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Abstract

We present a review of several column generation formulations for the Routing and Wavelength Assignment (RWA) problem with the objective of minimizing the blocking rate. Several improvements are proposed together with a comparison of the different formulations with respect to the quality of their continuous relaxation bounds and their computing solution ease. Experimental results are presented on several classical network and traffic instances.

Key Words: WDM network; network dimensioning; RWA problem; column generation; optimal solution.

Résumé

Nous présentons une synthèse de plusieurs formulations de type génération de colonnes pour le problème de routage et d'affectation de longueurs d'onde (RWA– Routing and Wavelength Assignment) avec l'objectif de minimiser le taux de blocage. Plusieurs améliorations sont proposées ainsi qu'une comparaison des formulations par rapport à la qualité de leurs bornes de relaxation continue et de leur facilité de résolution. Des résultats de calcul sont présentés pour plusieurs jeux de données.

1 Introduction

Many papers have already appeared on the RWA problem, i.e., the routing and wavelength assignment problem, one of the central problem in the dimensioning of optical WDM networks. As it is a highly combinatorial problem, various heuristic scheme solutions have been proposed under different traffic assumptions with static or dynamic patterns, with single or multi hops, and for various objectives, cf. the surveys of Dutta and Rouskas [1] and Zang, Jue and Mukherjee [2] for a summary of the works until 2000, and Jaumard, Meyer and Thiongane [3] for a recent survey on symmetrical systems under various objectives.

Several compact ILP formulations have been proposed for the RWA problem: see [4] and [3] for recent surveys in the asymmetrical and symmetrical cases respectively. They all share the drawback to be highly symmetrical with respect to wavelength permutations. As a consequence, even problems of moderate size can hardly be solved to optimality. In an attempt to overcome this drawback, column generation like formulations have been proposed (Ramaswami and Sivarajan [5], Lee et al. [6]). We review these formulations, improve and compare them and propose a new one.

The paper is organized as follows. In the next section, we present a more formal statement of the RWA problem and define the notations that will be used throughout the paper. The following sections are each devoted to a specific column generation formulation of the RWA problem: Section 4 to the maximal independent set formulation of Ramaswami and Sivarajan [5], Section 5 to the independent routing configuration formulation of Lee et al. [6], Section 6 to a new maximal independent routing configuration formulation. We then present in Section 7 a relaxation of the formulations presented in Sections 5 and 6 and compare the linear programming relaxation upper bound provided by the various column generation formulations. In Section 8 we propose a branch-and-bound algorithm to solve the new maximal independent routing configuration formulation presented in Section 6. Computational results are given in Section 9. Conclusions are drawn in the last section.

2 Statement of the max-RWA problem

Let us consider a WDM optical network represented by a multigraph $G = (V, E)$ with node set $V = \{v_1, v_2, \ldots, v_n\}$ where each node is associated with a node of the physical network, and with arc set $E = \{e_1, e_2, \ldots, e_m\}$ where each arc is associated with a fiber link of the physical network: the number of arcs from v_i to v_j is equal to the number of fibers supporting traffic from v_i to v_j. Connections and fiber links are assumed to be directional, and the traffic to be asymmetrical. The set of available wavelengths is denoted by $\Lambda = \{\lambda_1, \lambda_2, \ldots, \lambda_W\}$ with $W = |\Lambda|$. The traffic is defined by a $n \times n$ matrix T where T_{sd} defines the number of requested connections from v_s to v_d. All wavelengths are assumed to have the same capacity. Let $\mathcal{SD} = \{(v_s, v_d) \in V \times V : T_{sd} > 0\}$. Denote by $\mathcal{P}_{sd}$ the set of elementary paths from v_s to v_d for $(v_s, v_d) \in \mathcal{SD}$ and by $\mathcal{P}$ the overall collection of elementary paths, i.e., $\mathcal{P} = \bigcup_{(v_s,v_d)\in\mathcal{SD}} \mathcal{P}_{sd}$. Let $\omega^+(v_i)$ (resp. $\omega^-(v_i)$) be the set of outgoing (resp. incoming) fiber links at node v_i.

We consider only single-hop connections, i.e., the same wavelength is used from the source to the destination for all connection requests. Note that it has been shown (see [7]) that wavelength conversion (i.e., multiple-hop connections) does not help very much in order to reduce the blocking rate.

The RWA problem can then be formally stated as follows: given a multigraph G corresponding to a WDM optical network, and a set of requested connections, find a suitable lightpath (p, λ) for each (accepted) connection where p is a routing path and λ a wavelength, so that no two paths sharing an arc of G are assigned the same wavelength. We study the objective of minimizing the blocking rate, that is equivalent to maximizing the number of accepted connections, leading to the so-called max-RWA problem.

3 Path modeling

A rather straightforward formulation for the RWA problem, often mentioned in the literature, corresponds to the so-called PATH formulation: see, e.g., [5, 4]. This formulation suffers from the drawback of exhibiting a wavelength symmetry: one can deduce $W!$ alternate solutions for any given solution through wavelength permutations. It is therefore not suited for practical computation taking in account that, in addition, the number of paths in a general network is exponential in the number of nodes. Nevertheless we mention it for several reasons. Firstly, due to its exponential number of variables, it fits naturally in the column generation framework. Secondly, it provides a bridge between the other column generation procedures studied later in this paper and the compact formulations reviewed in [4] with respect to their linear programming LP relaxation. Although we do not consider particular constraints (e.g., limit on the number of hops) on the lighpaths in this paper, it is worth noting that such constraints are usually much easier to handle with path formulations than with arc or link formulations. At last, let us recall that it has been shown in [4] that the LP relaxation of the PATH formulation and of the compact formulations have the same optimal value.

3.1 The master problem

Let us define the parameters

$$\delta_e^p = \begin{cases} 1 & \text{if arc } e \text{ belongs to path } p \\ 0 & \text{otherwise} \end{cases}$$

for all $p \in \mathcal{P}$ and $e \in E$. The PATH formulation can be written as follows:

$$\max \quad z_{\text{PATH}}(x) = \sum_{\lambda\in\Lambda} \sum_{p\in\mathcal{P}} x_p^\lambda$$

subject to:

$$\sum_{p \in \mathcal{P}} \delta_e^p x_p^\lambda \leq 1 \qquad e \in E, \lambda \in \Lambda \tag{1}$$

$$\sum_{\lambda \in \Lambda} \sum_{p \in \mathcal{P}_{sd}} x_p^\lambda \leq T_{sd} \qquad (v_s, v_d) \in \mathcal{SD} \tag{2}$$

$$x_p^\lambda \in \{0, 1\} \qquad p \in \mathcal{P}, \lambda \in \Lambda. \tag{3}$$

Constraints (1) correspond to the clash constraints, i.e., they express that there is at most one lightpath going through each pair (e, λ). Constraints (2) are the demand constraints: one must ensure that the number of accepted connections for a given pair source-destination does not exceed the demand, while we attempt to maximize the number of accepted connections in the objective function.

3.2 The auxiliary problems

The LP relaxation, denoted by LP_PATH, is obtained by replacing the binary constraints (3) by $0 \leq x_p^\lambda \leq 1$ for all $p \in \mathcal{P}$ and $\lambda \in \Lambda$. As the number of paths can be exponential, let us consider the LP_PATH formulation with a variable subset of $\{x_p^\lambda : p \in \mathcal{P},\ \lambda \in \Lambda\}$, leading to the so-called *restricted master problem*. To check whether the optimal solution of the restricted master problem is also optimal for the original LP_PATH, we need to verify whether there exists a variable x_p^λ with a positive reduced cost that could be added to the restricted master problem, see, e.g., Nemhauser and Wolsey [8] for an introduction to column generation. If such a variable exists, it is added to the variable subset of the restricted master problem that is solved again. Otherwise the LP_PATH has been solved optimally.

Let $u_{e\lambda}^0$ be the dual value associated with constraint (1) for a given $(e, \lambda) \in E \times \Lambda$, and u_{sd}^1 the dual value associated with constraints (2) for a given $(v_s, v_d) \in \mathcal{SD}$. Note that the constraints $x_p^\lambda \leq 1$ are implied by (1), so we do not need to consider them explicitly. The reduced cost of variable x_p^λ is

$$\overline{c}(x_p^\lambda) = 1 - \sum_{e \in p} u_{e\lambda}^0 - u_{s(p)d(p)}^1$$

where $s(p)$ and $d(p)$ denote respectively the source and the destination node of path p.

For a given $\lambda \in \Lambda$, a variable with positive reduced cost can be found by solving the following ILP problem:

$$\max \quad \overline{c}_{\text{AUX-PATH},\lambda}(\alpha) = 1 - \sum_{e \in E} u_{e\lambda}^0 \alpha_e - \sum_{(v_s, v_d) \in \mathcal{SD}} u_{sd}^1 \alpha_{sd}$$

subject to:

$$\sum_{(v_s,v_d)\in\mathcal{SD}} \alpha_{sd} = 1 \tag{4}$$

$$\sum_{v_s:(v_s,v_i)\in\mathcal{SD}} \alpha_{si} + \sum_{e\in\omega^+(v_i)} \alpha_e = \sum_{e\in\omega^-(v_i)} \alpha_e + \sum_{v_d:(v_i,v_d)\in\mathcal{SD}} \alpha_{id} \qquad v_i \in V \tag{5}$$

$$\alpha_{sd} \in \{0,1\} \qquad (v_s, v_d) \in \mathcal{SD} \tag{6}$$

$$\alpha_e \in \{0,1\} \qquad e \in E \tag{7}$$

with $\alpha_{sd} = 1$ if a path from v_s to v_d is selected, and $\alpha_{sd} = 0$ otherwise; and $\alpha_e = 1$ if arc e is used for the path, and $\alpha_e = 0$. Note that constraint (4) ensures that exactly one source and one destination are selected. Once these source and destination nodes are selected, constraints (5) are ordinary flow conservation equations. The flow-conservation formulation is valid for finding a shortest path since there can be no cycle with strictly positive cost as $u^0_{e\lambda} \geq 0$ for all e. Note that since this shortest-path formulation has the integrality property, the integrality constraints on the variables α_e can be dropped (recall that a problem has the integrality property if its solution is unchanged when the integrality restriction is removed).

Following the observation that the weights on the arcs do not depend on the pair (v_s, v_d), a simple solution scheme can be based on solving an all-pair shortest path problem in $O(n^3)$ using the Floyd-Warshall algorithm (see, e.g., [9]) and adding to each path the corresponding cost u^1_{sd} in $O(|\mathcal{SD}|)$. Hence, an overall complexity is $O(n^3)$.

3.3 Discussion

Several compact formulations, i.e., with a polynomial number of variables and constraints, have been proposed for the max-RWA problem, with LP relaxations that yield the same optimal value than that given by LP_PATH: see [4]. Therefore, the drawback of having an exponential number of variables is not compensated by a gain in the quality of the upper bound for the PATH formulation. Even worse, this column generation formulation also exhibits the same symmetry with respect to the permutations of the wavelengths than that showed by the compact formulations. We will show in the following sections that there are alternative column generation formulations with more attractive properties.

4 Maximal independent set modeling

A first alternative column generation formulation was proposed by Ramaswami and Sivarajan [5], overcoming the wavelength symmetry problem. In order to express it, let us first define the wavelength clash (or conflict) graph $G_W = (V_W, E_W)$. The set of nodes is a union of node sets

$$V_W = \bigcup_{(v_s,v_d)\in\mathcal{SD}} V_W^{sd},$$

where $V_W^{sd} = \{r_p : p \in \mathcal{P}_{sd}\}$ is a set of route nodes, i.e., of nodes associated with potential routes for connections from v_s to v_d for all $(v_s, v_d) \in \mathcal{SD}$, and $E_W = \{\{r_p, r_{p'}\} \in V_W \times V_W :$ paths p and p' have at least one common fiber link$\}$. Let $\mathcal{I}_{\max}$ be the overall set of maximal independent sets of G_W, and let w_I be the number of wavelengths associated with I for each $I \in \mathcal{I}_{\max}$.

4.1 MAX_IS mathematical formulation

Let us define the following set of coefficients:

$$\delta_{pI} = |\{r_p\} \cap I| = \begin{cases} 1 & \text{if path } p \text{ is such that } r_p \text{ belongs to independent set } I \\ 0 & \text{otherwise} \end{cases}$$

and observe that

$$\sum_{p\in\mathcal{P}_{sd}} \delta_{pI} = |I \cap V_W^{sd}| \qquad I \in \mathcal{I}_{\max}, \quad (v_s, v_d) \in \mathcal{SD}. \tag{8}$$

The Ramaswami and Sivarajan [5] formulation amounts to find a set of $q \leq W$ maximal independent sets subject to some constraints. It is formally expressed as follows :

$$\max \qquad \sum_{(v_s,v_d)\in\mathcal{SD}} y_{sd}$$

subject to:

$$\sum_{I\in\mathcal{I}_{\max}} w_I \leq W \tag{9}$$

$$x_p \leq \sum_{I\in\mathcal{I}_{\max}} w_I \delta_{pI} \qquad p \in \mathcal{P} \tag{10}$$

$$y_{sd} \leq \sum_{p\in\mathcal{P}_{sd}} x_p \qquad (v_s, v_d) \in \mathcal{SD} \tag{11}$$

$$0 \leq y_{sd} \leq T_{sd} \qquad (v_s, v_d) \in \mathcal{SD} \tag{12}$$

$$x_p \geq 0 \qquad p \in \mathcal{P} \tag{13}$$

$$w_I \in \mathbb{N} \qquad I \in \mathcal{I}_{\max}. \tag{14}$$

The variable y_{sd} counts the number of accepted connections from v_s to v_d for all (v_s, v_d) in $\mathcal{SD}$, while the variable x_p counts the number of times a given path is selected for a lightpath for all $p \in \mathcal{P}$. Note that we may a priori allow more lightpaths between a pair of source-destination than required, but constraints (11)–(12) ensure that we grant no more than the number of requested connections. The variables x_p can be eliminated by combining constraints (10) and (11). Moreover the nonnegativity constraints on the variables y_{sd} can be eliminated because of the objective function. Using (8), we obtain the following MAX_IS formulation:

$$\max \qquad z_{\text{MAX\_IS}}(w, y) = \sum_{(v_s,v_d) \in \mathcal{SD}} y_{sd}$$

subject to:

$$\sum_{I \in \mathcal{I}_{\max}} w_I \leq W \tag{15}$$

$$y_{sd} - \sum_{I \in \mathcal{I}_{\max}} w_I |I \cap V_W^{sd}| \leq 0 \qquad (v_s, v_d) \in \mathcal{SD} \tag{16}$$

$$y_{sd} \leq T_{sd} \qquad (v_s, v_d) \in \mathcal{SD} \tag{17}$$

$$w_I \in \mathbb{N} \qquad I \in \mathcal{I}_{\max}. \tag{18}$$

The most important feature of the MAX_IS formulation lies in the fact that wavelengths are assigned only once an optimal solution has been found, therefore eliminating the symmetry problem arising from equivalent solutions up to a wavelength permutation in the classical ILP formulations and in the PATH formulation, see, e.g., Jaumard, Meyer and Thiongane [3]. Let w^* be an optimal solution of the MAX_IS formulation and let $I_1, I_2, \ldots, I_q, q \leq W$ be the independent sets such that $w_I^* \geq 1$. One can then distribute the wavelengths over the independent sets as follows: assign $\lambda_t, t = 1 + \sum_{i=1}^{\tau-1} w_{I_i}^*, ..., \sum_{i=1}^{\tau} w_{I_i}^*$ to the independent set I_τ for $\tau = 1, 2, \ldots, q$ with the convention that $\sum_{i=1}^{0} w_{I_i}^* = 0$.

4.2 Solution of the LP relaxation of MAX_IS

Although Ramaswami and Sivarajan [5] proposed the above MAX_IS formulation, i.e., a so-called column generation formulation, they did not solve it using column generation techniques even if those techniques usually lead to a much more efficient solution scheme than the Simplex algorithm as it avoids considering explicitly all potential variables/columns. Let us study how to solve the LP relaxation, denoted by LP_MAX_IS, using column generation techniques.

The LP_MAX_IS relaxation is obtained by replacing the integer constraints (18) by $w_I \geq 0$ for all $I \in \mathcal{I}_{\max}$. As the number of maximal independent sets can be exponential, let us consider the LP_MAX_IS formulation with all variables y_{sd} such that $(v_s, v_d) \in \mathcal{SD}$ and a variable subset of $\{w_I : I \in \mathcal{I}_{\max}\}$, leading to the so-called *restricted master problem.* To check whether the optimal solution of the restricted master problem is also optimal for the original LP_MAX_IS, we need to verify whether there exists a variable w_I with a positive reduced cost. If such a variable exists, it is added to the variable subset of the restricted master problem that is solved again. We iterate until no variable w_I with a positive reduced cost can be found: the LP_MAX_IS has then been solved optimally.

Let u^0 be the dual value associated with constraint (15) and u^1_{sd} the dual value associated with constraint (16) in an optimal solution of the current restricted master problem. Then the reduced cost for variable w_I is

$$\overline{c}(w_I) = -u^0 + \sum_{(v_s,v_d)\in\mathcal{SD}} |I \cap V_W^{sd}| u^1_{sd}.$$

Checking the existence of a variable with positive reduced cost corresponds to a weighted independent set problem that can be written:

$$\max \quad \overline{c}_{\text{AUX\_MAX\_IS}}(\alpha) = -u^0 + \sum_{(v_s,v_d)\in\mathcal{SD}} \sum_{p: r_p \in V_W^{sd}} u^1_{sd}\, \alpha_p$$

subject to:

$$\begin{aligned} &\alpha_p + \alpha_{p'} \leq 1 && (r_p, r_{p'}) \in E_W \\ &\alpha_p \in \{0,1\} && r_p \in V_W \end{aligned}$$

where $\alpha_p = 1$ if node r_p belongs to the independent set and 0 otherwise. Many exact methods have been proposed for the weighted independent set problem, see, e.g., Mehrotra and Trick [10], Balas and Xue [11]. It is usual in column generation methods not to solve exactly the auxiliary problem but rather to stop when one or more columns with the appropriate sign for their reduced cost, are found. Note that here, there is no guarantee that such columns would correspond to a *maximal* independent set. Even if we solve the subproblem at optimality, there is no guarantee that the optimal solution found would correspond to a maximal independent set when some u^1_{sd} are equal to 0. This minor difficulty can be solved in two ways. Firstly, the independent set can be completed to a maximal independent set. Clearly this operation cannot decrease the objective value. Secondly, by looking more carefully at the formulation, we see that it remains valid even if non-maximal independent sets are considered. Indeed the formulation is valid whether the independent sets are maximal independent sets or not.

4.3 Quality of the LP bound of the MAX_IS formulation

The LP relaxation upper bound obtained with this formulation can be strictly better than the LP relaxation bound of the PATH formulation:

Example 1 *Consider the KK Network of Kleiberg and Kumar's [12] shown in Figure 1(a). Let* $W = 1$. *Assume that the traffic matrix is given by* $T_{13} = T_{14} = T_{25} = T_{63} = T_{65} = 1$, *all other entries of* T *being* 0. *The paths of interest are* $p_1 = (v_1, v_2, v_3)$, $p_2 = (v_1, v_2, v_4)$, $p_3 = (v_2, v_4, v_5)$, $p_4 = (v_6, v_4, v_2, v_3)$ *and* $p_5 = (v_6, v_4, v_5)$, *see Figure 1(b). The maximal independent sets in the conflict graph are* $I_1 = \{r_1, r_3\}$, $I_2 = \{r_1, r_5\}$, $I_3 = \{r_2, r_4\}$, $I_4 = \{r_2, r_5\}$ *and* $I_5 = \{r_3, r_4\}$, *see Figure 1(c). Solving the linear programs yields* $z^{\text{LP}}_{\text{MAX\_IS}} = 2$ *(obtained by selecting any one of the independent sets) and* $z^{\text{LP}}_{\text{PATH}} = \frac{5}{2}$ *(obtained for* $x^1_p = \frac{1}{2}$ *for all* p*).*

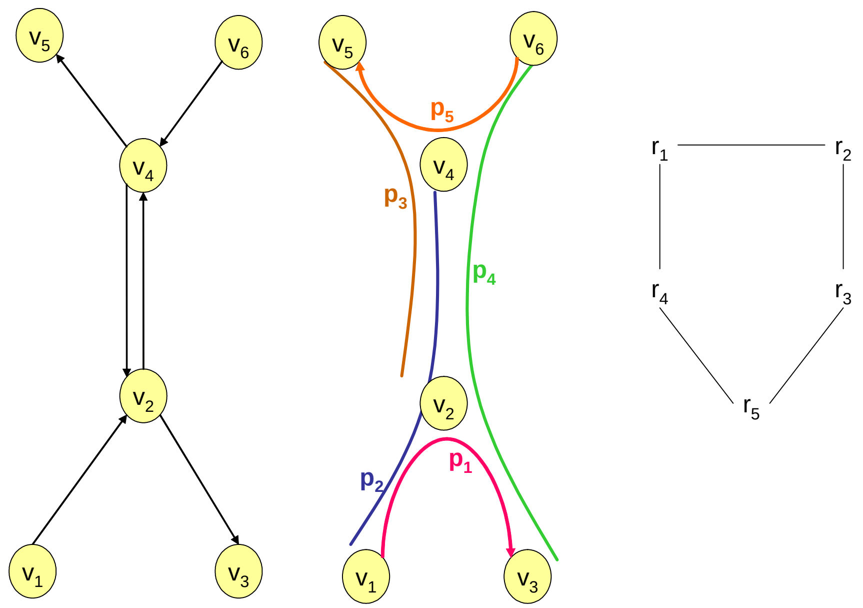

Figure 1: (a) KK Network (b) Lightpaths (c) Conflict Graph

We will show in Section 7 (see Corollary 6) that $z^{\text{LP}}_{\text{MAX\_IS}} \leq z^{\text{LP}}_{\text{PATH}}$.

A major drawback of the MAX_IS formulation is however that the AUX_MAX_IS auxiliary problem needs to be solved on the wavelength clash graph that may involve an exponential number of vertices as each vertex is associated with an elementary path for a given pair of source and destination nodes. The formulations proposed in the next sections do not suffer from this drawback as the solution algorithm of their auxiliary problems requires to consider only implicitly the wavelength clash graph, see Section 7 for more details.

5 Independent routing configuration modeling

Lee et al. [6] (see also [13, 14]) have introduced the concept of independent routing configuration where an independent routing configuration is implicitly associated with a set of paths, not necessarily unique, that can be used for satisfying a given fraction of the connections with the same wavelength. An independent routing configuration C is represented by a non-negative vector a^C such that

$$a_{sd}^C = \text{number of connection requests from } v_s \text{ to } v_d \text{ that are supported by configuration } C$$
$$a_{sd}^C \leq T_{sd} \qquad (v_s, v_d) \in \mathcal{SD}.$$

We denote by $\mathcal{C}$ the set of all possible independent routing configurations.

5.1 IRC mathematical formulation

We define the variables w_C that indicate how many occurrences of a given independent routing configuration are used simultaneously, each occurrence with a different wavelength. The independent routing configuration formulation, denoted by IRC, can be expressed as follows for the max-RWA problem:

$$\max \qquad z_{\text{IRC}}(w) = \sum_{C \in \mathcal{C}} \sum_{(v_s,v_d) \in \mathcal{SD}} a_{sd}^C w_C$$

subject to:

$$\sum_{C \in \mathcal{C}} w_C \leq W \tag{19}$$
$$\sum_{C \in \mathcal{C}} a_{sd}^C w_C \leq T_{sd} \qquad (v_s, v_d) \in \mathcal{SD} \tag{20}$$
$$w_C \in \mathbb{N} \qquad C \in \mathcal{C}. \tag{21}$$

5.2 Solution of the LP relaxation of the IRC formulation

The LP relaxation, denoted by LP_IRC, is obtained by replacing the integrality constraints (21) in IRC by $w_C \geq 0$ for all $C \in \mathcal{C}$. As the number of independent routing configurations can be exponential, we consider again a so-called restricted master problem on a subset of the variables and examine the reduced cost to determine whether or not we have reached the optimal solution of LP-IRC. Let (u^0, u_{sd}^1) be an optimal solution of the dual of the current restricted master problem. Then the reduced cost $\bar{c}(w_C)$ of column w_C can be written

$$\bar{c}(w_C) = -u^0 + \sum_{(v_s,v_d) \in \mathcal{SD}} (1 - u_{sd}^1) a_{sd}^C.$$

To find whether there exists a configuration with a positive reduced cost, Lee et al. [14] consider the following auxiliary problem:

$$\max \quad \bar{c}_{\text{AUX1-IRC}}(\alpha) = -u^0 + \sum_{(v_s,v_d) \in \mathcal{SD}} \sum_{p \in \mathcal{P}_{sd}} (1 - u_{sd}^1) \alpha_p$$

subject to:

$$\sum_{p \in \mathcal{P}} \delta_e^p \alpha_p \leq 1 \qquad e \in E \tag{22}$$

$$\sum_{p \in \mathcal{P}_{sd}} \alpha_p \leq T_{sd} \qquad (v_s, v_d) \in \mathcal{SD} \tag{23}$$

$$\alpha_p \in \{0, 1\} \qquad p \in \mathcal{P} \tag{24}$$

where $\alpha_p = 1$ if path p is selected and 0 otherwise, and δ_e^p is defined in Section 3.1. The auxiliary problem corresponds here again to a weighted independent set problem (using the clique formulation, see e.g., Grötschel et al. [15]), but with some cardinality constraints.

Lee et al. [14] solve it using column generation and a branch-and-price algorithm, or in other words they have a column generation algorithm for solving the auxiliary problems embedded in the column generation (heuristic) algorithm for solving the master problem.

In order to overcome the need of an embedded column generation solution, an interesting alternative is to reformulate the auxiliary problem as a multi-flow problem:

$$\max \quad \bar{c}_{\text{AUX2-IRC}}(\alpha) = -u^0 + \sum_{(v_s,v_d) \in \mathcal{SD}} \sum_{e \in \omega^+(v_s)} \alpha_e^{sd}(1 - u_{sd}^1)$$

subject to:

$$\sum_{(v_s,v_d) \in \mathcal{SD}} \alpha_e^{sd} \leq 1 \qquad e \in E \tag{25}$$

$$\sum_{e \in \omega^+(v_i)} \alpha_e^{sd} = \sum_{e \in \omega^-(v_i)} \alpha_e^{sd} \qquad (v_s, v_d) \in \mathcal{SD}, \quad v_i \in V \setminus \{v_s, v_d\} \tag{26}$$

$$\sum_{e \in \omega^+(v_s)} \alpha_e^{sd} \leq T_{sd} \qquad (v_s, v_d) \in \mathcal{SD} \tag{27}$$

$$\sum_{e \in \omega^-(v_s)} \alpha_e^{sd} = 0 \qquad (v_s, v_d) \in \mathcal{SD} \tag{28}$$

$$\alpha_e^{sd} \in \{0, 1\} \qquad (v_s, v_d) \in \mathcal{SD}, \quad e \in E \tag{29}$$

where $\alpha_e^{sd} = 1$ if a path from v_s to v_d goes through fiber link e, and 0 otherwise. Constraints (25) and (26) define a set of disjoint paths, i.e., a configuration. If $\bar{c}_{\text{AUX2-IRC}}(\alpha) \leq 0$ then LP_IRC has been solved to optimality. Otherwise the routing configuration C defined by the vector (a_{sd}^C) with $a_{sd}^C = \sum_{e \in \omega^+(v_s)} \alpha_e^{sd}$ for $(v_s, v_d) \in \mathcal{SD}$ is added to the restricted master problem, which is solved again.

6 Maximal independent routing configuration modeling

By combining the ideas of the formulations of the two previous sections, we obtain a new formulation that requires only maximal independent routing configurations where an independent routing configuration C is maximal if there does not exist another independent routing configuration C' such that $a^{C'} \geq a^C$.

6.1 MAX_IRC mathematical formulation

Let $\mathcal{C}_{\max}$ be the set of all maximal independent routing configurations and let again w_C the number of occurrences of the independent routing configuration C that are used, each with a different wavelength. Then MAX_IRC can be formulated as follows:

$$\max \qquad z_{\text{MAX\_IRC}}(w, y) = \sum_{(v_s, v_d) \in \mathcal{SD}} y_{sd} \tag{30}$$

subject to:

$$\sum_{C \in \mathcal{C}_{\max}} w_C \leq W \tag{31}$$

$$y_{sd} \leq \sum_{C \in \mathcal{C}_{\max}} a_{sd}^C w_C \qquad (v_s, v_d) \in \mathcal{SD} \tag{32}$$

$$y_{sd} \leq T_{sd} \qquad (v_s, v_d) \in \mathcal{SD} \tag{33}$$

$$w_C \in \mathbb{N} \qquad C \in \mathcal{C}_{\max}. \tag{34}$$

6.2 Solution of the LP relaxation of MAX_IRC

Let u^0 be the dual value associated with constraint (31) and u^1_{sd} the dual value associated with constraint (32) in the optimal solution of the restricted master problem. The reduced cost for variable w_C is $-u^0 + \sum_{(v_s, v_d) \in \mathcal{SD}} a^C_{sd} u^1_{sd}$. The auxiliary problem is then defined by:

$$\max \quad \bar{c}_{\text{AUX\_MAX\_IRC}}(\alpha) = -u^0 + \sum_{(v_s, v_d) \in \mathcal{SD}} \sum_{e \in \omega^+(v_s)} \alpha_e^{sd} u^1_{sd}$$

subject to:

$$\sum_{(v_s, v_d) \in \mathcal{SD}} \alpha_e^{sd} \leq 1 \qquad e \in E \tag{25}$$

$$\sum_{e \in \omega^+(v_i)} \alpha_e^{sd} = \sum_{e \in \omega^-(v_i)} \alpha_e^{sd} \qquad (v_s, v_d) \in \mathcal{SD}, \quad v_i \in V \setminus \{v_s, v_d\} \tag{26}$$

$$\sum_{e \in \omega^+(v_s)} \alpha_e^{sd} \leq T_{sd} \qquad (v_s, v_d) \in \mathcal{SD} \tag{27}$$

$$\sum_{e \in \omega^-(v_s)} \alpha_e^{sd} = 0 \qquad (v_s, v_d) \in \mathcal{SD} \tag{28}$$

$$\alpha_e^{sd} \in \{0, 1\} \qquad (v_s, v_d) \in \mathcal{SD}, \quad e \in E. \tag{29}$$

Note that the constraints are the same than for the auxiliary problem of the IRC formulation, but the objective function differs.

6.3 Comparison of formulations IRC and MAX_IRC

Let $z_{\text{IRC}}^{\text{LP}}$ and $z_{\text{MAX\_IRC}}^{\text{LP}}$ be the optimal values of the LP relaxation of formulation IRC and MAX_IRC respectively. We have the following result:

Proposition 1 $z_{\text{IRC}}^{\text{LP}} = z_{\text{MAX\_IRC}}^{\text{LP}}$.

Proof. Let w^* be an optimal solution of the LP relaxation of formulation (IRC). With each independent routing configuration $C \in \mathcal{C}$, we associate a maximal independent routing configuration $m(C) \in \mathcal{C}_{\max}$ such that $a^C \leq a^{m(C)}$. Define $\hat{w}$ as $\hat{w}_g = \sum_{C \in \mathcal{C}: m(C) = g} w_C^*$ for each $g \in \mathcal{C}_{\max}$ and let $\hat{y}_{sd} = \min\{\sum_{C \in \mathcal{C}_{\max}} a_{sd}^C \hat{w}_C, T_{sd}\}$ for $(v_s, v_d) \in \mathcal{SD}$. Note that $\hat{y}_{sd} \geq \sum_{C \in \mathcal{C}} a_{sd}^C w_C^*$ for $(v_s, v_d) \in \mathcal{SD}$. Therefore $(\hat{w}, \hat{y})$ is a feasible solution to the LP relaxation of (MAX_IRC) with value $\geq z_{\text{IRC}}^{\text{LP}}$, hence $z_{\text{MAX\_IRC}}^{\text{LP}} \geq z_{\text{IRC}}^{\text{LP}}$.

Conversely let (w^*, y^*) be an optimal solution of the LP relaxation of formulation (MAX_IRC). Clearly at the optimum it holds

$$y_{sd}^* = \min\{\sum_{C \in \mathcal{C}_{\max}} a_{sd}^C w_C^*, T_{sd}\}, \qquad (v_s, v_d) \in \mathcal{SD}.$$

If for all $(v_s, v_d) \in \mathcal{C}_{\max}$, $y_{sd}^* = \sum_{C \in \mathcal{C}_{\max}} a_{sd}^C w_C^*$, then $\hat{w}$ defined by

$$\hat{w}_C = \begin{cases} w_C^* & \text{if } C \in \mathcal{C}_{\max} \\ 0 & \text{if } C \in \mathcal{C} \setminus \mathcal{C}_{\max} \end{cases}$$

is a feasible solution to the LP relaxation of (IRC) with value $z_{\text{MAX\_IRC}}^{\text{LP}}$, hence the inequality $z_{\text{IRC}}^{\text{LP}} \geq z_{\text{MAX\_IRC}}^{\text{LP}}$.

Otherwise there exists $(v_{\tilde{s}}, v_{\tilde{d}}) \in \mathcal{SD}$ such that $\sum_{C \in \mathcal{C}_{\max}} a_{\tilde{s}\tilde{d}}^C w_C^* > T_{\tilde{s}\tilde{d}}$. For each $C \in \mathcal{C}_{\max}$, we define the configuration $m(C)$ such that $a_{sd}^{m(C)} = a_{sd}^C$ for $(v_s, v_d) \in \mathcal{SD} \setminus \{(v_{\tilde{s}}, v_{\tilde{d}})\}$ and $a_{\tilde{s}\tilde{d}}^{m(C)} = 0$. Clearly the configuration $m(C)$ is still an independent routing configuration, although not necessarily maximal. Let $\lambda = \frac{T_{\tilde{s}\tilde{d}}}{\sum\limits_{C \in \mathcal{C}_{\max}} a^C_{\tilde{s}\tilde{d}} w^*_C}$. We define $\hat{w}$ as follows:

$$\hat{w}_C = \begin{cases} w^*_C & \text{if } C \in \mathcal{C}_{\max} \text{ and } C = m(C) \\ \lambda w^*_C & \text{if } C \in \mathcal{C}_{\max} \text{ and } C \neq m(C) \\ (1-\lambda) w^*_{C'} & \text{if } C \in \mathcal{C} \setminus \mathcal{C}_{\max} \text{ and } \exists C' \in \mathcal{C}_{\max} : C = m(C') \\ 0 & \text{if } C \in \mathcal{C} \setminus \mathcal{C}_{\max} \text{ and } \nexists C' \in \mathcal{C}_{\max} : C = m(C') \end{cases}$$

By construction

$$\begin{aligned} \sum_{C \in \mathcal{C}} \hat{w}_C a^C_{\tilde{s}\tilde{d}} &= \sum_{C \in \mathcal{C}_{\max}} (\lambda w^*_C) a^C_{\tilde{s}\tilde{d}} = T_{\tilde{s}\tilde{d}} \\ \sum_{C \in \mathcal{C}} \hat{w}_C a^C_{sd} &= \sum_{C \in \mathcal{C}_{\max}} \Big(\lambda w^*_C a^C_{sd} + (1-\lambda) w^*_C a^{m(C)}_{sd} \Big) \\ &= \sum_{C \in \mathcal{C}_{\max}} w^*_C a^C_{sd} \qquad (v_s, v_d) \in \mathcal{SD} \setminus \{(v_{\tilde{s}}, v_{\tilde{d}})\} \\ \sum_{C \in \mathcal{C}} \hat{w}_C &= \sum_{C \in \mathcal{C}_{\max}} (\lambda w^*_C + (1-\lambda) w^*_C) = \sum_{C \in \mathcal{C}_{\max}} w^*_C. \end{aligned}$$

Hence $(\hat{w}, y^*)$ is a feasible solution of the formulation derived from (MAX_IRC) by replacing $\mathcal{C}_{\max}$ by $\mathcal{C}$, with value $z^{\text{LP}}_{\text{MAX\_IRC}}$. Note that the number of pairs (v_s, v_d) such that $y^*_{sd} \neq \sum\limits_{C \in \mathcal{C}} a^C_{sd} \hat{w}_C$ has been decreased by 1. Repeating this procedure, we eventually reach the case where $y^*_{sd} = \sum\limits_{C \in \mathcal{C}} a^C_{sd} \hat{w}_C$ for all $(v_s, v_d) \in \mathcal{SD}$, which concludes the proof. □

One of the advantages of the MAX_IRC formulation over IRC is that the former generally requires less columns. Indeed, consider a network with 4 nodes v_1, v_2, v_3, v_4 such that there exists a pair of fiber links between the following pair of nodes: (v_1, v_2), (v_1, v_3) and (v_1, v_4). Assume that the traffic matrix is $T_{12} = 3$, $T_{13} = 2$ and $T_{14} = 1$, and that 3 wavelengths are available. There is an unique maximal independent routing configuration: this maximal configuration satisfies $a^C_{v_1 v_2} = a^C_{v_1 v_3} = a^C_{v_1 v_4} = 1$. An optimal solution of the MAX_IRC formulation is therefore defined by this configuration with weight $w^*_C = 3$. In contrast, an optimal solution of the IRC configuration will require at least 2 columns: a first column C defined by $a^C_{v_1 v_2} = a^C_{v_1 v_3} = 1$ with weight $w^*_C = 2$ and a second column C' defined by $a^{C'}_{v_1 v_2} = a^{C'}_{v_1 v_4} = 1$ with weight $w^*_{C'} = 1$. Note however that the MAX_IRC formulation requires the additional variables y_{sd}, that are however in polynomial number.

7 Relaxation of the cardinality constraints in the auxiliary problems of formulations IRC and MAX_IRC

In the formulations IRC and MAX_IRC presented in Sections 5 and 6 respectively, we required the columns corresponding to the routing configurations to satisfy the cardinality constraints

$$a_{sd}^C \leq T_{sd}, \qquad (v_s, v_d) \in \mathcal{SD}.$$

In this section, we eliminate this requirement. This amounts to the elimination of constraints (27) in the corresponding auxiliary problems. The relaxed IRC and MAX_IRC formulations are called respectively IRC_RL and MAX_IRC_RL, and we denote by $\mathcal{C}_{\text{RL}}$ (respectively $\mathcal{C}_{\max\text{\_RL}}$) the modified set of independent routing configurations (respectively maximal independent routing configurations). Note that the formulations IRC_RL and MAX_IRC_RL remain valid ILP formulations, as the cardinality constraints are still present in the master problems.

7.1 Comparison of the upper bounds provided by the formulations IRC, MAX_IRC, IRC_RL and MAX_IRC_RL

By looking carefully at the proof of Proposition 1, we see that the cardinality constraints in the auxiliary problem do not play any role in the proof. Hence we have

Proposition 2 $z^{\text{LP}}_{\text{IRC\_RL}} = z^{\text{LP}}_{\text{MAX\_IRC\_RL}}$.

Proof. Same as the proof of Proposition 1. □

Proposition 3 *The following inequality holds:*

$$z^{\text{LP}}_{\text{MAX\_IRC}} \leq z^{\text{LP}}_{\text{MAX\_IRC\_RL}}.$$

Moreover this inequality is strict for some instances.

Proof. Let (w^*, y^*) be an optimal solution of the LP relaxation of MAX_IRC. With any $C \in \mathcal{C}_{\max}$ we associate a $m(C) \in \mathcal{C}_{\max\text{\_RL}}$ such that $a^{m(C)} \geq a^C$. We define

$$\tilde{w}_g = \sum_{C \in \mathcal{C}_{\max}: m(C) = g} w_C^*, \qquad g \in \mathcal{C}_{\max\text{\_RL}}.$$

We have

$$y_{sd}^* \leq \sum_{C \in \mathcal{C}_{\max}} a_{sd}^C w_C^* \leq \sum_{g \in \mathcal{C}_{\max\text{\_RL}}} a_{sd}^g \tilde{w}_g, \qquad (v_s, v_d) \in \mathcal{SD}$$

$$\sum_{g \in \mathcal{C}_{\max\text{\_RL}}} \tilde{w}_g = \sum_{C \in \mathcal{C}_{\max}} w_C^* \leq W.$$

This shows that $(\tilde{w}, y^*)$ is a feasible solution to the LP relaxation of MAX_IRC_RL with value $z^{\text{LP}}_{\text{MAX\_IRC}}$. Hence the inequality $z^{\text{LP}}_{\text{MAX\_IRC\_RL}} \geq z^{\text{LP}}_{\text{MAX\_IRC}}$.

To see that the inequality can be strict for some instances, consider the instance defined by the network of Figure 2, the traffic matrix T such that $T_{14} = 4$, $T_{23} = 1$ and $T_{ij} = 0$ for the other entries, and $W = 2$. Observe that there is exactly 1 path from v_1 to v_4:

$v_1 \to v_3 \to v_2 \to v_4$, whereas from v_2 to v_3 there are 2 paths, which in addition turn out to be disjoint: $v_2 \to v_1 \to v_3$ and $v_2 \to v_4 \to v_3$. Hence there are 2 maximal independent routing configurations

$$C_1 = \begin{pmatrix} 0 \\ a \end{pmatrix}, \qquad C_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

where a is equal to 1 if the cardinality constraints $a_{sd} \leq T_{sd}$ are present in the auxiliary problem, and to 2 if they are not present. The LP relaxations of formulations MAX_IRC and MAX_IRC_RL are therefore

$$\begin{array}{ll} \max & y_{14} + y_{23} \\ \text{s.t.} & \left\{ \begin{array}{l} w_1 + w_2 \leq 2 \\ y_{14} \leq w_2 \\ y_{23} \leq a w_1 \\ y_{14} \leq 4 \\ y_{23} \leq 1 \\ w_1, w_2 \geq 0 \end{array} \right. \end{array}$$

Clearly $z^{\text{LP}}_{\text{MAX\_IRC}} = 2$ while $z^{\text{LP}}_{\text{MAX\_IRC\_RL}} = 2.5$, obtained for $(w_1^*, w_2^*) = (0.5, 1.5)$. Note that in this case, the extreme points of the LP relaxation are integral, hence $z^{\text{LP}}_{\text{MAX\_IRC}}$ coincides with the optimal value of the integer problem. □

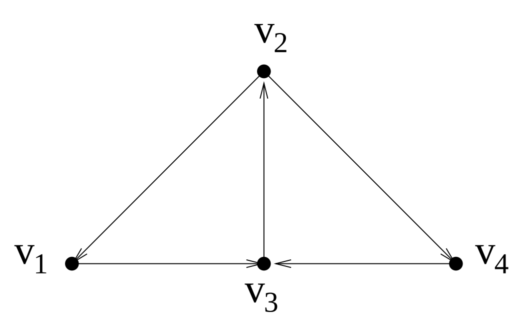

Figure 2: Network showing that $z^{\text{LP}}_{\text{MAX\_IRC}} \neq z^{\text{LP}}_{\text{MAX\_IRC\_RL}}$

7.2 Comparison of the formulations MAX_IS and MAX_IRC_RL

We now compare the formulations MAX_IS and MAX_IRC_RL. Recall that the MAX_IS auxiliary problem can be written

$$\begin{array}{ll} \max & -u^0 + \displaystyle\sum_{(v_s,v_d)\in\mathcal{SD}} \sum_{p: r_p \in V_W^{sd}} \alpha_p u^1_{sd} \\ \text{s.t.} & \left\{ \begin{array}{ll} \alpha_p + \alpha_{p'} \leq 1 & (r_p, r_{p'}) \in E_W \\ \alpha_p \in \{0,1\} & r_p \in V_W. \end{array} \right. \end{array} \quad (35)$$

By definition of the wavelength clash graph G_W defined in Section 4, $(r_p, r_{p'}) \in E_W$ if and only if the paths p and p' share at least one arc $e \in E$. Hence inequalities (35) can be written

$$\alpha_p + \alpha_{p'} \leq 1 \qquad r_p, r_{p'} \in V_W : e \in p \cap p', \qquad e \in E$$

which in turn can be written

$$\sum_{r_p \in V_W : e \in p} \alpha_p \leq 1 \qquad e \in E. \tag{36}$$

Using path notation, the auxiliary problem can then be rewritten:

$$\max \quad -u^0 + \sum_{(v_s,v_d) \in \mathcal{SD}} \sum_{p \in \mathcal{P}_{sd}} \alpha_p u^1_{sd}$$

$$\text{s.t.} \quad \begin{cases} \sum\limits_{p \in \mathcal{P} : e \in p} \alpha_p \leq 1 & e \in E \quad (37) \\ \alpha_p \in \{0,1\} & p \in \mathcal{P}. \quad (38) \end{cases}$$

Since $\mathcal{P}_{sd}$ is the set of all elementary paths from v_s to v_d, the constraints (37)–(38) can be replaced by flow constraints: we then obtain the AUX_MAX_IRC_RL auxiliary problem of formulation MAX_IRC_RL.

Note that although the auxiliary problems are identical for the two formulations, it does not imply that the set of columns in the master problem are identical. Indeed, a column corresponding to a maximal independent routing configuration in the MAX_IRC_RL formulation identifies the number of disjoint paths for each pair of source-destination nodes. It does not explicitly provide the paths; the only information we have is that there exist paths that can support the maximal independent routing configuration. In contrast, a column corresponding to an independent set in formulation MAX_IS identifies a set of disjoint paths. Since a given maximal independent routing configuration may be associated with several different sets of disjoint paths, it follows that for one column of MAX_IRC_RL, we may have many corresponding columns of MAX_IS. In other words, MAX_IRC_RL eliminates a second type of symmetry.

The following result is a direct consequence of the relation between the 2 formulations:

Proposition 4 $z^{\text{LP}}_{\text{MAX\_IS}} = z^{\text{LP}}_{\text{MAX\_IRC\_RL}}$.

7.3 Comparison of the upper bounds provided by formulations PATH and IRC_RL

We now compare the upper bounds provided by the linear relaxations of formulations IRC_RL and PATH.

Proposition 5 $z^{\text{LP}}_{\text{IRC\_RL}} \leq z^{\text{LP}}_{\text{PATH}}$.

Proof. Let us recall first the linear relaxation of the PATH aggregated formulation according to the wavelength λ:

$$\max \quad z_{\text{PATH-AGGR}}(x) = \sum_{p \in \mathcal{P}} x_p$$

subject to:

$$\sum_{p \in \mathcal{P}} \delta_{ep} x_p \leq W \qquad e \in E \tag{39}$$

$$\sum_{p \in \mathcal{P}_{sd}} x_p \leq T_{sd} \qquad (v_s, v_d) \in \mathcal{SD} \tag{40}$$

$$x_p \geq 0 \qquad p \in \mathcal{P}. \tag{41}$$

Note that the result $z^{\text{LP}}_{\text{PATH-AGGR}} = z^{\text{LP}}_{\text{PATH}}$ has been shown in [5].

Let w^*_C be an optimal solution of the LP relaxation of formulation IRC_RL. With each independent routing configuration $C \in \mathcal{C}$, we associate a set $\mathcal{P}^C$ of paths realizing C. Denote by $F(\mathcal{P}^C)$ the forest induced by $\mathcal{P}^C$. Let us define

$$\hat{x}_p = \sum_{C \in \mathcal{C}: p \in \mathcal{P}^C} w^*_C, \qquad p \in \mathcal{P}$$

and let us show that $\hat{x}$ is a feasible solution to (39)–(41). Consider first constraints (39). By definition of an independent routing configuration, an arc can be used at most once. Hence

$$\sum_{p \in \mathcal{P}} \delta_{ep} \hat{x}_p = \sum_{C \in \mathcal{C}: e \in F(\mathcal{P}^C)} w^*_C \leq \sum_{C \in \mathcal{C}} w^*_C \leq W$$

by (19). This shows that (39) are satisfied.
Now

$$\sum_{p \in \mathcal{P}_{sd}} \hat{x}_p = \sum_{p \in \mathcal{P}_{sd}} \sum_{C \in \mathcal{C}: p \in \mathcal{P}^C} w^*_C = \sum_{C \in \mathcal{C}} |\mathcal{P}_{sd} \cap \mathcal{P}^C| w^*_C = \sum_{C \in \mathcal{C}} a^C_{sd} w^*_C \leq T_{sd}$$

by (21). Hence constraints (40) are satisfied. Since (41) are clearly satisfied, we conclude that $\hat{x}$ is a feasible solution of the linear programming relaxation of the formulation PATH-AGGR. Now $\sum_{p \in \mathcal{P}} \hat{x}_p = \sum_{C \in \mathcal{C}} w^*_C = z^{\text{LP}}_{\text{IRC\_RL}}$, hence $z^{\text{LP}}_{\text{PATH}} \geq z^{\text{LP}}_{\text{IRC\_RL}}$. □

7.4 Summary of the comparison of the upper bounds

Let us now summarize the upper bound comparisons.

Proposition 6

$$z^{\text{LP}}_{\text{IRC}} = z^{\text{LP}}_{\text{MAX\_IRC}} \leq z^{\text{LP}}_{\text{MAX\_IRC\_RL}} = z^{\text{LP}}_{\text{IRC\_RL}} = z^{\text{LP}}_{\text{MAX\_IS}} \leq z^{\text{LP}}_{\text{PATH}}.$$

Moreover each inequality can be strict for some instances.

Proof. In Section 6 and in Sections 7.1 through 7.3, we established the following relations:

$$
\begin{aligned}
&z^{\mathrm{LP}}_{\mathrm{IRC}} = z^{\mathrm{LP}}_{\mathrm{MAX\_IRC}} && \text{(Proposition 1)}\\
&z^{\mathrm{LP}}_{\mathrm{MAX\_IRC}} \leq z^{\mathrm{LP}}_{\mathrm{MAX\_IRC\_RL}} && \text{(Proposition 3)}\\
&z^{\mathrm{LP}}_{\mathrm{MAX\_IRC\_RL}} = z^{\mathrm{LP}}_{\mathrm{IRC\_RL}} && \text{(Proposition 2)}\\
&z^{\mathrm{LP}}_{\mathrm{MAX\_IRC\_RL}} = z^{\mathrm{LP}}_{\mathrm{MAX\_IS}} && \text{(Proposition 4)}\\
&z^{\mathrm{LP}}_{\mathrm{IRC\_RL}} \leq z^{\mathrm{LP}}_{\mathrm{PATH}} && \text{(Proposition 5),}
\end{aligned}
$$

which imply the first part of Proposition 6. The existence of an instance for which the first inequality is strict follows from Proposition 3, while an instance for which $z^{\mathrm{LP}}_{\mathrm{MAX\_IS}} < z^{\mathrm{LP}}_{\mathrm{PATH}}$ was given in Example 1. □

Even if the upper bound provided by the column generation formulations IRC and MAX_IRC is stronger than the one provided by the compact formulations and by the PATH formulations, there exist instances of the max-RWA problem for which the upper bound is not equal to the optimal value.

Example 2 *Consider again the network of Figure 2 and assume that the traffic matrix is now given by* $T_{14} = 4$ *and* $T_{23} = 7$*, the other entries being equal to* 0*. Let the number of wavelengths* W *be equal to* 7*. The maximal IRCs are again*

$$C_1 = \begin{pmatrix} 0 \\ 2 \end{pmatrix}, \qquad C_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$

An optimal solution of the LP *relaxation of* MAX_IRC *is* $(\tilde{w}, \tilde{y})$*, with* $\tilde{w}_1 = \tilde{w}_2 = 3.5$*,* $\tilde{y}_{14} = 3.5$ *and* $\tilde{y}_{23} = 7$*. Its value is* $z^{\mathrm{LP}}_{\mathrm{MAX\_IRC}} = 10.5$*, while the optimal value of problem* MAX_IRC *is 10.*

8 Branch-and-bound algorithm

In the previous sections, we have shown that among the column generation formulations reviewed, the formulations IRC and MAX_IRC present the strongest LP relaxation. Both formulations are based on independent routing configurations. However by working on the restricted set of maximal IRCs, the formulation MAX_IRC presents a theoretical advantage over the IRC one in terms of the maximum number of columns (variables) that could be needed to solve the formulation. Based on these considerations, we present in this section a branch-and-bound algorithm for solving the formulation (MAX_IRC). Recall that this formulation is as below:

$$\max \qquad z_{\mathrm{MAX\_IRC}}(w, y) = \sum_{(v_s, v_d) \in \mathcal{SD}} y_{sd} \tag{30}$$

subject to:

$$\sum_{C \in \mathcal{C}_{\max}} w_C \leq W \tag{31}$$

$$y_{sd} \leq \sum_{C \in \mathcal{C}_{\max}} a_{sd}^C w_C \qquad (v_s, v_d) \in \mathcal{SD} \tag{32}$$

$$y_{sd} \leq T_{sd} \qquad (v_s, v_d) \in \mathcal{SD} \tag{33}$$

$$w_C \in \mathbb{N} \qquad C \in \mathcal{C}_{\max}. \tag{34}$$

Observe that instead of (31), we could take the corresponding equality constraints:

$$\sum_{C \in \mathcal{C}_{\max}} w_C = W. \tag{31b}$$

We recall some notions. At a given node of the branching tree, only a subset of columns is explicitly available. The problem (30)–(34) is referred to as the *master problem*, the problem (30)–(34) with only the subset of (explicit) columns as the *restricted master problem*. The LP relaxation of formulation (MAX_IRC) is obtained by replacing constraints (34) by $w_C \geq 0$ for all $C \in \mathcal{C}_{\max}$.

The choice of an initial set of columns is discussed in Section 8.1; the branching is presented in Section 8.2; the bounding, which consists in solving the LP relaxation, is presented in Section 8.3 and an heuristic is proposed in Section 8.4.

8.1 Initial set of columns

When constraints (31) are used, it is possible to start the algorithm without any columns $C \in \mathcal{C}_{max}$. Indeed the solution $(w, y) = (0, 0)$ is feasible for the problem.

When constraints (31b) are used, there must be at least one column $C \in \mathcal{C}_{max}$ to start the algorithm. Such an initial column is generated by solving the auxiliary problem (see Section 8.3 or Section 6.2) with all u_{sd}^1 equal to 1: this amounts to generate an IRC that satisfies the largest number of connections.

8.2 Branching

The branching is an adaptation of the branching scheme proposed by Vanderbeck [16] (see also [17]).

Let $(\hat{w}, \hat{y})$ be an optimal solution of the LP relaxation of the current master problem and let $\hat{\mathcal{C}} \subset \mathcal{C}_{\max}$ be the subset of explicitly available columns. If $\hat{w}$ is fractional, there must exist a subset $\tilde{\mathcal{C}} \subseteq \mathcal{C}_{\max}$ such that $\alpha = \sum_{C \in \tilde{\mathcal{C}} \cap \hat{\mathcal{C}}} \hat{w}_C$ is fractional (take for example $\tilde{\mathcal{C}} = \{C^f\}$ where C^f is a column of $\hat{\mathcal{C}}$ such that $\hat{w}_{C^f}$ is fractional). We create two branches, one in which we add the inequality

$$\sum_{C \in \tilde{\mathcal{C}}} w_C \leq \lfloor \alpha \rfloor \tag{42}$$

in the master problem and one in which we add the inequality:

$$\sum_{C \in \tilde{\mathcal{C}}} w_C \geq \lceil \alpha \rceil. \tag{43}$$

The question is now how to define the set $\tilde{\mathcal{C}}$ in order that the solution of auxiliary problem remains tractable in order to find new promising columns. Observe first that if $\sum\limits_{C \in \hat{\mathcal{C}}} \hat{w}_C$ is fractional, we can choose $\tilde{\mathcal{C}} = \mathcal{C}_{\max}$. The modification in the auxiliary problem is simple: it amounts to add a constant to the objective function, which corresponds to the dual value associated with this constraint. Note that this case can only happen if constraints (31) are used.

In the sequel we consider the case where $\sum\limits_{C \in \hat{\mathcal{C}}} \hat{w}_C$ is integral. Vanderbeck [16] has proposed different ways to define the column subset to be used for the branching. We choose the option based on a set of bounds on the components of the columns.

Recall that the components of column $C \in \mathcal{C}_{\max}$ are the $a_{sd}^C, (v_s, v_d) \in \mathcal{SD}$. A component bound constraint $a_{sd}^C > a_f$ or $a_{sd}^C < a_f$, where a_f is assumed to be fractional, is defined by a triple $\beta \equiv (s, d, a_f)$ and the direction of the inequality. The notation $\beta^>$ refers to the lower bound constraint $a_{sd}^C > a_f$ while the notation $\beta^<$ refers to the upper bound constraint $a_{sd}^C < a_f$. Let $\mathcal{C}(\beta^>) = \{C \in \mathcal{C}_{\max} : a_{sd}^C > a_f\} = \{C \in \mathcal{C}_{\max} : a_{sd}^C \geq \lceil a_f \rceil\}$ be the set of columns $C \in \mathcal{C}_{\max}$ that satisfy the component bound constraint $\beta^>$. We define similarly $\mathcal{C}(\beta^<) = \{C \in \mathcal{C}_{\max} : a_{sd}^C \leq \lfloor a_f \rfloor\}$. With this notation,

$$\mathcal{C}_{\max} = \mathcal{C}(\beta^>) \cup \mathcal{C}(\beta^<) \qquad \text{and} \qquad \mathcal{C}(\beta^>) \cap \mathcal{C}(\beta^<) = \emptyset.$$

Let $B = (B^>, B^<)$ be a set of component lower and upper bound constraints. We define

$$\mathcal{C}(B) = \left(\bigcap_{\beta^> \in B^>} \mathcal{C}(\beta^>) \right) \bigcap \left(\bigcap_{\beta^< \in B^<} \mathcal{C}(\beta^<) \right).$$

For a set of component bound constraints B, let us define

$$f(B) = \sum_{C \in \mathcal{C}(B)} \left(\hat{w}_C - \lfloor \hat{w}_C \rfloor \right).$$

The following result is proved in [16]:

Proposition 7 *(Vanderbeck [16])*
Given a non-integral feasible solution $(\hat{w}, \hat{y})$ of the LP *relaxation of the current master problem such that $\sum\limits_{C \in \mathcal{C}_{\max}} \hat{w}_C$ is integral, there exists a set of component bounds B with $|B^>| +$*

$|B^<| \leq \lfloor \log f \rfloor + 1$ *such that* $\sum_{C \in \mathcal{C}(B)} \hat{w}_C$ *is fractional, where* $f(B) = \sum_{C \in \mathcal{C}_{\max}} (\hat{w}_C - \lfloor \hat{w}_C \rfloor)$ *represents the fractional characteristic of the current solution.*

The proof of Proposition 7 is based on the following lemma.

Lemma 1 *(Vanderbeck [16])*
If, for a given component bound set B*,* $f(B) \geq 1$*, there exists a component bound* $\beta \notin B$ *such that* $0 < f(B \cup \{\beta\}) \leq \frac{f(B)}{2}$.

Proof. If $f(B) \geq 1$, there exist C^1 and $C^2 \in \mathcal{C}(B)$ with $w_{C^1} - \lfloor w_{C^1} \rfloor > 0$ and $w_{C^2} - \lfloor w_{C^2} \rfloor > 0$. As $C^1 \neq C^2$, $\exists (v_s, v_d) \in \mathcal{SD}$ such that $a_{sd}^{C^1} \neq a_{sd}^{C^2}$. Without loss of generality, assume that $a_{sd}^{C^1} < a_{sd}^{C^2}$. Let $v = \frac{a_{sd}^{C^1} + a_{sd}^{C^2}}{2} + \frac{1}{3}$ and $\beta \equiv (s, d, v)$. Then, as $a_{sd}^{C^1} < v < a_{sd}^{C^2}$, $f(B \cup \{\beta^>\}) \geq w_{C^1} - \lfloor w_{C^1} \rfloor > 0$ and $f(B \cup \{\beta^<\}) \geq w_{C^2} - \lfloor w_{C^2} \rfloor > 0$. Moreover $f(B) = f(B \cup \{\beta^>\}) + f(B \cup \{\beta^<\})$. Thus either $f(B \cup \{\beta^>\}) \leq \frac{f(B)}{2}$ or $f(B \cup \{\beta^<\}) \leq \frac{f(B)}{2}$.
□

Proof of Proposition 7 : Let $B^0 = \emptyset$. By assumption, we have $f(B^0) \geq 1$. If $f(B^k) \geq 1$, we apply Lemma 1, obtaining a set of component bounds B^{k+1} that satisfies $f(B^{k+1}) \leq \frac{f(B^k)}{2}$. Hence $f(B^k) \leq \left(\frac{1}{2}\right)^k f(B^0)$. The greatest k such that $f(B^k) < 1$ is therefore $k = \lfloor \log f \rfloor + 1$. ■

In practice, we first look if there exists a component bound set B of cardinality 1. If not, we apply the procedure suggested in the proof of Proposition 7 to find a component bound set B of cardinality ≥ 2.

8.3 Bounding

The upper bound at a given node of the branching tree is obtained by solving the LP relaxation of the master problem, by using the column generation technique to generate new columns when needed. The difficulty is that the master problem may contain branching constraints. Assume that at node N, the master problem contains the following branching constraints

$$\varepsilon_j \sum_{C \in \mathcal{C}^j} w_C \leq \varepsilon_j K^j, \qquad j \in G_N \tag{44}$$

where $\varepsilon_j = 1$ or $\varepsilon_j = -1$ depending whether the constraint (44_j) is of the form (42) or (43) for $j \in G_N$. Let $\mu_j, j \in G_N$ be the dual value associated with inequalities (44). The objective function of the modified auxiliary problem is

$$\max \quad c^1_{\text{AUX-MAX\_IRC}}(\alpha, g) = -u^0 + \sum_{v_s, v_d \in V} \sum_{e \in \omega^+(v_s)} \alpha_e^{sd} u^1_{sd} - \sum_{j \in G_N} \mu_j \varepsilon_j g_j \tag{45}$$

where $g_j, j \in G_N$ are additional binary variables that indicate if the column defined by α is in $\mathcal{C}^j$ (value 1) or not (value 0).

For each $\beta \in \bigcup_{j \in G_N} B^j$, we introduce the binary variable η^β defined as follows. Assume that β is defined by the triple (s, d, a_f). Then

$$\eta^\beta = \begin{cases} 1 & \text{if column } C \text{ satisfies } a^C_{sd} < a_f \\ 0 & \text{otherwise} \end{cases}$$

Let $B^j = B^{j>} \cup B^{j<}$. The constraints to be added to the auxiliary problem for the component bound $B_j, j \in G_N$ are the following:

$$g_j \geq 1 - \sum_{\beta \in B^{j<}} (1 - \eta^\beta) - \sum_{\beta \in B^{j>}} \eta^\beta \tag{46}$$

$$g_j \leq \eta^\beta \qquad \beta \in B^{j<} \tag{47}$$

$$g_j \leq 1 - \eta^\beta \qquad \beta \in B^{j>} \tag{48}$$

$$(\overline{a}_{sd} - \lceil a_f \rceil + 1)\eta^\beta \leq \overline{a}_{sd} - \sum_{e \in \omega^+(v_s)} \alpha_e^{sd} \qquad \beta \equiv (s, d, a_f) \in B^{j>} \tag{49}$$

$$\lceil a_f \rceil \eta^\beta \geq \lceil a_f \rceil - \sum_{e \in \omega^+(v_s)} \alpha_e^{sd} \qquad \beta \equiv (s, d, a_f) \in B^{j>} \tag{50}$$

$$(\lfloor a_f \rfloor + 1)\, \eta^\beta \geq \lfloor a_f \rfloor + 1 - \sum_{e \in \omega^+(v_s)} \alpha_e^{sd}, \qquad \beta \equiv (s, d, a_f) \in B^{j<} \tag{51}$$

$$(\overline{a}_{sd} - \lfloor a_f \rfloor)\eta^\beta \leq \overline{a}_{sd} - \sum_{e \in \omega^+(v_s)} \alpha_e^{sd} \qquad \beta \equiv (s, d, a_f) \in B^{j<} \tag{52}$$

$$\eta^\beta \in \{0, 1\} \qquad \beta \in B^j \tag{53}$$

$$g_j \geq 0 \tag{54}$$

where $\overline{a}_{sd}$ is an upper bound on the number of connections from v_s to v_d that can be accepted with only one wavelength.

When $\varepsilon_j = -1$, constraints (46), (49) and (51) can be omitted (observe that $\mu_j \geq 0$), while when $\varepsilon_j = 1$, constraints (47), (48), (50) and (52) can be omitted. Also when $|B^j| = 1$, there is a simplification because constraints (46)–(48) are equivalent to $g_j = \eta^\beta$, hence the unique variable η^β can be eliminated. In practice, we transform the inequalities (46) or (47)–(48) to equalities.

8.4 Heuristic

8.4.1 Rounding-off heuristic

Let $(\hat{w}, \hat{y})$ be the optimal solution of the LP relaxation of the current master problem found when computing the upper bound (see Section 8.3) and denote by $\hat{\mathcal{C}}$ the subset of explicitly available columns.

An initial solution (w^0, y^0) is built by rounding the fractional solution, i.e., $w^0_C = \lfloor \hat{w}_C \rfloor$ for all $C \in \hat{\mathcal{C}}$, while y^k $(k = 0)$ is given by the formula

$$y^k_{sd} = \min \left\{ \sum_{C \in \hat{\mathcal{C}}} w^k_C a^C_{sd}, T_{sd} \right\}, \qquad (v_s, v_d) \in \mathcal{SD}. \tag{55}$$

This solution is then iteratively improved using 2 operations:

Increase : this operation is executed when the following two conditions are met: a) $\sum_{C \in \hat{\mathcal{C}}} w^k_C < W$ and b) at least one connection is rejected in the current solution. We increase the value of a column $\tilde{C} \in \hat{\mathcal{C}}$ by 1 such that the increase of the objective function is maximized. As secondary criterion for the choice of the column, we minimize the absolute difference between its value in the greedy solution and its value in the fractional solution. Note that as long $\hat{\mathcal{C}}$ contains at least one column with $a^C_{sd} \geq 1$ for every $(v_s, v_d) \in \mathcal{SD}$, we are guaranteed that the objective value increases by at least 1 at every execution of this operation. In particular the number of successive executions of this operation is bounded by W.

Decrease : this operation is executed when $\sum_{C \in \hat{\mathcal{C}}} w^k_C = W$. We attempt to find a column for which it is possible to decrease its value without reducing the objective value. If such a column exists, we do the decrease; otherwise we stop the heuristic.

8.4.2 MIP heuristic

We solve the MAX_IRC formulation with the current set of columns using Cplex-MIP. The branching constraints generated so far are not included. A limit on the number of nodes in the branch-and-bound tree is used as stopping criteria.

9 Computational results

As already observed in [4, 3], for most of the classical traffic and network instances, the optimal value of the linear relaxation of the compact formulations, rounded to the next integer value, is equal to the optimal (integer) value. Therefore, even if theoretically, the optimal values of the linear relaxation of the IRC or the MAX_IRC column generation formulations can be better than the optimal values of the LP relaxation of the compact relaxations, column generation formulations are of little help for solving those instances more efficiently as the problem means finding a feasible (integer) solution that matches the upper bound, a task that is well performed by heuristics for the RWA problem, see, e.g., [7].

Therefore we focus on instances that we could not be solved using the compact formulations [3]. It is believed that it was due to the fact that there is a gap for those instances, and that the gap was responsible for the difficulty to solve them exactly in a reasonable amount of time. Using the column generation formulation, we were able to solve exactly all these instances but one.

Most of these instances are with symmetrical traffic. This means that a request is between 2 nodes rather than from one node to the other and that the links of the network are assumed to be bidirectional (or that the links between two nodes are unidirectional but come by pairs, one link for each direction). In order to use the model for asymmetrical traffic that we developed in this paper for solving the instances with symmetrical traffic, we proceed as follows. First the underlying graph must be bidirected, i.e., the number of arcs from u to v is equal to the number of arcs from v to u for any pair (u, v) of nodes. A request between nodes v_i and v_j is subdivided into 2 subrequests, one from v_i to v_j and the other from v_j to v_i. The two requests must be accepted simultaneously, or rejected simultaneously. Moreover, if the two subrequests are accepted, they must be so with the same wavelength and the two paths differ from each other by reversing the directions of their arcs. In practice, we consider explicitly only one of the two subrequests for each request, the other one being considered implicitly. By doing so, the only constraints that need to be changed are the clash constraints in the auxiliary problem:

$$\sum_{(v_s,v_d)\in\mathcal{SD}} \left(\alpha_e^{sd} + \alpha_{\overline{e}(e)}^{sd}\right) \leq 1 \qquad e \in E$$

where it is assumed that with each arc $e \in E$ we associate one opposite arc, that we denote by $\overline{e}(e) \in E$.

We considered two families of such difficult instances. The first family, NSF3, was introduced in [3]. All instances of this family are with symmetrical traffic. NSF3 is a variant of the NSF network [18], which was obtained by removing some links. The traffic matrix was obtained by making the traffic matrix of Khrisnaswamy [19] symmetric, see e.g. [20] for details. The number of available wavelengths varies between 10 and 32. Half of the 12 instances could not be solved to optimality using the compact formulations.

The second family of difficult instances was introduced in [21], see also [7]. These instances were constructed by generating traffic matrices in a special way, on two classical optical networks, NSF and EON. The NSF network is a network with 14 nodes and 21 links, with a maximum of 4 links per node and is described in, e.g., Krishnaswamy and Sivarajan [18]. The EON network has 20 nodes and 39 optical links. A description of it can be found e.g., in Mahony *et al* [22]. The traffic matrices were constructed in such a way that a gap exists. In order to make the instances more realistic, we embedded the special structure into additional traffic (leading to the so-called noisy instances), i.e., we added some connection requests at random. The resulting traffic matrices have 652 connection requests for NSF (respectively 1576 for EON) in the asymmetrical model, and 428 (resp. 1172) in the symmetrical model. The number of available wavelengths is $W = 32$.

We compare the solutions obtained using the compact formulation with the ones obtained when using two column generation formulations, MAX_IRC and IRC. For both MAX_IRC and IRC, we solved the reduced master problem to optimality (with the help of the MIP heuristic described in Section 8.4) at every nodes. The computational results are given in Table 1 where the number at the end of the instance names corresponds to the number of wavelengths. z^{LP} and z^* are respectively the optimal value of the LP relaxation and the optimal value of the RWA problem for the compact and the column generation formulations. For instances that could not be solved to optimality, we provide an interval on z^*. $\underline{z}^{root}$ is the value of the best solution found at the root node, #nodes is the number of nodes in the branching tree, *depth* is the maximum depth of the tree, #col is the total number of variables w_C generated and finally *cpu* is the computational time. The results for MAX_IRC are given in the first line, and those for IRC, when available, are given in the second line.

We observe that for all instances the optimal value is equal to the optimal value of the LP relaxation of the column generation formulation, hence the problem again means finding a feasible solution whose value matches the LP bound. For these instances this turned out to be easy as very few backtracks were needed. Notice that the optimal solution has always been found by the CPLEX heuristic in the compact formulation, although CPLEX alone fails to prove its optimality while the MAX_IRC column generation formulation always found it with a proof of its optimality. We also observe that the MAX_IRC formulation outperforms the IRC one as expected. Last, the largest gap we observed between the optimal values of the LP relaxations of the column generation vs. the compact formulations was 7.6 % for the NSF_Sym_noise/32, while the optimal LP was again equal to the optimal integer solution for the column generation formulations.

We also tested one instance for which there seems to be a gap for the column generation formulation. This instance corresponds to the NSF optical network, with the symmetric traffic matrix, adapted from the asymmetric traffic matrix of Krishnaswamy [19], using the symmetric model and with $W = 12$ available wavelengths (the instance was incorrectly announced to be solved using the compact formulation in [3]). We observed a similar behavior than for the compact formulation, namely that the branching tree becomes very deep (maximum depth on the order of 100), which prevents from solving the instance to optimality in reasonable time. This shows the need of more work on the branching (an interesting step in this direction could be the recent paper by Vanderbeck [23]) and/or on the bounding.

Table 1: Computational Results on the Comparison among Compact and Column Generation Formulations

instance	compact form. z^{LP}	compact form. z^*	column generation formulations z^{LP}	z^*	$\underline{z}^{root}$	#nodes	depth	#col.	cpu
NSF3_sym/10	78	78	78	78	76	17	9	70	48s
					76	55	28	137	2mn27s
NSF3_sym/12	87	87	87	87	85	37	19	75	68s
					83	51	26	132	2mn05s
NSF3_sym/14	96	96	96	96	94	47	24	100	1mn42
					91	63	29	157	3mn24
NSF3_sym/16	105	105	105	105	104	37	19	105	1mn34
					103	25	13	149	2mn38
NSF3_sym/18	112	[111,112]	111	111	111	1	1	46	25s
					109	55	24	151	3mn03
NSF3_sym/20	119	[117,119]	117	117	115	23	12	60	51s
					112	69	34	150	4mn34
NSF3_sym/22	126	[123,126]	123	123	121	37	19	60	58s
					119	83	41	160	5mn08
NSF3_sym/24	133	[129,133]	129	129	128	29	15	59	54s
					123	73	36	168	4mn38
NSF3_sym/26	138	[135,138]	135	135	133	33	17	69	69s
					129	91	44	168	15mn47s
NSF3_sym/28	142	[141,142]	141	141	139	37	19	90	1mn37
					135	91	46	185	28mn46
NSF3_sym/30	146	146	146	146	145	13	7	74	58s
					142	133	47	243	4h42mn
NSF3_sym/32	150	150	150	150	149	23	12	83	1mn14
					145	137	52	222	1h16mn
NSF_sym_noise/32	317.5	[295, 313]	295	295	294	11	6	47	6mn11
NSF_asym_noise/32	551	[536, 550]	536	536	536	1	1	36	47s
EON_sym_noise/32	876.5	[844, 862]	844	844	843	87	44	127	5h28mn
EON_asym_noise/32	1325.6	[1303, 1320]	1303	1303	1300	141	71	228	9h51mn

10 Conclusions

We have described and compared four column generation formulations for the max-RWA problem. Although the column generation formulations allowed to solve to optimality for the first time several instances, much work needs to be done to make these methods efficient, particularly on instances with a gap. Another direction of research is to adapt these methods to different objectives such as the minimization of the congestion or of the network load, or to adapt them to solve the GRWA problem, i.e., the RWA problem with traffic grooming that is not easily solved by compact formulations due to a too large number of variables. Finally additional constraints should be added, such as for example a limit of the number of hops on the lightpaths. Such constraints are often easier to deal with in a column generation framework compared to a compact formulation.

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