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Abstract

The well-known method of stochastic programming in extensive form is used on the large scale, partial equilibrium, technology rich global 15-region TIMES Integrated Assessment Model (ETSAP-TIAM), to assess climate policies in a very uncertain world. The main uncertainties considered are those of the Climate Sensitivity parameter, and of the rate of economic development. In this research, we argue that the stochastic programming approach is well adapted to the treatment of major uncertainties, in spite of the limitation inherent to this technique due to increased model size when many outcomes are modeled. The main advantage of the approach is to obtain a single hedging strategy while uncertainty prevails, contrary to classical scenario analysis. Furthermore, the hedging strategy has the very desirable property of attenuating the (in)famous 'razor edge' effect of Linear Programming, and thus to propose a more robust mix of technologies to attain the desired climate target. Although the example treated uses the classical expected cost criterion, the paper also presents, and argues in favor of, altering this criterion to introduce risk considerations, by means of a linearized semi-variance term, or by using the Savage criterion. Risk considerations are arguably even more important in situations where the random events are of a 'one-shot' nature and involve large costs or payoffs, as is the case in the modeling of global climate strategies. The article presents methodological details of the modeling approach, and uses realistic instances of the ETSAP-TIAM model to illustrate the technique and to analyze the resulting hedging strategies. The instances modeled and analyzed assume several alternative global temperature targets ranging from less than 2°C to 3°C. The 2.5°C target is analyzed in some more details.

The paper makes a distinction between random events that induce anticipatory actions, and those that do not. The first type of event deserves full treatment via stochastic programming, while the second may be treated via ordinary sensitivity analysis. The distinction between the two types of event is not always straightforward, and often requires experimentation via trial-and-error. Some examples of such sensitivity analyses are provided as part of the TIAM application.

Key Words: Energy modeling; Uncertainty; Stochastic programming; Hedging strategies; Climate policies; Technology.

Résumé

La technique de la programmation stochastique sous forme extensive est appliquée au modèle énergétique mondial de grande taille, ETSAP-TIAM, pour l'analyse des politiques climatiques sous incertitude. Les principaux aléas considérés sont ceux de la sensibilité du climat et du taux de développement économique. Dans cet article, nous montrons que la programmation stochastique est bien adaptée au traitement des principaux aléas en dépit des limites de traitement imposées par la taille des problèmes qui en résulte. Le principal avantage de la technique est l'obtention d'une stratégie de sauvegarde (hedging strategy) unique durant la période d'incertitude, contrairement aux techniques classiques d'analyse par scénarios indépendants. De plus, la stratégie de sauvegarde a l'avantage d'atténuer l'effet 'bang-bang' de la programmation linéaire, et constitue donc un mixte technologique plus robuste pour l'atteinte d'une cible climatique.

L'exemple traité utilise comme critère d'optimisation l'espérance mathématique du coût total, mais l'article recommande l'introduction de deux concepts d'aversion au risque, l'un par l'adjonction d'un terme de semi-variance, l'autre par l'utilisation du critère Minimax-Regret de Savage. L'article présente les détails méthodologiques de ces approches et leur implantation dans un programme linéaire de grande taille, et utilise des instances réalistes du modèle ETSAP-TIAM appliqué à la recherche de stratégies climatiques robustes. Les cibles climatiques visées sont exprimées en termes de changement de la température globale n'excédant pas 3°C en 2100. La cible de 2.5°C est étudiée en détail.

L'article distingue deux types d'aléas : ceux qui induisent des actions anticipées, et les autres. Le premier type mérite un traitement explicite de l'incertitude, alors que le second peut être traité par l'analyse de sensibilité classique. La détection de ces deux types d'aléas n'est pas toujours facile, et nécessite souvent une exploration empirique plus ou moins rigoureuse. Elle est cependant jugée importante. Des exemples d'analyse de sensibilité sont montrés pour des paramètres jugés essentiels.

Acknowledgments: This work is the main contribution of the Energy Technology Systems Analysis Programme (ETSAP) to Work Group 1 of the EMF-22 program of research. ETSAP is the principal sponsor of the development of the TIMES Integrated Assessment Model (ETSAP-TIAM) used to conduct our analysis.

1 Introduction

The Stochastic Programming (S.P.) paradigm [2, 19] is now a fully mature one, and has been used in many applications. It is recognized as a rigorous way to account for risky events while optimizing a particular system. One of the well recognized computational drawbacks of S.P. (at least in its extensive form) is that it quickly leads to large scale instance of the original problem, whenever the number of random events grows and/or the number of outcomes of each event becomes too large. In this article, we argue for a reasoned use of Stochastic Programming in large scale integrated Energy-Climate models such as the ETSAP-TIAM model [10, 12], in which only the main uncertainties are modeled, while others are treated via sensitivity analysis. More precisely, the paper makes a distinction between random events that induce anticipatory actions, and those that do not. The first type of event deserves full treatment via stochastic programming, while the second may be treated via ordinary sensitivity analysis. The distinction between the two types of event is not always straightforward, and often requires experimentation via trial-and-error. Some examples of such sensitivity analyses are provided as part of the TIAM application.

One of the main advantages of the S.P. approach is to obtain an explicit single hedging strategy while uncertainty prevails, contrary to classical scenario analysis. Furthermore, the hedging strategy has the very desirable property of attenuating the (in)famous 'razor edge' effect of Linear Programming, and thus to propose a more robust mix of technologies to attain the desired climate target. In a nutshell, a good hedging strategy takes into account the possible outcomes, and strikes an optimal compromise between the negative effects of the many ways of "guessing wrong" [11].

Although the examples treated use the classical expected cost criterion, the paper also presents – and argues in favor of, altering this criterion to introduce risk considerations, by means of a linearized semi-variance term, or by using the Savage criterion. Risk considerations are arguably even more important in situations where the random events are of a 'one-shot' nature and involve large costs or payoffs, as is the case in the modeling of global climate strategies.

This article presents methodological details of the modeling approach, and uses realistic instances of the ETSAP-TIAM model to illustrate the technique and to analyze the resulting hedging strategies. The instances modeled and analyzed propose several alternative global temperature targets ranging from less than 2°C to 3°C. The 2.5°C target is analyzed in some more details.

In brief, the main objectives of this work are:

a) to demonstrate the power of stochastic programming in calculating optimal hedging strategies using a large scale, realistic energy-climate integrated model, in the presence major uncertainties on climate policies (climate sensitivity and future economic growth),
b) to analyze hedging strategies, i.e. a set of early robust actions capable of maintaining the global temperature within specified bounds, in spite of the uncertainty. Robust actions are those actions chosen in the hedging strategy but not in the Base case. In fact, hedging is deemed relevant if decisions made prior the resolution of uncertainty are different from those in the base case (otherwise, "*wait and see*" is a good policy). Hedging is even more useful when it is not identical to *any* of the perfect forecast strategies, since such a situation clearly shows that the optimal technology and energy decisions are not easily predictable without an explicit treatment of uncertainty.

Among the results obtained, the fact that no perfect forecast is able to reproduce the hedging strategy confirms the relevance of using stochastic programming in order to analyze preferred climate policies in an uncertain world.

c) To formulate alternate criteria for use with the S.P. approach, such as the adjunction of a semi-variance term or the use of the Savage criterion.

Section 2 contains a discussion of climate uncertainties. Section 3 describes the TIAM model and the methodology used to represent the uncertainties and to compute hedging strategies with stochastic programming. Sections 4 and 5 present results, including several sensitivity analyses, Section 6 presents alternate criteria, and Section 7 concludes the article.

2 Uncertainty in Energy-Climate Studies

The impacts of greenhouse gas (GHG) emissions on climate may be sketched as a chain of causal relationships, where GHG emissions provoke an increase in the concentration of GHG's in the atmosphere and in oceans; the increased concentrations provoke an increase of the atmospheric radiative forcing (RF) by the various gases, which in turn has an impact on the global temperature of the atmosphere and oceans. Nordhaus and Boyer [15] proposed simple and well-documented linear recursive equations for calculating CO_2 concentrations and global temperature changes. The climate module of ETSAP-TIAM is based on these equations for the carbon cycle and on two one-box models (simple exponential decay) for the CH_4 and N_2O cycles. The other substances that induce radiative atmospheric forcing (other Kyoto gases, Montreal Protocol gases, aerosols, etc.) are treated via an exogenous forcing trajectory.

In this article, two parameters of the climate equations are considered as highly uncertain: the climate sensitivity (C_s), defined as the equilibrium response of global surface temperature to a doubling of the equivalent CO_2 concentration; and the inverse of the thermal capacity of the atmospheric layer and the upper oceans, also called "lag parameter", key determinant of transient temperature change. While C_s has received a great deal of attention, its value is still highly uncertain [1, 3]. Until recently, a range between 1.5°C and 4.5°C was commonly quoted [6]. More recent studies have strongly argued for a wider range of 0.5°C to 9°C or even 10°C [1, 8]. Regarding the lag parameter, its value may either be considered to be approximately independent of C_s, or it may be assumed to vary inversely with C_s.[1] The latter case results in higher transient temperature increases than with a fixed value of the lag parameter (for example, in our results, we observed that, when using a fixed lag parameter, the smallest achievable temperature increase is 0.5°C lower that when assuming a variable lag value). In the main analyses presented here, we make the prudent assumption of a variable lag, and we adopt the values adopted by the EMF-22 group[2] for the purpose of conducting comparative analyses of climate stabilization strategies with different models (Table 1). It is also assumed that the uncertainty on C_s and on the lag parameter will be fully resolved in 2040,[3] and that no significant additional knowledge will be obtained before that resolution date. In addition, sensitivity analysis on the date of resolution is presented in Section 5.

Table 1: Uncertain values of the climate sensitivity and the lag parameter

Climate Sensitivity	**Likelihood**	**Corresponding Lag Parameter**
1.5°C	0.25	0.065742
3°C	0.45	0.014614
5°C	0.15	0.010278
8°C	0.15	0.008863

Another potential source of uncertainty besides C_s is the annual rate at which the World economy develops, as this has a direct impact on economic demands and thus on GHG emissions. In this research, we also use the EMF-22 assumption that the base case global annual GDP growth rate is known until 2040. At that date, all future annual global growth rates until 2100 are assumed to be revealed and may have one of two equally probable values: a high value (equal to 4/3 of the base case rate), and a Low value (equal to 2/3 of the base case rate). The same simple-to-double growth rate assumption is used for the GDP growth rate

[1] By linking C_s and σ_1, Yohe et al. [20] assume a deterministic relationship between the two parameters. Fussel [4] criticizes this relationship, since it results in values for the thermal capacity of the atmosphere and the upper oceans that are outside the physically plausible range. Moreover, the probabilistic relationship underestimates the true uncertainty about the transient climate response.

[2] The Energy Modelling Forum is an international forum on energy and environmental markets. The EMF-22 ongoing study, "Climate Policy Scenarios for Stabilization and in Transition", focuses on comprehensive analyses of long-run climate stabilization policies under uncertainty as well as intermediate-term transition policies.

[3] A recent paper by Weizmann [18] argues the case for the impossibility to fully resolve the uncertainty on Cs in finite time. If this were the case, the Stochastic Programming approach would be much simplified.

of each region of ETSAP-TIAM. Regional GDP growth rates affect the growth rates of each energy service demand having GDP as a driver. World GDP starts from 32 trillion $ in 2000 and reaches 260 trillion $ (Base), 181 trillion $ (Low) or 385 trillion $ (High) in 2100.

Year 2040 corresponds to the beginning of the period 2040–2060 of the TIMES model. This period is called "2050" in results provided by TIMES. Therefore, all the results presented for years 2050 and after correspond to the part of the event tree after uncertainty is resolved, while results presented for years 2030 and before correspond to the part of the event tree before uncertainty is resolved.

3 Modeling Uncertainty in a Large Scale Integrated Energy-Climate Model

3.1 The TIMES Integrated Assessment Model (ETSAP-TIAM)

ETSAP-TIAM (TIMES Integrated Assessment Model) is a detailed, technology-rich Global TIMES model. It is a multi-region partial equilibrium model of the energy systems of 15 regions covering the entire World. The 15 regional models are: Africa, Australia-New Zealand, Canada, Central and South America, China, Eastern Europe, Former Soviet Union, India, Japan, Mexico, Middle-East, Other Developing Asia, South Korea, United States, and Western Europe. In addition, the upstream and energy trade sectors in each country are split into OPEC/Non-OPEC. The regional modules are linked by trade variables of the main energy forms (coal, oil, gas) and of emission permits. Thus, impacts on trade (terms of trade) of environmental policies are taken into. ETSAP-TIAM's planning horizon extends from 2000 to 2100, divided into 7 periods of varying lengths, suitably chosen.

ETSAP-TIAM is a global instance of the TIMES model generator (full documentation is available from www.etsap.org/documentation.asp), where a bottom-up, detailed technological representation of each economic sector is combined with a key linkage to the rest of the economy via demands for energy services that are elastic to their own prices.

TIMES computes an inter-temporal dynamic partial equilibrium on energy markets, where demands for *energy services* are exogenously specified only in the reference case, and are sensitive to price changes (via a set of own-price elasticities) in all alternate scenarios. The equilibrium is driven by the maximization, via linear programming, of the total surplus (i.e. the sum of producers and suppliers surpluses), which acts as a proxy for welfare in each region of the model. Although TIMES does not encompass macroeconomic variables beyond the energy sector, accounting for price elasticity of demands captures a major element of feedback effects between the energy system and the economy. The surplus maximization is subject to many constraints, such as: supply bounds (in the form of detailed supply curves) for the primary resources, technical constraints governing the creation, operation, and abandonment of each technology, balance constraints for all energy forms and emissions, timing of investment payments and other cash flows, and the satisfaction of a set of demands for energy services in all sectors of the economy.

The construction of the base case demands for energy services is done by using the global General Equilibrium model GEM-E3 (http://www.gem-e3.net/), which provides a set of coherent *drivers* for each region and for the World as a whole, such as population, households, GDP, sectors outputs, and technical progress. These drivers are then transformed into growth rates for each of the 42 TIAM demands for energy services, via the generic relationship:

$$demand\_rate = driver\_rate \backslash times\ decoupling\_factor.$$

The decoupling factors account for phenomena such as saturation (factor is then less than 1) and suppressed demands (factor is then larger than 1), and are in part empirically based. Most demands have economic growth as their driver. As already mentioned, the demands of ETSAP-TIAM are user-specified only for the reference scenario, and are prone to endogenous changes in all alternate scenarios, in response to endogenously changing demand prices. The elasticities of demands to their own price range from 0 to -0.6, with a majority in the range -0.2 to -0.3.

ETSAP-TIAM comprises several thousand technologies in all sectors of the energy system (see sketch in Figure 1). A technology may represent any process that produces, transforms, conveys, and/or consumes energy and/or emissions (and some materials). It is characterized by several technical and economic parameters and by emission coefficients for the three main GHG's: CO_2, CH_4, and N_2O. Energy and industrial emissions of these three gases are all modeled. In addition, net CO_2 emissions from land use are represented, and non-energy CH_4 and N_2O emissions are also modeled (e.g. CH_4 and N_2O from landfills, manure, enteric fermentation, rice paddies, etc.). The model constructs a coherent image of the future energy system by choosing a mix of technologies to invest in and operate at each future period, with the objective of maximizing total surplus, while respecting the many constraints of the model. A complete description of ETSAP-TIAM's technological database is not possible within the limits of an article, but we wish to mention some options for GHG emission reductions available in the model: first, emission reductions may be done via the numerous fuel and technology switching options that are available in each sector, and via specific CH_4 and N_2O abatement options (e.g. suppression and/or combustion of fugitive methane from landfills, thermal destruction of N_2O in the adipic acid industry, etc.). CO_2 emissions may in some cases be captured and stored (CCS options) before their release into the atmosphere (e.g. CO_2 capture from the flue gas of fossil fueled power plants, from hydrogen production processes, and from oil extraction processes; storage in depleted oil fields, deep saline aquifers, deep oceans, etc.). Finally, atmospheric CO_2 may be partly absorbed and fixed by biological sinks such as forests; the model has six options for forestation and avoided deforestation, as described in [17] and adopted by the EMF-22 group. Note also that methane emissions from the agriculture sector are fully accounted for, even if no abatement options are considered.

3.2 Using the Model

As noted before, most of climate equations from Nordhaus and Boyer [15] have been adapted and integrated into the model. One difference with these authors, we have adopted a separate representation of the N_2O and CH_4 atmospheric cycles, via simple one-box models where the concentration decays exponentially at constant rate.

ETSAP-TIAM may be used to evaluate different kinds of climate targets: emission limits directly, concentration bounds, bounds on radiative forcing, and finally, limits on global temperature change. However, the non-convexity of the radiative forcing expressions (see e.g. the forcing expression for CO_2 as equation 1) precludes using the temperature equations as regular constraints of the ETSAP-TIAM model. Therefore, we have linearized the forcing expressions for CO_2, CH_4, and N_2O, within the useful concentration ranges. The result is an approximation that remains within 1% of the true forcing value within the range of interest for forcing values (i.e. between 300 and 600 ppmv CO_2-eq).

$$\Delta F(t) = \gamma^* \frac{In(M_{atm}(t)/M_0}{In2} + FEX(T) \tag{1}$$

where:

- $\Delta F(t)$ is the increase of the radiative forcing at period t relative to pre-industrial level
- M_0 is the pre-industrial (circa 1750) reference atmospheric concentration of CO_2
- γ is the radiative forcing sensitivity to the doubling of atmospheric CO_2 concentration (3.7 W/m^2)

3.3 The Computation of Hedging Strategies

3.3.1 Stochastic Programming

The treatment of uncertainty is done via Stochastic Linear Programming in extensive form [2, 19]. In this method, the model takes a single *hedging* strategy until resolution of uncertainty, so as to be best positioned

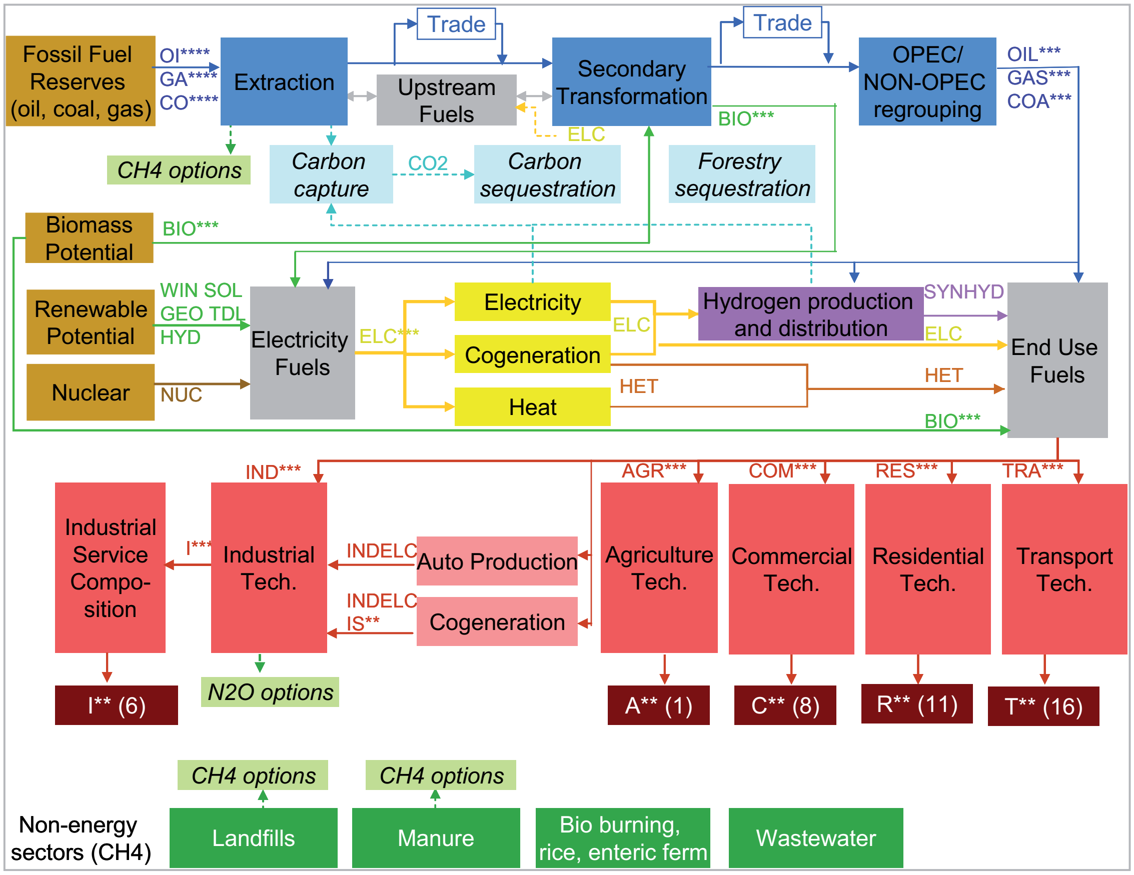


Figure 1: ETSAP-TIAM Reference Energy System

to adapt to any of the possible long term futures (after the resolution date). In the application described in Section 4, the optimization criterion is the *expected value* of the total surplus. Other optimizing criteria may be preferred. Loulou and Kanudia [11] presents an application using the Minimax Regret criterion. Another approach is available in TIMES, in which the criterion to maximize is a combination of the expected surplus and of a risk term calculated as the linearized semi-variance. Section 6 discusses these alternate criteria.

A typical stochastic LP with the expected value criterion is written as follows, in the simpler two-stage case where all uncertainties are resolved at a single date θ:

$$\textbf{Maximize} \sum_{t} \beta(t) \sum_{s=1,toS} C(t,s) \cdot X(t,s) \cdot p(s) \tag{2}$$

Subject to:

$$A(t,s) \times X(t,s) \geq b(t,s) \tag{3}$$

and $X(t,1) = X(t,2) = \ldots X(t,S)$, if $t <$ resolution date θ
where

- s represent the possible *states of the world* (sow), $s = 1, 2, \ldots, S$
- $p(s)$ is the probability that *sow* s realizes
- C and b are respectively the surplus and the RHS vectors of the LP
- A is the matrix of LP coefficients

- $X(t, s)$ is the vector of decision variables at time t, under state-of-the-world s
- $\beta(t)$ is the discounting factor that converts 1$ from time t to time O.

Remark: We insist on the fact that the main interest of a hedging strategy resides in its description of what to do prior to the resolution date. In contrast, traditional deterministic scenario analysis computes multiple strategies even prior to the resolution date, leaving the decision maker in a quandary as to which one should be followed. Once uncertainty is resolved, the decision maker no longer faces uncertainty, and her decisions result from optimizing a deterministic problem from θ onward. Nevertheless, the computation of the hedging strategy must also take into account *all possible outcomes after the resolution date*. In other words, short term hedging decisions are devised while taking the uncertain long term into consideration. This is the essence of decision under risk, and in particular of stochastic programming.

The above discussion leads to an interesting classification of uncertain events that can be very useful in deciding which event(s) to model explicitly via stochastic programming, and which not: suppose it may be established that a certain event does NOT induce anticipatory actions. By this we mean that the hedging strategy is insensitive to the various possible outcomes of the event. In such a case, it is clear that there is no advantage in explicitly modeling the event as part of the S.P. event tree. Hence, only those events that are likely (or proven) to influence the Hedging strategy should be an explicit part of the event tree. Of course, the question now is how to detect such events, and the answer to that question is quite empirical. Some experimentation, and some judgment is required in order to sort out the two types of event. We shall come back to this question in the next paragraph and also in Section 5.

3.3.2 The Choice of the Two Uncertain Parameters

For a given temperature target, the two selected uncertain parameters are, as illustrated by Figure 2: i) the climate sensitivity C_s (four possible values), and ii) the vector of energy service demands resulting from the future economic growth (two possible values). The combination of these two uncertainties leads to 8 possible States of the World (SoW). However, after conducting stochastic optimizations with the 8 SoW's, it was *observed* that the impact of economic uncertainty on the hedging strategy before 2040 was quite negligible. In other words, the hedging decisions taken *before* 2040 are quite insensitive to the values of economic demands *after* 2040. In still other words, there is no anticipatory effect for the economic growth. Therefore, we decided to eliminate economic growth as an explicit uncertainty in our main runs reported in Section 4, and to assess the impact of uncertain economic growth on the hedging strategy as one kind of sensitivity analysis in Section 5. The resulting event tree, with only Cs as the uncertain parameter, has 4 branches, as shown in Figure 3.[4]

4 A Treated Example: Hedging Strategy and Perfect Forecast Strategies for a 2.5°C Target on Global Temperature Change

4.1 Hedging and Perfect Forecast Strategies

Our initial objective was to calculate hedging strategies for two alternative scenarios, where the alternative targets for temperature change[5] are 2°C and 3°C. As it turned out, with the options present in the model, the 3°C target is achievable at very moderate cost, while the more severe 2°C target is only achievable at very high cost. Therefore, only the intermediate 2.5°C scenario will be discussed in detail in this paper. Moreover, additional model runs revealed that the smallest achievable temperature increase is close to 1.9°C, albeit at extremely large cost, given the options for GHG control present in the model and the GDP growth assumptions. This means that more severe temperature targets would require additional CO_2 abatement

[4] Reducing the number of sow's has a direct impact on the computational time to resolve the LP. Typical time for solving the 8 sow problem was 440 minutes versus only 80 minutes for the 4 sow problem.

[5] The targets actually set in the model are in year 2100. However, we are able to calculate the evolution of the global temperature after 2100, by assuming that emissions decline linearly to 0 from 2100 to 2200.

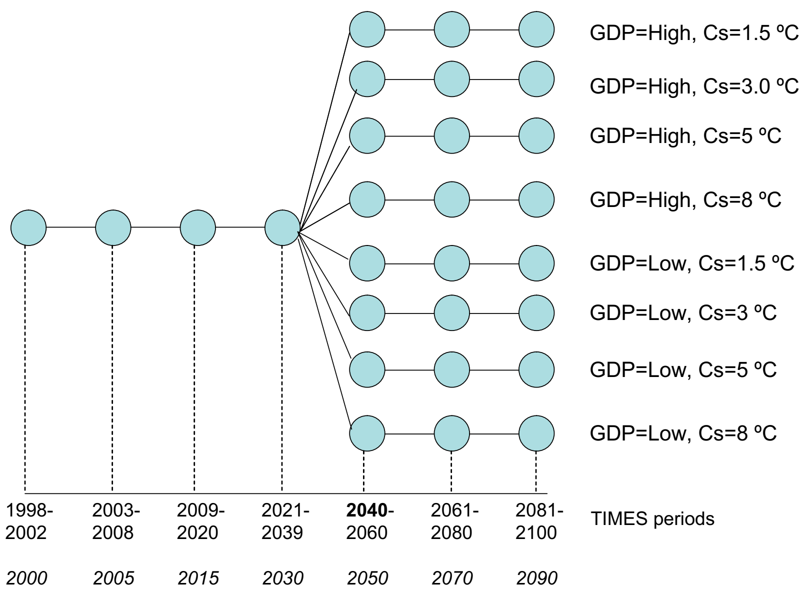


Figure 2: The event tree

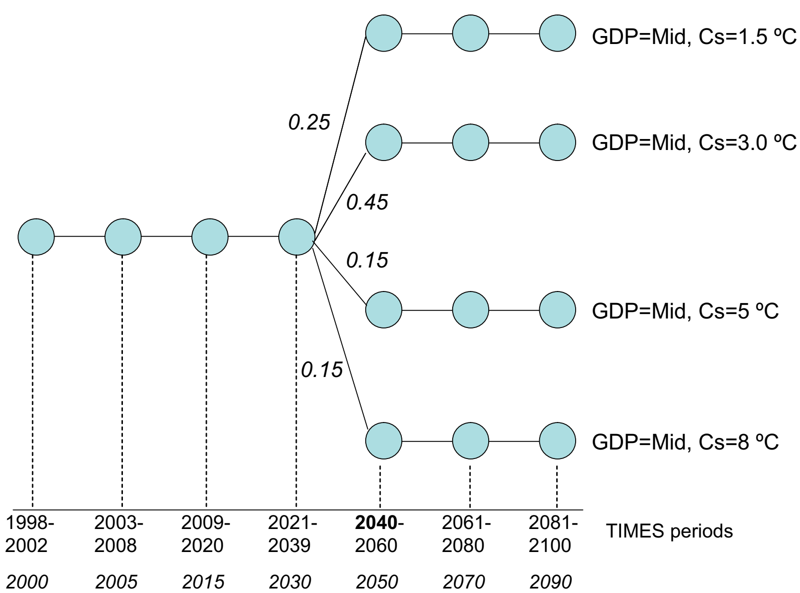


Figure 3: The reduced event tree

potential that is currently not yet seen as realistic.[6] Figure 5 shows that in order to keep global temperature change below the 1.9°C upper bound, the trajectory of CO_2-eq concentration must remain almost constant throughout the 21st century.

In addition to the hedging strategy, we also computed four (deterministic) perfect forecast strategies (noted PF), each assuming that the value of C_s is known as early as the first period. The theoretical interpretation of the four PF strategies is that of an optimal strategy if one assumes that the uncertainty is resolved at the beginning of the horizon. The PF's may be used to compute the Expected Value of Perfect

[6] No abatement options are available for rice production, enteric fermentation and biomass burning, whose CH_4 emissions are included in the model. This contributes to the infeasibility of any target smaller than 1.9°C.

Information (EVPI), which is the expected gain in welfare accrued if perfect information if available, i.e.:

$$EVPI = \sum_{s=1toS} p(s) \cdot \left[O_{PF(s)} - O_{HEDG}\right] \tag{4}$$

where

- $O_{PF(s)}$ is the surplus of the PFs strategy ($s = 1$ to S)
- O_{HEDG} is the expected surplus of the hedging strategy.

Another finding of the research is that when $C_s = 1.5°\text{C}$, the Base case satisfies the 2.5°C temperature constraint at all times, (provided emissions after 2100 decline linearly to 0 by 2200 as assumed here). Therefore, the $\text{PF}_{\text{Cs}=1.5°\text{C}}$ strategy is not different from the Base case.

4.2 Another Interpretation of the Perfect Forecast Strategies

In the detailed analysis of results for 2.5°C, we compare the results of the hedging strategy with those of a Base case where no climate target is imposed, but we also compare them with those of the Perfect Forecast strategies defined above. The rationale for this comparison stems from the following important remark: apart from its theoretical meaning as a perfect forecast strategy, any given PF strategy may be re-interpreted as a *heuristic* strategy *until the resolution date*. Indeed, a policy maker may wish to calculate a heuristic strategy that simply assumes a "guestimate" of Cs. This is easily done by optimizing a deterministic instance of the problem. Such a PF strategy is perfectly feasible (albeit not optimal) *until the resolution date*. After that date, decisions taken in the PF strategy are not realistic, since they ignore the fact that the value of C_s has indeed been revealed.

In this light, we shall discuss PF results only before 2040.[7] One finding is that $\text{PF}_{\text{Cs}=5°\text{C}}$ is the deterministic strategy that is closest to the optimal hedging one, although some significant differences between $\text{PF}_{\text{Cs}=5°\text{C}}$ and hedging exist in some areas, as we shall see. Therefore, when comparing Hedging with deterministic strategies, we shall always use $\text{PF}_{\text{Cs}=5°\text{C}}$ (and then only before 2040).

4.3 Cost Analysis as a Proxy for the Target's Attainability

We define the expected *cost* of a strategy as the Net Present value of the loss of expected surplus, relative to that of the base case. This provides a convenient indicator of the overall difficulty of reaching a particular target, and therefore a convenient way to compare various strategies. In addition to the NPV, we are interested in the marginal cost of one tonne of GHG.

4.3.1 Loss of Surplus and Expected Value of Perfect Information

The global net present value of the surplus attached to a climate strategy represents a compact measure of the social welfare associated with that strategy. Table 2 shows the expected loss of total surplus of the hedging strategy and of the perfect forecast strategy, relative to that of Base taken as reference. The loss of surplus when following Hedging is 35% higher than the expected loss for the perfect information strategy.[8] This difference represents the expected value of perfect information (210 B$ in NPV).

[7] With additional work, each PF strategy may also become a complete strategy as follows: freeze all PF decisions until 2040 at their observed values in the solution, and then re-optimize the system over periods post-2040 periods with each of the Cs values. In this way, each PF strategy gives birth to four post-2040 trajectories, which, taken together, constitute a bona fide complete strategy. This was not implemented in this research, but is illustrated [11].

[8] The corresponding annuities represent less than 0.1% of the World GDP (33000 B$ in year 2000). However, the stream of expenditures would clearly be lower in early years and higher in later years. Furthermore, equity issues might dictate an uneven imputation of the overall cost among the regions.

Table 2: Loss of surplus and expected value of perfect information

Strategy	**Loss of surplus** ($NPV_{5\%}$ in B$)	**Probability**	**Expected loss** (NPV in B$ and annuity in B$/year)	**EVPI** (NPV in B$ and annuity in B$/year)
BASE	0	1	–	–
PF Cs=1.5°C	0	0.25	–	–
PF Cs=3°C	43	0.45	–	–
PF Cs=5°C	580	0.15	–	–
PF Cs=8°C	3353	0.15	–	–
Total PF			610 (31)	–
HEDGING	820		820 (41)	210 (11)

EPVI = Expected loss $_{\text{HEDGING}}$ − Expected loss $_{\text{PERFECT FORECAST}}$

Table 3: Marginal cost of GHG ($/t$CO_2$)

Year	2000	2005	2015	2030	2050	2070	2090
TIMES periods	*1998–2002*	*2003–2008*	*2009–2020*	*2021–2039*	*2040–2060*	*2061–2080*	*2081–2100*
HEDGING Cs=1.5°C					0	0	0
HEDGING Cs=3°C	1	2	4	10	0	2	3
HEDGING Cs=5°C					11	40	80
HEDGING Cs=8°C					176	620	1236
PF Cs=3°C	0	0	0	1	2	7	14
PF Cs=5°C	0	1	2	4	12	43	86
PF Cs=8°C	3	7	12	28	84	296	589

4.3.2 Marginal Cost of GHG

We first recall that the environmental constraint is defined in terms of global atmospheric CO_2-equivalent concentration. Thus, CO_2, CH_4 and N_2O have the same marginal cost in all regions and all sectors of the model.

Before 2040, the marginal cost of GHG in the hedging strategy remains low (Table 3). The analysis of hedging abatement options before 2040 shows that relatively inexpensive forestry measures contribute to this low price. The fact that no abatement option is available for methane from rice production, enteric fermentation and biomass burning, contributes to the observed high GHG price in the late horizon (up to more than 1200$/t$CO_2$), when methane represents the most important remaining GHG due to the lack of reduction options for agricultural CH_4 emissions.

We also observe that none of the perfect forecast strategies is able to provide a good approximation of the expected GHG price under uncertainty, although $PF_{Cs=5^\circ C}$ is the closest to hedging in that respect.

4.4 Global Emissions and Climate Results

4.4.1 Base Case Emissions

The base case GHG emission trajectory (Figure 4) as well as the atmospheric GHG concentration reached in 2090 (Figure 5) are fairly close to the B2 Emission Scenario proposed by the Intergovernmental Panel on Climate Change [6, 14]. CO_2 remains the most important GHG (around 79%), followed by CH_4 (around 19%) and N_2O (less than 2%). As for sector emissions in the base case, the electricity and transportation sectors are the highest GHG contributors in 2000 (more than 40% of total GHGs), and the electricity and industry sectors become the highest contributors at the end of the horizon (more than 48% of total GHG).

4.4.2 Emissions in the Optimal Hedging Strategy with 2.5°C Temperature Constraint

The situation is radically different under the 2.5°C temperature constraint, since both the electricity and industry sectors are able to reduce to almost zero (less than 3% of total GHG) their emissions in the most

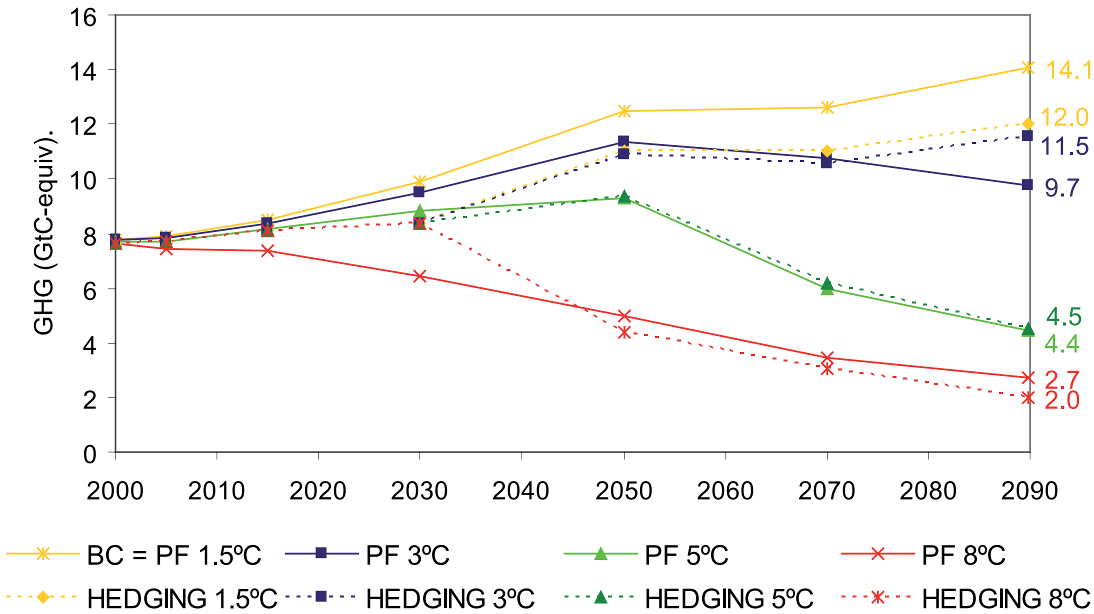


Figure 4: GHG emissions of hedging and perfect forecast strategies

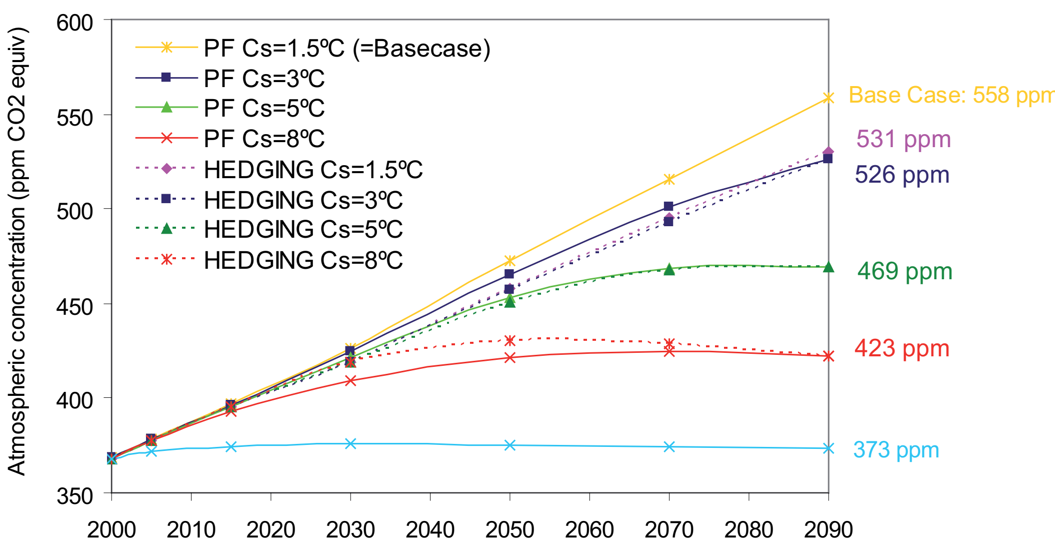


Figure 5: Atmospheric concentration (CO_2-equiv) under hedging and perfect forecast strategies

stringent branch, mainly thanks to CCS in the electricity sector, and switching to electricity in the industrial sector. In this most stringent branch, transport and agriculture are the highest remaining GHG contributors (30% and 41% of total GHG). No such drastic decrease of CH_4 emissions is possible because some non-energy agriculture-related sources have no abatement options in the model. Based on emissions, the $\mathrm{PF}_{\mathrm{Cs}=5^\circ\mathrm{C}}$ strategy is also the deterministic strategy that is closest to the optimal hedging strategy before 2040.

Atmospheric concentration obtained with the lowest value of C_s (Figure 5) is lower in Hedging than in Base although no target was imposed on this branch of the Hedging. This is because hedging actions taken pre-2040 push concentration downward. Again, $PF_{Cs=5°C}$ is the PF strategy that is closest to Hedging before 2040.

In the base case, the temperature increase in 2090 is in the range from 1.4°C to 2.4°C, depending on C_s (Figure 6). In all hedging branches, temperature peaks within he 22nd century, and then declines, so that the equilibrium temperature is always lower than the maximum observed temperature from 2000 to 2200. This might not necessarily be the case for other temperature scenarios, or if a slower emission decline was assumed after 2100.

4.5 Robust (Hedging) Actions

4.5.1 CO_2 Sequestration

Sequestration by forests appears to be a very robust abatement option, since it penetrates in the hedging strategy as early as 2005 (Table 4) and uses its full potential. In fact, it plays a transitional role until less expensive energy options become available. As regards CCS options (with sequestration in deep oceans, saline aquifers, coal bed methane recovery, depleted oil and gas fields, enhanced oil recovery), they are much less robust, as they penetrate only slightly in 2030 in the hedging strategy, while they are used much earlier (in 2005) and at a higher level in $PF_{Cs=8°C}$, and used only after 2040 in the other PF strategies. In other words, no perfect forecast strategiy is able to reproduce the hedging strategy.

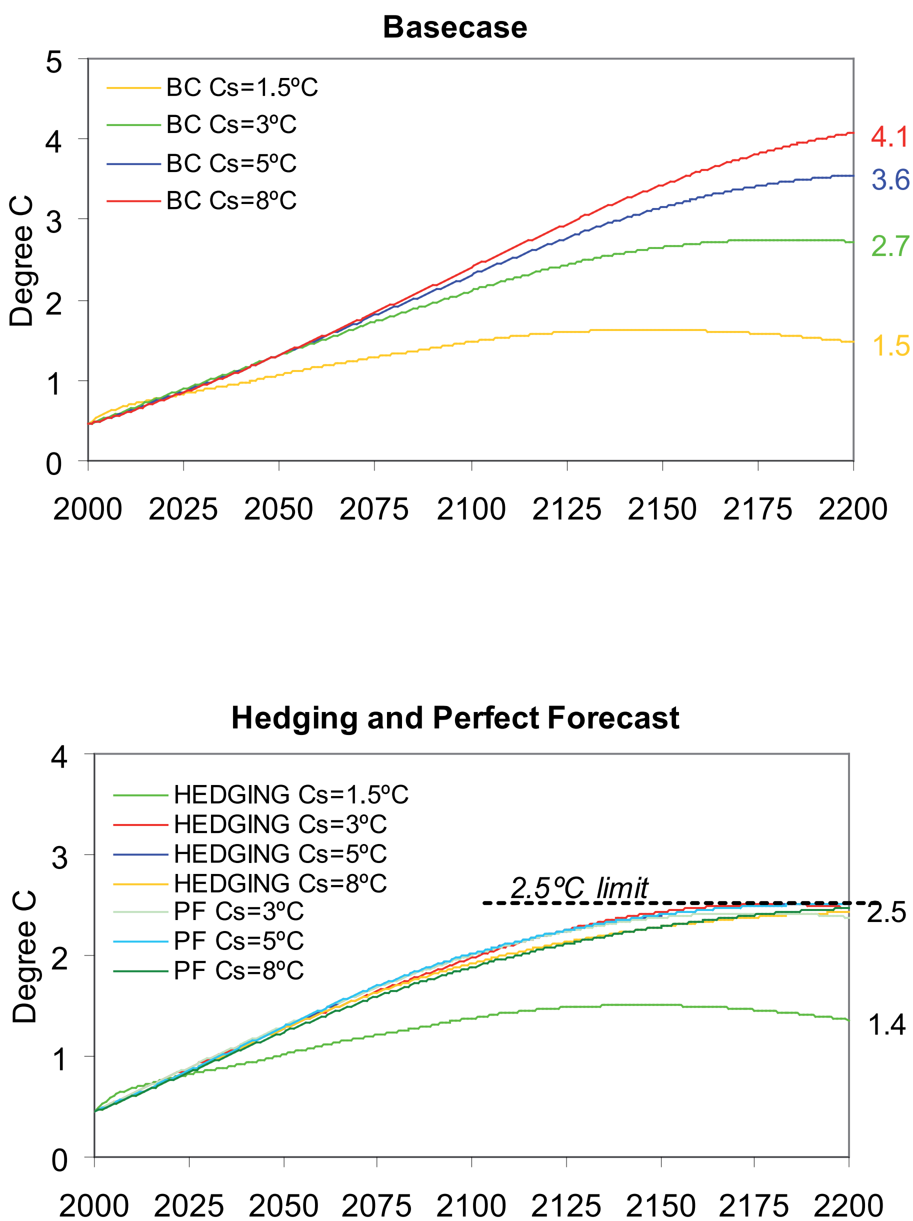
Table 4: Contribution of CCS and forestry to the total GHG reduction

Contribution of CCS to GHG (CO_2 equiv) reduction						
Year	2005	2015	2030	2050	2070	2090
TIMES periods	*2003–2008*	*2009–2020*	*2021–2039*	*2040–2060*	*2061–2080*	*2081–2100*
HEDGING Cs=1.5°C				0.0%	0.0%	0.0%
HEDGING Cs=3°C				0.0%	0.0%	0.0%
HEDGING Cs=5°C	0.0%	0.0%	2.9%	1.0%	5.3%	10.8%
HEDGING Cs=8°C				17.0%	10.6%	10.7%
PF Cs=1.5°C	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
PF Cs=3°C	0.0%	0.0%	0.0%	0.0%	1.9%	3.7%
PF Cs=5°C	0.0%	0.0%	0.0%	1.1%	6.6%	10.9%
PF Cs=8°C	7.3%	3.7%	5.6%	17.7%	13.3%	11.9%
Contribution of forestry sequestration to GHG (CO_2 equiv) reduction						
Year	2005	2015	2030	2050	2070	2090
TIMES periods	*2003–2008*	*2009–2020*	*2021–2039*	*2040–2060*	*2061–2080*	*2081–2100*
HEDGING Cs=1.5°C				65%	99%	97%
HEDGING Cs=3°C				61%	85%	78%
HEDGING Cs=5°C	35%	53%	29%	31%	27%	21%
HEDGING Cs=8°C				12%	18%	16%
PF Cs=1.5°C	0%	0%	0%	0%	0%	0%
PF Cs=3°C	85%	86%	77%	53%	61%	41%
PF Cs=5°C	44%	65%	43%	29%	26%	20%
PF Cs=8°C	25%	27%	16%	13%	19%	17%

4.5.2 Electricity Sector's Actions

Electricity production is shown in Table 5. The first observation is that, as expected, electricity production up to 2030 takes a middle-of-the-road course up to 2030, compared to the PF strategies.

In the pre-2040 periods, we note significant differences in the Hedging and $PF_{Cs=5°C}$ strategies mainly in two categories: first, the PF strategy widely overestimates the amount of coal based electricity production (with CCS) compared to Hedging. In contrast, it underestimates the optimal amount of biomass fueled electricity and also the amount of hydroelectricity, compared to Hedging,. For other types of electricity (from gas and nuclear), $PF_{Cs=5°C}$ production levels are quite close to the optimal hedging amounts over the entire pre-2040 period.



* Assuming emissions linearly decrease to 0 from 2100 to 2200

Figure 6: Temperature increase 2000–2200*

Table 5: Electricity production (EJ/year)

Plant Type		2000	2005	2015	2030	2050	2070	2090
COAL	BASE, PF Cs=1.5°C	18	17	17	15	28	22	24
FIRED	PF Cs=3°C	18	17	17	15	25	18	15
	PF Cs=5°C	17	17	17	9	5	3	8
	PF Cs=8°C	16	17	11	0	12	12	14
	HEDGING Cs=1.5°C					25	24	23
	HEDGING Cs=3°C					25	21	20
	HEDGING Cs=5°C	16	17	17	5	5	3	8
	HEDGING Cs=8°C					14	10	12
OIL + GAS	BASE, PF Cs=1.5°C	5	10	18	34	54	57	61
FIRED	PF Cs=3°C	5	10	18	34	52	56	54
	PF Cs=5°C	6	10	18	34	56	39	29
	PF Cs=8°C	8	10	20	29	34	27	30
	HEDGING Cs=1.5°C					51	58	61
	HEDGING Cs=3°C					51	56	62
	HEDGING Cs=5°C	7	10	18	36	59	38	29
	HEDGING Cs=8°C					29	23	25
NUCLEAR	BASE, PF Cs=1.5°C	9	8	10	11	20	59	109
	PF Cs=3°C	9	8	10	11	20	59	109
	PF Cs=5°C	9	8	10	11	20	73	128
	PF Cs=8°C	9	8	10	13	28	74	136
	HEDGING Cs=1.5°C					20	59	109
	HEDGING Cs=3°C					20	59	109
	HEDGING Cs=5°C	9	8	10	11	20	73	128
	HEDGING Cs=8°C					28	74	138
HYDRO	BASE, PF Cs=1.5°C	9	9	10	11	13	22	26
	PF Cs=3°C	9	9	10	11	19	26	38
	PF Cs=5°C	9	9	10	15	30	39	44
	PF Cs=8°C	9	9	12	25	35	42	49
	HEDGING Cs=1.5°C					19	19	27
	HEDGING Cs=3°C					19	24	28
	HEDGING Cs=5°C	9	9	10	17	28	39	44
	HEDGING Cs=8°C					35	44	53
BIOMASS	BASE, PF Cs=1.5°C	0	0	0	0	1	1	1
	PF Cs=3°C	0	0	0	0	1	1	1
	PF Cs=5°C	0	0	0	0	1	1	1
	PF Cs=8°C	0	0	0	1	6	4	3
	HEDGING Cs=1.5°C					1	1	1
	HEDGING Cs=3°C					1	1	1
	HEDGING Cs=5°C	0	0	0	0	1	1	1
	HEDGING Cs=8°C					8	7	7
OTHER	BASE, PF Cs=1.5°C	0	0	0	0	1	1	1
RENEWWABLES	PF Cs=3°C	0	0	0	0	1	1	1
	PF Cs=5°C	0	0	0	0	1	1	2
	PF Cs=8°C	0	0	0	1	1	3	7
	HEDGING Cs=1.5°C					1	1	1
	HEDGING Cs=3°C					1	1	1
	HEDGING Cs=5°C	0	0	0	0	1	1	2
	HEDGING Cs=8°C					3	7	22
TOTAL	BASE, PF Cs=1.5°C	41	45	56	72	118	162	222
	PF Cs=3°C	41	45	56	72	118	161	219
	PF Cs=5°C	42	45	56	70	114	155	214
	PF Cs=8°C	42	44	54	68	115	162	239
	HEDGING Cs=1.5°C					117	162	222
	HEDGING Cs=3°C					117	162	222
	HEDGING Cs=5°C	42	45	56	70	114	155	214
	HEDGING Cs=8°C					117	166	257

Moreover, hydroelectricity (and, to a lesser extent, wind power too) and the shutdown of coal plants without CCS might qualify as hedging actions, since they appear before 2040. This is not the case of either power plants with CCS or nuclear plants.

In conclusion, the hedging strategy is significantly different from any of the PF strategies, which confirms the relevance of using stochastic programming.

4.5.3 End-Use Sectors

In *transportation*, Hedging stays close to the PF strategies and even to the Base case before 2040. This is due to two causes: first, vehicles have a rather limited technical life, so that pre-2040 decisions do not have a lasting effect after resolution time. The other important cause of the observed insensitivity of this sector is that the CO_2 price signal is simply not strong enough before 2040 to warrant a large departure from traditional fuels. After resolution time, of course, the strategies do differ, and they do so in a fairly predictable way: the larger C_s values entail smaller market shares for RPP's and larger for alcohols and natural gas. Electricity keeps a very limited market share, and hydrogen (mainly produced by plants with CCS) makes a belated and small appearance in 2090 only in the most extreme branch of the Hedging).

The hedging strategy in *residential and commercial* buildings is characterized by very few energy changes compared to base case before 2040, and by an increase of electricity after 2040 (replacing natural gas and RPPs) in the most severe branches of the Hedging, mainly for space heating purposes.

In *industry*, differences between Hedging and Base case actions are slight before 2040. The exception being that N_2O abatement options in adipic and nitric acid industries penetrate as early as 2005 in the hedging strategy and thus are are robust decisions. After 2040, natural gas and, to a lesser extent, electricity, replace coal after 2040 in the most stringent branches of the hedging, mainly in chemical and other industry sub-sectors.

Demands (and thus economic activity) are affected by the introduction of the climate target, since the rising GHG price induces a rise in demand prices, and thus a decrease in demand levels. Their reduction starts as soon as 2005, remaining small until 2030 and reaching up to 5% in buildings and 6% in industry in the longer term.

The reduction of *upstream* emissions until 2030 is the result of both changes in the primary energy structure driven by final energy changes (for example, CO_2 and CH_4 reduction in coal extraction), and of specific GHG abatement measures (for example, degasification and pipeline injection of CH_4 in coal sector, inspection and maintenance of gas distribution facilities, flaring instead of venting, etc.) In fact, a few CH_4 reduction options appear to be non-regret measures and penetrate even in the base case.

Finally, CH_4 capture options in *landfill* and, to a lesser extent, *manure* emission abatement measures also appear to be either non-regret or robust (penetration before 2040 in the hedging strategy). In fact, we observe that the relative CH_4 reduction is more important than the CO_2 reduction in the short term, due to the availability of these low-cost CH_4 capture options in upstream and landfills. This result is in line with the literature (e.g. [7]).

4.6 Super-Hedging Actions

A "super-hedging" action is an action that penetrates more in the hedging strategy than in *any* of the perfect forecast strategies. The existence of such an action seems counter-intuitive, since it lies outside limits defined by the perfect forecast strategies, but such actions nevertheless do exist, which confirms that stochastic analysis of future climate strategies may propose decisions that are beyond any combination of the deterministic strategies [9].

Electricity production from renewables, fuel switches in industry (to biomass and gas), consumption of ethanol in several subsectors, consumption of geothermal in commercial buildings and biomass in residential buildings, and finally CH_4 abatement actions are all super-hedging actions.

5 Sensitivity Analyses

Sensitivity analyses were undertaken on: the exogenous radiative forcing, the very long term emission profile, the date of resolution of uncertainties, the pace of development of nuclear power plants, and the own price elasticities of demands.

In our main experiment, the assumed value of *exogenous forcing* in ETSAP-TIAM is 0.4 W/m^2 indefinitely, a fairly large value. Such forcing accounts for substances in the atmosphere that are not explicitly modeled in TIAM, and also for a possible increase in solar activity. We conducted a simple calculation: keeping the Hedging strategy calculated in Section 4, we simply varied the value of the residual exogenous forcing from 0 to 0.8 W/m^2, and we re-calculated (outside the model) the resulting temperature changes. We found that the resulting equilibrium temperature of the Hedging strategy remains less than 2.5°C across most of the range, reaching 2.8°C for the highest value (0.8) of the exogenous forcing. Although these temperature shifts are not negligible, they do not drastically depart from the temperature changes observed in the main hedging strategy.

Changing the assumption about the *post-2100 emission curve* (for instance extending the period of emission decrease to 200 years instead of 100 years), has of course no impact on the equilibrium temperature, but has an impact on the peak temperature. However, this impact remains very small. This analysis is most reassuring, as it tends to confirm that emission policies beyond 2100 have a small impact on temperature increase, as long as a policy of eradicating all emissions is followed, irrespective of the speed of that eradication.

Advancing the date at which the climate uncertainty is resolved to 2020 (instead of 2040) results in welfare savings of 159 B$, i.e. a full 3/4 of the EVPI. Such an analysis may provide a useful guide in deciding research expenditures in the climate change domain.

There may be societal and political reasons that may warrant limiting the degree of *penetration of nuclear power*. Therefore, we have undertaken sensitivity analyses on both the level of nuclear power in the base case and on the maximum allowed level of nuclear energy. In both cases, other reduction options (wind, solar, biomass etc.) penetrate to replace the nuclear loss, and the loss of surplus of the new hedging strategy is moderately increased. This confirms that *nuclear per se does not qualify as a robust abatement option* but also that the limitation of nuclear penetration does not seriously compromise the possibilities to satisfy a 2.5°C target at an "acceptable" cost.

Finally, if *demand elasticities* are set to 0, the expected loss of total surplus of the hedging strategy increases by almost 15%, and the marginal cost of GHG reduction is around 19% higher compared to the hedging strategy with elastic demands (higher electricity consumption, higher penetration of hydrogen and natural gas in the transportation sector, higher penetration of low emitting power plants etc.). Moreover, the reduction of emissions starts earlier, so that emissions are smaller before 2040 and higher in the long term compared to the hedging strategy with elastic demands.

6 Alternate Criteria for Stochastic Programming

The Expected Cost criterion, although widely used, is justified only if the policy maker is risk neutral. Risk neutrality usually applies when the payoffs attached to each outcome stay within a reasonably narrow range, and when they do not represent very large losses or gains as compared to the mean payoff. In all other cases, considerations of risk aversion or risk preference should be introduced in the criterion to be optimized. In the most general terms, the policy maker should choose a criterion that represents her utility function. We review here two such utility functions, both well-known, although not often utilized in large scale models.

6.1 Expected Cost Criterion with Risk Aversion

6.1.1 E-V Approach

The E-V model (an abbreviation for Expected Value-Variance) was pioneered by Harry Markowitz [13] for applications in financial portfolios.

In the E-V approach, it is assumed that the variance of the cost is an acceptable measure of the risk attached to a strategy in the presence of uncertainty. The variance of the cost $\boldsymbol{C_s}$ of a given strategy $\boldsymbol{s}$ is computed as follows:

$$Var(C_s) = \sum_j p_j \cdot (Cost_{j|s} - EC_s)^2$$

where $\boldsymbol{Cost_{j|s}}$ is the cost when strategy $\boldsymbol{s}$ is followed and the $\boldsymbol{j^{th}}$ state of nature prevails, and $\boldsymbol{EC_s}$ is the expected cost of strategy $\boldsymbol{s}$, defined as usual by:

$$EC_s = \sum_j p_j \cdot Cost_{j|s}$$

The E-V approach thus replaces the expected cost criterion by the following utility function to minimize:

$$U = EC + \lambda \cdot \sqrt{Var(C)}$$

where $\lambda > 0$ is a measure of the risk aversion of the decision maker. For $\lambda = 0$, the usual expected cost criterion is obtained. Larger values of λ indicate increasing risk aversion.

Taking risk aversion into account by this formulation would lead to a non-linear, non-convex model, with all its ensuing computational restrictions. These would impose serious limitations on model size. We therefore propose a linearized version that is more suitable for large scale applications.

6.1.2 Utility Function with Linearized Risk Aversion

This criterion mimics the E-V approach while eliminating two drawbacks. First, it eliminates the lower part of the range of variation of the cost, which indeed should not be considered as a risk. Second, it is linear and thus compatible with the overall LP model. To avoid non-linearity, it is possible to replace the semi-variance by the Upper-absolute-deviation, defined by:

$$UpAbsDev\ (Cost_s) = \sum_j p_j \cdot \left\{Cost_{j|s} - EC_s\right\}^+$$

where $y = \{x\}^+$ is defined by the following two *linear* constraints:

$$y \geq x, y \geq 0$$

and the utility function is now written as the following *linear* expression:

$$U = EC + \lambda \cdot UpsAbsDev\ (C)$$

This is the expected utility formulation implemented into the TIMES model generator.

6.2 The Savage Criterion (Minimax Regret)

The Minimax Regret Criterion, also known as Savage Criterion [16], is one of the more credible criteria for selecting public policies when the likelihoods of the various possible outcomes are not known with sufficient precision to use the classical expected value or expected utility criteria. In order to fix ideas, we shall assume in what follows that a certain decision problem is couched in terms of a cost to minimize (a symmetric formulation is obtained in the case of a payoff to maximize). We may thus denote by $C(z, s)$ the cost incurred when strategy s is used, and outcome z occurs. The Regret $R(z, s)$ is defined as the difference between the cost incurred with the pair (z, s) and the least cost achievable under outcome perfect information on z, i.e.:

$$R(z,s) = C(z,s) - \underset{t \in S}{Min}\, C(z,t), \qquad \forall z \in Z, \quad s \in S$$

where Z is the set of possible outcomes and S is the set of feasible strategies. Note that, by construction, a regret $R(z, s)$ is always non negative, and that it has value 0 for one outcome at least. Note also that the term 'regret' is particularly well chosen as it does quantify how much the policy maker would regret having chosen strategy $\boldsymbol{s}$ when outcome $\boldsymbol{z}$ occurs.

A *Minimax Regret (MMR) strategy* is any $s*$ that minimizes the worst regret:

$$s* \in \underset{s \in S}{ArgMin}\{\underset{z \in Z}{Max}\, R(z,s)\}$$

and the corresponding Minimax Regret is equal to:

$$MMR = \underset{s \in S}{Min}\{\underset{z \in Z}{Max}\, R(z,s)\}$$

6.3 Application of Minimax Regret in Large Scale Linear Programs

We now turn to the application of the above definition to the case when the cost $C(z, s)$ is not an explicit expression. Rather, it is implicitly computed via an optimization program. This is the case in particular when using a model such as TIMES, where the long term energy system least cost (i.e. max surplus) is computed by solving a large scale linear program. The notation of Section 3 is used again, with appropriate changes. In the formulation 5 below, the A matrix defines the very large number of techno-economic constraints of the model, and the last group of constraints have some uncertain parameters as coefficients or as right hand sides. We assume for simplicity of notation, that all uncertainties are in the RHS, D, and that they are all resolved at the same date denoted by t^*. In the absence of uncertainty, the TIMES linear program has the following structure:

$$\begin{aligned} &\textbf{Maximize } C \times X \\ &\textbf{Subject to:} \\ &A \times X \geq b \\ &E \times X \leq D \end{aligned} \tag{5}$$

Assuming now that the RHS, D, is uncertain, and that the uncertainty is resolved at time t^*, we observe that prior to t^*, all decisions are taken under uncertainty, whereas at t^* and later, decisions are taken under perfect knowledge of the value of D. It is convenient to decompose the vector X of decision variables into two vectors (X_1 and X_2), X_1 representing the decisions to be taken prior to t^*, and X_2 those decisions at t^* and later. We shall assume that the uncertain vector D may take an arbitrary but finite number n of distinct values: $D_1, D_2, \ldots, D_n$.

We denote by $M(D)$ the minimum value obtained from the minimization of the above LP when vector D is known. This is the same as calculating the minimum cost under perfect information on D. We therefore may now theoretically calculate the *Regret* of strategy X as follows:

$$R(X, D_i) = C^t X - M(D_i)$$

And the maximum regret of strategy X as:

$$Max_i \left\{ C^t X - M(D_i) \right\}$$

Finally, the Minimax Regret strategy is an optimal solution to the following optimization program:

$$\begin{aligned} MMR = & \underset{X_1, X_2}{Min} \underset{i}{Max} \left[C_1{}^t X_1 + C_2{}^t X_2^i - M(D_i)\right] \\ & s.t. \quad A_1 X_1 + A_2 X_2^i \geq b, \qquad i = 1, 2, \ldots, n \\ & \qquad E_1{}^t X_1 + E_2{}^t X_2^i \leq D_i, \qquad i = 1, 2, \ldots, n \end{aligned} \tag{6}$$

The above program is not quite an L.P., but may be converted into one by introducing a new variable:

$$\begin{aligned} MMR = & \underset{X_1, X_2, \phi}{Min} [\phi] \\ & s.t. \quad \phi \geq C_1{}^t X_1 + C_2{}^t X_2^i - M(D_i), \quad \forall i \\ & \qquad A_1 X_1 + A_2 X_2^i \geq b, \quad \forall i \\ & \qquad E_1 X_1 + E_2^i \leq D_i, \quad \forall i \end{aligned} \tag{7}$$

Note carefully that a *bona fide* strategy X is such that X_1 is common to all outcomes D, whereas there is a different vector X_2^i for each outcome D_i. This is so because decisions made at t^* and later take into account the knowledge of the true value of D that realizes at t^*. Hence, the LP (8) has up to n replications of the constraints, and of the X_2 variables (to be more precise, all constraints which do not involve X_2 variables are not replicated, and therefore, the size of (7) may be significantly smaller than n times the size of LP (5).

Important remark: An unfortunate phenomenon occurs when (7) is solved: since all that matters when computing the *MMR* strategy is indeed the value of the Minimax Regret, all other regrets are left free to take any values, as long as these values remain below the *MMR*. This remark is equivalent to saying that (7) is highly degenerate. For example, in one instance reported in [11], the *MMR* is equal to 3,311 M$, but when (7) is solved, it turns out that *each* of the n regrets (i.e. each right-hand-side of the first constraints of (7)), is found to be also equal to 3,311 M$. This is undesirable, as in practice, depending upon the actual value of z which realizes, the regret can be quite much lower than *MMR*. In order to remove the dual degeneracy, it is useful to proceed in two phases: first, consider (8) as essentially only a way of computing the partial strategy up to the resolution date, i.e. X_1. Next, when this is done, each X_2^i may be computed independently by (a) fixing X_1 at its optimal value (call it X_1^*), and (b) for each i, solving the following linear program:

$$\begin{aligned} & \underset{x_2}{Min} \left[c_1{}^t X_1^* + c_2{}^t X_2 - M(D_i)\right] \\ & s.t. \qquad A_1 X_1^* + A_2 X_2 \geq b \\ & \qquad E_1{}^t X_1^* + E_2{}^t X_2 \leq D_i \end{aligned} \tag{8}$$

The largest LP to solve is clearly (7), which has the same approximate size as a classical stochastic LP defined on the same problem instance, and using the expected value criterion. In addition, $n-1$ smaller problems (4) must be solved, in order to compute the $n-1$ non degenerate strategies after date t.

7 Conclusion

In this article, the classical Stochastic Programming technique is presented and applied to large scale instances of the Integrated Assessment Model (ETSAP-TIAM). The instances solved and discussed lead to the long term analysis of climate stabilization strategies under high uncertainty of climate sensitivity *Cs* (in the range 1.5 to 8°C) and of economic growth (simple-to-double GDP growth rates from 2040). Both uncertainties are assumed to be resolved in 2040. The methodology relies on the computation of a hedging strategy based on the maximization under uncertainty (via Stochastic Programming) of total World surplus over the 21st century. The properties of the resulting strategies are stressed, and a class of hedging and super-hedging actions is identified.

Amongst the most noticeable results, the model reveals that the smallest achievable temperature increase is close to 1.9°C, albeit at a very large cost, by a combination of energy switching, capture and storage of

CO_2, CO_2 sequestration by forests and non-CO_2 emission reduction options. This means that more severe temperature targets would require additional GHG abatement potential that is currently not yet seen as realistic. Moreover, the impact of uncertainty of the climate sensitivity parameter C_s is major, requiring the implementation of early actions (before 2040) in order to reach the temperature target. In other words, the "wait and see" approach is not recommended. Robust abatement options include: substitution of coal power plants by hydroelectricity, sequestration by forests, CH_4 and N_2O reduction. Nuclear power plants, electricity production with CCS, and end-use fuel substitution do not belong to early actions. Among them, several options appear also to be super-hedging actions i.e. they penetrate more in the hedging strategy than in any of the perfect forecast strategies (e.g. hydroelectricity, CH_4 reduction), proving that stochastic analyze of future climate strategies might give insights that are beyond any combination of the deterministic strategies. In contrast, the uncertainty of the GDP growth rates has very little impact on pre-2040 decisions. This insensitivity is a pleasant surprise, as it shows that the hedging strategy for only one random parameter (C_s) is also a quasi-optimal strategy when the two types of uncertainty are present.

The comparison of hedging with perfect forecast strategies shows that a deterministic strategy with Cs=5°C is closest to the hedging strategy. However, the two differ in several key aspects, and this confirms the relevance of using stochastic programming in order to analyze preferred climate policies in an uncertain world where the correct climate response is known only far into the future. In particular, the perfect forecast strategy provides a poor approximation of the optimal electricity production mix, of the price of carbon, and of the penetration of several sequestration options.

Among the more sensitive parameters of the problem, resolving the uncertainties in 2020 rather than 2040 induces a 19% reduction in the loss of expected surplus, and keeping the same hedging strategy while assuming a doubling of the exogenous forcing has a non negligible (although moderate) raises global temperature by 0.3°C.

Two modifications of the criterion expected cost used in Stochastic Programming are explicited: the first one is the Expected value-semi-variance criterion, linearized in order to be integrated into the ETSAP-TIAM model. The second is the Savage criterion consisting in minimizing the maximum regret. This criterion requires a major modification of the original LP underlying the TIAM model.

Future work will operationalize the Minimax Regret criterion as an integral option of the TIAM model.

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