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Abstract

This paper addresses the structural optimization of a next generation optical, lattice based, yottabit-per-second transport network topology, called the *YottaWeb* [1, 2, 3]. One of the key issues for this type of structure is to assess what is the number of agile cores that each edge node has to be attached to. This is what is called the *dimension* of the structure. In this article, several optimization metrics are explored and it is found that the optimal solution corresponds to a structure of dimension two.

Key Words: Agile Optical Core, YottaWeb, PetaWeb, Next Generation Internet, Optical Internet, Lattice Structure.

Résumé

Cet article traite le problème de design topologique d'un réseau transportant des capacités de yottabits par seconde, appelé le *Yottaweb* [1, 2, 3]. Une des notions fondamentales de ce réseau est de trouver quel est le nombre de nœuds cœurs agiles auxquels chaque nœud d'accès doit être attaché. Ceci est appelé la *dimension* de la structure. Dans cet article nous explorons différentes métriques d'optimisation et trouvons que la dimension optimale est deux.

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1 Introduction

Advances in optical networking technologies have led to the proposal of a distributed, dynamic channel-switching optical core that scales to the capacity of several terabits (10^{12}) per second [4]. The resulting agile optical core with thousands of edge nodes has been coined the Petaweb. Given the size of the network, an efficient grouping of the edge nodes into a gracefully expanding multidimensional regular lattice structure, called *Yottaweb* has been proposed [5]. The topology simplifies routing and addressing as it favors quick connections between edge node pairs.

Figure 1(retrieved from [3]) shows the two ways of visualizing the Yottaweb architecture. On the left-hand side the Yottaweb is seen as a collection of edge nodes that communicate through one or several agile cores. On the right-hand side, the lattice structure of the Yottaweb is underlined. In the regular Yottaweb, all the edge nodes are connected to exactly the same number of Agile Cores, which corresponds to the *dimension* of the lattice structure. The figure portrays two cases, one in which all the edge nodes are connected to a single agile core, thus yielding a one-dimension lattice, and one where each edge node presents two agile core connections.

Previous studies [1, 2, 3] have investigated how to efficiently arrange the edge nodes into the regular lattice structure, given different types of traffic, different dimensions and different capacities of the agile cores. However an issue that has not been tackled up to know is what is the best dimension for this structure. This is an important problem because once the dimension is chosen, efficient ways to design and update the lattice structure can be foreseen. Therefore, the object of this paper is to find the ideal dimension that the structure should present once different performance or design metrics are studied and assessed.

This paper is organized as follows. In Section 2, we provide some concepts and notations. The most suitable objective functions are presented and analyzed in Section 3. A new combined metric is proposed in Section 4 and Conclusions and Remarks are summarized in Section 5.

2 Concepts and notations

Let us introduce some useful notation.

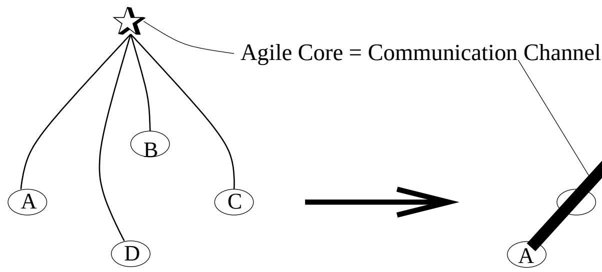
$N =$ the edge node set,

$M =$ the set of agile cores,

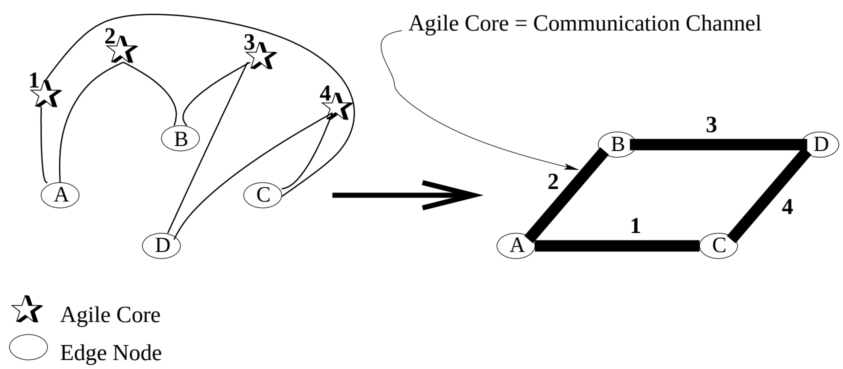
$D =$ the dimension of the network, that is, the number of agile cores every single edge node is connected,

$\mathcal{K} =$ the capacity of the edge nodes,

$\mathcal{H}_1 =$ the number of one hop communications.



(a) **One Dimension Lattice**



(b) **Two-Dimension Lattice**

Figure 1: YottaWeb Arrangement into Lattice Structure.

In [1], a proposition states that the proportionality between the number of access nodes $|N|$, the number of agile cores $|M|$, the capacity of the access nodes $\mathcal{K}$ and the dimension of the network D in a regular lattice structure of the YottaWeb is governed by the following equalities:

$$|N| = \mathcal{K}^D \tag{1}$$

and

$$|M| = D \times \mathcal{K}^{D-1} \tag{2}$$

The concept of the lattice topology stated in [1] and [5], gives a broad choice of possible lattice dimensions. Thus, in what follows, we underline the various aspects which intervene in the choice of a given dimension. Beshai et al. [5] estimated that with 1000 access nodes by sub-network, formed by an agile core, and an entrance capacity to each access node of a terabit per second (10^{12} bit/sec), one could reach total external capacities of yottabits per second with a lattice of dimension 4. Thus, without much restrictions, we limit this study to dimensions two to four.

3 Objectives of the design

For a given number $|N|$ of nodes, the basic objective of the design of the YottaWeb is to find the optimal dimension D_{opt}, in order to:

(a) minimize the number of agile cores $|M| = D \times \frac{|N|}{\mathcal{K}} = D \times |N|^{\frac{D-1}{D}}$;
(b) minimize the capacity $\mathcal{K}$ of the ACs;
(c) maximize the one hop communications $\mathcal{H}_1 = |N| \times D \times (\mathcal{K} - 1) = |N| \times D \times (N^{\frac{1}{D}} - 1)$;
(d) minimize the edge nodes capacity.

3.1 Minimization of the number of Agile Cores

This objective is directly related to the cost structure since agile cores are expensive. Concerning the relationship of this objective with the dimension of the network, it can be seen that the required number of ACs, $|M|$, grows exponentially with D. Thus, the minimum is obtained for $D = 2$.

3.2 Minimization of the Agile Core capacity

This is an interesting objective to reduce the agile core management complexity. The Minimization of the Agile Core Capacity (MACC) can be defined as:

$$[MACC] \quad Z_{MACC} = \min |N|^{\frac{1}{D}} \tag{3}$$

$$\text{subject to} \qquad 2 \leq D \leq 4 \tag{4}$$
$$\text{with} \quad D \quad \text{positive integers} \tag{5}$$

where the objective (3) is the individual capacity of the agile cores, and the constraints (4) define the bound of the studies from dimension 2 to 4.

The objective derivative is:

$$Z'_{MACC} = (|N|^{1/D})' = -\frac{\ln(|N|)}{D^2}|N|^{\frac{1}{D}},$$

then, the optimal objective function is reached for $D = 4$.

3.3 Maximize the one hop communications

The Yottaweb was specifically designed to allow as many one hop communications as possible to simplify routing and addressing. By inspection of the derivative with respect to D of the expression for $\mathcal{H}_1$, it is clear that the number of one-hop connections increases with the dimension. Thus, the optimal dimension for this case would be 4.

3.4 Minimize the edge node capacity

This objective relates to the cost and complexity of the edge nodes. The communication ports inside the edge nodes are generally divided into $D+1$ groups. The first group of dual ports (input/output) m_0 is devoted to local communications between end nodes connected to the same edge node. The other groups, annotated as m_i, $i = 1, \ldots, D$, represent the group of ports associated to the communication in each of the D different dimensions.

In the regular lattice structure, the numbers $|m_i|$, $i = 0, 1, \ldots, D$ are directly proportional to the nodal connectivity that is $D(\mathcal{K} - 1)$. Thus, for $|N|$ nodes, the total number of ports at the edge nodes level is directly proportional to $|N| \times (D(\mathcal{K} - 1) + 1)$ replacing $\mathcal{K}$ from (1) we have that the minimization of the edge node capacity can be written as :

$$[\textbf{MENC}] \quad Z_{MENC} = \min |N| \times (D(|N|^{\frac{1}{D}} - 1) + 1) \tag{6}$$

$$\text{subject to:} \qquad 2 \leq D \leq 4 \tag{7}$$

$$\mathcal{K}, D \quad \text{positive integers.} \tag{8}$$

It can be seen that Z_{MENC} is a decreasing function of D, with the following negative derivative:

$$\begin{aligned} Z'_{MENC} &= |N|\,(|N|^{\frac{1}{D}} - 1 - \frac{|N|^{\frac{1}{D}}\,\ln(|N|)}{D}) && (9)\\ &= |N|\,[|N|^{\frac{1}{D}}\underbrace{(1 - \frac{\ln(|N|)}{D})}_{\leq 0} - 1] && (10)\\ &\leq 0 && (11) \end{aligned}$$

Thus, the ideal dimension when the edge nodes capacity is minimized is 4.

3.5 Analysis

From the previous analysis, we can make the following observation:

(i) the number of agile cores $|M|$ that is equal to $D \times \frac{|N|}{\mathcal{K}} = D \times |N|^{\frac{D-1}{D}}$, increases with the dimension. Thus, the objective (a) is met for the *smallest* D;

(ii) The capacity of an agile core decreases with the dimension. Then the objective (b) is met for the *largest* D;

(iii) The number of one hop connections $\mathcal{H}_1$ increases with the dimension. The objective (c) is then met for the *largest* D;

(iv) The edge nodes capacity is also minimized with the *largest* D.

According to statements (ii), (iii) and (iv), it can be concluded that the fulfillment of criteria (b) also leads to the fulfillment of (c) and (d) and vice versa. However, all of the objectives are irreconcilable, because of objective (a). Thus, there is a need for a trade-off between objective (a) and the others.

4 A New Metric

To conciliate all the objectives of the YottaWeb design, let us define a new metric that combines the number of ACs $|M|$ of objective (a), with $\mathcal{K}$ which is one of the metrics of objectives (b), (c) and (d). The new metric, coined *capacity number*, is $\mathcal{N}_K = M \times \mathcal{K}$. It is obvious that in order to optimize the whole lattice structure, the *capacity number* should be minimized. Thus, Minimization of the Capacity Number Problem (**MCN**) can be defined as:

$$\mathcal{N}_{K,opt} = \min(D \times N^{\frac{D-1}{D}}) \times N^{\frac{1}{D}} = D \times N \tag{12}$$

$$\text{subject to} \quad 2 \leq D \leq 4 \tag{13}$$

$$D \quad integer. \tag{14}$$

$\mathcal{N}_K$ is a strictly increasing function of D, and its minimum is reached for $D = 2$. This results shows that the need to reduce $|M|$ outweighs the other considerations, since the variation induced by the dimension on the number of agile cores is more important than any other design optimization concern. Therefore, $D = 2$ is a good candidate for an efficient YottaWeb. On the other hand, compared to higher dimension lattices, the two dimension lattice presents low alternate inter nodal paths between every pair of nodes. However, this problem could be easily circumvented by increasing the capacity between the edge nodes.

5 Remarks and Conclusions

The Yottaweb is a novel architecture that facilitates communication between high capacitated edge nodes. The upgradability of the structure lays on the notion of dimension that can be defined as the number of agile cores each edge node is attached to. This paper has found an answer to the question of what is the most suitable dimension for the Yottaweb. For this, we firstly outlined the most important objective functions that could be foreseen for an effective design. Then, when some of the metrics were found to be contradictory to each other, we proposed a combined objective that we called the capacity number. Under such a metric, the ideal dimension was found to be $D = 2$. The importance of this result is that we now have all the elements to explore efficient methods to optimize the upgrade and expansion of the Yottaweb architecture.

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