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Abstract

This paper deals with the class of continuous-time singular linear Markovian jump systems with totally and partially known transition jump rates. The filtering problem of this class of systems is tackled. New sufficient conditions for $\mathscr{H}_\infty$ filtering are developed. A design procedure for the $\mathscr{H}_\infty$ filter which guarantees that the dynamics of the filter error will be piecewise regular, impulse-free and stochastically stable with γ-disturbance rejection is proposed. It is shown that the addressed problem can be solved if the corresponding developed linear matrix inequalities (LMIs) with some constraints are feasible. A numerical example is employed to show the usefulness of the proposed results.

Key Words: Singular systems, descriptor systems, continuous-time linear systems, linear matrix inequality, stochastic stability, $\mathscr{H}_\infty$ filtering, disturbance rejection.

Résumé

Cet article traite de la classe des systèmes continus singuliers à sauts markoviens avec des taux de transitions, totalement ou partiellement, inconnus. Le problème de filtrage de cette classe de système est étudié. De nouvelles conditions en forme d'inégalités matricielles linéaires sont développées qui assurent que l'erreur de filtrage est régulière, sans impulsion et stochastiquement stable et en même temps assure le rejet de perturbation avec un niveau donné $\gamma > 0$. Un exemple numérique pour montrer l'utilité des résultats développés est proposé.

Mots clés : Systèmes singuliers, systèmes linéaires continus, inégalités matricelles linéaires, stabilité stochastique, filtrage, rejet de perturbations.

1 Introduction

Singular continuous-time linear systems represent an important class of systems that has attracted a lot of researchers from mathematics and control communities. Singular systems are also referred to as descriptor systems, implicit systems, generalized state-space systems, differential-algebraic systems or semi-state systems [1, 6, 10]. The class of singular systems is more appropriate to describe the behavior of some practical systems like power systems [6], electrical systems [1], chemical systems [12]. Many problems for this class of systems either in the continuous-time and discrete-time have been tackled and interesting results have been reported in the literature and the references therein. Among these contributions we quote those of [4, 7, 8, 9, 11, 12, 13, 14, 15, 16, 19, 20, 21] where the reader can found the interesting results developed on the different tackled problems.

On the other hand the class of Markovian jump systems has been found more appropriate to describe practical systems with random abrupt changes in their structures such as components failures or repairs, sudden environment disturbance, interconnections changing and operating in different point of a nonlinear plant. This fact was the cause of the tremendous interest to the Markovian jump systems. For more details of this class of systems or on what has been done of the different problems, we refer the reader to [2, 5] and the references therein.

This paper deals with the class of singular systems with random abrupt changes and which combines the two previous classes of systems. To the best of the author's knowledge this class of systems has not been fully investigated so far and only few references have been reported in the literature on the subject [1, 4]. We have to mention that this class of systems may have discontinuities in the states when the mode jumps from one value to another that we can not avoid for autonomous systems and which makes them more complicated compared to the singular deterministic systems. For more details on this phenomena we refer the reader to [1].

The filtering has been applied for many years and continues to be used in many industrial applications ranging from aerospace to economics including engineering, biology, geoscience, management, etc. This problem has attracted a lot of researcher from different communities and interesting results have been reported in the literature. For more details on the filtering we refer the reader to Boukas and Liu [2, 3] and the references therein for the filtering of Markovian jump systems and to [1, 17] for the filtering of the class of linear singular systems. For the class of systems we are considering, the filtering problem has never been tackled before and this is our concern in this paper. Two cases will be addressed in the paper. The first one considers that the transition jump rates of the Markov process that describes the behavior of the system are totally known while the second relaxes this assumption and considers them as partially known.

The goal of this paper consists of developing results on $\mathscr{H}_\infty$ filtering for the class of linear singular systems with random abrupt changes when the transition jump rates are totally and partially known. We are mainly interested by the results that are in the LMI setting that make them more tractable using the tools in the marketplace.

The rest of this paper is organized as follows. In Section 2, the problem is stated and the goal of the paper is given. In Section 3, the main results are developed and they include results on the design of a $\mathscr{H}_\infty$ filter that makes the error piecewise regular, impulse-free and stochastically stable and at the same time guarantees the disturbance rejection of level $\gamma > 0$. Section provides a numerical example to show the usefulness of the proposed results either in completely and partially known transition jump rates.

Notation: Throughout this paper, $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote, respectively, the n dimensional Euclidean space and the set of all $n \times m$ real matrices. The superscript "$^\mathsf{T}$" denotes matrix transposition and the notation $X \geq Y$(respectively, $X > Y$) where X and Y are symmetric matrices, means that $X - Y$ is positive semi-definite (respectively, positive definite). $\mathbb{I}$ is the identity matrices with compatible dimensions. $\mathscr{L}_2$ is the space of integral vector over $[0, \infty)$. $\|\cdot\|$ will refer to the Euclidean vector norm whereas $\|\cdot\|$ denotes the $\mathscr{L}_2$-norm over $[0, \infty)$ defined as $\|f\|^2 = \int_0^\infty f^T(t)f(t)\, dt$.

2 Problem statement

Let us assume that the system behavior be described by the following differential-algebraic equations:

$$\begin{cases} E(r_t)\dot{x}(t) = A(r_t)x(t) + B(r_t)w(t), x(0) = x_0, \\ y(t) = C_y(r_t)x(t) + D_y(r_t)w(t), \\ z(t) = C_z(r_t)x(t) + D_z(r_t)w(t), \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $y(t) \in \mathbb{R}^k$ is the measurement, and $z(t) \in \mathbb{R}^p$ is the signal to be estimated, $w(t) \in \mathbb{R}^m$ is the disturbance input which is assumed belong to $\mathscr{L}_2[0, \infty)$, $A(i)$, $B(i)$, $C_y(i)$, $C_z(i)$, $D_y(i)$ and $D_z(i)$ are known real matrices with appropriate dimensions, the matrix $E(i)$ may be singular, and we assume that $rank(E(i)) = n_E \leq n$.

The Markov process $\{r_t, t \geq 0\}$ beside taking values in the finite set $\mathscr{S}$, represents the switching between the different modes and its behavior is described by the following probability transitions:

$$\begin{aligned} &\mathbb{P}\left[r_{t+h} = j | r_t = i\right] \\ &= \begin{cases} \lambda_{ij} h + o(h) & \text{when } r_t \text{ jumps from } i \text{ to } j \\ 1 + \lambda_{ii} h + o(h) & \text{otherwise} \end{cases} \end{aligned} \tag{2}$$

where λ_{ij} is the transition jump rate from mode i to mode j with $\lambda_{ij} \geq 0$ when $i \neq j$ and $\lambda_{ii} = -\sum_{j=1, j\neq i}^N \lambda_{ij}$ and $o(h)$ is such that $\lim_{h\to 0} \frac{o(h)}{h} = 0$.

The system disturbance, $w(t)$, is assumed to belong to $\mathscr{L}_2[0,\infty)$ which means that the following holds:

$$\int_0^\infty w^\top(t)w(t)dt < \infty \tag{3}$$

This implies that the disturbance has finite energy.

The following definitions will be used in the rest of this paper. For more details on the singular systems properties, we refer the reader to [6] and the references therein.

Definition 2.1 *[6]*

i. *System (1) with $w(t) = 0$ for $t \geq 0$ is said to be regular if the characteristic polynomial,* $\det(sE(i) - A(i))$ *is not identically zero.*

ii. *System (1) with $w(t) = 0$ for $t \geq 0$ is said to be impulse free, i.e. the* $\deg(\det(sE(i) - A(i))) = rank(E(i))$.

For more details on other properties and the existence of the solution of system (1), we refer the reader to [16, 18], and the references therein. In general, the regularity is often a sufficient condition for the analysis and the synthesis of singular systems.

Definition 2.2 *System (1) is said to be stochastically stable (SS) if there exists a constant $M(x_0, r_0) > 0$ such that the following holds for any initial conditions (x_0, r_0):*

$$\mathbb{E}\left[\int_0^\infty x^\top(t)x(t)|x_0, r_0\right] \leq M(x_0, r_0). \tag{4}$$

The filtering problem consists of computing an estimate, $\hat{z}(t)$, of the signal, $z(t)$, via a causal Markovian jump linear filter which provides a uniformly small estimation error, $z(t) - \hat{z}(t)$, for all ω satisfying some properties (finite energy or finite power). There exist in the literature different approaches for designing a filter that estimates the system states. In this paper we will restrict ourself to the $\mathscr{H}_\infty$ filtering.

In order to put the $\mathscr{H}_\infty$ filtering problem of the class of systems (1) we are considering here in the stochastic setting, let us introduce the space $\mathscr{L}_2\left[\Omega, \mathcal{F}, \mathbb{P}\right]$ of $\mathcal{F}$-measurable processes, $z(t) - \hat{z}(t)$, for which the following holds:

$$\|z - \hat{z}\|_2 \stackrel{\Delta}{=} \left\{\mathbb{E}\left[\int_0^\infty \left[z(t) - \hat{z}(t)\right]^\top \left[z(t) - \hat{z}(t)\right] dt\right]\right\}^{\frac{1}{2}} < \infty. \tag{5}$$

The goal of this paper is to design a linear n-order filter of the following form:

$$\begin{cases} E(r_t)\dot{\hat{x}}(t) = K_A(r_t)\hat{x}(t) + K_B(r_t)y(t), \hat{x}(0) = 0, \\ \hat{z}(t) = K_C(r_t)\hat{x}(t), \end{cases} \tag{6}$$

which gives an estimate of the state vector, $\hat{x}(t)$ at time, t, and can ensure that the extended system $(x(t), x(t) - \hat{x}(t))$ is piecewise regular, impulse-free and stochastically stable and the estimation error, $z(t) - \hat{z}(t)$, is bounded for all noises $\omega(t) \in \mathscr{L}_2[0, \infty)$. The matrices $K_A(i)$, $K_B(i)$ and $K_C(i)$, $i \in \mathscr{S}$ are design parameters that should be determined in order to estimate the state vector properly.

If we combine the dynamical system's state equation (1) with the filter's state equation (6), we get the following extended one:

$$\tilde{E}(r_t)\dot{\tilde{x}}(t) = \tilde{A}(r_t)\tilde{x}(t) + \tilde{B}(r_t)\omega(t), \tilde{x}(0) = (x_0^\top, x_0^\top)^\top, \tag{7}$$

where

$$\begin{aligned}
\tilde{x}(t) &= \begin{bmatrix} x(t) \\ x(t) - \hat{x}(t) \end{bmatrix}, \tilde{E}(r_t) = \begin{bmatrix} E(r_t) & 0 \\ 0 & E(r_t) \end{bmatrix}, \\
\tilde{A}(r_t) &= \begin{bmatrix} A(r_t) & 0 \\ A(r_t) - K_B(r_t)C_y(r_t) - K_A(r_t) & K_A(r_t) \end{bmatrix}, \\
\tilde{B}(r_t) &= \begin{bmatrix} B(r_t) \\ B(r_t) - K_B(r_t)D_y(r_t) \end{bmatrix}.
\end{aligned}$$

The estimation error, $e(t) = z(t) - \hat{z}(t)$, satisfies the following:

$$e(t) = \tilde{C}(r_t)\tilde{x}(t) + \tilde{D}(r_t)\omega(t) \tag{8}$$

with

$$\begin{aligned}
\tilde{C}(r_t) &= \begin{bmatrix} C_z(r_t) - K_C(r_t) & K_C(r_t) \end{bmatrix}, \\
\tilde{D}(t) &= D_z(r_t).
\end{aligned}$$

More often the transition jump rates can not be easily accessible and an alternate to overcome this case is required. The following assumption will be used in this paper to develop new results for the case of partially known transition jump rates.

Assumption 2.1 *The jump rates are assumed to satisfy the following:*

$$0 < \underline{\lambda}_i \leq \lambda_{ij} \leq \bar{\lambda}_i, \forall i, j \in \mathscr{S}, j \neq i \tag{9}$$

where $\underline{\lambda}_i$ and $\bar{\lambda}_i$ are known parameters for each mode or may represent the lower and upper bounds when all the jump rates are known, i.e.: $0 < \underline{\lambda}_i = \min_{j\in\mathscr{S}} \{\lambda_{ij}, i \neq j\}$, $0 < \bar{\lambda}_i = \max_{j\in\mathscr{S}} \{\lambda_{ij}, i \neq j\}$, with $\underline{\lambda}_i \leq \bar{\lambda}_i$.

Remark 2.1 *We have to mention that some alternatives have been proposed to handle such case by considering uncertainties on the jump rates. Our approach is totally different and requires only lower and upper bounds in each mode to establish the results we propose in this paper.*

In the rest of the paper we will assume that system (1) is stochastically stable. Notice that this is not a restriction since if our system not stochastically stable, we can firstly design a controller that makes it stable.

The goal of this paper is to design an n-order filter of the form (6) for the cases of completely and partially known transition jump rates that makes the system error piecewise regular, impulse-free and stochastically stable and guarantees the $\mathscr{H}_\infty$ performance

$$\|z(t) - \hat{z}(t)\|_2 \leq \gamma \left[\|\omega\|_2^2 + x_0^\top R x_0 \right]^{\frac{1}{2}}, \tag{10}$$

where $\gamma > 0$ and R is a symmetric and positive-definite matrix.

3 Main results

Before presenting the procedure to design the filter (6) in the two cases, we recall in the case of totally known transition jump rates the following result which gives the conditions that the filter error should satisfy to guarantee to be piecewise regular, impulse-free and stochastically stable and at the same time assures a desired disturbance rejection level. For the proof of this lemma we refer the reader to Boukas [1].

Lemma 3.1 *(see Boukas [1]) Let* $K_A = (K_A(1), \cdots, K_A(N))$, $K_A(i) \in \mathbb{R}^{n\times n}$, $K_B = (K_B(1), \cdots, K_B(N))$, $K_B(i) \in \mathbb{R}^{n\times k}$, *and* $K_C = (K_C(1), \cdots, K_C(N)))$, $K_C(i) \in \mathbb{R}^{p\times n}$, *be given sets of gains. Let* γ *be given positive constant and* R *is a given symmetric and positive-definite matrix representing the weighting of the initial conditions. If there exists a set of nonsingular matrices* $P = (P(1), \cdots, P(N))$, $P(i) \in \mathbb{R}^{n\times n}$, *such that the following set of the coupled LMIs holds:*

$$\begin{bmatrix} \tilde{J}_1(i) & P^\top(i)\tilde{B}(i) & \tilde{C}^\top(i) \\ \tilde{B}^\top(i)P(i) & -\gamma^2\mathbb{I} & \tilde{D}^\top(i) \\ \tilde{C}(i) & \tilde{D}(i) & -\mathbb{I} \end{bmatrix} < 0, \tag{11}$$

$$\begin{bmatrix} \mathbb{I} & \mathbb{I} \end{bmatrix} \tilde{E}^\top(r_0)P(r_0) \begin{bmatrix} \mathbb{I} \\ \mathbb{I} \end{bmatrix} \leq \gamma^2 R, \tag{12}$$

where $\tilde{J}_1(i) = \tilde{A}^\top(i)P(i) + P^\top(i)\tilde{A}(i) + \sum_{j=1}^N \lambda_{ij} E^\top(j)P(j)$, *with the following constraints:*

$$E^\top(i)P(i) = P^\top(i)E(i) \geq 0 \tag{13}$$

then the extended system is piecewise regular, impulse-free and stochastically stable and, moreover the estimation error satisfies the following:

$$\|z(t) - \hat{z}(t)\|_2 \leq \gamma \left[\|\omega\|_2^2 + x_0^\top R x_0 \right]^{\frac{1}{2}}. \tag{14}$$

For a given set of gains of the filter, we can compute the minimum disturbance rejection by solving the following convex optimization problem:

$$
\text{P:}\begin{cases}
\min_{\substack{v>0,\\ P=(P(1),\cdots,P(N))}} v \\
\text{s.t:} \\
E^\top(i)P(i) = P^\top(i)E(i) \geq 0, \\
\begin{bmatrix} \tilde{J}_1(i) & P^\top(i)\tilde{B}(i) & \tilde{C}^\top(i) \\ \tilde{B}^\top(i)P(i) & -v\mathbb{I} & \tilde{D}^\top(i) \\ \tilde{C}(i) & \tilde{D}(i) & -\mathbb{I} \end{bmatrix} < 0, \\
\begin{bmatrix} \mathbb{I} & \mathbb{I} \end{bmatrix} \tilde{E}^\top(r_0)P(r_0) \begin{bmatrix} \mathbb{I} \\ \mathbb{I} \end{bmatrix} \leq vR,
\end{cases}
$$

where $v = \gamma^2$.

But since we don't have yet developed a way to compute the filter gains, this optimization problem is useless. The design of the filter's gains should be included in an optimization problem similar to this one that can help us to determine simultaneously the filter's gains and the minimum disturbance rejection.

Notice that the condition (11) is nonlinear in $P(i)$ and the design filter parameters. To cast the design of the $\mathscr{H}_\infty$ filter in the LMI framework, let us transform this condition in order to compute the gains $K_A(i)$, $K_B(i)$ and $K_C(i)$.

Let us first of all compute $\tilde{J}_1(i)$, $P^\top(i)\tilde{B}(i)$, $\tilde{C}^\top(i)$, and $\tilde{D}^\top(i)$ in function of $A(i)$, $B(i)$, $C_y(i)$, $D_y(i)$, $C_z(i)$ and $D_z(i)$. Using the expression of $\tilde{A}(i)$, $\tilde{B}(i)$, $\tilde{C}(i)$ and $\tilde{D}(i)$, and assuming that $P(i) = \text{diag}[X_1(i), X_2(i)]$ we get:

$$
\begin{aligned}
\tilde{J}_1(i) &= \tilde{A}^\top(i)P(i) + P^\top(i)\tilde{A}(i) + \sum_{j=1}^{N} \lambda_{ij} E^\top(j)P(j) \\
&= \begin{bmatrix} \tilde{J}_{X_1}(i) & \begin{bmatrix} A^\top(i)X_2(i) \\ -C_y^\top(i)K_B^\top(i)X_2(i) \\ -K_A^\top(i)X_2(i) \end{bmatrix} \\ \begin{bmatrix} X_2^\top(i)A(i) \\ -X_2^\top(i)K_B(i)C_y(i) \\ -X_2^\top(i)K_A(i) \end{bmatrix} & \tilde{J}_{X_2}(i) \end{bmatrix},
\end{aligned}
$$

$$
\begin{aligned}
\tilde{P}^\top(i)\tilde{B}(i) &= \begin{bmatrix} X_1^\top(i) & 0 \\ 0 & X_2^\top(i) \end{bmatrix} \begin{bmatrix} B(i) \\ B(i) - K_B(i)D_y(i) \end{bmatrix} \\
&= \begin{bmatrix} X_1^\top(i)B(i) \\ X_2^\top(i)B(i) - X_2^\top(i)K_B(i)D_y(i) \end{bmatrix},
\end{aligned}
$$

$$\tilde{C}(i) = \left[\begin{array}{cc} C_z(i) - K_C(i) & K_C(i) \end{array} \right],$$
$$\tilde{D}(i) = D_z(i),$$

with

$$\tilde{J}_{X_1}(i) = A^\top(i)X_1(i) + X_1^\top(i)A(i) + \sum_{j=1}^{N} \lambda_{ij} E^\top(j)X_1(j),$$
$$\tilde{J}_{X_2}(i) = K_A^\top(i)X_2(i) + X_2^\top(i)K_A(i) + \sum_{j=1}^{N} \lambda_{ij} E^\top(j)X_2(j).$$

Using these relations, (11) becomes:

$$\left[\begin{array}{cccc} \tilde{J}_{X_1}(i) & \left[\begin{array}{c} A^\top(i)X_2(i) \\ -C_y^\top(i)K_B^\top(i)X_2(i) \\ -K_A^\top(i)X_2(i) \end{array} \right] & X_1^\top(i)B(i) & C_z^\top(i) - K_C^\top(i) \\ \left[\begin{array}{c} X_2^\top(i)A(i) \\ -X_2^\top(i)K_B(i)C_y(i) \\ -X_2^\top(i)K_A(i) \end{array} \right] & \tilde{J}_{X_2}(i) & \left[\begin{array}{c} X_2^\top(i)B(i) \\ -X_2^\top(i)K_B(i)D_y(i) \end{array} \right] & K_C^\top(i) \\ B^\top(i)X_1(i) & \left[\begin{array}{c} B^\top(i)X_2(i) \\ -D_y^\top(i)K_B^\top(i)X_2(i) \end{array} \right] & -\gamma^2\mathbb{I} & D_z^\top(i) \\ C_z(i) - K_C(i) & K_C(i) & D_z(i) & -\mathbb{I} \end{array} \right] < 0.$$

Letting $Y(i) = X_2^\top(i)K_A(i)$, $Z(i) = X_2^\top(i)K_B(i)$, and $W(i) = K_C(i)$, we get:

$$\left[\begin{array}{cc} \tilde{J}_{X_1}(i) & \left[\begin{array}{c} A^\top(i)X_2(i) \\ -C_y^\top(i)Z^\top(i) \\ -Y^\top(i) \end{array} \right] \\ \left[\begin{array}{c} X_2^\top(i)A(i) \\ -Z(i)C_y(i) \\ -Y(i) \end{array} \right] & J_{X_2}(i) \\ B^\top(i)X_1(i) & \left[\begin{array}{c} B^\top(i)X_2(i) \\ -D_y^\top(i)Z^\top(i) \end{array} \right] \\ C_z(i) - W(i) & W(i) \end{array} \right.$$

$$\begin{array}{cc} X_1^\top(i)B(i) & \begin{bmatrix} C_z^\top(i) \\ -W^\top(i) \end{bmatrix} \\ \begin{bmatrix} X_2^\top(i)B(i) \\ -Z(i)D_y(i) \end{bmatrix} & W^\top(i) \\ -\gamma^2\mathbb{I} & D_z^\top(i) \\ D_z(i) & -\mathbb{I} \end{array} \Bigg] < 0, \quad (15)$$

with

$$J_{X_2}(i) = Y^\top(i) + Y(i) + \sum_{j=1}^{N} \lambda_{ij} E^\top(j) X_2(j).$$

Notice also that the condition, $\tilde{E}^\top(i)P(i) = P^\top(i)\tilde{E}(i) \geq 0$, becomes:

$$E^\top(i)X_1(i) = X_1^\top(i)E(i) \geq 0, \quad (16)$$
$$E^\top(i)X_2(i) = X_2^\top(i)E(i) \geq 0. \quad (17)$$

For the last relation of the theorem, we have:

$$\begin{bmatrix} \mathbb{I} & \mathbb{I} \end{bmatrix} \begin{bmatrix} E^\top(r_0) & 0 \\ 0 & E^\top(r_0) \end{bmatrix} \begin{bmatrix} X_1(r_0) & 0 \\ 0 & X_2(r_0) \end{bmatrix} \begin{bmatrix} \mathbb{I} \\ \mathbb{I} \end{bmatrix}$$
$$= E^\top(r_0)X_1(r_0) + E^\top(r_0)X_2(r_0) < \gamma^2 R. \quad (18)$$

The following theorem gives the results for the design of the gains of the $\mathscr{H}_\infty$ filter.

Theorem 3.1 *Let γ and R be respectively given positive constant and a symmetric and positive-definite matrix representing the weighting of the initial conditions. If there exist sets of nonsingular matrices $X_1 = (X_1(1), \cdots, X_1(N))$, $X_1(i) \in \mathbb{R}^{n\times n}$ and $X_2 = (X_2(1), \cdots, X_2(N))$, $X_2(i) \in \mathbb{R}^{n\times n}$ and matrices $Y = (Y(1), \cdots, Y(N))$, $Y(i) \in \mathbb{R}^{n\times n}$ $Z = (Z(1), \cdots, Z(N))$, $Z(i) \in \mathbb{R}^{n\times k}$ and $W = (W(1), \cdots, W(N))$ $W(i) \in \mathbb{R}^{p\times n}$ satisfying the set of coupled LMIs (15) and (18) with the constraints (16)-(17), then there exists a filter of the form (6) such that the estimation error is piecewise regular, impulse-free and stochastically stable and bounded by:*

$$\|z - \hat{z}\|_2 \leq \gamma \left[\|\omega\|_2^2 + x_0^\top R x_0\right]^{\frac{1}{2}}. \quad (19)$$

The filter's gains are given by:

$$\begin{cases} K_A(i) = X_2^{-\top}(i)Y(i), \\ K_B(i) = X_2^{-\top}(i)Z(i), \\ K_C(i) = W(i). \end{cases} \quad (20)$$

If the initial conditions are equal to zero, the previous results becomes easier and are given by the following corollary.

Corollary 3.1 *Let the initial conditions of system (7) be zero. Let γ be given positive constant. If there exist sets of nonsingular matrices $X_1 = (X_1(1), \cdots, X_1(N))$, $X_1(i) \in \mathbb{R}^{n\times n}$ and $X_2 = (X_2(1), \cdots, X_2(N))$, $X_2(i) \in \mathbb{R}^{n\times n}$ and matrices $Y = (Y(1), \cdots, Y(N))$, $Y(i) \in \mathbb{R}^{n\times n}$ $Z = (Z(1), \cdots, Z(N))$, $Z(i) \in \mathbb{R}^{n\times k}$ and $W = (W(1), \cdots, W(N))$ $W(i) \in \mathbb{R}^{p\times n}$ satisfying the LMIs (15)-(18) for every $i \in \mathscr{S}$, then there exists a filter of the form (6) such that the estimation error is piecewise regular, impulse-free and stochastically stable and bounded by:*

$$\|z - \hat{z}\|_2 \leq \gamma \|\omega\|_2.$$

The filter's gains are given by (20).

The minimal noise attenuation level, γ, that can be verified by the filter of the form of (6) can be obtained by solving the following optimization problem:

$$\mathcal{P}_0 : \begin{cases} \min\limits_{\substack{v>0,\\ X_1=(X_1(1),\cdots,X_1(N)),\\ X_2=(X_2(1),\cdots,X_2(N)),\\ Y=(Y(1),\cdots,Y(N)),\\ Z=(Z(1),\cdots,Z(N)),\\ W=(W(1),\cdots,W(N))}} v \\ s.t. \\ E^\top(i)X_1(i) = X_1^\top(i)E(i) \geq 0, \\ E^\top(i)X_2(i) = X_2^\top(i)E(i) \geq 0, \\ \Theta_v(i) < 0, \\ E^\top(r_0)X_1(r_0) + E^\top(r_0)X_2(r_0) < vR, \end{cases}$$

where $\Theta_v(i)$ is obtained from (15) by replacing γ^2 by v. Thus, if the convex optimization problem $\mathcal{P}_0$ has a solution, v, then by using Theorem 3.1, the corresponding error of the filter (6) is stable with noise attenuation level $\sqrt{v}$.

We can also get other sufficient conditions that can be used to design the filter of the form (6). Now if there exists a positive scalar $\varepsilon(i)$ for $i \in \mathscr{S}$, such that the following holds:

$$E^\top(i)P(i) \leq \left[\frac{1}{4}\varepsilon(i)\mathbb{I} + \varepsilon^{-1}(i)E^\top(i)P(i)P^\top(i)E(i)\right].$$

Notice that this condition is always true.

Proceeding as before and by using this inequality, we get the following theorem that summarizes the results for the design of the gains of the $\mathscr{H}_\infty$ filter.

Theorem 3.2 *Let γ and R be respectively given positive constants and a symmetric and positive-definite matrix representing the weighting of the initial conditions. If there exist sets of nonsingular matrices $X_1 = (X_1(1), \cdots, X_1(N))$, $X_1(i) \in \mathbb{R}^{n\times n}$ and $X_2 = (X_2(1), \cdots, X_2(N))$, $X_2(i) \in \mathbb{R}^{n\times n}$ and matrices $Y = (Y(1), \cdots, Y(N))$, $Y(i) \in \mathbb{R}^{n\times n}$ $Z = (Z(1), \cdots, Z(N))$, $Z(i) \in \mathbb{R}^{n\times k}$, $W = (W(1), \cdots, W(N))$ $W(i) \in \mathbb{R}^{p\times n}$ and a set of positive scalars $\varepsilon = (\varepsilon(1), \cdots, \varepsilon(N))$ satisfying the following set of coupled LMIs:*

$$\left[\begin{array}{cccccc}
J_{X_1}(i) & \begin{bmatrix} A^\top(i)X_2(i) \\ -C_y^\top(i)Z^\top(i) \\ -Y^\top(i) \end{bmatrix} & X_1^\top(i)B(i) & \begin{bmatrix} C_z^\top(i) \\ -W^\top(i) \end{bmatrix} & \mathcal{S}_i(X_1) & 0 \\
\begin{bmatrix} X_2^\top(i)A(i) \\ -Z(i)C_y(i) \\ -Y(i) \end{bmatrix} & J_{X_2}(i) & \begin{bmatrix} X_2^\top(i)B(i) \\ -Z(i)D_y(i) \end{bmatrix} & W^\top(i) & 0 & \mathcal{S}_i(X_2) \\
B^\top(i)X_1(i) & \begin{bmatrix} B^\top(i)X_2(i) \\ -D_y^\top(i)Z^\top(i) \end{bmatrix} & -\gamma^2\mathbb{I} & D_z^\top(i) & 0 & 0 \\
C_z(i) - W(i) & W(i) & D_z(i) & -\mathbb{I} & 0 & 0 \\
\mathcal{S}_i(X_1) & 0 & 0 & 0 & -\mathcal{X}_i(\varepsilon) & 0 \\
0 & \mathcal{S}_i(X_2) & 0 & 0 & 0 & -\mathcal{X}_i(\varepsilon)
\end{array}\right] < 0, \tag{21}$$

$$E^\top(r_0)X_1(r_0) + E^\top(r_0)X_2(r_0) < \gamma^2 R \tag{22}$$

where

$$\begin{aligned}
J_{X_1}(i) &= A^\top(i)X_1(i) + X_1^\top(i)A(i) + \lambda_{ii}E^\top(i)X_1(i) \\
&\quad + \sum_{j=1, j\neq i}^{N} \lambda_{ij}\frac{1}{4}\varepsilon(j)\mathbb{I}, \\
J_{X_2}(i) &= Y^\top(i) + Y(i) + \lambda_{ii}E^\top(i)X_2(i) + \sum_{j=1, j\neq i}^{N} \lambda_{ij}\frac{1}{4}\varepsilon(j)\mathbb{I}, \\
\mathcal{S}_i(X_1) &= \Big(\sqrt{\lambda_{i1}}E^\top(1)X_1(1), \cdots, \sqrt{\lambda_{ii-1}}E^\top(i-1)X_1(i-1), \\
&\quad \sqrt{\lambda_{ii+1}}E^\top(i+1)X_1(i+1), \cdots, \sqrt{\lambda_{iN}}E^\top(N)X_1(N)\Big),
\end{aligned}$$

$$\mathcal{S}_i(X_2) = \Big(\sqrt{\lambda_{i1}}E^\top(1)X_2(1), \cdots, \sqrt{\lambda_{ii-1}}E^\top(i-1)X_2(i-1), \\ \sqrt{\lambda_{ii+1}}E^\top(i+1)X_2(i+1), \cdots, \sqrt{\lambda_{iN}}E^\top(N)X_2(N)\Big),$$
$$\mathcal{X}_i(\varepsilon) = diag\left[\varepsilon(1)\mathbb{I}, \cdots, \varepsilon(i-1)\mathbb{I}, \varepsilon(i+1)\mathbb{I}, \cdots, \varepsilon(N)\mathbb{I}\right],$$

with the following constraints:

$$E^\top(i)X_1(i) = X_1^\top(i)E(i) \geq 0, \tag{23}$$
$$E^\top(i)X_2(i) = X_2^\top(i)E(i) \geq 0, \tag{24}$$

then there exists a filter of the form (6) such that the estimation error is piecewise regular, impulse-free and stochastically stable and bounded by:

$$\|z - \hat{z}\|_2 \leq \gamma\left[\|\omega\|_2^2 + x_0^\top R x_0\right]^{\frac{1}{2}}. \tag{25}$$

The filter's gains are given by (20).

Let us now focus on the case of partially known transition jump rates, which is the case of almost all the practical systems, and develop results that allow us to design the filter of the form (6). First of all notice that from Theorem 3.1, the term $\sum_{j=1}^N \lambda_{ij}E^\top(j)X_k(j), k = 1, 2$ can be rewritten as follows:

$$\sum_{j=1}^N \lambda_{ij}E^\top(j)X_k(j) = \lambda_{ii}E^\top(i)X_k(i) + \sum_{j=1, j\neq i}^N \lambda_{ij}E^\top(j)X_k(j)$$

Based now on the Assumption 2.1, we get:

$$\sum_{j=1, j\neq i}^N \lambda_{ij}E^\top(j)X_k(j) \leq \bar{\lambda}_i \sum_{j=1, j\neq i}^N E^\top(j)X_k(j)$$
$$\lambda_{ii}E^\top(i)X_k(i) = -\sum_{j=1, j\neq i}^N \lambda_{ij}E^\top(i)X_k(i) \leq -(N-1)\underline{\lambda}_i E^\top(i)X_k(i)$$

Using these relations and the results of Theorem 3.1, we get the following design procedure for the filter (6).

Theorem 3.3 *Let γ and R be respectively given positive constant and a symmetric and positive-definite matrix representing the weighting of the initial conditions. If there exist sets of nonsingular matrices $X_1 = (X_1(1), \cdots, X_1(N))$, $X_1(i) \in \mathbb{R}^{n\times n}$ and $X_2 = (X_2(1), \cdots, X_2(N))$, $X_2(i) \in \mathbb{R}^{n\times n}$ and matrices $Y = (Y(1), \cdots, Y(N))$, $Y(i) \in \mathbb{R}^{n\times n}$*

$Z = (Z(1), \cdots, Z(N))$, $Z(i) \in \mathbb{R}^{n\times k}$ and $W = (W(1), \cdots, W(N))$ $W(i) \in \mathbb{R}^{p\times n}$ satisfying the following set of coupled LMIs:

$$\begin{bmatrix} J_{X_1}(i) & \begin{bmatrix} A^\top(i)X_2(i) \\ -C_y^\top(i)Z^\top(i) \\ -Y^\top(i) \end{bmatrix} \\ \begin{bmatrix} X_2^\top(i)A(i) \\ -Z(i)C_y(i) \\ -Y(i) \end{bmatrix} & J_{X_2}(i) \\ B^\top(i)X_1(i) & \begin{bmatrix} B^\top(i)X_2(i) \\ -D_y^\top(i)Z^\top(i) \end{bmatrix} \\ C_z(i) - W(i) & W(i) \end{bmatrix}$$

$$\begin{matrix} X_1^\top(i)B(i) & \begin{bmatrix} C_z^\top(i) \\ -W^\top(i) \end{bmatrix} \\ \begin{bmatrix} X_2^\top(i)B(i) \\ -Z(i)D_y(i) \end{bmatrix} & W^\top(i) \\ -\gamma^2\mathbb{I} & D_z^\top(i) \\ D_z(i) & -\mathbb{I} \end{matrix} \Bigg] < 0, \tag{26}$$

$$E^\top(r_0)X_1(r_0) + E^\top(r_0)X_2(r_0) < \gamma^2 R \tag{27}$$

where

$$J_{X_1}(i) = A^\top(i)X_1(i) + X_1^\top(i)A(i) - (N-1)\underline{\lambda}_i E^\top(i)X_1(i) + \bar{\lambda}_i \sum_{j=1, j\neq i}^{N} E^\top(j)X_1(j)),$$

$$J_{X_2}(i) = Y^\top(i) + Y(i) - (N-1)\underline{\lambda}_i E^\top(i)X_2(i) + \bar{\lambda}_i \sum_{j=1, j\neq i}^{N} E^\top(j)X_2(j),$$

with the following constraints:

$$E^\top(i)X_1(i) = X_1^\top(i)E(i) \geq 0, \tag{28}$$
$$E^\top(i)X_2(i) = X_2^\top(i)E(i) \geq 0, \tag{29}$$

then there exists a filter of the form (6) such that the estimation error is piecewise regular, impulse-free and stochastically stable and bounded by:

$$\|z - \hat{z}\|_2 \leq \gamma \left[\|\omega\|_2^2 + x_0^\top R x_0\right]^{\frac{1}{2}}. \tag{30}$$

The filter's gains are given by (20).

Similarly we can the following results in case of partially known transition jump rates.

Theorem 3.4 *Let γ and R be respectively given positive constants and a symmetric and positive-definite matrix representing the weighting of the initial conditions. If there exist sets of nonsingular matrices $X_1 = (X_1(1), \cdots, X_1(N))$, $X_1(i) \in \mathbb{R}^{n\times n}$ and $X_2 = (X_2(1), \cdots, X_2(N))$, $X_2(i) \in \mathbb{R}^{n\times n}$ and matrices $Y = (Y(1), \cdots, Y(N))$, $Y(i) \in \mathbb{R}^{n\times n}$ $Z = (Z(1), \cdots, Z(N))$, $Z(i) \in \mathbb{R}^{n\times k}$, $W = (W(1), \cdots, W(N))$ $W(i) \in \mathbb{R}^{p\times n}$ and a set of positive scalars $\varepsilon = (\varepsilon(1), \cdots, \varepsilon(N))$ satisfying the following set of coupled LMIs:*

$$\left[\begin{array}{cccccc} J_{X_1}(i) & \begin{bmatrix} A^\top(i)X_2(i) \\ -C_y^\top(i)Z^\top(i) \\ -Y^\top(i) \end{bmatrix} & X_1^\top(i)B(i) & \begin{bmatrix} C_z^\top(i) \\ -W^\top(i) \end{bmatrix} & \mathcal{S}_i(X_1) & 0 \\ \begin{bmatrix} X_2^\top(i)A(i) \\ -Z(i)C_y(i) \\ -Y(i) \end{bmatrix} & J_{X_2}(i) & \begin{bmatrix} X_2^\top(i)B(i) \\ -Z(i)D_y(i) \end{bmatrix} & W^\top(i) & 0 & \mathcal{S}_i(X_2) \\ B^\top(i)X_1(i) & \begin{bmatrix} B^\top(i)X_2(i) \\ -D_y^\top(i)Z^\top(i) \end{bmatrix} & -\gamma^2\mathbb{I} & D_z^\top(i) & 0 & 0 \\ C_z(i) - W(i) & W(i) & D_z(i) & -\mathbb{I} & 0 & 0 \\ \mathcal{S}_i(X_1) & 0 & 0 & 0 & -\mathcal{X}_i(\varepsilon) & 0 \\ 0 & \mathcal{S}_i(X_2) & 0 & 0 & 0 & -\mathcal{X}_i(\varepsilon) \end{array}\right] < 0, \tag{31}$$

$$E^\top(r_0)X_1(r_0) + E^\top(r_0)X_2(r_0) < \gamma^2 R \tag{32}$$

where

$$\begin{aligned} J_{X_1}(i) &= A^\top(i)X_1(i) + X_1^\top(i)A(i) - (N-1)\underline{\lambda}_i E^\top(i)X_1(i) \\ &\quad + \sum_{j=1, j\neq i}^{N} \bar{\lambda}_i \frac{1}{4}\varepsilon(j)\mathbb{I}, \\ J_{X_2}(i) &= Y^\top(i) + Y(i) - (N-1)\underline{\lambda}_i E^\top(i)X_2(i) + \sum_{j=1, j\neq i}^{N} \bar{\lambda}_i \frac{1}{4}\varepsilon(j)\mathbb{I}, \\ \mathcal{S}_i(X_1) &= \Big(\sqrt{\bar{\lambda}_i}E^\top(1)X_1(1), \cdots, \sqrt{\bar{\lambda}_i}E^\top(i-1)X_1(i-1), \\ &\quad \sqrt{\bar{\lambda}_i}E^\top(i+1)X_1(i+1), \cdots, \sqrt{\bar{\lambda}_i}E^\top(N)X_1(N)\Big), \end{aligned}$$

$$\mathcal{S}_i(X_2) = \Big(\sqrt{\bar{\lambda}_i}E^\top(1)X_2(1), \cdots, \sqrt{\bar{\lambda}_i}E^\top(i-1)X_2(i-1), \sqrt{\bar{\lambda}_i}E^\top(i+1)X_2(i+1), \cdots, \sqrt{\bar{\lambda}_i}E^\top(N)X_2(N)\Big),$$
$$\mathcal{X}_i(\varepsilon) = diag\left[\varepsilon(1)\mathbb{I}, \cdots, \varepsilon(i-1)\mathbb{I}, \varepsilon(i+1)\mathbb{I}, \cdots, \varepsilon(N)\mathbb{I}\right],$$

with the following constraints:

$$E^\top(i)X_1(i) = X_1^\top(i)E(i) \geq 0, \tag{33}$$
$$E^\top(i)X_2(i) = X_2^\top(i)E(i) \geq 0, \tag{34}$$

then there exists a filter of the form (6) such that the estimation error is piecewise regular, impulse-free and stochastically stable and bounded by:

$$\|z - \hat{z}\|_2 \leq \gamma \left[\|\omega\|_2^2 + x_0^\top R x_0\right]^{\frac{1}{2}}. \tag{35}$$

The filter's gains are given by (20).

4 Numerical example

To show the validness of our results, let us consider a numerical example of a singular system with state space in $\mathbb{R}^3$. The data of this system are as follow:

$$A(1) = \begin{bmatrix} -3.0 & 1.0 & 0.0 \\ 0.3 & -2.5 & 1.0 \\ -0.1 & 0.3 & -3.8 \end{bmatrix}, A(2) = \begin{bmatrix} -4.0 & 1.0 & 0.0 \\ 0.3 & -3.0 & 1.0 \\ -0.1 & 0.3 & -4.8 \end{bmatrix},$$
$$B(1) = \begin{bmatrix} 0.0 \\ 0.0 \\ 1.0 \end{bmatrix}, B(2) = \begin{bmatrix} 0.0 \\ 0.0 \\ 1.0 \end{bmatrix},$$
$$C_y(1) = \begin{bmatrix} 0.1 & 0.2 & 0.0 \end{bmatrix}, C_y(2) = \begin{bmatrix} 0.2 & 0.1 & 0.0 \end{bmatrix},$$
$$C_z(1) = \begin{bmatrix} 0.2 & 0.1 & 0.0 \end{bmatrix}, C_z(2) = \begin{bmatrix} 0.1 & 0.2 & 0.0 \end{bmatrix},$$
$$D_y(1) = \begin{bmatrix} 0.1 \end{bmatrix}, D_y(2) = \begin{bmatrix} 0.2 \end{bmatrix},$$
$$D_z(1) = \begin{bmatrix} 0.2 \end{bmatrix}, D_z(2) = \begin{bmatrix} 0.1 \end{bmatrix}.$$

The singular matrices $E(1)$ and $E(2)$ are given by:

$$E(1) = E(2) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

The totally known transition jump rates are given by:

$$\Lambda = \begin{bmatrix} -1 & 1 \\ 1.1 & -1.1 \end{bmatrix}.$$

In case of partially known transition jump rates, we assume that we have the following bounds:

$$\text{mode } \# \ 1{:}\underline{\lambda}_1 = 0.8, \bar{\lambda}_1 = 1.2,$$
$$\text{mode } \# \ 2{:}\underline{\lambda}_2 = 0.9, \bar{\lambda}_2 = 1.3.$$

Notice that these values represent well a lower and an upper bounds respectively for the jump rates 1 and 1.1 of the considered transition jump rates matrix Λ.

Let us focus on the results of Theorems 3.1 and 3.3. When we assume that the transition jump rates are totally known and given by the matrix Λ, solving the LMIs of Theorem 3.1, we get:

$$K_A(1) = \begin{bmatrix} -1.6783 & -0.0828 & 0.5228 \\ -0.1146 & -2.1891 & 0.7330 \\ -2.8184 & -3.5545 & -3.6841 \end{bmatrix},$$
$$K_B(1) = \begin{bmatrix} 1.0287 \\ 1.3702 \\ 13.7120 \end{bmatrix}, K_C(1) = \begin{bmatrix} 0.0624 & 0.0888 & 0.2113 \end{bmatrix},$$
$$K_A(2) = \begin{bmatrix} -3.3779 & 0.5256 & -0.1336 \\ 0.3011 & -1.6788 & 0.8553 \\ -4.1267 & -1.2989 & -4.8583 \end{bmatrix},$$
$$K_B(2) = \begin{bmatrix} 0.5077 \\ -0.2788 \\ 11.0406 \end{bmatrix}, K_C(2) = \begin{bmatrix} 0.0413 & 0.1200 & 0.1345 \end{bmatrix}.$$

Assuming now that the transition jump rates are partially known and solving the LMIs of Theorem 3.3, we get:

$$K_A(1) = \begin{bmatrix} -2.2296 & 0.1517 & 0.0112 \\ -0.4427 & -2.9076 & 1.1675 \\ -1.7309 & -3.4890 & -3.1064 \end{bmatrix},$$
$$K_B(1) = \begin{bmatrix} 0.8524 \\ 1.9441 \\ 11.7931 \end{bmatrix}, K_C(1) = \begin{bmatrix} 0.1046 & 0.2198 & 0.1212 \end{bmatrix},$$
$$K_A(2) = \begin{bmatrix} -3.4264 & 0.4949 & -0.1320 \\ 0.3794 & -1.6943 & 0.8746 \\ -4.5055 & -1.3536 & -4.8755 \end{bmatrix},$$
$$K_B(2) = \begin{bmatrix} 0.5220 \\ -0.3818 \\ 11.1904 \end{bmatrix}, K_C(2) = \begin{bmatrix} 0.0388 & 0.1250 & 0.1491 \end{bmatrix}.$$

As it can be seen both of the theorems assure the existence of the filter that makes the system error piecewise, regular, impulse-free and stochastically stable and at the same time guarantees the minimal disturbance rejection level. The gains of the filter in the two cases are different while $\gamma^\star$ are almost the same.

Using now the other proposed approach and proceeding similarly as for the previous one we obtain the following results when the transition jump rates are totally and partially known. For the totally known case, solving the LMIs of Theorem 3.2, we get:

$$K_A(1) = \begin{bmatrix} -2.2633 & -0.0079 & -0.1875 \\ -0.2408 & -2.2899 & 0.6336 \\ -2.3280 & -3.3270 & -4.7839 \end{bmatrix},$$

$$K_B(1) = \begin{bmatrix} 2.4122 \\ 2.0643 \\ 16.5265 \end{bmatrix}, K_C(1) = \begin{bmatrix} 0.1304 & 0.1411 & 0.2686 \end{bmatrix},$$

$$K_A(2) = \begin{bmatrix} -3.5296 & 0.7743 & -0.2269 \\ 0.1676 & -2.4317 & 0.9026 \\ -3.0803 & -0.7313 & -5.3676 \end{bmatrix},$$

$$K_B(2) = \begin{bmatrix} 0.5847 \\ -0.0898 \\ 11.3202 \end{bmatrix}, K_C(2) = \begin{bmatrix} 0.0478 & 0.1308 & 0.2036 \end{bmatrix}.$$

Assuming now that the transition jump rates are partially known and solving the LMIs of Theorem 3.4, we get:

$$K_A(1) = \begin{bmatrix} -2.4654 & 0.0504 & -0.2486 \\ -0.1981 & -2.4448 & 0.6406 \\ -2.0907 & -3.4834 & -4.8499 \end{bmatrix},$$

$$K_B(1) = \begin{bmatrix} 2.4609 \\ 2.2744 \\ 16.8636 \end{bmatrix}, K_C(1) = \begin{bmatrix} 0.1245 & 0.1578 & 0.2810 \end{bmatrix},$$

$$K_A(2) = \begin{bmatrix} -3.3699 & 0.6812 & -0.2708 \\ 0.1267 & -2.0079 & 0.8530 \\ -3.4466 & -0.8205 & -5.3582 \end{bmatrix},$$

$$K_B(2) = \begin{bmatrix} 0.6863 \\ -0.0445 \\ 11.3009 \end{bmatrix}, K_C(2) = \begin{bmatrix} 0.0451 & 0.1159 & 0.2034 \end{bmatrix}.$$

For the purpose of comparison of the different procedure we proposed to design the $\mathscr{H}_\infty$ filter we have computed the minimum disturbance rejection level, $\gamma^\star$, for each one in the

Table 1: Comparison between the four procedures

Comparison				
Minimum rejection level	Theorem 3.1	Theorem 3.2	Theorem 3.3	Theorem 3.4
$\gamma^\star$	0.2	0.2	0.2	0.2

case of totally and partially known transition jump rates. The results are given in Table 1. As it can be seen from the results all the procedure give the same minimum disturbance rejection level and the computed gains are almost the same which shows the efficiency of the proposed methods.

5 Conclusion

In this paper we dealt with the class of continuous-time singular linear systems with random abrupt changes. Under the complete and partial knowledge of the transition jump rates a design procedure for the design of an $\mathscr{H}_\infty$ filter is developed. The proposed $\mathscr{H}_\infty$ filter guarantees that the system error is piecewise regular, impulse-free, stochastically stable and at the same time assures the disturbance rejection of a certain level $\gamma > 0$. The results we developed can easily be solved using any LMI toolbox like the one of Matlab or the one of Scilab. A numerical example is provided to show the usefulness of the developed results.

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