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On Stability and Stabilization of Continuous-Time Singular Markovian Switching Systems

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Abstract

This paper deals with the class of continuous-time singular linear systems with Markovian switching. Under full and partial knowledge of the jump rates of the continuous-time Markov process sufficient conditions in the LMI setting for the system to be piecewise regular, impulse-free and stochastically stable are developed. A state feedback controller that makes the closed-loop system piecewise regular, impulse-free and stochastically stable is also designed. Numerical examples are proposed to show the validness of the developed results.

Key Words: Singular systems, Descriptor systems, Jump linear systems, Linear matrix inequality, Stochastic stability, Stochastic stabilizability, Partial knowledge of the jump rates.

Résumé

Cet article traite de la classe des systèmes singuliers à sauts markoviens. Sous les hypothèses des connaissances partielles ou totales des taux de transitions du système, des conditions suffisantes en forme de LMI sont proposées pour garantir que le système soit par morceaux régulier, sans impulsion et stochastiquement stable. Une procédure de design du contrôleur par retour d'état est aussi proposée pour garantir le même objectif pour la boucle fermée du système. Des exemples numériques sont employés pour montrer la validité des résultats développés.

1 Introduction

During the last decades, the class of stochastic systems driven by continuous-time Markov chains has been used to model many practical systems, where random failures and repairs and sudden environment changes may occur. For more detail on what it has been done on the subject, we refer the reader to [1, 2], and the references therein. This class of systems has also attracted a lot of researchers from both mathematical and control community. Many results on stochastic stability and stochastic stabilization have been reported to the literature. For more details on these results we refer the reader to [1, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12] and the references therein, where different approaches have been used. The $\mathscr{H}_\infty$ control problem was investigated in [13, 14], where sufficient conditions for the solvability of this problem was proposed. For Markovian jump system with time-delays, the results on stability analysis and $\mathscr{H}_\infty$ control were also reported in [15], [16] and [17] for different types of time delays. For more detail on Markovian jumping systems with time delay, we refer the reader to [2] and the references therein.

In parallel, there have been also considerable research efforts on the study of singular systems. This is due to the extensive applications of singular systems in many practical systems, such as circuits boundary control systems, chemical processes, and other areas for more details of this, we refer the reader to [18, 19, 20, 21] and the references therein. Singular systems are also referred to as descriptor systems, implicit systems, generalized state-space systems, differential-algebraic systems or semi-state systems (see [18, 20]). A great number of fundamental notions and results in control and systems theory based on state-space systems have been successfully extended to singular systems; see, e.g., [22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32], and the references therein.

However, up to date and to the best of our knowledge, the class of singular systems with Markovian switching has not yet been fully investigated and this will be the goal of this paper. On the top of this, most of the results reported in the literature even for normal Markov jump systems require the complete knowledge of the Markov process that describes the behavior the system mode. Most of the results developed for this class of systems are not easily tractable for more details we refer the reader to [33, 34, 35] where different approaches have been used to get the established results that are totally different from the one of this paper. In this paper, we will mainly concentrate on the stochastic stability, the robust stochastic stability and stabilization of such class of systems. Firstly a sufficient condition, in the linear matrix inequality (LMI) setting is developed to check if a given system of this class of systems, is piecewise regular, impulse-free and stochastically stable. Based on this, a sufficient condition for the robust stochastic stability that can also been used to check if a given uncertain system of the class under consideration is piecewise regular, impulse-free and stochastically stable is also proposed. A design state feedback controller such that the closed-loop system is piecewise regular, impulse-free and stochastically stable. Complete and partial knowledge of the jump rates are considered. Finally, numerical examples are provided to demonstrate the effectiveness of the proposed methods.

The rest of this paper is organized as follows. In Section 2, the problem is formulated and the goal of the paper is stated. In Section 3, the main results are given and these include results on stochastic stability, robust stochastic stability and stochastic stabilization. Section 4 presents numerical examples to show the usefulness of the developed theoretical results.

Notation. Throughout this paper, $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote, respectively, the n dimensional Euclidean space and the set of all $n \times m$ real matrices. The superscript "$^\top$" denotes matrix transposition and the notation $X \geq Y$ (respectively, $X > Y$) where X and Y are symmetric matrices, means that $X - Y$ is positive semi-definite (respectively, positive definite). $\mathbb{I}$ is the identity matrices with compatible dimensions. $\mathbb{E}\{\cdot\}$ denotes the expectation operator with respective to some probability measure $\mathcal{P}$.

2 Problem statement

Consider a stochastic switching system with N modes, i.e., $\mathscr{S} = \{1, 2, \ldots, N\}$. The mode switching is assumed to be governed by a continuous-time Markov process $\{r_t, t \geq 0\}$ taking values in the state space $\mathscr{S}$ and having the following infinitesimal generator

$$\Lambda = (\lambda_{ij}), i, j \in \mathscr{S},$$

where $\lambda_{ij} \geq 0, \forall j \neq i, \lambda_{ii} = -\sum_{j\neq i} \lambda_{ij}$.

The mode transition probabilities are described as follows:

$$P[r_{t+\Delta} = j | r_t = i] = \begin{cases} \lambda_{ij}\Delta + o(\Delta), & j \neq i \\ 1 + \lambda_{ii}\Delta + o(\Delta), & j = i \end{cases} \tag{1}$$

where $\lim_{\Delta\to 0} o(\Delta)/\Delta = 0$.

Let the class of Markovian switching singular systems be defined in a fundamental probability space $(\Omega, \mathcal{F}, \mathbb{P})$ and assume that its behavior is described by the following dynamics:

$$\begin{cases} E(r_t)\dot{x}(t) = A(r_t, t)x(t) + B(r_t, t)u(t), \\ x(s) = x_0 \end{cases} \tag{2}$$

where $x(t) \in \mathbb{R}^n$ is the system state, $u(t) \in \mathbb{R}^m$ is the input system, $A(r_t, t)$ and $B(r_t, t)$ are assumed to have uncertainties, i.e.: $A(r_t, t) = A(r_t) + D_A(r_t)F_A(r_t)E_A(r_t)$ and $B(r_t, t) = B(r_t) + D_B(r_t)F_B(r_t)E_B(r_t)$ with $A(r_t)$, $D_A(r_t)$, $E_A(r_t)$, $B(r_t)$, $D_B(r_t)$ and $E_B(r_t)$ are known real matrices with appropriate dimensions for each $r_t \in \mathscr{S}$, $F_A(r_t)$ and $F_B(r_t)$ satisfies $F_A^\top(i)F_A(i) \leq \mathbb{I}$ and $F_B^\top(i)F_B(i) \leq \mathbb{I}$ for each $i \in \mathscr{S}$, the matrix $E(i)$ may be singular, and we assume $0 \leqslant \text{rank}(E(i)) = n_E < n$.

Remark 2.1 *When the uncertainties are equal to zero the system will be referred to as nominal system. The uncertainties that satisfies the previous conditions are referred to as admissible. The uncertainties we are considering in this paper are known in the literature as norm bounded uncertainties. When the matrix $E(i)$, $i \in \mathscr{S}$ is nonsingular the system (2) is referred to as normal system.*

Definition 2.1 *[18]*

i. Nominal system (2) is said to be regular if the characteristic polynomial, $\det(sE - A(i))$ *is not identically zero for each mode $i \in \mathscr{S}$.*

ii. Nominal system (2) is said to be impulse-free, i.e. the $\deg(\det(sE - A(i))) = \text{rank}(E)$ *for each mode $i \in \mathscr{S}$.*

For more details on other properties and the existence of the solution of system (2), we refer the reader to [36, 24], and the references therein. In general, the regularity is often a sufficient condition for the analysis and the synthesis of singular systems.

For the system (2), we have the following definitions:

Definition 2.2 *Nominal system (2) with $u(t) = 0, t \geq 0$ is said to be stochastically stable if there exists a constant $T(x_0, r_0)$ such that*

$$\mathbb{E}\left[\int_0^\infty \|x(t)\|^2 dt \middle| r_0, x(0) = x_0\right] \leq T(x_0, r_0); \tag{3}$$

Definition 2.3 *Uncertain system (2) with $u(t) = 0, t \geq 0$ is said to be robust stochastically stable if there exists a constant $T(x_0, r_0)$ such that (3) holds for all admissible uncertainties.*

The controller we will use in this paper is assumed to have the following structure:

$$u(t) = K(r_t)x(t), \tag{4}$$

with $K(i) \in \mathbb{R}^{m \times n}$, $i \in \mathscr{S}$, a constant matrix to be determined.

Definition 2.4 *Nominal system (2) is said to be stochastically stabilizable if there exists a control of the form (4) such that the closed-loop system is stochastically stable.*

The definition of robust stochastic stabilizability is given by:

Definition 2.5 *System (2) is said to be robust stochastically stabilizable if there exists a control of the form (4) such that the closed-loop system is stochastically stable for all admissible uncertainties.*

Remark 2.2 *Notice that the class of singular Markovian jump systems we are considering in this paper presents discontinuities when the mode jumps. For more details on the subject we refer the reader to Boukas [36].*

Combining the system dynamics and the controller expression, we get the following closed-loop dynamics:

$$E(r_t)\dot{x}(t) = A_{cl}(r_t)x(t), \tag{5}$$

where $A_{cl}(r_t) = A(t_t) + D_A(r_t)F_A(r_t)E_A(r_t) + B(r_t)K(r_t) + D_B(r_t)F_B(r_t)E_B(r_t)K(r_t)$ with $K(r_t)$ is the controller gain that we have to compute.

Assumption 2.1 *The jump rates are assumed to satisfy the following:*

$$0 < \underline{\lambda}_i \leq \lambda_{ij} \leq \bar{\lambda}_i, \forall i, j \in \mathscr{S}, j \neq i \tag{6}$$

where $\underline{\lambda}_i$ and $\bar{\lambda}_i$ are known parameters for each mode or may represent the lower and upper bounds when all the jump rates are known, i.e.: $0 < \underline{\lambda}_i = \min_{j \in \mathscr{S}} \{\lambda_{ij}, i \neq j\}$, $0 < \bar{\lambda}_i = \max_{j \in \mathscr{S}} \{\lambda_{ij}, i \neq j\}$, with $\underline{\lambda}_i \leq \bar{\lambda}_i$.

Remark 2.3 *For most of the practical systems, the transition rates can not easily be obtained and in general require more time and a huge amount of money that we should pay to accomplish the experiment that will give such rates. Therefore, results that relax the knowledge of the transition rates are more appropriate for practical systems.*

In this paper we are interested in developing LMI conditions that can be used to check if a given system is piecewise regular, impulse-free and stochastically stable. The stabilization problem is also considered. The robust problems in case of the presence on norm bounded uncertainties are also tackled. Full and partial knowledge of the jump rate are considered in this paper. The conditions we will develop here will be in terms of the solutions to linear matrix inequalities that can be easily obtained using LMI control toolbox.

Before closing this section, let us recall some lemmas that we will be using in the rest of the paper.

Lemma 2.1 *[2] Let H and G be given matrices with appropriate dimensions and F satisfying $F^\top F \leq \mathbb{I}$. Then, we have for any $\varepsilon > 0$,*

$$HFG + G^\top F^\top H^\top \leq \varepsilon H H^\top + \frac{1}{\varepsilon} G^\top G.$$

Lemma 2.2 *(Schur complement Lemma) [2] The linear matrix inequality*

$$\begin{bmatrix} H & S^\top \\ S & R \end{bmatrix} > 0$$

is equivalent to

$$R > 0, H - S^\top R^{-1} S > 0$$

where $H = H^\top$, $R = R^\top$ and S is a matrix with appropriate dimension.

3 Main results

In this section, we start by developing results that assure that the nominal system (2) is piecewise regular, impulse-free and stochastically stable. Then, these results are extended to the case of uncertain systems. LMI conditions are established to check either if a nominal system or an uncertain system is piecewise regular, impulse-free and stochastically stable. The stabilization problem is also tackled using a state feedback controller. LMI conditions to design the state feedback that makes the closed-loop system piecewise regular, impulse-free and stochastically stable when the jump rates are completely known or partially known.

3.1 Complete knowledge of the jump rates

Let us now consider the nominal system and see under which conditions the corresponding dynamics will be piecewise regular, impulse-free and stochastically stable when we have complete knowledge of the jump rates of the Markov process $\{r_t, t \geq 0\}$. The following lemma gives such results.

Lemma 3.1 *(see Boukas [36]) The nominal singular Markovian jump system (2) with $u(t) = 0, t \geq 0$ is piecewise regular, impulse-free and stochastically stable if there exists a set of nonsingular matrices $P = (P(1), \cdots, P(N))$ such that the following set of coupled LMIs holds for each $i \in \mathscr{S}$:*

$$P^\top(i)A(i) + A^\top(i)P(i) + \sum_{j=1}^{N} \lambda_{ij} E^\top(j)P(j) < 0. \tag{7}$$

with the following constraints:

$$E^\top(i)P(i) = P^\top(i)E(i) \geq 0. \tag{8}$$

Let us now concentrate on the robust stochastic stability of our unforced system and develop sufficient conditions that guarantee that the uncertain system will be piecewise regular, impulse-free and stochastically stable for all admissible uncertainties. For this purpose, using the results of Lemma 3.1, the dynamics will be piecewise regular, impulse free and stochastically stable if there exists a set of nonsingular matrices $P = (P(1), \cdots, P(N))$ such the following hold for each $i \in \mathscr{S}$:

$$\begin{aligned} &E^\top(i)P(i) = P^\top(i)E(i) \geq 0 \\ &P^\top(i)A(i) + A^\top(i)P(i) + P^\top(i)D_A(i)F_A(i)E_A(i) + E_A^\top(i)F_A^\top(i)D^\top(i)P(i) \\ &\qquad + \sum_{j=1}^N \lambda_{ij}E^\top(j)P(j) < 0. \end{aligned}$$

Using Lemma 2.1, for any $\varepsilon_A(i) > 0$, $i \in \mathscr{S}$ we have:

$$\begin{aligned} &P^\top(i)D_A(i)F_A(i)E_A(i) + E_A^\top(i)F_A^\top(i)D^\top(i)P(i) \\ &\qquad \leq \varepsilon_A^{-1}(i)P^\top(i)D_A(i)D_A^\top(i)P(i) + \varepsilon_A(i)E_A^\top(i)E_A(i) \end{aligned}$$

Using now this inequality and Lemma 2.2, we get the following results for the robust stochastic stability.

Lemma 3.2 *The uncertain singular Markovian switching system (2) with $u(t) = 0, t \geq 0$ is piecewise regular, impulse-free and stochastically stable if there exist a set of nonsingular matrices $P = (P(1), \cdots, P(N))$ and a set of positive scalars $\varepsilon_A = (\varepsilon_A(1), \cdots, \varepsilon_A(N))$ such that the following set of coupled LMIs holds for each $i \in \mathscr{S}$ and for all admissible uncertainties:*

$$\begin{bmatrix} J(i) & P^\top(i)D_A(i) \\ D_A^\top(i)P(i) & -\varepsilon_A(i)\mathbb{I} \end{bmatrix} < 0. \tag{9}$$

where $J(i) = P^\top(i)A(i) + A^\top(i)P(i) + \sum_{j=1}^N \lambda_{ij}E^\top(j)P(j) + \varepsilon_A(i)E^\top(i)E_A(i)$; with the constraints (8)

In Lemma 3.1 and Lemma 3.2, we have developed results that allow us to check if a given unforced system of the class we are considering is piecewise regular, impulse-free and stochastically stable either for nominal and uncertain cases. In the rest of this section we will focus of the design of the state feedback controller of the form (4). To design this state feedback controller, we need to transform the results of the previous lemmas since the term $\sum_{j=1}^N \lambda_{ij}E^\top(j)P(j)$ poses some problems. For this purpose, notice that using Lemma 2.1, we have:

$$E^\top(j)P(j) \leq \frac{\varepsilon^{-1}(j)}{4}\mathbb{I} + \varepsilon(j)E^\top(j)P(j)P^\top(j)E(j)$$

for any $\varepsilon(i) > 0$.

Let $X(i) = P^{-1}(i)$. Firstly, for the nominal system pre- and post-multiply respectively (7)–(8) by $X^\top(i)$ and $X(i)$, we get:

$$\begin{bmatrix} J_1(i) & \mathscr{Z}_i(X) & \mathscr{S}_i(X) \\ \mathscr{Z}_i^\top(X) & -\mathscr{X}_i(\varepsilon) & 0 \\ \mathscr{S}_i^\top(X) & 0 & -\mathscr{W}_i(X) \end{bmatrix} < 0$$

$$X^\top(i)E^\top(i) = X(i)E^\top(i) \geq 0$$

where

$$
\begin{aligned}
J_1(i) &= A(i)X(i) + X^\top(i)A^\top(i) + \lambda_{ii}X^\top(i)E^\top(i) \\
\mathscr{Z}_i(X) &= \left[\begin{array}{cccccc} \sqrt{\lambda_{i1}}X^\top(i) & \cdots & \sqrt{\lambda_{ii-1}}X^\top(i) & \sqrt{\lambda_{ii+1}}X^\top(i) & \cdots & \sqrt{\lambda_{iN}}X^\top(i) \end{array}\right] \\
\mathscr{S}_i(X) &= \left[\begin{array}{ccc} \sqrt{\lambda_{i1}}X^\top(i)E^\top(1) & \cdots & \sqrt{\lambda_{ii-1}}X^\top(i)E^\top(i-1) \end{array}\right. \\
&\qquad \left.\begin{array}{ccc} \sqrt{\lambda_{ii+1}}X^\top(i)E^\top(i+1) & \cdots & \sqrt{\lambda_{iN}}X^\top(i)E^\top(N) \end{array}\right] \\
\mathscr{X}_i(\varepsilon) &= \mathrm{diag}\left[4\varepsilon(1)\mathbb{I}, \cdots, 4\varepsilon(i-1)\mathbb{I}, 4\varepsilon(i+1)\mathbb{I}, \cdots, 4\varepsilon(N)\mathbb{I}\right] \\
\mathscr{W}_i(X) &= \mathrm{diag}\Big[\varepsilon^{-1}(1)X^\top(1)X(1), \cdots, \varepsilon^{-1}(i-1)X^\top(i-1)X(i-1), \\
&\qquad \varepsilon^{-1}(i+1)X^\top(i+1)X(i+1), \cdots, \varepsilon^{-1}(N)X^\top(N)X(N)\Big]
\end{aligned}
$$

Using now the fact that:

$$
\varepsilon^{-1}(i)X^\top(i)X(i) \le X^\top(i) + X(i) - \varepsilon(i)\mathbb{I}
$$

we get the following results:

Lemma 3.3 *The nominal singular Markovian jump unforced system (2) is piecewise regular, impulse-free and stochastically stable if there exist a set of nonsingular matrices $X = (X(1), \cdots, X(N))$ and a set of positive scalars $\varepsilon = (\varepsilon(1), \cdots, \varepsilon(N))$ such that the following set of coupled LMIs holds for each $i \in \mathscr{S}$:*

$$
\begin{bmatrix} J_1(i) & \mathscr{Z}_i(X) & \mathscr{S}_i(X) \\ \mathscr{Z}_i^\top(X) & -\mathscr{X}_i(\varepsilon) & 0 \\ \mathscr{S}_i^\top(X) & 0 & -\mathscr{X}_i(X) \end{bmatrix} < 0 \tag{10}
$$

where

$$
\begin{aligned}
J_1(i) &= A(i)X(i) + X^\top(i)A^\top(i) + \lambda_{ii}X^\top(i)E^\top(i) \\
\mathscr{Z}_i(X) &= \left[\begin{array}{cccccc} \sqrt{\lambda_{i1}}X^\top(i) & \cdots & \sqrt{\lambda_{ii-1}}X^\top(i) & \sqrt{\lambda_{ii+1}}X^\top(i) & \cdots & \sqrt{\lambda_{iN}}X^\top(i) \end{array}\right] \\
\mathscr{S}_i(X) &= \left[\begin{array}{ccc} \sqrt{\lambda_{i1}}X^\top(i)E^\top(1) & \cdots & \sqrt{\lambda_{ii-1}}X^\top(i)E^\top(i-1) \end{array}\right. \\
&\qquad \left.\begin{array}{ccc} \sqrt{\lambda_{ii+1}}X^\top(i)E^\top(i+1) & \cdots & \sqrt{\lambda_{iN}}X^\top(i)E^\top(N) \end{array}\right] \\
\mathscr{X}_i(\varepsilon) &= \mathrm{diag}\left[4\varepsilon(1)\mathbb{I}, \cdots, 4\varepsilon(i-1)\mathbb{I}, 4\varepsilon(i+1)\mathbb{I}, \cdots, 4\varepsilon(N)\mathbb{I}\right] \\
\mathscr{X}_i(X) &= \mathrm{diag}\Big[-\varepsilon(1)\mathbb{I} + X^\top(1) + X(1), \cdots, -\varepsilon(i-1)\mathbb{I} + X^\top(i-1) + X(i-1), \\
&\qquad -\varepsilon(i+1)\mathbb{I} + X^\top(i+1) + X(i+1), \cdots, -\varepsilon(N)\mathbb{I} + X^\top(N) + X(N)\Big]
\end{aligned}
$$

with the following constraints:

$$
X^\top(i)E^\top(i) = X(i)E^\top(i) \ge 0. \tag{11}
$$

Similarly, we can follow the same steps as for the nominal case and get the following results for the robust stochastic stability.

Lemma 3.4 *The uncertain singular Markovian switching unforced system (2) with $u(t) = 0, t \ge 0$ is piecewise regular, impulse-free and stochastically stable if there exist a set of nonsingular matrices $X = (X(1), \cdots, X(N))$ and sets of positive scalars $\varepsilon = (\varepsilon(1), \cdots, \varepsilon(N))$ and*

$\varepsilon_A = (\varepsilon_A(1), \cdots, \varepsilon_A(N))$ such that the following set of coupled LMIs holds for each $i \in \mathscr{S}$ and for all admissible uncertainties:

$$\begin{bmatrix} J_1(i) & X^\top(i)E_A^\top(i) & \mathscr{Z}_i(X) & \mathscr{S}_i(X) \\ E_A(i)X(i) & -\varepsilon_A(i)\mathbb{I} & 0 & 0 \\ \mathscr{Z}_i^\top(X) & 0 & -\mathscr{X}_i(\varepsilon) & 0 \\ \mathscr{S}_i^\top(X) & 0 & 0 & -\mathscr{X}_i(X) \end{bmatrix} < 0 \tag{12}$$

where

$$J_1(i) = A(i)X(i) + X^\top(i)A^\top(i) + \lambda_{ii}X^\top(i)E^\top(i) + \varepsilon_A(i)D_A(i)D_A^\top(i)$$

$$\mathscr{Z}_i(X), \mathscr{S}_i(X), \mathscr{X}_i(\varepsilon), \text{ and } \mathscr{X}_i(X) \text{ keep the same definitions as in Lemma 3.3}$$

with the constraints (11).

With the results of Lemma 3.3 and Lemma 3.4, we are now ready to establish the conditions that will allow us to design the state feedback controller of the form (4) either for the nominal and the uncertain cases. For this purpose, let us now consider the closed-loop dynamics of the nominal system and focus on the design of the controller gain. Using the results of Lemma 3.3 and after letting $Y(i) = K(i)X(i)$ and some algebraic manipulations, we get the following theorem.

Theorem 3.1 *There exists a state feedback of the form (4) such that the closed-loop nominal singular Markovian jump system (2) is piecewise regular, impulse-free and stochastically stable if there exist a set of nonsingular matrices $X = (X(1), \cdots, X(N))$, a set of matrices $Y = (Y(1), \cdots, Y(N))$ and a set of positive scalars $\varepsilon = (\varepsilon(1), \cdots, \varepsilon(N))$ such that the following set of coupled LMIs holds for each $i \in \mathscr{S}$:*

$$\begin{bmatrix} J_2(i) & \mathscr{Z}_i(X) & \mathscr{S}_i(X) \\ \mathscr{Z}_i^\top(X) & -\mathscr{X}_i(\varepsilon) & 0 \\ \mathscr{S}_i^\top(X) & 0 & -\mathscr{X}_i(X) \end{bmatrix} < 0 \tag{13}$$

where

$$J_1(i) = A(i)X(i) + X^\top(i)A^\top(i) + B(i)Y(i) + Y^\top(i)B^\top(i) + \lambda_{ii}X^\top(i)E^\top(i)$$

$$\mathscr{Z}_i(X), \mathscr{S}_i(X), \mathscr{X}_i(\varepsilon), \text{ and } \mathscr{X}_i(X) \text{ keep the same definitions as in Lemma 3.3}$$

with the constraints (11). The controller gain $K(i)$ is given by $K(i) = Y(i)X^{-1}(i)$.

For the uncertain case, we proceed similarly as for the nominal case using instead the results of Lemma 3.4 and we get the following results for the robust stochastic stabilization.

Theorem 3.2 *There exists a state feedback of the form (4) such that the closed-loop uncertain singular Markovian switching system (2) is piecewise regular, impulse-free and stochastically stable if there exist a set of nonsingular matrices $X = (X(1), \cdots, X(N))$, a set of matrices $Y = (Y(1), \cdots, Y(N))$ and sets of positive scalars $\varepsilon = (\varepsilon(1), \cdots, \varepsilon(N))$, $\varepsilon_A = (\varepsilon_A(1), \cdots, \varepsilon_A(N))$ and $\varepsilon_B = (\varepsilon_B(1), \cdots, \varepsilon_B(N))$ such that the following set of coupled LMIs holds for each $i \in \mathscr{S}$ and for all admissible uncertainties:*

$$\begin{bmatrix} J_3(i) & X^\top(i)E_A^\top(i) & Y^\top(i)E_B^\top(i) & \mathscr{Z}_i(X) & \mathscr{S}_i(X) \\ E_A(i)X(i) & -\varepsilon_A(i)\mathbb{I} & 0 & 0 & 0 \\ E_B(i)Y(i) & 0 & -\varepsilon_B(i)\mathbb{I} & 0 & 0 \\ \mathscr{Z}_i^\top(X) & 0 & 0 & -\mathscr{X}_i(\varepsilon) & 0 \\ \mathscr{S}_i^\top(X) & 0 & 0 & 0 & -\mathscr{X}_i(X) \end{bmatrix} < 0 \tag{14}$$

where

$$J_3(i) = A(i)X(i) + X^\top(i)A^\top(i) + B(i)Y(i) + Y^\top(i)B^\top(i) + \lambda_{ii}X^\top(i)E^\top(i) + \varepsilon_A(i)D_A(i)D_A^\top(i) + \varepsilon_B(i)D_B(i)D_B^\top(i)$$

$\mathscr{Z}_i(X), \mathscr{S}_i(X), \mathscr{X}_i(\varepsilon),$ *and* $\mathscr{X}_i(X)$ *keep the same definitions as in Lemma 3.3*

with the constraints (11). The controller gain $K(i)$ *is given by* $K(i) = Y(i)X^{-1}(i)$*.*

In this subsection we developed results that required complete access to the jump rates. Practically this will not always be the case and sometimes we have only partial knowledge of the jump rates of the process $\{r_t, t \geq 0\}$ that we can get from some experiments. The results of the next section needs only the knowledge in each mode of two values (the lower and upper bounds of the jump rates). The results we will develop present some conservatism compared to the case of complete knowledge of the jump rates. But it is the price that we should pay.

3.2 Partial knowledge of the jump rates

The aim of this section is to extend the results we developed in the previous section for the stochastic stability or the stochastic stabilization either for the nominal or the uncertain cases when we have only partial knowledge of the transition jump rates. For this purpose, let us now consider firstly the nominal system and see under which conditions the corresponding dynamics will be piecewise regular, impulse-free and stochastically stable when we have only partial knowledge of the jump rates of the Markov process $\{r_t, t \geq 0\}$. Using Assumption 2.1 and proceeding similarly as we did previously we can easily get the following results:

Lemma 3.5 *The nominal singular Markovian jump system (2) with* $u(t) = 0, t \geq 0$ *is piecewise regular, impulse-free and stochastically stable if there exist a set of nonsingular matrices* $X = (X(1), \cdots, X(N))$ *and a set of positive scalars* $\varepsilon = (\varepsilon(1), \cdots, \varepsilon(N))$ *such that the following set of coupled LMIs holds for each* $i \in \mathscr{S}$*:*

$$\begin{bmatrix} J_1(i) & \mathscr{Z}_i(X) & \mathscr{S}_i(X) \\ \mathscr{Z}_i^\top(X) & -\mathscr{X}_i(\varepsilon) & 0 \\ \mathscr{S}_i^\top(X) & 0 & -\mathscr{X}_i(X) \end{bmatrix} < 0 \tag{15}$$

where

$$J_1(i) = A(i)X(i) + X^\top(i)A^\top(i) - (N-1)\underline{\lambda}_i X^\top(i)E^\top(i)$$

$$\mathscr{Z}_i(X) = \begin{bmatrix} \sqrt{\bar{\lambda}_i}X^\top(i) & \cdots & \sqrt{\bar{\lambda}_i}X^\top(i) & \sqrt{\bar{\lambda}_i}X^\top(i) & \cdots & \sqrt{\bar{\lambda}_i}X^\top(i) \end{bmatrix}$$

$$\mathscr{S}_i(X) = \Big[\sqrt{\bar{\lambda}_i}X^\top(i)E^\top(1) \quad \cdots \quad \sqrt{\bar{\lambda}_i}X^\top(i)E^\top(i-1) \quad \sqrt{\bar{\lambda}_i}X^\top(i)E^\top(i+1) \quad \cdots \quad \sqrt{\bar{\lambda}_i}X^\top(i)E^\top(N) \Big]$$

$$\mathscr{X}_i(\varepsilon) = \text{diag}\left[4\varepsilon(1)\mathbb{I}, \cdots, 4\varepsilon(i-1)\mathbb{I}, 4\varepsilon(i+1)\mathbb{I}, \cdots, 4\varepsilon(N)\mathbb{I}\right]$$

$$\mathscr{X}_i(X) = \text{diag}\Big[-\varepsilon(1)\mathbb{I} + X^\top(1) + X(1), \cdots, -\varepsilon(i-1)\mathbb{I} + X^\top(i-1) + X(i-1), -\varepsilon(i+1)\mathbb{I} + X^\top(i+1) + X(i+1), \cdots, -\varepsilon(N)\mathbb{I} + X^\top(N) + X(N)\Big]$$

with the constraints (11).

For the uncertain unforced system, we can follows the same steps as for the nominal case and get the following results for the robust stochastic stability.

Lemma 3.6 *The uncertain singular Markovian switching system (2) with $u(t) = 0, t \geq 0$ is piecewise regular, impulse-free and stochastically stable if there exist a set of nonsingular matrices $X = (X(1), \cdots, X(N))$ and sets of positive scalars $\varepsilon = (\varepsilon(1), \cdots, \varepsilon(N))$ and $\varepsilon_A = (\varepsilon_A(1), \cdots, \varepsilon_A(N))$ such that the following set of coupled LMIs holds for each $i \in \mathscr{S}$ and for all admissible uncertainties:*

$$\begin{bmatrix} J_1(i) & X^\top(i)E_A^\top(i) & \mathscr{Z}_i(X) & \mathscr{S}_i(X) \\ E_A(i)X(i) & -\varepsilon_A(i)\mathbb{I} & 0 & 0 \\ \mathscr{Z}_i^\top(X) & 0 & -\mathscr{X}_i(\varepsilon) & 0 \\ \mathscr{S}_i^\top(X) & 0 & 0 & -\mathscr{X}_i(X) \end{bmatrix} < 0 \tag{16}$$

where

$$J_1(i) = A(i)X(i) + X^\top(i)A^\top(i) - (N-1)\underline{\lambda}_i X^\top(i)E^\top(i) + \varepsilon_A(i)D_A(i)D_A^\top(i)$$

$\mathscr{Z}_i(X), \mathscr{S}_i(X), \mathscr{X}_i(\varepsilon)$ *and* $\mathscr{X}_i(X)$ *keep the same definitions as in Lemma 3.5*

with the constraints (11).

With the results of Lemma 3.5 and Lemma 3.6, we are ready to design the stabilizing state feedback controller of the form (4) for our class of systems when we have partial knowledge of the transition jump rates. For this purpose, let us now consider the closed-loop dynamics of the nominal system and focus on the design of the controller gain. Based on the results of Theorem 3.6 and after letting $Y(i) = K(i)X(i)$ with some appropriate algebraic manipulations, we get the following theorem.

Theorem 3.3 *There exists a state feedback of the form (4) such that the closed-loop nominal singular Markovian jump system (2) is piecewise regular, impulse-free and stochastically stable if there exist a set of nonsingular matrices $X = (X(1), \cdots, X(N))$, a set of matrices $Y = (Y(1), \cdots, Y(N))$ and a set of positive scalars $\varepsilon = (\varepsilon(1), \cdots, \varepsilon(N))$ such that the following set of coupled LMIs holds for each $i \in \mathscr{S}$:*

$$\begin{bmatrix} J_2(i) & \mathscr{Z}_i(X) & \mathscr{S}_i(X) \\ \mathscr{Z}_i^\top(X) & -\mathscr{X}_i(\varepsilon) & 0 \\ \mathscr{S}_i^\top(X) & 0 & -\mathscr{X}_i(X) \end{bmatrix} < 0 \tag{17}$$

where

$$J_2(i) = A(i)X(i) + X^\top(i)A^\top(i) + B(i)Y(i) + Y^\top(i)B^\top(i) - (N-1)\underline{\lambda}_i X^\top(i)E^\top(i)$$

$\mathscr{Z}_i(X), \mathscr{S}_i(X), \mathscr{X}_i(\varepsilon)$ *and* $\mathscr{X}_i(X)$ *keep the same definitions as in Lemma 3.5*

with the constraints (11). The controller gain $K(i)$ is given by $K(i) = Y(i)X^{-1}(i)$.

For the uncertain system the design of the gain controller can be done in the same manner. In fact, based on the results of Lemma 3.6 and following steps as for the nominal case, we get the following results for the robust stabilization.

Theorem 3.4 *There exists a state feedback of the form (4) such that the closed-loop uncertain singular Markovian switching system (2) is piecewise regular, impulse-free and stochastically stable if there exist a set of nonsingular matrices $X = (X(1), \cdots, X(N))$, a set of* *matrices* $Y = (Y(1), \cdots, Y(N))$ *and sets of positive scalars* $\varepsilon = (\varepsilon(1), \cdots, \varepsilon(N))$, $\varepsilon_A = (\varepsilon_A(1), \cdots, \varepsilon_A(N))$ *and* $\varepsilon_B = (\varepsilon_B(1), \cdots, \varepsilon_B(N))$ *such that the following set of coupled LMIs holds for each* $i \in \mathscr{S}$ *and for all admissible uncertainties:*

$$\begin{bmatrix} J_3(i) & X^\top(i)E_A^\top(i) & Y^\top(i)E_B^\top(i) & \mathscr{Z}_i(X) & \mathscr{S}_i(X) \\ E_A(i)X(i) & -\varepsilon_A(i)\mathbb{I} & 0 & 0 & 0 \\ E_B(i)Y(i) & 0 & -\varepsilon_B(i)\mathbb{I} & 0 & 0 \\ \mathscr{Z}_i^\top(X) & 0 & 0 & -\mathscr{X}_i(\varepsilon) & 0 \\ \mathscr{S}_i^\top(X) & 0 & 0 & 0 & -\mathscr{X}_i(X) \end{bmatrix} < 0 \tag{18}$$

where

$$J_3(i) = A(i)X(i) + X^\top(i)A^\top(i) + B(i)Y(i) + Y^\top(i)B^\top(i) - (N-1)\underline{\lambda}_i X^\top(i)E^\top(i) + \varepsilon_A(i)D_A(i)D_A^\top(i) + \varepsilon_B(i)D_B(i)D_B^\top(i)$$

$\mathscr{Z}_i(X), \mathscr{S}_i(X), \mathscr{X}_i(\varepsilon)$ *and* $\mathscr{X}_i(X)$ *keep the same definitions as in Lemma 3.5*

with the constraints (11). The controller gain $K(i)$ *is given by* $K(i) = Y(i)X^{-1}(i)$.

Remark 3.1 *The conditions* $E(i)X(i) = X^\top(i)E^\top(i)$ *may be difficult to solve with some commercial toolboxes that don't handle equalities. To overcome this, we recommend to solve the following optimization problem instead:*

$$P : \begin{cases} \min \beta \\ s.t. : \\ \beta \geq 0 \\ \left[E(i)X(i) - X^\top(i)E^\top(i)\right]^\top \left[E(i)X(i) - X^\top(i)E^\top(i)\right] \leq \beta\mathbb{I} \\ \text{with the appropriate conditions} \end{cases}$$

that can be solved with any existing commercial toolbox.

4 Numerical example

To show the validness of our results, let us consider some numerical examples of two-mode singular system and state space in $\mathbb{R}^3$. The system we consider in these examples is stochastically unstable.

Example 4.1 *The first example is used to show the usefulness of the results in case of complete knowledge of the jump rates. The data of this system are as follows:*

- *mode # 1:*

$$A(1) = \begin{bmatrix} 0.0 & 1.0 & 0.0 \\ 0.0 & 0.0 & 1.0 \\ 1.0 & 2.0 & 3.0 \end{bmatrix}, \qquad B(1) = \begin{bmatrix} 0.0 \\ 0.0 \\ 1.0 \end{bmatrix},$$

$$D_A(1) = \begin{bmatrix} 0.0 \\ 0.0 \\ 0.1 \end{bmatrix}, \qquad E_A(1) = \begin{bmatrix} 0.1 & 0.2 & 0.1 \end{bmatrix},$$

$$D_B(1) = \begin{bmatrix} 0.0 \\ 0.0 \\ 0.1 \end{bmatrix}, \qquad E_B(1) = \begin{bmatrix} 0.1 \end{bmatrix},$$

- *mode # 2:*

$$A(2) = \begin{bmatrix} 0.0 & 1.0 & 0.0 \\ 0.0 & 0.0 & 1.0 \\ 3.0 & 2.0 & 1.0 \end{bmatrix}, \qquad B(2) = \begin{bmatrix} 0.0 \\ 0.0 \\ 2.0 \end{bmatrix},$$

$$D_A(2) = \begin{bmatrix} 0.0 \\ 0.0 \\ -0.1 \end{bmatrix}, \qquad E_A(2) = \begin{bmatrix} 0.1 & 0.2 & -0.1 \end{bmatrix},$$

$$D_B(2) = \begin{bmatrix} 0.0 \\ 0.0 \\ 0.2 \end{bmatrix}, \qquad E_B(2) = \begin{bmatrix} 0.1 \end{bmatrix}.$$

The transition matrix rates, Λ*, and the singular matrices,* $E(1)$ *and* $E(2)$*, are given by:*

$$\Lambda = \begin{bmatrix} -1 & 1 \\ 1.1 & -1.1 \end{bmatrix}, \qquad E(1) = E(2) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

Solving the LMIs of Theorem 3.1, we get:

$$\varepsilon(1) = 0.2604, \varepsilon(2) = 0.2573,$$

$$X(1) = \begin{bmatrix} 0.6925 & -0.2592 & 0.0 \\ -0.2592 & 0.4329 & 0.0 \\ -0.2149 & -0.2405 & 1.0091 \end{bmatrix}, \quad X(2) = \begin{bmatrix} 0.6749 & -0.2508 & 0.0 \\ -0.2508 & 0.4184 & 0.0 \\ -0.2043 & -0.2255 & 0.9603 \end{bmatrix},$$

$$Y(1) = \begin{bmatrix} 0.6821 & -0.6572 & -4.0176 \end{bmatrix}, \quad Y(2) = \begin{bmatrix} -0.5555 & -0.2949 & -0.9719 \end{bmatrix}.$$

which gives the following gains

$$K(1) = \begin{bmatrix} -2.1224 & -5.0012 & -3.9815 \end{bmatrix}, \quad K(2) = \begin{bmatrix} -2.0507 & -2.4797 & -1.0121 \end{bmatrix}$$

The system is simulated using Matlab and the results are illustrated by Figure 1 which shows that the closed-loop system under the computed controller is piecewise regular, impulse-free and stochastically stable.

For the design of the robust controller that makes the closed-loop system piecewise regular, impulse-free and stochastically stable, solving the LMIs Theorem 3.2, we get:

$$\varepsilon(1) = 0.2411, \varepsilon(2) = 0.2357,$$

$$\varepsilon_A(1) = 1.0098, \varepsilon_A(2) = 1.0978,$$

$$\varepsilon_B(1) = 1.3226, \varepsilon_B(2) = 1.0546,$$

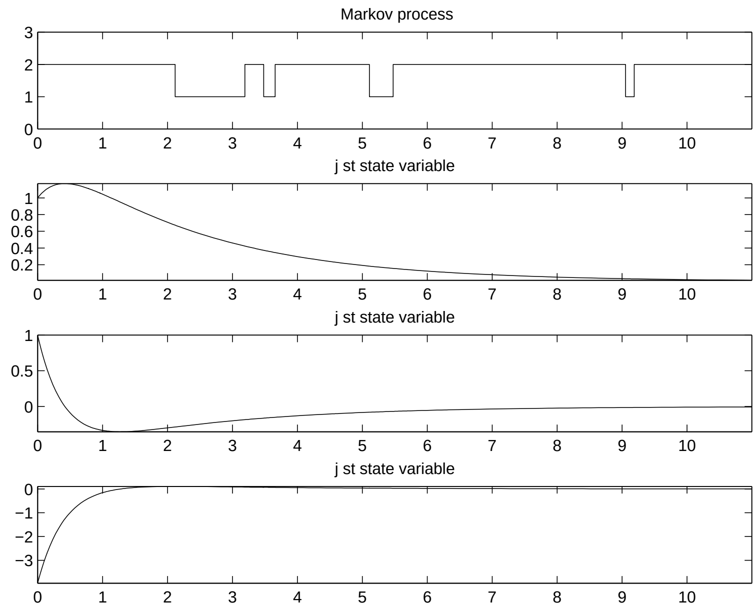

Figure 1: The behaviors of the system states in function of time t

$$X(1) = \begin{bmatrix} 0.6592 & -0.2556 & 0.0 \\ -0.2556 & 0.4119 & 0.0 \\ -0.1652 & -0.2410 & 0.8695 \end{bmatrix}, \quad X(2) = \begin{bmatrix} 0.6425 & -0.2458 & 0.0 \\ -0.2458 & 0.3960 & 0.0 \\ -0.1898 & -0.2210 & 0.9451 \end{bmatrix},$$

$$Y(1) = \begin{bmatrix} 0.5459 & -0.5383 & -3.6851 \end{bmatrix}, \quad Y(2) = \begin{bmatrix} -0.5199 & -0.2668 & -0.9944 \end{bmatrix}.$$

which gives the following gains

$$K(1) = \begin{bmatrix} -2.2414 & -5.1776 & -4.2383 \end{bmatrix}, \quad K(2) = \begin{bmatrix} -2.1015 & -2.5656 & -1.0522 \end{bmatrix}$$

The system is simulated using Matlab and the results are illustrated by Figure 2 which shows that the closed-loop system with the computed controller is piecewise regular, impulse-free and stochastically stable for all admissible uncertainties.

Example 4.2 *The second example is used to show the usefulness of the results in case of partial knowledge of the jump rates. The data of this system are the same as for the previous example except for the jump rates that we assume that we don't know the exact values but only bounds given by the following:*

- *mode # 1:*
$$\underline{\lambda}_1 = 0.8\bar{\lambda}_1 = 1.2$$
- *mode # 2:*
$$\underline{\lambda}_2 = 0.9\bar{\lambda}_2 = 1.3$$

Solving the LMIs of Theorem 3.3, we get:

$$\varepsilon(1) = 0.0263, \varepsilon(2) = 0.0260,$$

$$X(1) = \begin{bmatrix} 0.0773 & -0.0478 & 0.0 \\ -0.0478 & 0.0650 & 0.0 \\ -0.0041 & -0.0711 & 0.8565 \end{bmatrix}, \quad X(2) = \begin{bmatrix} 0.0757 & -0.0466 & 0.0 \\ -0.0466 & 0.0630 & 0.0 \\ -0.0040 & -0.0663 & 0.8120 \end{bmatrix},$$

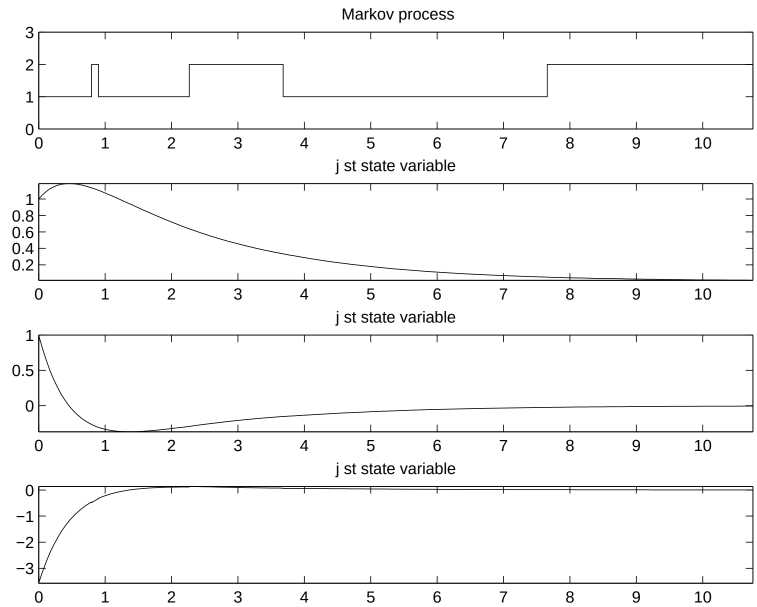

Figure 2: The behaviors of the system states in function of time t

$$Y(1) = \begin{bmatrix} 0.0695 & -0.0255 & -7.2930 \end{bmatrix}, \qquad Y(2) = \begin{bmatrix} -0.0454 & -0.0338 & -2.6931 \end{bmatrix}.$$

which gives the following gains

$$K(1) = \begin{bmatrix} -10.1872 & -17.1988 & -8.5149 \end{bmatrix}, \quad K(2) = \begin{bmatrix} -5.9671 & -8.4415 & -3.3166 \end{bmatrix}$$

The system is simulated using Matlab with the real jump rates and computed gains and the results are illustrated by Figure 3 which shows that the closed-loop system under the computed controller is piecewise regular, impulse-free and stochastically stable.

For the design of the robust controller that makes the closed-loop system piecewise regular, impulse-free and stochastically stable, solving the LMIs Theorem 3.4, we get:

$$\varepsilon(1) = 0.0216, \varepsilon(2) = 0.0214,$$
$$\varepsilon_A(1) = 1.0061, \varepsilon_A(2) = 1.1960,$$
$$\varepsilon_B(1) = 1.8137, \varepsilon_B(2) = 1.1261,$$
$$X(1) = \begin{bmatrix} 0.0636 & -0.0394 & 0.0 \\ -0.0394 & 0.0535 & 0.0 \\ -0.0022 & -0.0588 & 0.7048 \end{bmatrix}, \quad X(2) = \begin{bmatrix} 0.0623 & -0.0383 & 0.0 \\ -0.0383 & 0.0518 & 0.0 \\ -0.0032 & -0.0553 & 0.7038 \end{bmatrix},$$
$$Y(1) = \begin{bmatrix} 0.0496 & -0.0180 & -6.4840 \end{bmatrix}, \qquad Y(2) = \begin{bmatrix} -0.0372 & -0.0261 & -2.4876 \end{bmatrix}.$$

which gives the following gains

$$K(1) = \begin{bmatrix} -11.0639 & -18.6031 & -9.1992 \end{bmatrix}, \quad K(2) = \begin{bmatrix} -6.2655 & -8.9135 & -3.5347 \end{bmatrix}$$

The system is simulated using Matlab with the same data and the computed controller gains and the results are illustrated by Figure 4 which shows that the closed-loop system with the computed controller is piecewise regular, impulse-free and stochastically stable.

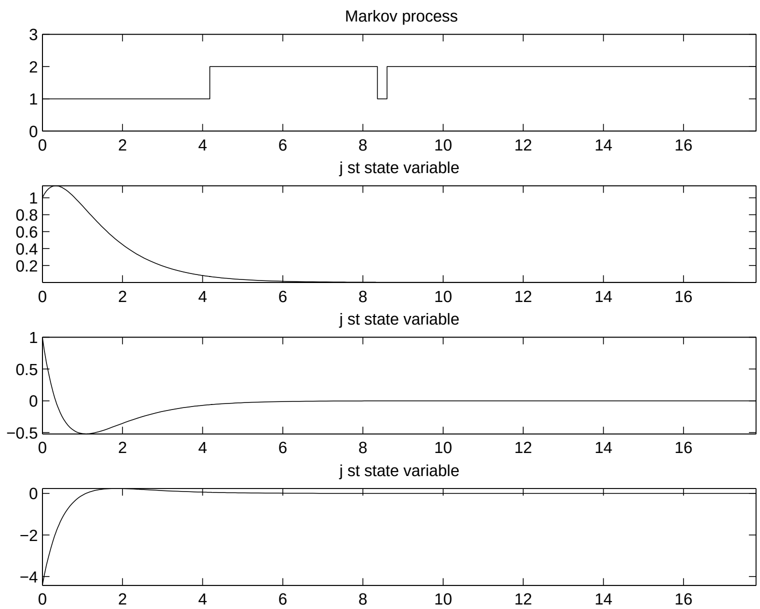

Figure 3: The behaviors of the system states in function of time t

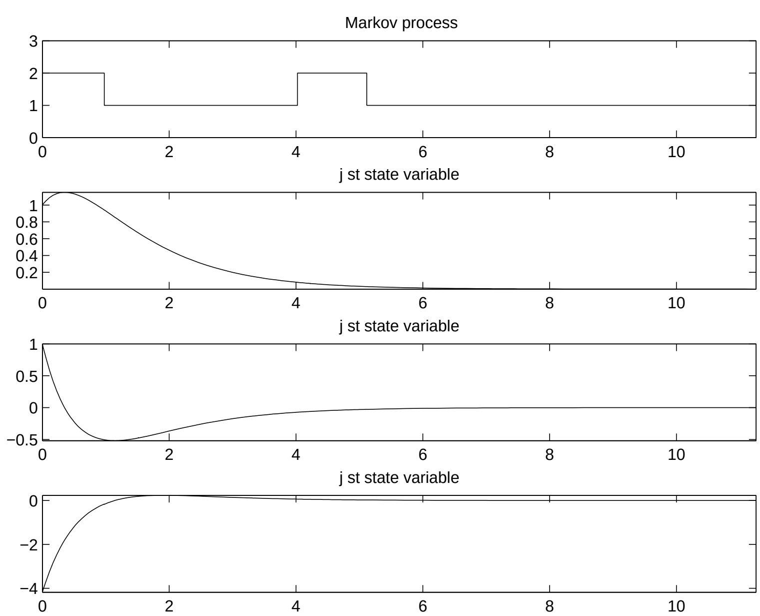

Figure 4: The behaviors of the system states in function of time t

5 Conclusion

This paper dealt with a class of continuous-time singular linear systems with Markovian switching. Results on stochastic stability and its robustness, and the stochastic stabilization and its robustness are developed. The LMI framework is used to establish the different results on stability, stabilization and their robustness. The results we developed here can easily be solved using any LMI toolbox like the one of Matlab or the one of Scilab.

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