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Controller Design for Discrete-Time Nonlinear Markovian Jump Systems Using Fuzzy Logic

El-Kébir Boukas

*GERAD and Mechanical Engineering Department
École Polytechnique de Montréal
P.O. Box 6079, Succ. “Centre-ville”
Montréal (Québec) Canada H3C 3A7
el-kebir.boukas@polymtl.ca*

Huaping Liu

*Department of Computer Science and Technology
State Key Laboratory of Intelligent Technology and Systems
Tsinghua University
Beijing, P.R. China, 100084*

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Abstract

This paper considers the class of discrete-time nonlinear Markovian jump systems. The stochastic stability and stabilization problems are tackled. A model-based fuzzy stabilization design utilizing the concept of the so-called parallel distributed compensation (PDC) is employed to stochastically stabilize the class of systems under consideration. LMI-based sufficient conditions are developed to synthesize the state feedback controller that stochastically stabilizes the fuzzy stochastic system. Two examples are worked out to show the validness of the theoretical results.

Key Words: Discrete-time nonlinear Markovian jump system, LMI, state feedback control, fuzzy control.

Résumé

Ce papier traite de la commande des systèmes non linéaires à sauts markoviens. Les problèmes de stabilité et de stabilisation stochastique sont considérés. Une approche floue utilisant le modèle TS est employée. Des conditions en forme d'inégalités matricielles linéaires sont proposées pour faire le design du contrôleur. Deux exemples numériques sont traités pour montrer l'efficacité de l'approche utilisée.

1 Introduction

Discrete-time nonlinear Markovian jump system is a hybrid one with state comprised of two components: a jumping mode and a state vector. The jumping mode is a discrete-time Markov chain representing the mode of the system. The state vector evolves according to a difference equation when the mode is fixed. The control of discrete-time linear Markovian jump systems has received considerable interests and interesting results have been reported in the literature. For more information on discrete-time Markovian jump linear systems, the reader is referred to [7, 9] and the references therein.

The fuzzy control of the continuous-time systems with Markovian jumps has been tackled mainly by Nguang and his coauthors. For more details on what it has been done of this class of systems we refer the reader to Assawinchaichote et al. [1] and the references therein.

To the best of our knowledge the control of nonlinear discrete-time Markovian jump systems has not been fully investigated. The goal of this paper is to study the stability and stabilization of the class of discrete-time nonlinear Markovian jump systems using the Takagi-Sugeno (T-S) approach. A model-based fuzzy stabilization design utilizing the concept of the so-called parallel distributed compensation (PDC) is employed to stochastically stabilize the class of systems under consideration. The sufficient conditions we will establish are all in LMI formalism which makes their resolution easy. The rest of this paper is organized as follows. Section 2 describes the system model. Section 3 addresses the stability and stabilization problems. Section 4 provides numerical examples to show the validness of the proposed results.

Notation. Throughout this paper, \mathbb{R}^n and $\mathbb{R}^{n \times m}$ denote, respectively, the n dimensional Euclidean space and the set of all $n \times m$ real matrices. The superscript “ T ” denotes matrix transposition and the notation $X \geq Y$ (respectively, $X > Y$) where X and Y are symmetric matrices, means that $X - Y$ is positive semi-definite (respectively, positive definite). \mathbb{I} is the identity matrices with compatible dimensions. $diag\{\}$ stands for a block-diagonal matrix. For a symmetric block matrix, we use “ $*$ ” as an ellipsis for the terms that are introduced by symmetry. $\mathbb{E}\{\cdot|\cdot\}$ stands for the conditional mathematical expectation.

2 Model Description

Let $\{r(t), t \geq 0\}$ be a Markov chain with state space $\mathcal{S} = \{1, \dots, N\}$ and probability transition matrix $\mathbb{P} = [p_{ij}]_{i,j \in \mathcal{S}}$, i.e. the transition probabilities of $\{r(t), t \geq 0\}$ are as follows:

$$\text{Prob}[r(t+1) = j | r(t) = i] = p_{ij}, \forall i, j \in \mathcal{S},$$

with $p_{ij} \geq 0, \forall i, j \in \mathcal{S}$ and $\sum_{j=1}^N p_{ij} = 1$, for $i \in \mathcal{S}$.

Consider a discrete-time nonlinear hybrid system with N modes and suppose that the system mode switching is governed by $\{r(t), t \geq 0\}$. Let the system dynamics be described by the following:

$$\begin{cases} x(t+1) = f(x(t), i) + g(x(t), i)u(t), \\ x(0) = x_0, \end{cases} \quad (1)$$

for $r(t) = i$, where $x(t) \in \mathbb{R}^n$ and $u(t) \in \mathbb{R}^m$ are respectively the state and the control input of the system. The functions $f(x(t), i)$ and $g(x(t), i)$, for any $x(t) \in \mathbb{R}^n$ and each $i \in \mathcal{S}$, are assumed to satisfy all the appropriate assumptions for the existence and uniqueness of the solution.

To the best of knowledge the control of the class of systems described by (1) has not been fully investigated and it remains a very difficult class to control. In this paper we will use the fuzzy logic approach to study the stochastic stability and the stochastic stabilization. For this purpose, the approach proposed by Takagi-Sugeno is used (see [12]). This fuzzy dynamic model is a piecewise interpolation of several linear models through membership functions. It is described by fuzzy rules of the type IF-THEN that represent local input output models for a nonlinear system. In the rest of this paper we will assume that the behavior of the class of stochastic fuzzy systems is described by the following:

Plant rule i : IF $z_1(t)$ is M_{i1} , \dots and $z_p(t)$ is M_{ip}

Then

$$\begin{cases} x(t+1) = \tilde{A}_i(r(t))x(t) + \tilde{B}_i(r(t))u(t), \\ x(0) = x_0, \end{cases} \quad (2)$$

where M_{ij} is the fuzzy set; $z_j(t)$ is the premise variable; $\tilde{A}_i(l)$ and $\tilde{B}_i(l)$ are real matrices with appropriate dimensions for each $i \in \mathcal{S}_q = \{1, 2, \dots, q\}$ and each $l \in \mathcal{S}$ and can be represented as:

$$\tilde{A}_i(l) = A_i(l) + \Delta A_i(l), \tilde{B}_i(l) = B_i(l) + \Delta B_i(l),$$

where $A_i(l)$ and $B_i(l)$ are known matrix, $\Delta A_i(l)$ and $\Delta B_i(l)$ are unknown but can be decomposed as:

$$[\Delta A_i(l) \quad \Delta B_i(l)] = D_i(l)\Delta_i(l, t)[E_i(l) \quad F_i(l)],$$

where $D_i(l)$, $E_i(l)$ and $F_i(l)$ are known and $\Delta_i(l, t)$ is unknown but satisfies

$$\Delta_i^T(l, t)\Delta_i(l, t) \leq \mathbb{I}.$$

Using the standard fuzzy inference method that uses a singleton fuzzifier, product of fuzzy inference and weighted average defuzzifier, the stochastic fuzzy systems we are considering are described by:

$$\begin{cases} x(t+1) = \sum_{i=1}^q h_i(z(t)) [\tilde{A}_i(r(t))x(t) + \tilde{B}_i(r(t))u(t)], \\ x(0) = x_0, \end{cases} \quad (3)$$

where $h_i(z(t)) = \frac{w_i(z(t))}{\sum_{i=1}^q w_i(z(t))}$ with $w_i(z(t)) = \prod_{j=1}^p M_{ij}(z_j(t))$, where $M_{ij}(z_j(t))$ is the grade of membership of $z_j(t)$ in the set M_{ij} .

Remark 1 Based on the definitions of $w_i(z(t))$ and $h_i(z(t))$, we assume $w_i(z(t)) \geq 0$, $\sum_{i=1}^q w_i(z(t)) > 0$ and therefore we have $h_i(z(t)) \geq 0$ and $\sum_{i=1}^q h_i(z(t)) = 1$.

Remark 2 The premise variable $z(t)$ can be one part of the state variable $x(t)$, or some combination of different components of $x(t)$. Therefore the terms $\sum_{i=1}^q h_i(z(t))A_i(r(t))x(t)$ and $\sum_{i=1}^q h_i(z(t))B_i(r(t))u(t)$ can be used to approximate the nonlinear representation $f(x(t), r(t)) + g(x(t), r(t))u(t)$. In addition, the terms $\Delta A_i(l)$ and $\Delta B_i(l)$ can be used to represent the modelling uncertainties effectively. For more details, please refer to [10].

Definition 1 System (3) with $u(t) = 0$ is said to be robustly stochastically stable if for any finite initial state $x_0 \in \mathbb{R}^n$ and initial mode r_0

$$\sum_{t=0}^{\infty} \mathbb{E}[\|x(t)\|^2 | x_0, r_0] < \infty. \quad (4)$$

Definition 2 System (3) is said to be robustly stabilizable in the stochastic sense if there exists a controller such that the closed-loop system is robustly stochastically stable.

To stabilize the class of nonlinear stochastic system (3) when the state is available for feedback for each mode $l \in \mathcal{S}$, we can use the PDC controller that we assume to be described by the following:

Controller rule i : IF $z_1(t) \in M_{i1}$ and \dots and $z_p(t) \in M_{ip}$, $l \in \mathcal{S}$

Then

$$u(t) = K_i(r(t))x(t),$$

where $K_i(l)$ is a gain to be determined for each $i \in \mathcal{S}_q$ and each $l \in \mathcal{S}$.

The overall state feedback fuzzy controller is represented by:

$$u(t) = \sum_{i=1}^q h_i(z(t))K_i(r(t))x(t), \quad (5)$$

which is also nonlinear since the premise variable $z(t)$ is dependent on the state variable $x(t)$.

Plugging the controller expression (5) in the system dynamics (3) gives the following closed-loop system:

$$x(t+1) = \sum_{i=1}^q \sum_{j=1}^q h_i(z(t))h_j(z(t)) \left[\tilde{A}_i(r(t))x(t) + \tilde{B}_i(r(t))K_j(r(t))x(t) \right]$$

$$\begin{aligned}
&= \sum_{i=1}^q h_i^2(z(t)) \left[\tilde{A}_{ii}(r(t))x(t) \right] \\
&\quad + 2 \sum_{i=1}^q \sum_{i < j}^q h_i(z(t))h_j(z(t)) \left[\frac{\tilde{A}_{ij}(r(t)) + \tilde{A}_{ji}(r(t))}{2} \right] x(t)
\end{aligned} \tag{6}$$

with $\tilde{A}_{ij}(l) = \tilde{A}_i(l) + \tilde{B}_i(l)K_j(l)$.

In this paper we are interested in developing sufficient conditions for the design of a state feedback controller for the class of stochastic fuzzy systems. LMI-based sufficient conditions are needed to synthesize the gain of the state feedback controller that stochastically stabilizes the class of systems we are dealing with. Our methodology is mainly based on the Lyapunov theory and some algebraic results.

Before closing this section, let us recall some lemmas that we will use in the rest of the paper.

Lemma 1 ([6]) *The linear matrix inequality*

$$\begin{bmatrix} H & S^\top \\ S & R \end{bmatrix} < 0$$

is equivalent to $R < 0$ and $H - S^\top R^{-1}S < 0$, where $H = H^\top$, $R = R^\top$ and S is a matrix with appropriate dimension.

Lemma 2 ([20]) *Let U , V , and Δ be matrices with appropriate dimensions. Suppose $\Delta^\top \Delta \leq \mathbb{I}$, then we have*

$$U\Delta V + (U\Delta V)^\top \leq \varepsilon U U^\top + \varepsilon^{-1} V^\top V$$

for any $\varepsilon > 0$.

3 Stability and Stabilization

In this section, we consider the stochastic stability and the stochastic stabilization of system (3). The following theorem gives the sufficient conditions on stochastic stability.

Theorem 1 *The fuzzy stochastic system (3) is robustly stochastically stable if there exists a set of symmetric and positive-definite matrices $P(1), \dots, P(N)$ such that the following set of coupled LMIs holds:*

$$\tilde{A}_i^\top(l)G(l)\tilde{A}_i(l) - P(l) < 0, \tag{7}$$

where $G(l) = \sum_{m=1}^N p_{lm}P(m)$.

Proof: Define a Lyapunov function candidate $V(x(t), r(t))$ as follows:

$$V(x(t), r(t)) = x^\top(t)P(r(t))x(t), \quad (8)$$

where $P(l)$ is a symmetric and positive-definite matrix for each $l \in \mathcal{S}$. Then, simple computation gives the following

$$\begin{aligned} \mathbb{E}[V(x(t+1), r(t+1))|x_0, r_0] - V(x(t), r(t) = l) \\ = \sum_{i=1}^q \sum_{j=1}^q h_i(z(t))h_j(z(t))x^\top(t) \left[\tilde{A}_i^\top(l)G(l)\tilde{A}_j(l) - P(l) \right] x(t). \end{aligned} \quad (9)$$

Thus, we have

$$\begin{aligned} \mathbb{E}[V(x(t+1), r(t+1))|x_0, r_0] - V(x(t), l) \\ \leq -\lambda_{\min}(-\Theta(l))x^\top(t)x(t) \\ \leq -\beta x^\top(t)x(t), \end{aligned} \quad (10)$$

where $\Theta(l) = \tilde{A}_i^\top(l)G(l)\tilde{A}_i(l) - P(l)$, and $\lambda_{\min}(-\Theta(l))$ denotes the minimal eigenvalue of $-\Theta(l)$ for all $i \in \mathcal{S}_r$ and $\beta = \inf\{\lambda_{\min}(-\Theta(l)), l \in \mathcal{S}\}$, $\beta > 0$. From (10), we obtain that for any $T \geq 1$

$$\mathbb{E}[V(x(T+1), r(T+1))|x_0, r_0] - \mathbb{E}[V(x_0, r_0)|x_0, r_0] \leq -\beta \sum_{t=0}^T \mathbb{E}[x^\top(t)x(t)|x_0, r_0],$$

This yields the following for any $T \geq 1$,

$$\begin{aligned} \sum_{t=0}^T \mathbb{E}[x^\top(t)x(t)|x_0, r_0] &\leq \frac{1}{\beta} \left[\mathbb{E}[V(x_0, r_0)|x_0, r_0] - \mathbb{E}[V(x(T+1), r(T+1))|x_0, r_0] \right] \\ &\leq \frac{1}{\beta} \mathbb{E}[V(x_0, r_0)|x_0, r_0], \end{aligned}$$

which implies

$$\sum_{t=0}^{\infty} \mathbb{E}[x^\top(t)x(t)|x_0, r_0] \leq \frac{1}{\beta} \mathbb{E}[V(x_0, r_0)|x_0, r_0] < \infty.$$

This means that system (3) is robustly stochastically stable and thus completes the proof of Theorem 1. \square

Remark 3 Note that the LMI (7) can be rewritten as follows:

$$\begin{bmatrix} -P(l) & \tilde{A}_i^\top(l)W(l) \\ W^\top(l)\tilde{A}_i(l) & -\Gamma \end{bmatrix} < 0,$$

where $\Gamma = \text{diag}(P^{-1}(1), \dots, P^{-1}(N))$ and $W(l) = [\sqrt{p_{l1}}\mathbb{I}, \dots, \sqrt{p_{lN}}\mathbb{I}]$.

Remark 4 When the number of fuzzy rules is limited to 1 (i.e.: the system is linear), the results of this theorem are the ones already developed for linear discrete-time systems with Markovian jumps.

Now let us concentrate on the design of a state feedback controller of the following form (5) that robustly stochastically stabilizes system (3). Using the closed-loop system we get:

$$\begin{aligned} \Delta V(x(t), r(t) = l) &= \mathbb{E}[V(x(t+1), r(t+1)) | x_0, r_0] - V(x(t), r(t) = l) \\ &= \sum_{i=1}^q \sum_{j=1}^q h_i(z(t), l) h_j(z(t), l) x^\top(t) \tilde{A}_{ij}^\top(l) G(l) \\ &\quad \sum_{\alpha=1}^q \sum_{\beta=1}^q h_\alpha(z(t), l) h_\beta(z(t), l) \tilde{A}_{\alpha\beta}(l) x(t) - x^\top(t) P(l) x(t) \\ &= \sum_{i=1}^q \sum_{j=1}^q \sum_{\alpha=1}^q \sum_{\beta=1}^q h_i(z(t), l) h_j(z(t), l) h_\alpha(z(t), l) \\ &\quad h_\beta(z(t), l) x^\top(t) \left[\tilde{A}_{ij}^\top(l) G(l) \tilde{A}_{\alpha\beta}(l) - P(l) \right] x(t) \\ &\leq \sum_{i=1}^q \sum_{j=1}^q h_i(z(t), l) h_j(z(t), l) x^\top(t) \left[\tilde{A}_{ij}^\top(l) G(l) \tilde{A}_{ij}(l) - P(l) \right] x(t) \\ &= \sum_{i=1}^q h_i^2(z(t), l) x^\top(t) \left[\tilde{A}_{ii}^\top(l) G(l) \tilde{A}_{ii}(l) - P(l) \right] x(t) \\ &\quad + 2 \sum_{i=1}^q \sum_{i < j}^q h_i(z(t), l) h_j(z(t), l) x_t^\top \\ &\quad \left[\left[\frac{\tilde{A}_{ij}(l) + \tilde{A}_{ji}(l)}{2} \right]^\top G(l) \left[\frac{\tilde{A}_{ij}(l) + \tilde{A}_{ji}(l)}{2} \right] - P(l) \right] x(t), \end{aligned}$$

Using the results of Kim and Lee [11], if we let

$$\tilde{A}_{ii}^\top(l) G(l) \tilde{A}_{ii}(l) - P(l) < -X_{ii}(l), i = 1, \dots, q, l = 1, \dots, N,$$

$$\left[\frac{\tilde{A}_{ij}(l) + \tilde{A}_{ji}(l)}{2} \right]^\top G(l) \left[\frac{\tilde{A}_{ij}(l) + \tilde{A}_{ji}(l)}{2} \right] - P(l) \leq -X_{ij}(l), i < j, l = 1, \dots, N,$$

where $X_{ij}(l)$ is a matrix to be determined for each i, j and l ; we get:

$$\begin{aligned} \Delta V(x(t), l) &< - \left[\sum_{i=1}^N h_i^2(z(t), l) x^\top(t) X_{ii}(l) x(t) \right. \\ &\quad \left. + 2 \sum_{i=1}^N \sum_{i < j}^N h_i(z(t), l) h_j(z(t), l) x^\top(t) X_{ij}(l) x(t) \right] \\ &= - \begin{bmatrix} h_1(z(t), l) x(t) \\ h_2(z(t), l) x(t) \\ \vdots \\ h_q(z(t), l) x(t) \end{bmatrix}^\top \begin{bmatrix} X_{11}(l) & X_{12}(l) & \cdots & X_{1q}(l) \\ X_{12}(l) & X_{22}(l) & \cdots & X_{2q}(l) \\ \vdots & \vdots & \ddots & \vdots \\ X_{1q}(l) & X_{2q}(l) & \cdots & X_{qq}(l) \end{bmatrix} \begin{bmatrix} h_1(z(t), l) x(t) \\ h_2(z(t), l) x(t) \\ \vdots \\ h_q(z(t), l) x(t) \end{bmatrix} \\ &= H^\top(z(t), l) [-\tilde{X}(l)] H(z(t), l), \end{aligned}$$

with

$$H(z(t), l) = \begin{bmatrix} h_1(z(t), l) x(t) \\ h_2(z(t), l) x(t) \\ \vdots \\ h_q(z(t), l) x(t) \end{bmatrix}, \quad \tilde{X}(l) = \begin{bmatrix} X_{11}(l) & X_{12}(l) & \cdots & X_{1q}(l) \\ X_{12}(l) & X_{22}(l) & \cdots & X_{2q}(l) \\ \vdots & \vdots & \ddots & \vdots \\ X_{1q}(l) & X_{2q}(l) & \cdots & X_{qq}(l) \end{bmatrix}.$$

Finally, we get the following sufficient conditions for robustly stochastic stability of the closed-loop system:

$$\tilde{A}_{ii}^\top(l) G(l) \tilde{A}_{ii}(l) - P(l) < -X_{ii}(l), i = 1, \dots, q, l = 1, \dots, N, \quad (11)$$

$$\left[\frac{\tilde{A}_{ij}(l) + \tilde{A}_{ji}(l)}{2} \right]^\top G(l) \left[\frac{\tilde{A}_{ij}(l) + \tilde{A}_{ji}(l)}{2} \right] - P(l) \leq -X_{ij}(l), i < j, l = 1, \dots, N, \quad (12)$$

$$\begin{bmatrix} X_{11}(l) & X_{12}(l) & \cdots & X_{1q}(l) \\ X_{12}(l) & X_{22}(l) & \cdots & X_{2q}(l) \\ \vdots & \vdots & \ddots & \vdots \\ X_{1q}(l) & X_{2q}(l) & \cdots & X_{qq}(l) \end{bmatrix} > 0, l = 1, \dots, N. \quad (13)$$

Letting $X(l) = P^{-1}(l)$, $Y_i(l) = K_i(l)X(l)$ and $Q_{ij}(l) = X(l)X_{ij}(l)X(l)$ and after pre- and post-multiply (11)–(12) and (13) respectively by $\text{diag}(X(l), \mathbb{I})$ and $\text{diag}(X(l), \dots, X(l))$, we get:

$$\begin{bmatrix} -X(l) + Q_{ii}(l) & \star \\ W^\top(l) [\tilde{A}_i(l)X(l) + \tilde{B}_i(l)Y_i(l)] & -\mathcal{B} \end{bmatrix} < 0, i = 1, \dots, q, l = 1, \dots, N, \quad (14)$$

$$\begin{bmatrix} -X(l) + Q_{ij}(l) & \star \\ \frac{1}{2}W^\top(l) \begin{bmatrix} \tilde{A}_i(l)X(l) + \tilde{B}_i(l)Y_j(l) \\ +\tilde{A}_j(l)X(l) + \tilde{B}_j(l)Y_i(l) \end{bmatrix} & -\mathcal{B} \end{bmatrix} \leq 0, i < j, l = 1, \dots, N, \quad (15)$$

$$\begin{bmatrix} Q_{11}(l) & Q_{12}(l) & \cdots & Q_{1q}(l) \\ Q_{12}(l) & Q_{22}(l) & \cdots & Q_{2q}(l) \\ \vdots & \vdots & \ddots & \vdots \\ Q_{1q}(l) & Q_{2q}(l) & \cdots & Q_{qq}(l) \end{bmatrix} > 0, l = 1, \dots, N, \quad (16)$$

where $\mathcal{B} = \text{diag}(X(1), \dots, X(N))$.

Since the conditions (14) and (15) incorporate uncertain matrices $\tilde{A}_i(l)$ and $\tilde{B}_i(l)$, they can not be used to design controller. To tackle this difficulty, we proposed the following theorem:

Theorem 2 *The closed-loop stochastic fuzzy system (3) is robustly stochastically stable under the state feedback controller of the form (5) if there exist matrices $X(l) > 0$, $Q_{ij}(l)$, $Y_i(l)$, and a set of scalars $\epsilon_i(l) > 0$, for $i, j = 1, 2, \dots, q$ and $l = 1, 2, \dots, N$, such that the set of coupled LMIs (17)–(19) hold. The gains of the controller are given respectively by $K_i(l) = Y_i(l)X^{-1}(l)$.*

$$\begin{bmatrix} -X(l) + Q_{ii}(l) & \star & \star & \star \\ W^\top(l) [A_i(l)X(l) + B_i(l)Y_i(l)] & -\mathcal{B} & \star & \star \\ 0 & \epsilon_i(l)D_i^T W(l) & -\epsilon_i(l) & \star \\ E_i(l)X(l) + F_i(l)Y_i(l) & 0 & 0 & -\epsilon_i(l) \end{bmatrix} < 0, \quad (17)$$

for $i = 1, \dots, q, l = 1, \dots, N$,

$$\begin{bmatrix} -X(l) + Q_{ij}(l) & \star & \star & \star & \star & \star \\ \begin{pmatrix} \frac{1}{2}W^\top(l)[A_i(l)X(l) + B_i(l)Y_j(l)] \\ +\frac{1}{2}W^\top(l)[A_j(l)X(l) + B_j(l)Y_i(l)] \end{pmatrix} & -\mathcal{B} & \star & \star & \star & \star \\ 0 & \epsilon_i(l)D_i^T W(l) & -2\epsilon_i(l) & \star & \star & \star \\ E_i(l)X(l) + F_i(l)Y_j(l) & 0 & 0 & -2\epsilon_i(l) & \star & \star \\ 0 & \epsilon_j(l)D_j^T W(l) & 0 & 0 & -2\epsilon_j(l) & \star \\ E_j(l)X(l) + F_j(l)Y_i(l) & 0 & 0 & 0 & 0 & -2\epsilon_j(l) \end{bmatrix} < 0, \quad (18)$$

for $i, j = 1, 2, \dots, q, i < j$, and $l = 1, \dots, N$,

$$\begin{bmatrix} Q_{11}(l) & Q_{12}(l) & \cdots & Q_{1q}(l) \\ Q_{12}(l) & Q_{22}(l) & \cdots & Q_{2q}(l) \\ \vdots & \vdots & \ddots & \vdots \\ Q_{1q}(l) & Q_{2q}(l) & \cdots & Q_{qq}(l) \end{bmatrix} > 0, l = 1, \dots, N. \quad (19)$$

Proof. By using Schur complements, (17) can be converted equivalently to

$$\begin{aligned} & \begin{bmatrix} -X(l) + Q_{ii}(l) & \star \\ W^\top(l) [A_i(l)X(l) + B_i(l)Y_i(l)] & -\mathcal{B} \end{bmatrix} + \epsilon_i(l) \begin{bmatrix} 0 \\ W^\top(l)D_i(l) \end{bmatrix} \begin{bmatrix} 0 & D_i^\top(l)W(l) \end{bmatrix} \\ & + \epsilon_i^{-1}(l) \begin{bmatrix} (E_i(l)X(l) + F_i(l)Y(l))^\top \\ 0 \end{bmatrix} \begin{bmatrix} E_i(l)X(l) + F_i(l)Y(l) & 0 \end{bmatrix} < 0. \end{aligned} \quad (20)$$

By using Lemma 2, we have

$$\begin{aligned} & \begin{bmatrix} 0 \\ W^\top(l)D_i(l) \end{bmatrix} \Delta_i(l, t) \begin{bmatrix} E_i(l)X(l) + F_i(l)Y(l) & 0 \end{bmatrix} \\ & + \begin{bmatrix} (E_i(l)X(l) + F_i(l)Y(l))^\top \\ 0 \end{bmatrix} \Delta_i(l, t) \begin{bmatrix} 0 & D_i^\top(l)W(l) \end{bmatrix} \\ & < \epsilon_i(l) \begin{bmatrix} 0 \\ W^\top(l)D_i(l) \end{bmatrix} \begin{bmatrix} 0 & D_i^\top(l)W(l) \end{bmatrix} \\ & + \epsilon_i^{-1}(l) \begin{bmatrix} (E_i(l)X(l) + F_i(l)Y(l))^\top \\ 0 \end{bmatrix} \begin{bmatrix} E_i(l)X(l) + F_i(l)Y(l) & 0 \end{bmatrix}. \end{aligned}$$

Therefore, we have

$$\begin{aligned} & \begin{bmatrix} -X(l) + Q_{ii}(l) & \star \\ W^\top(l) [A_i(l)X(l) + B_i(l)Y_i(l)] & -\mathcal{B} \end{bmatrix} \\ & + \begin{bmatrix} 0 \\ W^\top(l)D_i(l) \end{bmatrix} \Delta_i(l, t) \begin{bmatrix} E_i(l)X(l) + F_i(l)Y(l) & 0 \end{bmatrix} \\ & + \begin{bmatrix} (E_i(l)X(l) + F_i(l)Y(l))^\top \\ 0 \end{bmatrix} \Delta_i(l, t) \begin{bmatrix} 0 & D_i^\top(l)W(l) \end{bmatrix} < 0, \end{aligned}$$

which is

$$\begin{bmatrix} -X(l) + Q_{ii}(l) & \star \\ W^\top(l) [\tilde{A}_i(l)X(l) + \tilde{B}_i(l)Y_i(l)] & -\mathcal{B} \end{bmatrix} < 0. \quad (21)$$

Similarly, from (18) we can derive (15), and therefore the expected results are achieved. \square

Remark 5 When the number of local models is limited to 1 (i.e.: the system is linear), the results of the stabilization are the ones already developed for linear discrete-time systems with Markovian jumps.

4 Numerical Examples

To illustrate the effectiveness of the previous theoretical results, this section provides two numerical examples. The first is an academic one and the second is practical one. All of the computations are implemented using MATLAB with LMI Toolbox. We will use the following membership functions for the two examples:

$$\begin{aligned} h_1(x_1(t)) &= \frac{0.4}{1 + |x_1(t)|} \\ h_2(x_1(t)) &= \frac{0.5}{1 + |x_1(t)|} \\ h(x_1(t)) &= 1 - h_1(x_1(t)) - h_2(x_1(t)) \end{aligned}$$

The uncertainties in these examples are assumed to be zero. Notice that the presence of the uncertainties will be done in a similar way as we did for these two examples.

Example 1: For the academic example, let us consider a system with two states and three modes. The three local models that approximate the nonlinear dynamics in each mode are assumed to have the following data:

- mode # 1:

$$\begin{aligned} A_1(1) &= \begin{bmatrix} 1.0 & 1.0 \\ 0.4 & 0.2 \end{bmatrix}, & B_1(1) &= \begin{bmatrix} 0.0 \\ 0.1 \end{bmatrix}, \\ A_2(1) &= \begin{bmatrix} 1.0 & 0.1 \\ -0.25 & 0.2 \end{bmatrix}, & B_2(1) &= \begin{bmatrix} 0.0 \\ 0.2 \end{bmatrix}, \\ A_3(1) &= \begin{bmatrix} 1.0 & 0.1 \\ -0.3 & 0.2 \end{bmatrix}, & B_3(1) &= \begin{bmatrix} 0.0 \\ 0.5 \end{bmatrix}, \end{aligned}$$

- mode # 2:

$$\begin{aligned} A_1(2) &= \begin{bmatrix} 1.0 & 1.0 \\ 0.4 & 0.2 \end{bmatrix}, & B_1(2) &= \begin{bmatrix} 0.0 \\ 0.1 \end{bmatrix}, \\ A_2(2) &= \begin{bmatrix} 1.0 & 0.1 \\ -0.25 & 0.2 \end{bmatrix}, & B_2(2) &= \begin{bmatrix} 0.0 \\ 0.2 \end{bmatrix}, \\ A_3(2) &= \begin{bmatrix} 1.0 & 0.1 \\ -0.3 & 0.2 \end{bmatrix}, & B_3(2) &= \begin{bmatrix} 0.0 \\ 0.5 \end{bmatrix}, \end{aligned}$$

- mode # 3:

$$\begin{aligned} A_1(3) &= \begin{bmatrix} 1.0 & 1.0 \\ -0.4 & 0.1 \end{bmatrix}, & B_1(3) &= \begin{bmatrix} 0.0 \\ 0.11 \end{bmatrix}, \\ A_2(3) &= \begin{bmatrix} 1.0 & 1.0 \\ -0.2 & 0.2 \end{bmatrix}, & B_2(3) &= \begin{bmatrix} 0.0 \\ 0.21 \end{bmatrix}, \\ A_3(3) &= \begin{bmatrix} 1.0 & 0.1 \\ -0.1 & 0.2 \end{bmatrix}, & B_3(3) &= \begin{bmatrix} 0.0 \\ 0.3 \end{bmatrix}, \end{aligned}$$

The probability matrix between the different modes is given by:

$$P = \begin{bmatrix} 0.4 & 0.3 & 0.3 \\ 0.3 & 0.6 & 0.1 \\ 0.2 & 0.3 & 0.5 \end{bmatrix}.$$

Solving LMI (7) we can not find a feasible solution and this does not imply that the system is not stochastically stable since our conditions are only sufficient. Solving now the LMIs (17)–(19), we get:

$$\begin{aligned} X(1) &= \begin{bmatrix} 1.1744 & -0.6371 \\ -0.6371 & 0.9485 \end{bmatrix}, & X(2) &= \begin{bmatrix} 1.2407 & -0.6090 \\ -0.6090 & 1.0567 \end{bmatrix}, & X(3) &= \begin{bmatrix} 1.3610 & -0.6626 \\ -0.6626 & 0.9250 \end{bmatrix}, \\ Q_{11}(1) &= \begin{bmatrix} 0.3360 & -0.3524 \\ -0.3524 & 0.3896 \end{bmatrix}, & Q_{12}(1) &= \begin{bmatrix} 0.0077 & -0.0072 \\ -0.0072 & 0.0010 \end{bmatrix}, & Q_{13}(1) &= \begin{bmatrix} 0.0061 & -0.0114 \\ -0.0114 & -0.0154 \end{bmatrix}, \\ Q_{22}(1) &= \begin{bmatrix} 0.0826 & -0.0752 \\ -0.0752 & 0.3541 \end{bmatrix}, & Q_{23}(1) &= \begin{bmatrix} -0.0176 & 0.0014 \\ 0.0014 & -0.0170 \end{bmatrix}, & Q_{33}(1) &= \begin{bmatrix} 0.0841 & -0.0728 \\ -0.0728 & 0.3604 \end{bmatrix}, \\ Q_{11}(2) &= \begin{bmatrix} 0.4488 & -0.4143 \\ -0.4143 & 0.4163 \end{bmatrix}, & Q_{12}(2) &= \begin{bmatrix} 0.0044 & -0.0042 \\ -0.0042 & -0.0093 \end{bmatrix}, & Q_{13}(2) &= \begin{bmatrix} 0.0033 & -0.0034 \\ -0.0034 & -0.0099 \end{bmatrix}, \\ Q_{22}(2) &= \begin{bmatrix} 0.0506 & -0.0634 \\ -0.0634 & 0.4256 \end{bmatrix}, & Q_{23}(2) &= \begin{bmatrix} -0.0095 & -0.0005 \\ -0.0005 & -0.0126 \end{bmatrix}, & Q_{33}(2) &= \begin{bmatrix} 0.0507 & -0.0633 \\ -0.0633 & 0.4250 \end{bmatrix}, \\ Q_{11}(3) &= \begin{bmatrix} 0.4845 & -0.4033 \\ -0.4033 & 0.4236 \end{bmatrix}, & Q_{12}(3) &= \begin{bmatrix} 0.0484 & -0.0566 \\ -0.0566 & 0.0307 \end{bmatrix}, & Q_{13}(3) &= \begin{bmatrix} 0.0429 & -0.0508 \\ -0.0508 & 0.0246 \end{bmatrix}, \\ Q_{22}(3) &= \begin{bmatrix} 0.5654 & -0.4637 \\ -0.4637 & 0.4690 \end{bmatrix}, & Q_{23}(3) &= \begin{bmatrix} -0.2786 & 0.2025 \\ 0.2025 & -0.1431 \end{bmatrix}, & Q_{33}(3) &= \begin{bmatrix} 0.5646 & -0.4631 \\ -0.4631 & 0.4686 \end{bmatrix}, \\ Y_1(1) &= [-2.2040 \quad -1.4217], & Y_1(2) &= [0.8340 \quad -2.8534], & Y_1(3) &= [1.4181 \quad -3.4035], \\ Y_2(1) &= [-0.6098 \quad -0.3842], & Y_2(2) &= [-1.9786 \quad -0.5816], & Y_2(3) &= [0.1935 \quad -2.1370], \\ Y_3(1) &= [-0.1559 \quad -0.0408], & Y_3(2) &= [-4.0178 \quad 0.8305], & Y_3(3) &= [-0.3408 \quad -1.2385], \end{aligned}$$

which gives the following gains:

$$\begin{aligned} K_1(1) &= [-4.2322 \quad -4.3418], & K_1(2) &= [-0.9110 \quad -3.2254], & K_1(3) &= [-1.1508 \quad -4.5038], \\ K_2(1) &= [-1.1628 \quad -1.1862], & K_2(2) &= [-2.6006 \quad -2.0492], & K_2(3) &= [-1.5089 \quad -3.3911], \\ K_3(1) &= [-0.2457 \quad -0.2081], & K_3(2) &= [-3.9778 \quad -1.5066], & K_3(3) &= [-1.3855 \quad -2.3314]. \end{aligned}$$

Figures 1–3 give the simulation results, where the initial state is set to be $[1, -1]$. In Figure 1, solid line is for $x_1(t)$ and dashed line is for $x_2(t)$. Figure 3 and Figure 4 are for control effort and

jumping mode, respectively. We can see satisfactory results are obtained by this fuzzy jumping controller.

Example 2: For the second example, let us consider the model for the economic of the USA as described in [9] which is also known in the literature as Samuelson's multiplier model. This model is given by:

$$x(t+1) = F(x(t), u(t), s, \alpha), x(0) = x_0$$

where $x(t) \in \mathbb{R}^2$ with $x_2(t)$ stands for the national income ($x_1(t)$ differs from $x_2(t)$ only by a one-step lag), $u(t) \in \mathbb{R}$ represents the government expenditure, s is the marginal propensity to save ($\frac{1}{s}$ in the so-called multiplier) and α is an accelerator coefficient.

Based on historical data of the United States Department of Commerce the parameters s and α were grouped in three states called respectively, normal, boom and slump. In the rest of this paper we refer to states as the mode of our system. The table 1 gives the signification of these modes.

Table 1: System's mode

Mode	Name	Description
1	Normal	s and α are in mid-range
2	Boom	s is in low range (or α is in high range)
3	Slump	s is in high range (or α is in low range)

For this nonlinear system, we will try to represent it by three local models. The three local models that approximate the nonlinear dynamics in each mode are assumed to have the following data:

- mode # 1:

$$\begin{aligned} A_1(1) &= \begin{bmatrix} 0.0 & 1.0 \\ -2.5 & 3.2 \end{bmatrix}, & B_1(1) &= \begin{bmatrix} 0.0 \\ 1.0 \end{bmatrix}, \\ A_2(1) &= \begin{bmatrix} 0.0 & 1.0 \\ -2.0 & 2.2 \end{bmatrix}, & B_2(1) &= \begin{bmatrix} 0.0 \\ 1.0 \end{bmatrix}, \\ A_3(1) &= \begin{bmatrix} 0.0 & 1.0 \\ -1.5 & 2.5 \end{bmatrix}, & B_3(1) &= \begin{bmatrix} 0.0 \\ 1.0 \end{bmatrix}, \end{aligned}$$

- mode # 2:

$$\begin{aligned} A_1(2) &= \begin{bmatrix} 0.0 & 1.0 \\ -43.7 & 45.4 \end{bmatrix}, & B_1(2) &= \begin{bmatrix} 0.0 \\ 1.0 \end{bmatrix}, \\ A_2(2) &= \begin{bmatrix} 0.0 & 1.0 \\ -33.7 & 35.4 \end{bmatrix}, & B_2(2) &= \begin{bmatrix} 0.0 \\ 1.0 \end{bmatrix}, \\ A_3(2) &= \begin{bmatrix} 0.0 & 1.0 \\ -40.7 & 55.4 \end{bmatrix}, & B_3(2) &= \begin{bmatrix} 0.0 \\ 1.0 \end{bmatrix}, \end{aligned}$$

- mode # 3:

$$\begin{aligned} A_1(3) &= \begin{bmatrix} 0.0 & 1.0 \\ 5.3 & -5.2 \end{bmatrix}, & B_1(3) &= \begin{bmatrix} 0.0 \\ 1.0 \end{bmatrix}, \\ A_2(3) &= \begin{bmatrix} 0.0 & 1.0 \\ 4.3 & -4.2 \end{bmatrix}, & B_2(3) &= \begin{bmatrix} 0.0 \\ 1.0 \end{bmatrix}, \\ A_3(3) &= \begin{bmatrix} 0.0 & 1.0 \\ 3.3 & -3.2 \end{bmatrix}, & B_3(3) &= \begin{bmatrix} 0.0 \\ 1.0 \end{bmatrix}, \end{aligned}$$

The probability matrix between the different modes is given by:

$$P = \begin{bmatrix} 0.63 & 0.17 & 0.2 \\ 0.30 & 0.47 & 0.23 \\ 0.24 & 0.12 & 0.64 \end{bmatrix}.$$

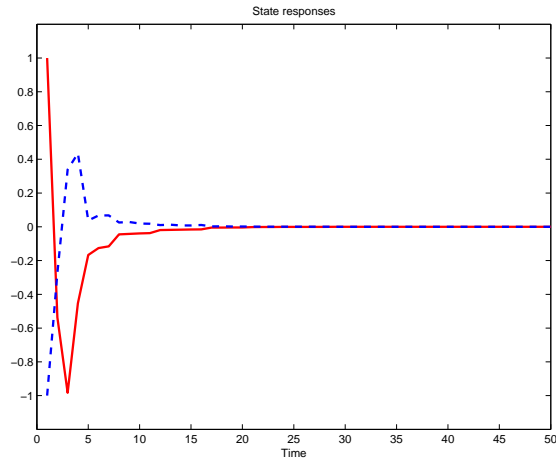
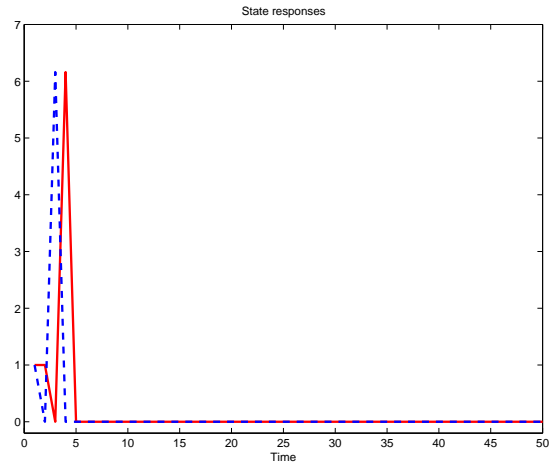
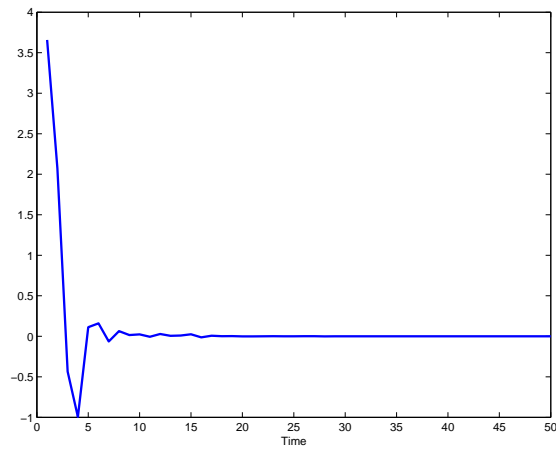
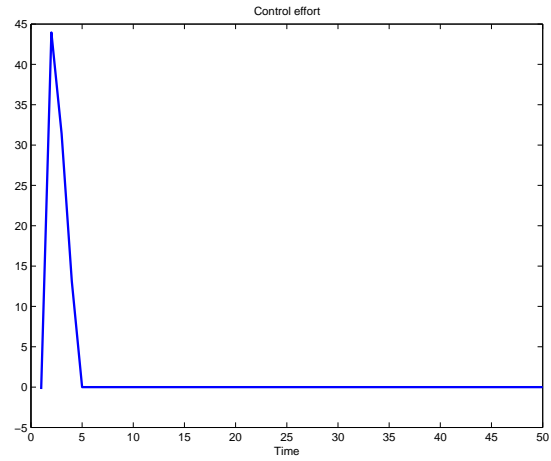
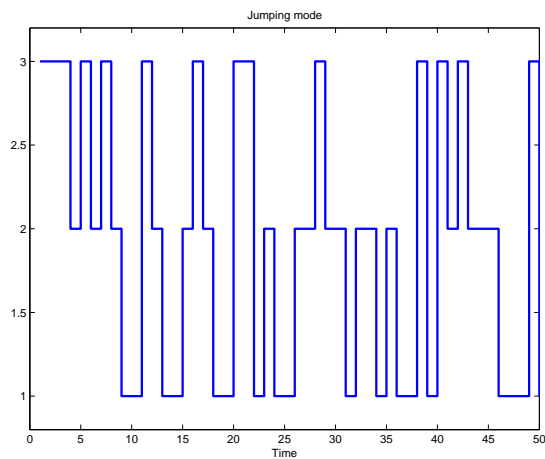
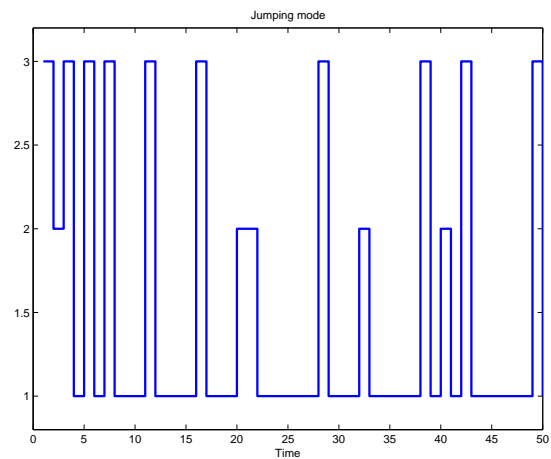
Solving LMI (7) we can not find a feasible solution and as before, this does not imply that the system is not stochastically stable. Solving now the LMIs (17)–(19), we get:

$$\begin{aligned} X(1) &= \begin{bmatrix} 1.1744 & -0.6371 \\ -0.6371 & 0.9485 \end{bmatrix}, & X(2) &= \begin{bmatrix} 1.2407 & -0.6090 \\ -0.6090 & 1.0567 \end{bmatrix}, & X(3) &= \begin{bmatrix} 1.3610 & -0.6626 \\ -0.6626 & 0.9250 \end{bmatrix}, \\ Q_{11}(1) &= \begin{bmatrix} 0.3360 & -0.3524 \\ -0.3524 & 0.3896 \end{bmatrix}, & Q_{12}(1) &= \begin{bmatrix} 0.0077 & -0.0072 \\ -0.0072 & 0.0010 \end{bmatrix}, & Q_{13}(1) &= \begin{bmatrix} 0.0061 & -0.0114 \\ -0.0114 & -0.0154 \end{bmatrix}, \\ Q_{22}(1) &= \begin{bmatrix} 0.0826 & -0.0752 \\ -0.0752 & 0.3541 \end{bmatrix}, & Q_{23}(1) &= \begin{bmatrix} -0.0176 & 0.0014 \\ 0.0014 & -0.0170 \end{bmatrix}, & Q_{33}(1) &= \begin{bmatrix} 0.0841 & -0.0728 \\ -0.0728 & 0.3604 \end{bmatrix}, \\ Q_{11}(2) &= \begin{bmatrix} 0.4488 & -0.4143 \\ -0.4143 & 0.4163 \end{bmatrix}, & Q_{12}(2) &= \begin{bmatrix} 0.0044 & -0.0042 \\ -0.0042 & -0.0093 \end{bmatrix}, & Q_{13}(2) &= \begin{bmatrix} 0.0033 & -0.0034 \\ -0.0034 & -0.0099 \end{bmatrix}, \\ Q_{22}(2) &= \begin{bmatrix} 0.0506 & -0.0634 \\ -0.0634 & 0.4256 \end{bmatrix}, & Q_{23}(2) &= \begin{bmatrix} -0.0095 & -0.0005 \\ -0.0005 & -0.0126 \end{bmatrix}, & Q_{33}(2) &= \begin{bmatrix} 0.0507 & -0.0633 \\ -0.0633 & 0.4250 \end{bmatrix}, \\ Q_{11}(3) &= \begin{bmatrix} 0.4845 & -0.4033 \\ -0.4033 & 0.4236 \end{bmatrix}, & Q_{12}(3) &= \begin{bmatrix} 0.0484 & -0.0566 \\ -0.0566 & 0.0307 \end{bmatrix}, & Q_{13}(3) &= \begin{bmatrix} 0.0429 & -0.0508 \\ -0.0508 & 0.0246 \end{bmatrix}, \\ Q_{22}(3) &= \begin{bmatrix} 0.5654 & -0.4637 \\ -0.4637 & 0.4690 \end{bmatrix}, & Q_{23}(3) &= \begin{bmatrix} -0.2786 & 0.2025 \\ 0.2025 & -0.1431 \end{bmatrix}, & Q_{33}(3) &= \begin{bmatrix} 0.5646 & -0.4631 \\ -0.4631 & 0.4686 \end{bmatrix}, \\ Y_1(1) &= [-2.2040 \quad -1.4217], & Y_1(2) &= [0.8340 \quad -2.8534], & Y_1(3) &= [1.4181 \quad -3.4035], \\ Y_2(1) &= [-0.6098 \quad -0.3842], & Y_2(2) &= [-1.9786 \quad -0.5816], & Y_2(3) &= [0.1935 \quad -2.1370], \\ Y_3(1) &= [-0.1559 \quad -0.0408], & Y_3(2) &= [-4.0178 \quad 0.8305], & Y_3(3) &= [-0.3408 \quad -1.2385], \end{aligned}$$

which gives the following gains:

$$\begin{aligned} K_1(1) &= [-4.2322 \quad -4.3418], & K_1(2) &= [-0.9110 \quad -3.2254], & K_1(3) &= [-1.1508 \quad -4.5038], \\ K_2(1) &= [-1.1628 \quad -1.1862], & K_2(2) &= [-2.6006 \quad -2.0492], & K_2(3) &= [-1.5089 \quad -3.3911], \\ K_3(1) &= [-0.2457 \quad -0.2081], & K_3(2) &= [-3.9778 \quad -1.5066], & K_3(3) &= [-1.3855 \quad -2.3314]. \end{aligned}$$

Also, we give the simulation results in Figures 4–6. The initial state is set to be $[1 \ 1]^T$. In Figure 4, solid line is for $x_1(t)$ and dashed line is for $x_2(t)$. Figures 5–6 give the control effort and the jumping mode.

Figure 1: State $x_1(t), x_2(t)$ Figure 2: State $x_1(t), x_2(t)$ Figure 3: Control effort $u(t)$ Figure 4: Control effort $u(t)$ Figure 5: Jumping mode $r(t)$ Figure 6: Jumping mode $r(t)$

5 Conclusion

This paper dealt with the control of the class of discrete-time nonlinear Markovian jump systems. The stochastic stability and stochastic stabilizability problems are studied and in each case sufficient conditions in the LMI setting have been established.

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