$\theta \left[2\omega - c \left(2\gamma \varphi_2 - a_3 \right) \right] - 2\gamma \varphi_1 a_3 + 2\varphi_1 \left(2\varsigma_2 + \gamma^2 \varphi_2 \right) + 2\varphi_2 \left(2\varsigma_1 + \gamma^2 \varphi_1 \right) - 4a_2 \varphi_2 = 0 \tag{31}$$

$$a_4 \left[4\omega - c \left(4\gamma \varphi_2 - a_3 \right) \right] + 4\varphi_2 \left(2\varsigma_2 + \gamma^2 \varphi_2 \right) - 8\rho \varphi_3 = 0 \tag{32}$$

$$\theta^2 + 2\beta\varsigma_1\left(\varsigma_1 + 2\gamma^2\varphi_1 - c\gamma\theta\right) - 2\beta a_1\varsigma_1 = 0 \tag{33}$$

$$a_4\theta + \beta\varsigma_1\left(\varsigma_2 + 2\gamma^2\varphi_2 - a_3\gamma\right) + \beta\varsigma_2\left(\varsigma_1 + 2\gamma^2\varphi_1 - c\gamma\theta\right) - 2\beta a_2\varsigma_2 = 0 \tag{34}$$

$$a_4^2 + 2\beta \varsigma_2 \left(\varsigma_2 + 2\gamma^2 \varphi_2 - a_3 \gamma\right) - 4\beta \rho \varsigma_3 = 0 \tag{35}$$

where the first three correspond to the manufacturer, whereas the second three correspond to the retailer, while we use the constant terms:

$$a_1 = \rho + 2\delta, a_2 = \rho + \delta, a_3 = c(\alpha - \omega\beta), a_4 = \alpha - \omega\beta.$$

We present the description of the procedure used to reduce the solution of that system into the solution of a system of one non-linear equation. The latter equation has been numerically solved by using Maple 10.

From (30), we can obtain ς_1 as a function of φ_1 : $\varsigma_1 = f(\varphi_1)$

where

$$f(\varphi_1) = \frac{c\theta \left(4\gamma\varphi_1 - c\theta\right) + 4a_1 - 4\gamma^2\varphi_1^2}{8\varphi_1} = \Omega_1 \tag{36}$$

Replacing (36) for (31) and (34), we can obtain both φ_2 and ς_2 as a function of φ_1 :

$$\varphi_2 = f(\varphi_1) = \frac{\Omega_2 \left(\Omega_3 \Omega_5 - 8\beta \Omega_1 \gamma^2 \varphi_1\right) - 4\varphi_1 \left(\Omega_3 \Omega_4 - 2\beta \Omega_1 \gamma^2 \Omega_2\right)}{\Omega_3 \left(\Omega_3 \Omega_5 - 8\beta \Omega_1 \gamma^2 \varphi_1\right)} = \Omega_6 \tag{37}$$

$$\varsigma_2 = f(\varphi_1) = \frac{\Omega_3 \Omega_4 - 2\beta \Omega_1 \gamma^2 \Omega_2}{\Omega_3 \Omega_5 - 8\beta \Omega_1 \gamma^2 \varphi_1} = \Omega_7 \tag{38}$$

with

$$\Omega_2 = f(\varphi_1) = 2\gamma \varphi_1 a_3 - \theta (2\omega + ca_3), \qquad \Omega_3 = f(\varphi_1) = 2 \left[\varphi_1 \gamma^2 + 2\Omega_1 + \gamma^2 \varphi_1 - c\theta \gamma - 2a_2 \right],$$

$$\Omega_4 = f(\varphi_1) = \beta \Omega_1 a_3 \gamma - a_4 \theta, \qquad \Omega_5 = f(\varphi_1) = \beta \left[2\Omega_1 + 2\gamma^2 \varphi_1 - c\gamma \theta - 2a_2 \right]$$

Similarly, we use (36)–(38) to derive φ_3 and ς_3 as function of φ_1 :

$$\varphi_3 = f(\varphi_1) = \frac{a_4 \left[4\omega - c \left(4\gamma\Omega_6 - a_3 \right) \right] + 4\Omega_6 \left(2\Omega_7 + \gamma^2 \Omega_6 \right)}{8\rho}$$
 (39)

$$\varsigma_3 = f(\varphi_1) = \frac{a_4^2 + 2\beta\Omega_7 \left(\Omega_7 + 2\gamma^2\Omega_6 - a_3\gamma\right)}{4\beta\rho} \tag{40}$$

Finally, replacing (36) into (48) gives a non-linear equation that unfortunately cannot be resolved analytically. We use the Maple function "fsolve" to obtain numerical solution.

Proof of Proposition 2. We need to establish the existence of bounded and continuously differentiable value functions $V_M^S(G)$, $V_R^S(G)$ such that there exists a unique solution G(t) to (1) and the HJB equations.

To obtain an equilibrium à la Stackelberg equilibrium we first determine the retailer's decision variables as a function of the manufacturer's controls. The retailer's HJB is

$$\rho V_R^S(G) = \max_{p \ge 0, A \ge 0} \left\{ \left(\alpha - \beta p^S + \theta G^S \right) \left(p^S - \omega \right) - \frac{(1 - B^S)}{2} A^{S^2} + V_R^{S'} \left(A^S + \gamma d^S - \delta G^S \right) \right\}$$
(41)

and the maximization provides the strategy

$$p^S = \frac{\alpha + \theta G^S + \omega \beta}{2\beta} \tag{42}$$

$$A^{S} = \frac{V_{R}^{S'}}{1 - B^{S}} \tag{43}$$

Substitute (42) and (43) into manufacturer's HJB equation to obtain

$$\rho V_{M}^{S}(G) = \max_{d \ge 0, B \ge 0} \left\{ \left(\frac{\alpha + \theta G^{S} - \omega \beta}{2} \right) \left(\omega - c d^{S} \right) - \frac{d^{S^{2}}}{2} - \frac{B^{S}}{2} \left(\frac{V_{R}^{S'}}{1 - B^{S}} \right)^{2} + V_{M}^{S'} \left(\frac{V_{R}^{S'}}{1 - B^{S}} + \gamma d^{S} - \delta G^{S} \right) \right\}$$
(44)

Performing the maximization of the right-hand side we obtain

$$d^{S} = \frac{2\gamma V_{M}^{S'} - c\left(\alpha + \theta G^{S} - \omega \beta\right)}{2} \tag{45}$$

$$B = \frac{2V_M^{S'} - V_R^{S'}}{2V_M^{S'} + V_R^{S'}} \tag{46}$$

Inserting (45) and (46) inside the HJB we obtain

$$(\alpha + \theta G^{S} - \omega \beta) \left(4\omega - 4c\gamma V_{M}^{S'} + c^{2} \left(\alpha + \theta G^{S} - \omega \beta \right) \right) + V_{R}^{S'^{2}} + 4V_{M}^{S'} \left(V_{R}^{S'} + \left(1 + \gamma^{2} \right) V_{M}^{S'} - 2\delta G^{S} \right) - 8\rho V_{M}^{S}(G) = 0 \quad (47)$$

$$\left(\alpha + \theta G^S - \omega \beta\right)^2 + \beta V_R^{S'} \left(2\left(1 + 2\gamma^2\right)V_M^{S'} + V_R^{S'} - 2\gamma c\left(\alpha + \theta G^S - \omega \beta\right) - 4\delta G^S\right) - 4\beta \rho V_R^S(G) = 0 \quad (48)$$

We may satisfy (47) and (48) by conjecturing quadratic value functions. We define $V_M^S(G) = \frac{\sigma_1}{2}G^2 + \sigma_2G + \sigma_3$ and $V_R^S(G) = \frac{\mu_1}{2}G^2 + \mu_2G + \mu_3$ where $\sigma_1, \sigma_2, \sigma_3, \mu_1, \mu_2$ and μ_3 are constant parameters. By inserting V_M^S and V_R^S and their derivatives into (47) and (48), we obtain the following six algebraic Ricatti equations,

$$\mu_1^2 + c\theta (c\theta - 4\gamma\sigma_1) + 4\sigma_1 \left[\mu_1 + (1 + \gamma^2) \sigma_1 \right] - 4a_1\sigma_1 = 0$$
(49)

$$\theta \left[2\omega - c \left(2\gamma\sigma_2 - a_3 \right) \right] + 2\sigma_1 \left(\mu_2 - \gamma a_3 \right) + \mu_1 \mu_2 + 2 \left[2 \left(1 + \gamma^2 \right) \sigma_1 + \mu_1 - 2a_2 \right] \sigma_2 = 0 \tag{50}$$

$$a_4 \left[4\omega - c \left(4\gamma \sigma_2 - a_3 \right) \right] + \mu_2^2 + 4\sigma_2 \left[\mu_2 + \left(1 + \gamma^2 \right) \sigma_2 \right] - 8\rho \sigma_3 = 0 \tag{51}$$

$$\theta^2 + \beta \mu_1 \left[2\sigma_1 (1 + 2\gamma^2) + \mu_1 - 2\gamma c\theta \right] - 2\beta a_1 \mu_1 = 0 \tag{52}$$

$$\theta a_4 + \beta \mu_1 \mu_2 + \beta \mu_1 \left[\sigma_2 (1 + 2\gamma^2) - \gamma a_3 \right] + \beta \mu_2 \left[\sigma_1 (1 + 2\gamma^2) - \gamma c \theta - 2a_2 \right] = 0$$
 (53)

$$a_4^2 + \beta \mu_2 \left[2\sigma_2 (1 + 2\gamma^2) + \mu_2 - 2\gamma a_3 \right] - 4\beta \rho \mu_3 = 0$$
 (54)

where the first three correspond to the manufacturer, whereas the second three correspond to the retailer. As for the previous scenario, we present the description of the procedure used to reduce the solution of that

system into the solution of a system of one non-linear equation. The latter equation has been numerically solved by using Maple 10.

From (52), we can obtain σ_1 as a function of $\mu_1 : \sigma_1 = f(\mu_1)$

where

$$\sigma_1 = f(\mu_1) = \frac{2\beta a_1 \mu_1 - \theta^2 + \mu_1 \beta \left(2\gamma c\theta - \mu_1\right)}{2\mu_1 \beta (1 + 2\gamma^2)} = \Psi_1 \tag{55}$$

Replacing (55) for (50) and (53), we can obtain both μ_2 and σ_2 as a function of μ_1 :

$$\mu_2 = f(\mu_1) = \frac{\Psi_3 \Psi_4 - \beta \mu_1 (1 + 2\gamma^2) \Psi_2}{\Psi_3 \Psi_5 - \beta \mu_1 (1 + 2\gamma^2) (\mu_1 + 2\Psi_1)} = \Psi_6$$
(56)

$$\sigma_2 = f(\mu_1) = \frac{\Psi_2 - (\mu_1 + 2\Psi_1)\Psi_6}{\Psi_3} = \Psi_7 \tag{57}$$

with

$$\Psi_{2} = f(\mu_{1}) = (2\Psi_{1}\gamma - c\theta) a_{3} - 2\theta\omega, \qquad \Psi_{3} = f(\mu_{1}) = 2\left[2\left(1 + \gamma^{2}\right)\Psi_{1} - \gamma c\theta + \mu_{1} - 2a_{2}\right],$$

$$\Psi_{4} = f(\mu_{1}) = \beta\mu_{1}\gamma a_{3} - \theta a_{4}, \qquad \Psi_{5} = f(\mu_{1}) = \beta\left(\Psi_{1}(1 + 2\gamma^{2}) - \gamma c\theta + \mu_{1} - 2a_{2}\right)$$

Similarly, we use (55)–(57) to derive φ_3 and ς_3 as function of φ_1 :

$$\sigma_3 = f(\mu_1) = \frac{a_4 \left[4\omega - c \left(4\gamma \Psi_7 - a_3 \right) \right] + \Psi_6^2 + 4\Psi_7 \left(\Psi_6 + 2\gamma^2 \Psi_7 \right)}{8\rho} \tag{58}$$

$$\mu_3 = f(\mu_1) = \frac{a_4^2 + \beta \Psi_6 \left(2\Psi_7 (1 + 2\gamma^2) + \Psi_6 - 2\gamma a_3\right)}{4\beta\rho}$$
(59)

Finally, replacing (55) into (49) gives a non-linear equation that unfortunately cannot be resolved analytically. Also in this case we use the Maple function "fsolve".

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