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Abstract

We consider an infinite-horizon differential game played by two direct marketers. Each player controls the number of emails sent to potential customers at each moment in time. There is a cost associated to the messages sent, as well as a potential reward. The latter is assumed to depend on the state variable defined as the level of the representative consumer's attention. Two features are included in the model, namely, marginal decreasing returns and bounded rationality. By the latter, we mean that the representative consumer has a limited capacity for processing the information received. The evolution of this capacity depends on its level, as well as on the emails sent by both players. This provides environmental flavour where, usually, one player's pollution emissions (here emails) also affect the payoff of the other player by damaging the common environment (here, the stock of consumer attention).

We characterize competitive equilibria for different scenarios based on each player's type, i.e., whether the player is a spammer or not. We define a spammer as a myopic player, that is, a player who cares only about short-term payoff and ignores the impact of her action on the state dynamics. In all scenarios, the game turns out to be of the linear-quadratic variety. Feedback Nash equilibria for the different scenarios are characterized and the equilibrium strategies and outcomes are compared.

Finally, we analyze the game in normal form, where each player has the option of choosing between being a spammer or not, and we characterize Nash equilibria.

Key Words: Electronic Business Dynamics; Electronic Mail; Direct Marketing; Differential Games; Spam.

Résumé

Un jeu différentiel sur horizon infini entre deux firmes ayant des activités de marketing direct est analysé. Chaque joueur décide du nombre de courriels par période qu'il va envoyer aux consommateurs éventuels, encourant ce faisant un coût certain d'envoi pour un gain espéré. Ce dernier est fonction de la variable d'état qui est ici l'attention du consommateur représentatif. Le modèle ainsi proposé a deux caractéristiques : gains marginaux décroissants et rationalité limitée. En effet, on suppose que le consommateur représentatif a une capacité limitée de traitement de l'information reçue. L'évolution de cette capacité dépend de son niveau courant ainsi que du nombre de courriels envoyés par chaque joueur à chaque instant. Ceci donne au modèle une teinture quelque peu environnementale où habituellement les émissions de pollution d'un joueur (ici les courriels) ont un impact sur les revenus de l'autre joueur par le truchement de la dégradation de l'environnement commun (le stock d'attention du consommateur dans le cas en l'espèce).

Différents scénarios sont analysés ainsi que les équilibres compétitifs qui en résultent selon si aucun, un ou les deux joueurs sont "spammers". Nous définissons un "spammer" en tant qu'un joueur myope, c'est-à-dire qui ne considère que les gains de court terme et ignore les conséquences de ses actions sur la dynamique de l'état. Quel que soit le scénario considéré, le modèle est du type linéaire quadratique. Les équilibres en rétroaction de Nash sont caractérisés pour tous les scénarios, ce qui nous permet par la suite une comparaison des résultats.

Finalement, nous analysons un jeu sous forme normal où chaque joueur a l'option de choisir d'être ou non un "spammer". Les équilibres de Nash sont caractérisés.

Mots clés : Dynamique de e-Business, courrier électronique, marketing direct, jeux différentiels, spam.

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1 Introduction

Assume your competitor is a spammer; how would you react to this in terms of emailing strategy to your potential customers? Would it drive you to behave like her, or would you remain a good citizen (non-spammer)? Basically, these are the research questions we wish to tackle in this paper. More specifically, we are interested in analyzing the strategic behavior of competing Internet marketers.

Direct marketers that are non-spammers are suffering their spamming colleagues for various reasons. First, spammers are fierce, low-cost competitors. Apparently, it is only necessary to complete one transaction per one million emails sent to be profitable.[1] Second, they are not only tapping into the pocketbook of the same consumer as the non-spammers, but also into the same consumer's attention stock. This implies a reduction in the efficiency of the non-spammers' marketing campaigns. Third, in their battle against spam, by e.g., changing their email addresses and installing filters, consumers are (intentionally or not, the result is the same) lowering the value of email databases, which are an important asset, for firms doing business on the Internet.[2] Actually, spam is an issue for all Internet stakeholders (Sipior et al., 2004). Spam is creating congestion, which is not cost free, on the networks of all Internet service providers. Firms have to deal with the management and safety of their computer systems and internal networks, and also with loss of productivity due to the time employees spend processing junk emails.[3] Individuals are irritated by the overwhelming volume of email they receive and are concerned about their privacy. Finally, governments and international bodies (OECD, EU, etc.) are interested in finding legal and technological solutions for reducing spam, in order to respond to the concerns of their constituencies.[4]

Many disciplines are interested in the spam phenomenon. For instance, computer scientists are devoting considerable effort to designing anti-spam protection systems and intelligent agents for managing email. Researchers in law and public decision-makers are looking for legal frameworks to reduce the burden of spam. Economists have been researching, among other things, the costs and the welfare implications of this form of pollution, as well as the problems caused by consumer information overload. This paper is mainly related to this last topic and to the strategic behavior of direct marketers.

We consider an email to be a communication emanating from a firm to a potential customer. The latter needs to process the email to obtain information from it, e.g. on the

[1] According to an article in the Spanish newspaper, *El País*, in its issue dated February 19, 2006.

[2] According to a study (Industry Canada (2005)), 16% of email address changes are due to spam.

[3] The cost for U.S. corporations has been estimated at \$8.9 billion (Industry Canada (2005)). Ferris Research, a consultancy, evaluates the loss to the European economy, due to spam, at \$2.5 billion (cited in *El País* in its issue dated February 19, 2006).

[4] Half of the states in the U.S.A. already have laws forbidding spamming.

product, the price, the delivery conditions, etc. If there are many senders (or even a few sending many emails), this potential customer may quickly become overloaded with information. Some papers, e.g. Jacoby et al. (1974), Malhorta (1982), Keller and Staelin (1987), Lurie (2004), Lee and Lee (2004), have looked into the impact of information overload, in terms of either quantity or quality and structure of information, on the evaluation of alternatives by customers, or on the optimality of consumer decisions. Actually, the problem of information overload would not exist if the consumer's cognitive capacity were unlimited. Therefore, if the consumer's bounded rationality is acknowledged, then too much information necessarily leads to a decrease in the customer's attention (Simon (1997)) and in the number of alternatives evaluated. Then, a lower response rate to direct-marketing offers is expected to follow.

Research devoted to information overload started well before the Internet (and spam) era. However, the low cost of designing electronic direct-marketing campaigns and the absence of entry barriers have emphasized, more than ever, the common (public-good) nature of the consumer's attention. McFadden (2001) argues that *the management of the digital commons is perhaps the most critical issue of market design that our society faces*. Van Zandt (2004) considers the competition for attention when senders interact strategically and face information-overloaded receivers, and proposes an attention allocation mechanism. Anderson and de Palma (2005) assess the tragedy of the commons associated with customer attention and evaluate the welfare implications of sending too much information. Shiman (1996, 1997) also deals with the welfare implications of the decrease in the cost of technologies for getting information about customers and for sending them messages. This burgeoning literature has lacked a definition of what a spammer is (which may have be seemed too obvious) and has not taken into account the heterogeneity in sender type: spammers or non-spammers. Clearly, information overload could occur even when all firms are good citizens. In this paper, we wish to test whether or not heterogeneity itself affects all senders' behavior.

To attempt to answer our research questions, we build a parsimonious model where two direct marketers compete for consumer attention. Our simple model includes the following features:

- *Dynamics:* The representative consumer is endowed with an attention's capacity, that evolves overtime. The adoption of a dynamic rather than static framework allows us to make the distinction between flows (emails) and stock (consumer capacity). Also, it also allows us to take into account the carry-over effects of emails and spam on this capacity. This means that the consumer remembers these events, which seems reasonable, simply by relying on her own experience.
- *Strategic competition:* All firms are drawing from the same pool of consumer attention (and the same pocketbooks); hence, it is mandatory that we incorporate competition

in the model. A response model can ignore competition, accommodate for it in a passive way, i.e., by fixing the values of the decision variables for the competitor and assuming no reaction to a firm's actions, or by considering strategic interactions.[5] Our research questions point quite naturally towards a model where players can react to each other.

- *Different types of players:* On addition to considering that each player influences the other's strategy, we wish to assess the impact of having heterogeneous players. Thus, we assume that each player can choose between being a spammer or not. This requires us to define precisely what is a spammer. The spammer is a player who optimizes her short-term (or current) payoff. This means, synonymously, that this player disregards the evolution of the system (the consumer's attention), or is myopic, i.e., having an infinite discount rate.

Our main results can be summarized as follows:

- The steady-state of the consumer's attention decreases with the number of spammers.
- A non-spammer sends more emails when facing a spammer than when her competitor is a non spammer firm.
- The Nash equilibrium of the game where each player chooses her type depends on the initial stock of attention.

The rest of the paper is organized as follows. In Section 2, we introduce the differential game model of competition for consumer attention. In Section 3, we characterize Nash equilibria in the different scenarios and compare the resulting strategies and steady states. In Section 4, we analyze the game in normal form, where each player chooses her type. In Section 5, we briefly conclude.

2 Model

To focus on strategic behavior, and following a long tradition in prisoner's dilemma literature, we shall assume away all sources of asymmetries. Put differently, we suppose that the players are symmetric in all of the problem's data except type, which is either spammer or non-spammer. Clearly, this symmetry assumption does not correspond to the above

[5]Note that there is an important direct-marketing literature interested in developing tools to describe customer response rates to direct offers (thus including emails) in terms of, e.g., the frequency and monetary value of the purchases. For instance, econometric models are designed to segment consumers and forecast their response rates (see, e.g., Bult and Wansbeek (1995)). Mathematical programming approaches, on the other hand, seek to optimize the frequency of mailing campaigns (see, e.g., Bitran and Mondschein (1996), Gönül and Ze Shi (1998), Piersma and Jonker (2004)). Strategic interaction between multiple senders on customers' response rates has been largely neglected in these approaches.

statement that a spammer is a low-cost competitor to a non-spammer. However, we prefer to keep a better control on the experiment we are conducting with this model.

Let time t be continuous. Denote by $x(t), t \in [0,\infty)$, the capacity of the representative consumer to process the information inflow (emails received). We assume that this capacity, called consumer attention,[6] is nonnegative and bounded, i.e., $0 \leq x(t) \leq x_{max}, \forall t \in [0,\infty)$, where x_{max} is the upper bound. The rate of variation of this stock depends on two factors, namely, the rate of depletion (or use) and the rate of regeneration. We assume that depletion results from the entering emails. To keep things simple, we suppose that the set of senders (players) is made up of two firms sending emails at rate $n_i(t), t \in [0,\infty)$. This leads to a loss of attention measured by $H(n_1(t), n_2(t))$, a nonnegative and increasing function in both arguments. On the other hand, the regeneration rate is given by a nonnegative function that we denote $G(x)$. As a result, the evolution of the consumer's attention is captured by the following differential equation

$$\dot{x}(t) = \frac{dx(t)}{dt} = G\left(x(t)\right) - H(n_1(t), n_2(t)), \quad x(0) = x_0,$$

where x_0 denotes the initial stock of attention.

Since not much insight can be obtained from the above general dynamics, we shall assume that both $G(x(t))$ and $H(n_1(t), n_2(t))$ can be well approximated by the following linear functions:

$$\begin{aligned} G(x(t)) &= L - \gamma x(t), \quad L, \gamma > 0, \\ H(n_1(t), n_2(t)) &= \alpha\left(n_1(t) + n_2(t)\right), \quad \alpha > 0. \end{aligned}$$

With these specifications, the evolution of the stock becomes

$$\dot{x}(t) = L - \gamma x(t) - \alpha\left(n_1\left(t\right) + n_2\left(t\right)\right), \quad x(0) = x_0. \tag{1}$$

The constant L is the regeneration rate when the stock converges to zero and γ is the natural decay rate of attention. The parameter α is a scaling factor transforming emails (sent and received) into loss of attention or depletion of the resource.

To help interpret the function $G(x)$, and as a benchmark, we state the following lemma, which characterizes the steady state of the resource if no emails were sent. The lemma also allows us to determine the value of the upper bound on the attention stock $x_{\max}$.

Lemma 1 *If no email is sent, then the attention stock converges to* $\frac{L}{\gamma}$.

[6] This concept of consumer attention could be related to the notion of *email acceptance* in Chen and Sudhir (2004).

Proof. If no email is sent, the attention trajectory over time is given by the solution of

$$\dot{x}(t) = L - \gamma x(t), \quad x(0) = x_0,$$

where x_0 denotes the initial attention level. The general solution to this first-order linear differential equation can be written as

$$x(t) = \frac{L}{\gamma} + C_1 e^{-t\gamma},$$

where C_1 denotes an integration constant. Using the initial condition, the integration constant is identified as $C_1 = \frac{x_0\gamma - L}{\gamma}$.

Finally, the time trajectory of the customer attention stock is

$$x(t) = \frac{L}{\gamma}\left(1 - e^{-t\gamma}\right) + x_0 e^{-t\gamma}.$$

Note that, in this case, when t goes to infinity, $x(t)$ converges to $\frac{L}{\gamma}$. ■

The above lemma shows that, even if no emails were sent, i.e., the capacity were not used, the stock is still bounded, thanks to the natural decay rate. Note that if the initial attention level, x_0, is lower than $\frac{L}{\gamma}$, then the attention stock converges to the upper bound $\frac{L}{\gamma}$ and $\forall t, x_0 \leq x(t) \leq \frac{L}{\gamma}$. However, if x_0 is greater than $\frac{L}{\gamma}$, then the attention stock converges to the lower bound $\frac{L}{\gamma}$ and $\forall t, \frac{L}{\gamma} \leq x(t) \leq x_0$. From now on, we focus on the first scenario[7] ($x_0 \leq x(t) \leq \frac{L}{\gamma} = x_{\max}$) and therefore, the natural growth of the attention is always positive on the interval $[0, \frac{L}{\gamma}]$, but the growth is decreasing with respect to x. Note that γ represents the speed of convergence to the (here upper) bound $\left(\frac{L}{\gamma}\right)$, i.e., the higher the value of γ, the faster the consumer "recovers" from past received messages.

The "production" of $n_i(t)$ emails by firm $i, i = 1, 2$, implies a cost, which is independent of the consumer's attention and denoted $C_i(n_i(t))$, and a revenue, assumed to depend on both the consumer's attention and the number of emails sent, $R_i\left(n_i(t), x(t)\right)$. We shall hereunder skip the time argument when no ambiguity may arise.

The (total) cost can be schematically decomposed into two components: the sending cost[8] and the preparation cost. The latter includes, e.g., the design, the message content, the targeting, the updating of the database, etc. For instance, the firm has to frequently change the design of its message to attract the consumer's attention. Also, given the

[7] A similar analysis could be done under the hypothesis $x_0 \geq \frac{L}{\gamma}$.

[8] Martin et al. (2003) evaluate the sending cost to be in the range of 5-7 $ per 1000 messages. Note that this cost is between \$500 to \$700 for traditional direct-marketing media.

frequency with which consumers change email addresses, the firm has to continuously invest in updating its database, a main asset for direct marketers. Further, the firm has to invest in competitive and technological intelligence to follow developments in new viruses, filters, etc. We believe that all these items can be captured by an increasing convex cost function that we take, for simplicity, to be quadratic:

$$C_i(n_i) = \frac{n_i^2}{2}. \tag{2}$$

Note that multiplying this cost by a positive constant, different from one, would not qualitatively change the results.

On the revenue side, we require the reward (i) to be zero if the attention's stock is (momentarily) exhausted, or if no mail is sent, (ii) to be increasing with the stock of attention and with the production (i.e., the number of emails sent), and (iii) to exhibit a positive interaction between the control n_i and the state x. This last item implies that, for a given attention level, the higher the number of emails sent, the higher is the revenue. Similarly, for a given number of emails sent, the higher the attention level, the higher the revenue. Although many functional forms could easily satisfy these requirements, we adopt for its simplicity and interpretability, the following multiplicatively separable function:

$$R_i\left(n_i, x\right) = \phi(x) g(n_i),$$

with

$$\phi(x) = \frac{x}{x_{\max}} = \frac{x\gamma}{L}, \quad g(n_i) = r_i n_i, \quad r_i > 0.$$

Clearly, $R_i\left(n_i, x\right)$ has the following properties:

$$\begin{aligned}
&R_i\left(n_i, 0\right) = R_i\left(0, x\right) = R_i\left(0, 0\right) = 0, \\
&\frac{\partial R_i}{\partial n_i}\left(n_i, x\right) \geq 0, \ \frac{\partial R_i}{\partial x}\left(n_i, x\right) \geq 0, \ \ \frac{\partial^2 R_i}{\partial n_i \partial x}\left(n_i, x\right) > 0.
\end{aligned}$$

Furthermore, note that $\phi(x)$ satisfies

$$0 \leq \phi(x) \leq 1, \quad \phi'(x) \geq 0, \quad \phi(0) = 0,$$

which confers to this function a propensity interpretation, which is appealing. Indeed, one expects the consumer to respond to an offer imbedded in an email with a certain "probability." Without rendering the model stochastic, and hence more complex, the idea of a "probabilistic" response is captured to some extent by $\phi(x)$. Further, given our assumption of strictly increasing convex costs, the linear specification of $g(n_i)$, instead of having a more classical concave one, is less severe.

Without any loss of generality, we shall normalize the maximum attention stock to be equal to one, hence taking $\gamma = L$. Assuming a profit-optimization behavior, the objective functional for player i then reads as follows:

$$\max_{n_i \geq 0} J_i = \int_0^\infty e^{-\rho t} \left\{ r_i n_i x - \frac{n_i^2}{2} \right\} dt, \tag{3}$$

where ρ denotes the constant discount rate.

Our objective is the study of the competition for consumer attention under different scenarios, depending on whether or not the firm is a spammer. As stated earlier, a spammer is assumed to behave as a myopic player, in the sense that she does not take into account the effect of her action on the dynamics of the attention stock. A spammer decides the optimal path of the number of emails to be sent in order to maximize her objective function (3). However, a no-spammer firm also cares about the long-term payoff and takes into account the dynamics of consumer attention, given by (1) when maximizing her objective function (3). We characterize the competitive equilibria for three different scenarios: first, neither player is a spammer; second, only one player behaves as a spammer; third, both players are spammers. The resulting equilibrium payoffs will form the entries in the matrix of the game in normal form, where each player chooses to be a spammer or not.

3 Equilibria

In the previous section, we defined by (1) and (3) an infinite-horizon differential game between two emailers. In the following propositions we characterize stationary feedback Nash equilibrium strategies for the different scenarios.[9]

PROPOSITION 1 *The symmetric stationary feedback Nash equilibrium emailing strategy is*

$$n^*(NS, NS; x) = rx - \alpha\,(A_1 x + A_2)\,, \quad \textit{if } x \geq \tfrac{A_2 \alpha}{r - A_1 \alpha}, \tag{4}$$

and zero otherwise, where NS, NS denotes that both player are non-spammers.

The symmetric firm's value function $V(NS, NS; x)$ is given by

$$V(NS, NS; x) = \frac{1}{2} A_1 x^2 + A_2 x + A_3, \tag{5}$$

where

$$A_1 \;=\; \frac{2\,L + 4\,r\,\alpha + \rho - \sqrt{(2\,L + 4\,r\,\alpha + \rho)^2 - 12\,r^2\,\alpha^2}}{6\,\alpha^2} > 0, \tag{6}$$

[9] The stationarity assumption is standard in infinite-horizon differential games.

$$A_2 \;=\; \frac{A_1\,L}{L + 2r\,\alpha - 3\,A_1\,\alpha^2 + \rho} > 0,\;\; A_3 = \frac{A_2\,\left(2\,L + 3\,A_2\,\alpha^2\right)}{2\,\rho} > 0. \qquad (7)$$

Proof. See Appendix A. ■

The optimal emailing strategy is a trade-off between the marginal reward and the marginal cost of sending an email. Indeed, the marginal revenue is given by rx. The marginal total loss is the sum of the marginal cost (n_i) and the loss in terms of the consumer's attention. The latter is given by the product of the marginal impact of sending an email on the evolution of the stock of attention, i.e., $\frac{\partial \dot{x}}{\partial n_i} = -\alpha$, valued at the shadow price of the stock, that is, $V'(x) = A_1 x + A_2$. An important, albeit obvious, observation for the sequel is that, in her strategy, a farsighted player takes into account both the direct (or immediate) impact and the indirect (or future/dynamic) effects of emailing. Actually, Mahajan and Venkatesh (2000) remark that accounting for both effects is a welcome move in e-business models.

Corollary 1 *The non-zero emailing strategy is strictly increasing with the attention stock.*

Proof. It is easy to prove that $r - \alpha A_1 > 0$ (see Appendix A), and hence the result. ■

An implication of this result is that the higher the attention, the higher the number of emails sent by the firm, i.e., $(n^*(NS, NS; x))' = r - \alpha A_1 > 0$.

The value of the attention at the steady state, $x_{ss}(NS, NS)$, is obtained after replacing the equilibrium strategies in (1), and solving for x when $\dot{x}(t) = 0$:

$$x_{ss}(NS, NS) = \frac{L + 2A_2\alpha^2}{L + 2\alpha r - 2A_1\alpha^2}. \qquad (8)$$

The steady state is always nonnegative and lower than one. Thus, excluding the uninteresting case where neither player sends any email, the consumer's attention is, as expected, not at its maximal value in the steady state. If we define information overload (IO) as the difference in the consumer's attention stock between its maximal value and the steady state, then

$$IO = 1 - x_{ss}(NS, NS) = \frac{2\alpha\,(r - \alpha\,(A_1 + A_2))}{L + 2\alpha r - 2A_1\alpha^2}.$$

Clearly, $IO \in [0, 1]$, and its actual level in steady state depends on the parameters' values. The next proposition provides some static comparative results.

Proposition 2

(i) Increasing L increases the steady state of the consumer's attention.

(ii) Increasing α, r or ρ decreases the steady state of the consumer's attention.

Proof. Replace in (8) the expressions of the parameters A_1 and A_2 given in (6) and (7). Take the partial derivatives with respect to the different parameters. After straightforward but tedious computations, the results are achieved. ■

The results are quite intuitive. Indeed, increasing L shifts the stock upward, all else being equal. When sending an email becomes more attractive, i.e., when the value of r is higher, the players increase their sending activities which in turn reduces the stock. The result of varying α can be explained as follows. Increasing the marginal damage cost α leads the players to decrease their sending activities but at a lower pace than the regeneration of the stock. Finally, the more impatient are the players (higher ρ), the more they use the resource and the lower is the steady-state value of the consumer's attention.[10]

Assume now that at least one of the two firms is a spammer. Recall that a spammer is a player who disregards the state dynamics. Therefore, the optimal spamming strategy is derived by solving, at each moment in time, a static optimization problem, i.e., maximizing the instantaneous profit. We shall suppose that player 1 is the non-spammer firm and player 2, the spammer. The latter optimization problem is thus

$$\max_{n_2 \geq 0} J_2 = \int_0^\infty e^{-\rho t}(rx - \frac{1}{2}n_2)n_2 \, dt. \tag{9}$$

The next proposition characterizes the non-spammer and spammer optimal strategies and value functions.

Proposition 3 *When player 1 is a-non spammer and player 2 is a spammer, the feedback Nash equilibrium strategies are given by*

$$n_1^*(NS, S; x) = rx - \alpha\left(B_1 x + B_2\right), \text{ if } x \geq \frac{B_2\alpha}{r - B_1\alpha} \geq 0, \tag{10}$$

and zero otherwise;

$$n_2^*(NS, S; x) = rx. \tag{11}$$

[10] The results in this proposition hold true for L, α and r in the other scenarios and will not be repeated. The result for ρ applies in the scenario with one spammer. In the case of two spammers, the steady state is independent of ρ.

The players' value functions are as follows:

$$V_1(NS, S; x) = \frac{1}{2}B_1 x^2 + B_2 x + B_3, \tag{12}$$

$$V_2(NS, S; x) = \frac{1}{2\rho}(rx)^2, \tag{13}$$

where arguments (NS, S) *denote that the first player is a non-spammer and the second player is a spammer, and*

$$\begin{aligned} B_1 &= \frac{2\,L + 4\,r\,\alpha + \rho - \sqrt{(2\,L + 4\,r\,\alpha + \rho)^2 - 4r^2\,\alpha^2}}{2\,\alpha^2} > 0, \\ B_2 &= \frac{B_1\,L}{L + 2r\,\alpha - B_1\,\alpha^2 + \rho} > 0, \;\; B_3 = \frac{B_2\,\left(2\,L + B_2\,\alpha^2\right)}{2\,\rho} > 0. \end{aligned}$$

Proof. See Appendix A. ■

Later, we shall compare the results (strategies and steady-state values) obtained under the different behavioral assumptions. For the moment, we observe that the spammer's strategy is independent of what the other player is doing (which is observable in reality). This is due to the fact that each firm's objective function depends only on her decision variable. Hence, by not seeing the interaction between the players' strategies in the state dynamics, the spammer is also ignoring the competitor when optimizing her payoff.

The steady-state level of the attention stock in this context is given by

$$x_{ss}(NS, S) = \frac{L + B_2\alpha^2}{L + 2\alpha r - B_1\alpha^2}. \tag{14}$$

It is easy to prove that $0 \leq x_{ss}(NS, S) \leq 1$.

The last scenario involves two myopic players. Given that a myopic player implements the strategy in (11) irrespective of what the other competitor is doing, the equilibrium strategies in this scenario are given by

$$n_i^*(S, S; x) = rx, \quad i = 1, 2. \tag{15}$$

The value function of any myopic player is

$$V_i(S, S; x) = \frac{1}{2\rho}(rx)^2, \quad i = 1, 2, \tag{16}$$

where arguments (S, S) denote that both players are spammers.

Inserting the two optimal spamming strategies given by (15) in (1), and solving for x when $\dot{x}(t) = 0$, we get the steady-state attention level

$$x_{ss}(S,S) = \frac{L}{L + 2r\alpha}. \tag{17}$$

Note that $0 \leq x_{ss}(S,S) \leq 1$.

3.1 Comparison

The next propositions compare the equilibrium emailing strategies and the consumer's attention levels, obtained under the different scenarios.

PROPOSITION 4 *The equilibrium emailing strategies compare as follows:*

$$n^*(NS,NS;x) \leq n_1^*(NS,S;x) \leq n_2^*(NS,S;x) = n^*(S,S;x), \quad \forall x \in [0,1],$$

where $n^*(NS,NS;x)$, $n_1^*(NS,S;x)$, $n_2^*(NS,S;x)$, $n^*(S,S;x)$ *are given by (4), (10) and (11), respectively.*

Proof. From (10) and (11), inequality $n_1^*(NS,S;x) \leq n_2^*(NS,S;x)$ $\forall x \in [0,1]$ follows immediately since $B_1, B_2 > 0$.

From (4) and (10), the following equivalence is deduced:

$$n^*(NS,NS;x) \leq n_1^*(NS,S;x) \;\Leftrightarrow\; (B_2 - A_2) + (B_1 - A_1)x \leq 0.$$

Some easy computations allow us to establish that $B_1 - A_1 < 0, B_2 - A_2 < 0$. Therefore, the above inequality is satisfied for any positive value of the attention stock. ■

This proposition shows that, independently of the level of the attention stock, the non-spammer firm sends always a fewer number of emails than the competing spammer firm: $(n_1^*(NS,S;x) \leq n_2^*(NS,S;x) = n^*(S,S;x))$. Moreover, a non-spammer firm sends more emails when it is competing against a spammer firm than it does when its competitor is also a non-spammer: $(n^*(NS,NS;x) \leq n_1^*(NS,S;x))$. This means that the presence of a spammer has a dual effect on the number of emails sent: one (tautological) direct effect, i.e., the spammer sends more emails than if she were not a spammer; and an indirect one, the non-spammer also sends more emails. Therefore, the good citizen is pushed into an escalation strategy. This points towards the result that the two emailing policies are strategic complements.[11] Therefore, if we extrapolate to the case where the spammer faces

[11]Strategic complementarity means that if one player increases the value of her strategic variable, the other player will do the same. Strategic substitutability works the other way around.

Table 1: Normal Form of the Game

(1;2)	NS	S
NS	$(V(NS,NS;x_0);\ V(NS,NS;x_0))$	$(V_1(NS,S;x_0);\ V_2(NS,S;x_0))$
S	$(V_2(NS,S;x_0);\ V_1(NS,S;x_0))$	$(V_2(NS,S;x_0);\ V_2(NS,S;x_0))$

a lower cost than does the non-spammer, then we can conjecture that everything would be worsened, i.e., even more emails sent and a lower steady-state value for the consumer's attention. The following proposition is somehow a direct consequence of the previous one.

PROPOSITION 5 *The steady-state attention stocks satisfy the following inequalities:*

$$1 \geq x_{ss}(NS,NS) \geq x_{ss}(NS,S) \geq x_{ss}(S,S) \geq 0.$$

Proof. It suffices to compare the expressions of the steady states for the different scenarios, given by (8), (14) and (17), taking into account the Ricatti equations that define the coefficients of the value functions. ■

4 To Spam or Not to Spam: A Strategic Choice

We suppose now that the two players have the opportunity to choose their type (spammer or non-spammer) and that this choice is based only on a comparison of the payoffs. The latter are given by the value functions evaluated at the initial attention stock x_0, under the different scenarios (SC), i.e., $V_i(SC,SC;x_0), i = 1,2, SC \in \{NS,S\}$. Table 1 shows the normal form of the game, where the decisions of player 1 are displayed in rows, and those of player 2, in columns.

More specifically, the quantities in this matrix are as follows:

- $V(NS,NS;x_0)$ is the value function of a non-spammer firm competing against another non-spammer firm, given in (5).
- $V_1(NS,S;x_0)$ is the value function of a non-spammer firm competing against a spammer, given in (12).
- $V_2(NS,S;x_0)$ is the value function of a spammer firm competing against a non-spammer, given in (13)
- Finally, since the value function of a spammer firm competing against another spammer is the same as that of a spammer competing against a non-spammer, the payoffs in cell $(2,2)$ are both equal to $V_2(NS,S;x_0)$.

To analyze this game, we have to compare two second-order polynomials, i.e., $P_1(x_0) = V(NS, NS; x_0) - V_2(NS, S; x_0)$ and $P_2(x_0) = V_1(NS, S; x_0) - V_2(NS, S; x_0)$. By the definition of a Nash equilibrium, we have

- (NS, NS) is a Nash equilibrium if and only if $P_1(x_0) \geq 0$.
- (S, S) is a Nash equilibrium if and only if $P_2(x_0) \leq 0$.
- (NS, S) and (S, NS) are Nash equilibria if and only if $P_1(x_0) \leq 0$ and $P_2(x_0) \geq 0$.

The expressions of the polynomials $P_1(x_0)$ and $P_2(x_0)$ involve the coefficients of the value functions and are very complicated to allow for clear-cut results regarding their signs. However, we are still able to prove the following proposition.

Proposition 6

1. *(NS, NS) is the unique Nash equilibrium if and only if $x_0 < \tilde{x}_0$.*
2. *(S, S) is the unique Nash equilibrium if and only if $x_0 > \hat{x}_0$.*
3. *(NS, NS) and (S, S) are Nash equilibria if and only if $x_0 \in [\tilde{x}_0, \hat{x}_0]$.*

The expressions of $\hat{x}_0$ and $\tilde{x}_0$ are given in Appendix B.

Proof. See Appendix B. ■

The proposition characterizes Nash equilibria in terms of the initial value of the attention stock, x_0. The pair of strategies (NS, NS) is the only Nash equilibrium if the initial attention stock is "low." Conversely, if the initial attention stock is sufficiently high, then the pair (S, S) is the unique Nash equilibrium of the game. If the initial attention stock is of "intermediate" value, then the pairs (NS, NS) and (S, S) are Nash equilibria. The somewhat surprising result is that, regardless of the parameters' values, are, the case where the two firms are of different types can never be an equilibrium. Clearly, this is not what we observe in reality. This apparent contradiction can be explained as a modelling issue, i.e., the symmetry assumption is not valid, or it reflects the fact that the direct-marketing industry has not yet reached an equilibrium with only one surviving type. As mentioned earlier, one way of leaving out symmetry would be to assume that the spammer firm faces an almost zero cost and a very low rate of return. The most likely impact is that the spammer will send even more emails, without however, having any impact on the characterization of the equilibrium.

5 Concluding Remarks

We analyzed a dynamic game between emailers who seek to capture consumer attention. Although the expressions of the strategies and outcomes are tedious, we are still able to obtain some qualitative insight from the results. Namely, we showed that the strategies used by a spammer and a non-spammer are very different, and that the steady-state attention stock decreases with the number of spammers. This suggests that if entry into the direct-marketing industry remains wide open, with almost no operations costs, then we can expect a long-term deterioration in the attention level. The corollary is a low response rate to offers made by firms via the Internet. This is one more reason to find a solution to the spam phenomenon. Finally, an important conclusion is that the scenario involving different types of firms is not part of a Nash equilibrium.

At the modelling level, one contribution of this paper is its clear definition of a spammer. This goes further than the previous characterizations of spammers based on email content. Unlike the economic literature dealing with information overload, our approach allows to explicitly assess the impact of spammers on consumer attention. Further, our model endogenizes the strategic choice regarding type made by a firm.

Our model suffers from several limitations. First, the response functions are linear in the state variable mainly to preserve mathematical tractability. Considering non-linear response functions, at the cost however of having to fully rely on numerical methods to obtain certain results, may lead to different insights. Second, we have assumed that the only players are the firms sending emails. One could introduce a third party that regulates business conduct and examine its impact on the emailing strategies. Third, we have assumed that only the number of messages matters, without considering content or quality, which could be used by non-spammers to differentiate themselves from spammers. Finally, the number of players is fixed. It would be interesting to extend the model by considering this number as endogenously determined by the gains and losses made by each category of player, using an evolutionary game theory approach. The model we proposed is a first step towards understanding the competition for attention in an information-rich environment such as the Internet. It is also a first attempt to assess the impacts of the spam phenomenon from a dynamic perspective. The above improvements are only some of the many interesting research questions still open for investigation.

Appendix A

A.1 Proof of Proposition 1

The sufficient condition for a stationary feedback Nash equilibrium requires us to find bounded and continuously differentiable functions denoted by $V_i(NS, NS; x)$, $i = 1, 2$, which satisfy, for all $x(t) \geq 0$, the Hamilton-Jacobi-Bellman (HJB) equations for player $i = 1, 2$. We first concentrate on finding solutions of the HJB equations. This equation for player i is given by

$$\rho V_i(NS,NS;x) = \max_{n_i\geq 0}\left\{(rx-\frac{1}{2}n_i)n_i + (V_i(NS,NS;x))'\left(L(1-x)-\alpha n_1-\alpha n_2\right)\right\}, \quad (18)$$

where $V_i(NS,NS;x)$ denotes player i' s value function in the scenario where neither player is a spammer.

The first-order optimality condition reads

$$n_i^*(NS,NS;x) = rx - \alpha\left(V_i(NS,NS;x)\right)'.$$

We are assuming that both players are symmetric; therefore, we omit the subscript denoting each player. The symmetric emailing strategy is

$$n^*(NS,NS;x) = rx - \alpha\left(A_1x + A_2\right), \quad \text{if } x \geq \tfrac{A_2\alpha}{r-A_1\alpha}, \quad (19)$$

and zero otherwise.

Inserting (19) in (18), and assuming that the value function is quadratic due to the linear-quadratic structure of the model, and given by

$$V(NS,NS;x) = \frac{1}{2}A_1x^2 + A_2x + A_3, \quad (20)$$

the coefficients A_1, A_2, A_3 are determined by identification, as follows:

$$A_1 = \frac{2\,L+4\,r\,\alpha+\rho \pm \sqrt{\left(2\,L+4\,r\,\alpha+\rho\right)^2-12\,r^2\,\alpha^2}}{6\,\alpha^2} > 0, \quad (21)$$

$$A_2 = \frac{A_1\,L}{L+2r\,\alpha-3\,A_1\,\alpha^2+\rho} > 0,$$

$$A_3 = \frac{A_2\left(2\,L+3\,A_2\,\alpha^2\right)}{2\,\rho} > 0.$$

The value inside the square root in the expression of A_1 is always positive and it is easy to prove that both roots in (21) are positive real numbers. Let $A_1^{'}$ be the root with the positive sign affecting the square root, and $A_1^{''}$ the root with the negative sign.

A sufficient condition guaranteeing that the expressions in (20) and (19) are firms' value functions and emailing strategies is given by

$$\lim_{t\to\infty} e^{-\rho t}V(NS,NS;x(NS,NS;t)) = 0, \quad (22)$$

where $x(NS, NS; t)$ is the solution of the closed-loop dynamics obtained after substitution of the optimal emailing strategies (19) into the attention stock dynamics given by (1). This solution can be written as

$$x(NS, NS; t) = (x_0 - x_{ss}(NS, NS))e^{\lambda_1 t} + x_{ss}(NS, NS), \tag{23}$$

where $x_{ss}(Ns, NS)$ refers to the steady state of the attention variable given by (8), and λ_1 is

$$\lambda_1 = -L - 2r\alpha + 2\alpha^2 A_1.$$

The quadratic functional specification in (20) allows condition (22) to be satisfied when the attention stock is bounded. This condition is guaranteed if the steady state is globally asymptotically stable.

The attention dynamics, once the optimal strategy (19) has been replaced, is

$$\dot{x}(t) = L(1 - x) - 2\alpha \left(rx - \alpha \left(A_1 x + A_2\right)\right).$$

Collecting with respect to x, we get

$$\dot{x}(t) = L + 2A_2\alpha^2 + x\left(-L - 2r\alpha + 2\alpha^2 A_1\right).$$

The steady state, $x_{ss}(Ns, NS)$, is globally asymptotically stable if and only if

$$\lambda_1 = -L - 2r\alpha + 2\alpha^2 A_1 < 0.$$

After some manipulations, it can be proved that if A_1' is chosen, the value of λ_1 is always positive, leading to an unbounded attention stock. However, if A_1'' is selected, we have

$$\lambda_1 = \rho - L - 2r\alpha - 2\sqrt{\left(\frac{\rho}{2} + L + 2r\alpha\right)^2 - 3\left(r\alpha\right)^2},$$

which can easily be proved negative. This choice leads to a globally stable steady-state, implying that, for any initial value of the attention stock, x_0, the optimal time path of the attention stock $x(t)$ converges to the steady state $x_{ss}(Ns, NS)$.

From (19) the optimal symmetric strategy is positive if and only if

$$rx - \alpha \left(A_1 x + A_2\right) \geq 0.$$

Collecting this expression with respect to x, we get

$$-A_2\alpha + x\left(r - A_1\alpha\right).$$

Since $A_2\alpha \geq 0$ and some straightforward manipulations show that $\frac{\gamma}{L} - A_1''\alpha > 0$, therefore the optimal strategy is nonnegative as long as

$$x \geq \frac{A_2\alpha}{r - A_1''\alpha}.$$

A.2 Proof of Proposition 3

Here we follow the same steps as in the proof of Proposition 1 to derive the equilibrium strategies under the assumption that firm 2 behaves as a spammer, while firm 1 is a non-spammer.

The HJB equation for player 1 is

$$\begin{aligned}\rho V_1(NS,S;x) &= \max_{n_1\geq 0}\Big\{(rx-\frac{1}{2}n_1)n_1 \\ &+ (V_1(NS,S;x))'\left(L(1-x)-\alpha n_1-\alpha n_2\right)\Big\}. \end{aligned} \tag{24}$$

The first-order optimality condition for player 1 is

$$n_1^*(NS,S;x) = rx-\alpha(V_1(NS,S;x))'. \tag{25}$$

Inserting (25) and (11) into (24), and assuming a quadratic value function such as

$$V_1(NS,S;x) = \frac{1}{2}B_1x^2+B_2x+B_3, \tag{26}$$

we identify the following coefficients:

$$\begin{aligned} B_1 &= \frac{2\,L+4\,r\,\alpha+\rho\pm\sqrt{(2\,L+4\,r\,\alpha+\rho)^2-4r^2\,\alpha^2}}{2\,\alpha^2} > 0, \\ B_2 &= \frac{B_1\,L}{L+2r\,\alpha-B_1\,\alpha^2+\rho} > 0, \\ B_3 &= \frac{B_2\,\left(2\,L+B_2\,\alpha^2\right)}{2\,\rho} > 0. \end{aligned} \tag{27}$$

The value inside the square root in the expression of B_1 is always positive and it is easy to prove that both roots in (27) are positive real numbers. Let $B_1^{'}$ be the root with the positive sign affecting the square root, and $B_1^{''}$ the root with the negative sign.

Along the same lines as in the proof of Proposition 1, we look for a globally asymptotically stable steady state, which implies bounded attention stock along its optimal time path given by

$$x(NS,S;t) = (x_0-x_{ss}(NS,S))e^{\lambda_2 t}+x_{ss}(NS,S), \tag{28}$$

where $x_{ss}(NS,S)$ refers to the steady state of the attention variable given by (14), and λ_2 is

$$\lambda_2 = -L+B_1\alpha^2-2r\alpha.$$

The steady state, $x_{ss}(NS, S)$, is globally asymptotically stable if and only if $\lambda_2 < 0$. It can be easily proved that this last condition can only be ensured if coefficient B_1'' is selected.

The non-spammer's optimal emailing strategy is positive if and only if

$$rx - \alpha(B_1 x + B_2) \geq 0.$$

Collecting the terms in x

$$x(-B_1\alpha + r) - B_2\alpha \geq 0.$$

Note that $B_2 \geq 0$ and after some manipulations, it can be proved that

$$-\alpha B_1'' + r \geq 0.$$

Therefore, $n_1^*(NS, S; x)$ is positive if and only if

$$x \geq \frac{B_2\alpha}{r - B_1''\alpha}.$$

The optimal spamming strategy is derived straightforwardly from the first-order optimality condition. To get the expression of the spammer's value function it suffices to replace the optimal spamming strategy given by (11) in the Hamilton-Jacobi-Bellman equation associated with the spammer's optimization problem.

Appendix B

B.1 Proof of Proposition 6

The polynomials $P_1(x_0)$ and $P_2(x_0)$ are given by

$$P_1(x_0) = \frac{1}{2}(A_1 - C_1)x_0^2 + A_2 x_0 + A_3, \quad P_2(x_0) = \frac{1}{2}(B_1 - C_1)x_0^2 + B_2 x_0 + B_3,$$

where

$$C_1 = \frac{r^2}{\rho} > 0.$$

Some tedious but easy computations allow us to establish that

$$A_1 - C_1 < 0, \quad C_1 - B_1 > 0.$$

Therefore, from the study of the second-order polynomials $P_1(x_0)$ and $P_2(x_0)$, the following results are derived:

$$P_1(x_0) \geq 0 \;\Leftrightarrow\; x_0 \leq \hat{x}_0 \tag{29}$$

where

$$\hat{x}_0 = \frac{-A_2 - \sqrt{A_2^2 - 2(A_1 - C_1)A_3}}{A_1 - C_1} > 0;$$

$$P_2(x_0) \leq 0 \;\Leftrightarrow\; x_0 > \tilde{x}_0, \tag{30}$$

where

$$\tilde{x}_0 = \frac{B_2 + \sqrt{B_2^2 + 2(C_1 - B_1)B_3}}{C_1 - B_1} > 0.$$

It can be proved that the coefficients of the value functions compare as follows

$$A_1 - B_1 > 0, \quad A_2 - B_2 > 0, \quad A_3 - B_3 > 0.$$

These inequalities allow us to establish that $\hat{x}_0 > \tilde{x}_0$.

From the above conditions, together with (29) and (30) and the definition of a Nash equilibrium, the different results in Proposition 6 follow.

References

[1] Anderson, S., A. de Palma. 2005. A Theory of Information Overload. Mimeo.

[2] Bitran, G., S. Mondschein. 1996. Mailing decisions in the catalog sales industry. *Management Science* **42**(9) 1364–381.

[3] Bult, J.R., T. Wansbeek. 1995. Optimal selection for direct marketing. *Marketing Science* **14**(4) 378–394.

[4] Chen, Y,, K. Sudhir. 2004. When shopbots meet Emails: Implications for price competition on the Internet. *Quantitative Marketing and Economics* **2**(3) 233–255.

[5] Gönül, F., M. Ze Shi. 1998. Optimal mailing of catalogs: A new methodology using estimable structural dynamic programming models. *Management Science* **44**(9) 1249–1262.

[6] Industry Canada, http://e-com.ic.gc.ca/epic/internet/inecic-ceac.nsf/en/h_gv00170e.html, consulted in April 2005.

[7] Jacoby, J., D. E. Speller, C. K. Berning. 1974. Brand choice behavior as a function of information load-replication and extension. *Journal of Consumer Research* **1** 33–41.

[8] Keller, K. L., R. Staelin. 1987. Effects of quality and quantity of information on decision effectiveness. *Journal of Consumer Research* **14** 200–213.

[9] Lee, B.K., W.N. Lee. 2004. The effect of information overload on consumer choice quality in an on-line environment. *Psychology and Marketing* **21**(3) 159–162.

[10] Lurie, N. 2004. Decision making in information-rich environments: the role of information structure. *Journal of Consumer Research* **30**(4) 473–486.

[11] MacFadden, D. 2001. The tragedy of the commons. *Forbes ASAP* **168** 61–62.

[12] Mahajan, V., Venkatesh, R. 2000. Marketing modeling of e-business. *International Journal of Research in Marketing* **17** 215–225.

[13] Malhorta, N. K. 1982. Information load and consumer decision making. *Journal of Consumer Research* **8** 419–430.

[14] Martin, B., J. Van Durme, M. Raulas, M. Merisavo. 2003. Email advertising: Exploratory insights from Finland. *Journal of Advertising Research* **43**(3) 293–306.

[15] Piersma, N., J. Jonker. 2004. Determining the optimal direct mailing frequency. *European Journal of Operational Research* **158** 173–182.

[16] Shiman, D. 1996. When Emails becomes junk mail: the welfare implications of the advancement of communications technology. *Review of Industrial Organization* **11**(1) 35–48.

[17] Shiman, D. 1997. The impact of firms' increased information about consumers on the volume and targeting of direct marketing. Available at SSRN: http://ssrn.com/abstract=555646.

[18] Simon, H. 1997. *Designing Organizations for an Information-Rich World: Models of Bounded Rrationality*, MIT Press.

[19] Sipior, J., B. Ward, G. Bonner. 2004. Should spam be on the menu? *Communications of the ACM* **47**(6) 59–63.

[20] Van Zandt, T. 2004. Information overload in a network of targeted communication. *Rand Journal of Economics* **35**(3) 542–560.