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Overlapping-Generations
Environmental Game**

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Abstract

We consider two overlapping generations that want to coordinate their strategies of working, consuming and controlling pollution. As the cooperative solution is not an equilibrium, and hence is not a self-enforcing contract, a mechanism is required to sustain it. We show how incentive strategies, and the resulting incentive equilibrium, could provide such a mechanism. We also derive the conditions that ensure the credibility of these strategies.

Key Words: Overlapping-Generations Models; Environment; Incentive Strategies; Credibility.

Résumé

Dans cet article, nous considérons un jeu entre deux générations qui cherchent à coordonner leurs stratégies de travail, de consommation et de contrôle de la pollution. Nous déterminons l'équilibre de Nash ainsi que la solution coopérative et proposons une manière de répartir le gain issu de la collaboration. Nous montrons que la solution coopérative est accessible à l'aide d'un équilibre incitatif crédible.

Mots clés :   générations imbriquées, environnement, stratégies incitatives, crédibilité.

1   Introduction

Environmental economics has been an active research area in the last couple of decades, and has considered a myriad of topics, such as, taxation, strategic interactions, international treaties and their stability, assessment of pollution cost, R&D and emissions reduction, migration, etc.[1] The game-theoretic subset of this literature can be characterized along methodological lines. One possible classification is to distinguish between static and dynamic (to allow for pollution accumulation) games, between cooperative and non-cooperative games and between games involving (implicitly) one generation of players and those (explicitly) assuming an overlapping-generations (OLG) framework. The arguments put forward to justify the latter approach, to which this paper belongs, include, e.g., that individuals take actions with consequences that outlive them, that intergenerational conflicts exist (on top of intra-generational ones), that time and generational dynamics are not equivalent, that the life spans of an economic agent and of the economy are different, etc.

The literature considering overlapping-generations (OLG) models with an environmental concern has focused on aspects such as sustainability, intergenerational distribution and assessment of environmental policies. John and Pecchenino (1994) use a discrete-time model to derive conditions for sustainable growth of both capital and environmental quality. Their model is extended in John et al. (1995), Ono (1996, 2003) and Ono and Maeda (2001, 2002) with a focus on determining and assessing environmental tax policies. Using a continuous-time model, Marini and Scaramozzino (1995) characterize the trade-off between the environment and capital accumulation. Bovenberg and Heijdra (1998) explore the impact of environmental taxation on the welfare of the different generations. In Bovenberg and Heijdra (2002), the authors analyze the effect of public abatement on private investment and the intergenerational distribution of welfare. Lines (2005), on the other hand, determines the conditions under which a sustainable intertemporal equilibrium with a pollution externality exists. Howarth (1998) uses a numerically calibrated OLG model of climate change and the world economy to examine efficient rates of greenhouse-gas-emissions abatement. Kavuncu and Knabb (2005) analyze the costs and benefits current and future generations incur as the result of climate change or of an environmental control policy. Jouvet et al. (2000) focus on the effect of the intergenerational degree of altruism on competitive steady-state consumption. Note that it is often the case in these papers that the analysis is restricted to the steady states. Further, they do not generally allow the old generation to take part in the investment for the maintenance of the environment.

Few papers adopt the formalism of differential games to study the strategic interactions between a sequence of OLG of agents. For instance, Haurie (2005) considers a multigenerational game model where the generations are altruistic and proposes criteria to perform a cost-benefit analysis for very long-term projects. Jørgensen and Yeung (1999) develop a differential game of inter- and intra-generational resource extraction. In Jørgensen and

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[1]Surveys are available in, e.g., Mäler and Vincent (2003, 2005, 2006).

Yeung (2005), the authors extend the model to a stochastic setting by introducing random disturbances in the stock dynamics and by considering an infinite number of OLG of players.

This paper explores the issue of implementing and sustaining an optimal cooperative solution in an overlapping-generations game. The setting is a discrete-time model where players maximize their welfare by choosing their labor supply, their consumption and their voluntary contribution to the maintenance of the environment. The model is in the spirit of those in Diamond (1965) and Grandmont (1985), with the additional feature that it considers pollution dynamics.

Our starting point is that the two generations wish to coordinate their strategies for working, for consumption and for investment in the quality of the environment in order to achieve a, possibly, higher collective welfare than the one that would result from acting non-cooperatively. It is well known that the cooperative solution is not, in general, an equilibrium and hence is not a self-enforcing contract. This means that a player may be tempted to cheat on the agreement, i.e., deviating unilaterally, whenever it is profitable for her to do so. Given this possibility, each generation would like to be reassured that the other will actually stick to her part of the cooperative agreement for as long as she, herself, is doing so. The objective of this paper is to show that this can be achieved by using incentive strategies and the resulting incentive equilibrium.

The idea of an incentive equilibrium has been developed and applied to resource problems by Ehtamo and Hämäläinen (1986, 1989, 1993). Recently this approach has been used by Jørgensen and Zaccour (2001) and by Martín-Herrán and Zaccour (2005) in differential games of pollution control. An incentive equilibrium has the property that when one player implements her (coordinated) strategy, the other player can do no better than to act in accordance with the agreement. One important question that must be addressed is whether a player should believe that the other will actually implement her strategy. Put differently, are incentive strategies credible? Informally speaking, credibility amounts to say that if, a player detects that the other player is deviating from the agreed-upon strategy, then it will indeed be in her (the cheated player) best interest to implement the incentive strategy and not to continue to cooperate.

Our approach can be summarized by the following algorithm:

1. Compute noncooperative (Nash equilibrium) and cooperative solutions. The cooperative solution is the collectively optimal one and the noncooperative equilibrium is the benchmark. The difference in total payoffs provides a measure of the dividend of cooperation to be shared by the two generations;

2. Share the cooperative outcome between the two generations. This can be done by using, e.g., the egalitarian principle, which leaves both generations at an equal distance from their noncooperative outcomes;

3. Characterize the incentive strategies and the resulting equilibrium and determine the conditions under which these strategies are credible.

To illustrate this approach, we shall consider a parsimonious model that nevertheless has the main characteristics of an OLG model and the environmental externality.

The rest of the paper is organized as follows. In Section 2, we present an OLG model with pollution externality. In Section 3, we compute the cooperative and noncooperative solutions. In Section 4, we construct an incentive equilibrium and establish conditions for the credibility of the incentive strategies. In Section 5, we briefly conclude.

2   The Model

Consider a two-period OLG model in discrete time. At each period, a generation of selfish consumers appears and lives for two periods, young and old. For simplicity, we assume that the population is constant and that each generation consists of a single representative individual. During her active period, the young generation works l_t and receives w_t as a wage. The total income, $w_t l_t^y$, is shared between consumption c_t and savings s_t. The income of the old generation corresponds to the yield of savings, which is given by Rs_t,where R is the interest factor. This total income is devoted to consumption c_{t+1} and to environmental maintenance m_{t+1}. This investment by the old generation in the quality of the environment can be justified essentially on health grounds. For instance, long-term exposure to air pollution can provoke respiratory problems or bronchitis (Xu et al. (1998), Williams (2002)).

We assume that pollution is a by-product of working activities and given by ϕl_t, with $\phi > 0$. Denote by P_t the stock of pollution at period t. It evolves according to the following simple dynamics:

$$P_{t+1} = (1 - \delta)P_t + \phi l_{t+1} - \rho m_{t+1},$$

where δ is a constant decay rate of pollution, and $\rho > 0$ is an efficiency parameter of the technology used for environmental maintenance. The term ρm_{t+1} measures the environmental improvement due to the old-generation contribution.

As is standard in this class of models, the representative agent derives a disutility from work $v(l_t)$ in the first period and a utility from consumption in the two periods given by $u(c_t)$ and $u(c_{t+1})$. We make the following:

Assumption 1
 (i) u and v are twice continuously differentiable on R^+;
 (ii) u and v have the following characteristics: $u'(c) > 0$, $u''(c) < 0$, $v'(l) > 0$, $v''(l) > 0$;
 (iii) $\lim_{l \to \infty} v'(l) = +\infty$, $\lim_{l \to 0} v'(l) = 0$ and $\lim_{c \to 0} u'(0) = +\infty$, i.e., zero consumption is impossible.

The young individual optimizes the discounted sum of welfare over the two periods, subject to the budget constraint in each period, and the evolution of the stock of pollution.

This optimization problem is given by

$$
\begin{aligned}
\max W^y &= u(c_t^y) - v(l_t^y) + \beta\left[u(c_{t+1}^y) - D(P_{t+1})\right], \\
\text{subject to} \quad &: \\
c_t^y + s_t^y &= w_t l_t^y, \\
c_{t+1}^y + m_{t+1}^y &= R s_t^y, \\
P_{t+1} &= (1-\delta)P_t + \phi l_{t+1} - \rho m_{t+1}^y,
\end{aligned}
$$

where β is the discount factor. After substitution for c_t^y, c_{t+1}^y and P_{t+1}, the above problem becomes

$$
\begin{aligned}
\max W^y = \; & u(w_t l_t^y - s_t^y) - v(l_t^y) + \beta\left\{u(R s_t^y - m_{t+1}^y) \right. \\
& \left. - b\left[(1-\delta)\left((1-\delta)P_{t-1} + \phi l_t^y - \rho m_t^o\right) + \phi l_{t+1} - \rho m_{t+1}^y\right]\right\}.
\end{aligned} \tag{P^y}
$$

The old-individual optimization problem consists in maximizing the second-period welfare subject to the budget constraint and the pollution stock, i.e.,

$$
\begin{aligned}
\max W^o &= u(c_t^o) - D(P_t), \\
\text{subject to} \quad &: \\
c_t^o + m_t^o &= R s_{t-1}^o, \\
P_t &= (1-\delta)P_{t-1} + \phi l_t^y - \rho m_t^o.
\end{aligned}
$$

Substituting for c_t^o and P, the above problem can be rewritten as follows:

$$
\max W^o = u\left(R s_{t-1}^o - m_t^o\right) - b\left[(1-\delta)P_{t-1} + \phi l_t^y - \rho m_t^o\right]. \tag{P^o}
$$

3   Nash Equilibrium and Cooperative Solution

In this section, we characterize both noncooperative and cooperative solutions to the game. In the latter, the assumption is that the two generations would agree to jointly optimize their welfare. In the noncooperative case, we shall look for Nash equilibria, which will be used as a benchmark to allocate the cooperation benefit. At least part of the analysis could be done without specifying the forms of the different functions involved in the model. However, to focus on the implementation of our methodological approach, and thus to show how incentive strategies can be used within the framework of overlapping generations to reach the cooperative solution, we shall adopt the following, very simple functional forms for utilities and costs:

$$
u(c_t) = \log(c_t), v(l_t) = \frac{l_t^2}{2}, D(P_t) = b S_t, \forall t, b > 0.
$$

Clearly, the above functional forms satisfy the assumptions made earlier.

The following proposition characterizes the unique Nash equilibrium identified by a $\sim$.

Proposition 1 *The unique Nash equilibrium is given by*

$$\tilde{m}^o_t = Rs^o_{t-1} - \frac{1}{b\rho}, \tilde{c}^o_t = \frac{1}{b\rho}, \tilde{c}^y_t = \frac{1}{\beta b\rho R},$$

$$\tilde{l}^y_t = X, \quad \tilde{s}^y_t = w_t X - \frac{1}{\beta b\rho R}, \quad \tilde{m}^y_{t+1} = Rw_t X - \left(\frac{1+\beta}{\beta b\rho}\right),$$

where

$$X = \beta b \left(w_t \rho R - \phi(1 - \delta)\right).$$

The equilibrium welfares are given by

$$\tilde{W}^o = \log(\frac{1}{b\rho}) - b\left[(1-\delta)P_{t-1} + \phi X - \rho\left(Rs^o_{t-1} - \frac{1}{b\rho}\right)\right]$$

$$\tilde{W}^y = \log(\frac{1}{\beta b\rho R}) - \frac{X^2}{2} + \beta\left\{\log\frac{1}{b\rho}\right.$$

$$-b(1-\delta)\left((1-\delta)P_{t-1} + \phi X - \rho\left(Rs^o_{t-1} - \frac{1}{b\rho}\right)\right)$$

$$\left. -b\phi l_{t+1} + b\rho\left(w_t RX - \left(\frac{1+\beta}{\beta b\rho}\right)\right)\right\}.$$

Proof. We start by solving the old-individual problem (P^o). Assuming an interior solution, the optimality condition is

$$\frac{\partial W^o}{\partial m^o_t} = \frac{-1}{Rs^o_{t-1} - m^o_t} + b\rho = 0 \Longleftrightarrow \tilde{m}^o_t = Rs^o_{t-1} - \frac{1}{b\rho}$$

$$\Rightarrow \tilde{c}^o_t = \frac{1}{b\rho}.$$

Substituting for $\tilde{m}^o_t$ and $\tilde{c}^o_t$ in W^o provides the equilibrium payoff in the Proposition.

Assuming an interior solution, first-order optimality conditions for the young-individual optimization problem are

$$\frac{\partial W^y}{\partial s^y_t} = \frac{-1}{w_t l^y_t - s^y_t} + \frac{\beta R}{Rs^y_t - m^y_{t+1}} = 0 \Longleftrightarrow c^y_t = \frac{c^y_{t+1}}{\beta R},$$

$$\frac{\partial W^y}{\partial l^y_t} = \frac{w_t}{w_t l^y_t - s^y_t} - l^y_t - \beta b\phi(1 - \delta) = 0 \Longleftrightarrow l^y_t = \frac{w_t}{c^y_t} - \beta b\phi(1 - \delta),$$

$$\frac{\partial W^y}{\partial m^y_{t+1}} = \frac{-\beta}{Rs^y_t - m^y_{t+1}} + \beta b\rho = 0 \Longleftrightarrow c^y_{t+1} = \frac{1}{b\rho}.$$

Using

$$c^y_t = \frac{c^y_{t+1}}{\beta R},$$

leads to

$$\tilde{c}^y_t = \frac{1}{\beta b \rho R} \quad \text{and} \quad \tilde{l}^y_t = \beta b \left(w_t \rho R - \phi(1-\delta) \right).$$

In addition, as we have

$$w_t l^y_t - s^y_t = \frac{1}{\beta b \rho R},$$

we get

$$\tilde{s}^y_t = w_t b \beta \left(w_t \rho R - \phi(1-\delta) \right) - \frac{1}{\beta b \rho R}.$$

Finally, the equality

$$c^y_{t+1} = R s^y_t - m^y_{t+1} = \frac{1}{b\rho},$$

gives the value of environmental maintenance, m^y_{t+1} :

$$\tilde{m}^y_{t+1} = R\tilde{s}^y_t - \frac{1}{b\rho} = R\beta w_t b \left(w_t \rho R - \phi(1-\delta) \right) - \left(\frac{1+\beta}{\beta b \rho} \right).$$

By straightforward substitution, we obtain the payoffs in the Proposition.   □

The contribution by the old generation to the maintenance of the environment is given by the difference between the yield of savings when young and the consumption. It is increasing in all the parameters, i.e.,

$$\frac{\partial \tilde{m}^o_t}{\partial s^o_{t-1}} = R > 0, \quad \frac{\partial \tilde{m}^o_t}{\partial R} = s^o_{t-1} > 0, \quad \frac{\partial \tilde{m}^o_t}{\partial b} = \frac{1}{b^2 \rho} > 0, \quad \frac{\partial \tilde{m}^o_t}{\partial \rho} = \frac{1}{b \rho^2} > 0.$$

This shows that a richer old generation will invest more in the environment. Also, the higher the marginal damage cost, and the efficiency of the emissions reduction technology, the higher the investment in the quality of the environment. The same observation can be made for the young generation. Indeed,

$$\begin{aligned}
\frac{\partial \tilde{m}^y_{t+1}}{\partial R} &= w_t X + R w_t \beta b w_t \rho > 0, \quad \frac{\partial \tilde{m}^y_{t+1}}{\partial w_t} = R\left(X + w_t \beta b \rho R \right) > 0, \\
\frac{\partial \tilde{m}^y_{t+1}}{\partial \beta} &= \frac{R w_t X}{\beta} + \frac{1}{\beta^2 b \rho} > 0, \quad \frac{\partial \tilde{m}^y_{t+1}}{\partial b} = \frac{R w_t X}{b} + \frac{1+\beta}{\beta \rho b^2} > 0, \\
\frac{\partial \tilde{m}^y_{t+1}}{\partial \rho} &= R^2 w_t^2 \beta b + \frac{1+\beta}{\beta b \rho^2} > 0.
\end{aligned}$$

Regarding the consumption policy, we observe that

$$c^y_t = \frac{c^y_{t+1}}{\beta R}, \quad \tilde{c}^o_t = c^y_{t+1} = \frac{1}{b\rho}.$$

The first part is the traditional result that the marginal rate of substitution between current and future consumption and the marginal rate of transformation must be equal. The second part states that the consumption policy is time-consistent.

The next propositions provide the cooperative solution and a comparison with the noncooperative equilibrium.

Proposition 2 *Assuming an interior solution, if the two generations cooperate by maximizing their joint payoffs, then the optimal solution is given by*

$$\hat{m}_t^o = Y, \quad \hat{c}_t^o = \frac{1}{b\rho\left(1 + \beta(1-\delta)\right)}, \quad \hat{c}_t^y = \frac{1}{\beta b\rho R},$$

$$\hat{l}_t^y = Z, \quad \hat{s}_t^y = w_t Z - \frac{1}{\beta b\rho R}, \qquad \hat{m}_{t+1}^y = Rw_t Z - \left(\frac{1+\beta}{\beta b\rho}\right),$$

where

$$Y = \left(Rs_{t-1}^o - \frac{1}{b\rho\left(1 + \beta(1-\delta)\right)}\right),$$
$$Z = b\left(w_t \beta\rho R - \phi\left(1 + \beta(1-\delta)\right)\right).$$

The total cooperative payoff is given by

$$\hat{W} = -\log(b\rho\left(1 + \beta(1-\delta)\right)) - b\left[(1-\delta)P_{t-1} + \phi Z - \rho Y\right]\left(1 + \beta(1-\delta)\right) - \frac{Z^2}{2}$$
$$- \log(\beta b\rho R) - \beta\left(\log(b\rho) + b\left(\phi l_{t+1} - \rho\left(Rw_t Z - \left(\frac{1+\beta}{\beta b\rho}\right)\right)\right)\right).$$

Proof. The cooperative solution is obtained by solving the following joint-optimization problem:

$$\max W = W^o + W^y = u\left(Rs_{t-1}^o - m_t^o\right) - b\left[(1-\delta)P_{t-1} + \phi l_t^y - \rho m_t^o\right]$$
$$+ u(w_t l_t^y - s_t^y) - v(l_t^y) + \beta\left\{u(Rs_t^y - m_{t+1}^y)\right.$$
$$\left. - b\left[(1-\delta)\left((1-\delta)P_{t-1} + \phi l_t^y - \rho m_t^o\right) + \phi l_{t+1} - \rho m_{t+1}^y\right]\right\}$$

Taking into account the functional forms for u and v, and assuming an interior solution, the optimality conditions are given by

$$\frac{\partial W}{\partial m_t^o} = \frac{-1}{Rs_{t-1}^o - m_t^o} + b\rho + \rho\beta b(1-\delta) = 0,$$

$$\Leftrightarrow \quad \hat{m}_t^o = Rs_{t-1}^o - \frac{1}{b\rho + \rho\beta b(1-\delta)} \Rightarrow \hat{c}_t^o = \frac{1}{b\rho + \rho\beta b(1-\delta)},$$

$$\frac{\partial W}{\partial s_t^y} = \frac{-1}{w_t l_t^y - s_t^y} + \frac{\beta R}{Rs_t^y - m_{t+1}^y} = 0 \Longleftrightarrow c_t^y = \frac{c_{t+1}^y}{\beta R},$$

$$\frac{\partial W}{\partial l_t^y} = -b\phi + \frac{w_t}{w_t l_t^y - s_t^y} - l_t^y - \beta b\phi(1-\delta) = 0 \Longleftrightarrow l_t^y = \frac{w_t}{c_t^y} - b\phi - \beta b\phi(1-\delta),$$

$$\frac{\partial W}{\partial m_{t+1}^y} = \frac{-\beta}{Rs_t^y - m_{t+1}^y} + \beta b\rho = 0 \Longleftrightarrow \hat{c}_{t+1}^y = \frac{1}{b\rho}.$$

Using

$$c_t^y = \frac{c_{t+1}^y}{\beta R},$$

we get

$$\hat{c}_t^y = \frac{1}{\beta b\rho R} \quad \text{and} \quad \hat{l}_{ct}^y = \beta b \left(w_t \rho R - \phi(1-\delta) \right) - b\phi.$$

In turn, this leads to

$$\hat{s}_t^y = w_t \hat{l}_t^y - \hat{c}_t^y = w_t b\beta \left(w_t \rho R - \phi(1-\delta) \right) - \frac{1}{\beta b\rho R} - w_t b\phi.$$

Since her consumption, when old, is

$$c_{t+1}^y = Rs_t^y - m_{t+1}^y = \frac{1}{b\rho},$$

her contribution to environmental improvement is thus:

$$\hat{m}_{t+1}^y = R\hat{s}_t^y - \frac{1}{b\rho}$$

$$= Rw_t b \left(w_t \beta \rho R - \phi \left(1 + \beta(1-\delta) \right) \right) - \left(\frac{1+\beta}{\beta b\rho} \right).$$

$\square$

Proposition 3 *Noncooperative and cooperative solutions compare as follows:*

$$\tilde{m}_t^o - \hat{m}_t^o = \frac{-\beta(1-\delta)}{b\rho \left(1 + \beta(1-\delta) \right)}, \quad \tilde{c}_t^o - \hat{c}_t^o = \frac{\beta(1-\delta)}{b\rho \left(1 + \beta(1-\delta) \right)},$$

$$\tilde{c}_t^y - \hat{c}_t^y = 0, \quad \tilde{l}_t^y - \hat{l}_t^y = b\phi, \quad \tilde{s}_t^y - \hat{s}_t^y = w_t b\phi,$$

$$\tilde{m}_{t+1}^y - \hat{m}_{t+1}^y = Rw_t b\phi.$$

Proof. Straightforward computations lead to the results. $\square$

Recalling that pollution emission is measured by ϕl_t, we encounter here a result that is traditional in environmental economics, i.e., that pollution is higher under a noncooperative mode of play than in the cooperative case. The young generation adopts the same consumption plan in the two regimes. This is achieved, on the revenue side, by working

and saving less in the cooperative case and, on the expenses side, by reducing the investment in the environment, which is consistent with the reduction in pollution. Note that the result is different for the old generation. Indeed, the latter reduces its consumption and increases its contribution to the maintenance of the environment when the game is played cooperatively. Looking at the welfare impact of cooperation, we state the following two propositions.

Proposition 4 *Nash equilibrium is strictly inefficient.*

Proof. Total difference is given by

$$\hat{W} - \left(\tilde{W}^o + \tilde{W}^y\right) = w_t \beta b^2 \rho R \phi + \frac{b^2 \phi^2}{2} + \beta(1 - \delta) - \log\left(1 + \beta(1 - \delta)\right).$$

Denote by $x = \beta(1 - \delta), 0 \leq x \leq 1$ and define

$$g(x) \triangleq x - \log(1 + x).$$

Clearly, $g'(x) > 0, \forall x \in [0, 1]$ and $g(0) = 0$ and $g(1) = 1 - \log 2 > 0$. Therefore, $g(x) > 0, x \in [0, 1]$. Hence the result. $\square$

Proposition 5 *Cooperation is Pareto-improving with respect to Nash equilibrium, if the following condition holds:*

$$b^2 \phi^2 + \frac{\beta(1 - \delta)}{1 + \beta(1 - \delta)} \geq \log\left(1 + \beta(1 - \delta)\right).$$

Proof. Denote by $\hat{W}^y$ and $\hat{W}^o$ the payoffs of the young and the old generations, respectively, when they implement the cooperative solution. Straightforward computations lead to the following difference for the young generation:

$$\hat{W}^y - \tilde{W}^y = \frac{1}{2\left(1 + \beta(1 - \delta)\right)} \left\{ b\phi \hat{l}_t^y + [\beta(1 - \delta)]^2 \right.$$
$$\left. w_t \beta b^2 \rho R \phi \left(1 + 2\beta(1 - \delta)\right) \right\},$$

which is clearly strictly positive.

For the old generation, the difference in welfare is given by

$$\hat{W}^o - \tilde{W}^o = -\log\left(1 + \beta(1 - \delta)\right) + b^2 \phi^2 + \frac{\beta(1 - \delta)}{1 + \beta(1 - \delta)},$$

which is nonnegative if

$$b^2 \phi^2 + \frac{\beta(1 - \delta)}{1 + \beta(1 - \delta)} \geq \log\left(1 + \beta(1 - \delta)\right).$$

$\square$

The message given by these two propositions is that it is collectively beneficial (as expected) to cooperate and that a side payment may be needed to ensure that both players are better off under cooperation. If one adopts, for instance, the egalitarian principle, that is, that both players improve their payoffs with respect to noncooperation by the same amount, then the side payment (SP) would be given by

$$SP = \frac{1}{2}\left[\hat{W}^y - \hat{W}^o + \tilde{W}^o - \tilde{W}^y\right].$$

With this transfer, the two generations will obtain the following net welfares:

$$
\begin{aligned}
NW^y &= \hat{W}^y - SP = \tilde{W}^y + \frac{\hat{W} - \left(\tilde{W}^o + \tilde{W}^y\right)}{2} > \tilde{W}^y,\\
NW^o &= \hat{W}^o + SP = \tilde{W}^o + \frac{W - \left(\tilde{W}^o + \tilde{W}^y\right)}{2} > \tilde{W}^o.
\end{aligned}
$$

It can readily be seen that each generation is getting its noncooperative payoff plus half of the surplus (or dividend) of cooperation given by $\hat{W} - \left(\tilde{W}^o + \tilde{W}^y\right)$. Since the latter is strictly positive, then the allocation (NW^y, NW^o) is Pareto-improving with respect to noncooperation.

4   Incentive Equilibrium

As has been shown, the coordinated solution is not an equilibrium. This section shows how the cooperative solution can be sustained as an incentive equilibrium.

The old generation has one control m_t^o and the young generation has five, namely, $c_t^y, s_t^y, l_t^y, c_{t+1}^y$ and m_{t+1}^y. Given the identities

$$
\begin{aligned}
c_t^y + s_t^y &= w_t l_t^y,\\
c_{t+1}^y + m_{t+1}^y &= R s_t^y,
\end{aligned}
$$

only three of them are actually free. Further, it has been shown in the previous section that the young generation keeps the same consumption pattern in both cooperative and noncooperative regimes, i.e.,

$$\hat{c}_t^y = \tilde{c}_t^y \text{ and } \hat{c}_{t+1}^y = \tilde{c}_{t+1}^y.$$

Then, it is easy to see that it is sufficient for the young generation to set its labor effort at the cooperative level to have all other variables set also at their coordinated values. In this context, the definition of an incentive equilibrium reduces to the following.

Definition 6 *Let* $\left(\hat{l}^y_t, \hat{m}^o_t\right) \in R^+ x R^+$ *be the coordinated solution. Let* $\Psi_y = \{\psi_y/\psi_y : R^+ \longrightarrow R^+\}$ *and* $\Psi_o = \{\psi_o/\psi_o : R^+ \longrightarrow R^+\}$ *be the sets of admissible incentive strategies.* $(\psi_y \in \Psi_y, \psi_o \in \Psi_o)$ *is called an incentive equilibrium at* $\left(\hat{l}^y_t, \hat{m}^o_t\right)$, *if*

$$
\begin{aligned}
W^y\left(\hat{l}^y_t, \hat{m}^o_t\right) &\geqslant W^y\left(l^y, \psi_o(l^y)\right), &\forall l^y \in R^+, \\
W^o\left(\hat{l}^y_t, \hat{m}^o_t\right) &\geqslant W^o\left(\psi_y(m^o), m^o\right), &\forall m^o \in R^+, \\
\psi_o(\hat{l}^y) &= \hat{m}^o, &\psi_y(\hat{m}^o) = \hat{l}^y.
\end{aligned}
$$

In order to determine an incentive equilibrium, one has to solve the problem of each generation assuming that the other is using the incentive strategy.

Proposition 7 *A strategy pair* $(\psi_y \in \Psi_y, \psi_o \in \Psi_o)$ *is an incentive equilibrium at* (l^y_c, m^o_c) *if*

$$
\begin{aligned}
\psi_y\left(\hat{m}^o_t\right) &= \hat{l}^y_t, &\psi'_y\left(\hat{m}^o_t\right) = -\frac{\rho\beta(1-\delta)}{\phi} = \frac{1}{\psi'_o(\hat{l}^y_t)}, \\
\psi_o\left(\hat{l}^y_t\right) &= \hat{m}^o_t, &\psi'_o(\hat{l}^y_t) = -\frac{\phi}{\rho\beta(1-\delta)} = \frac{1}{\psi'_y\left(\hat{m}^o_t\right)}.
\end{aligned}
$$

Proof. Substituting $\psi_y\left(m^o_t\right)$ for l^y_t, the old-generation problem becomes

$$
\max W^o = u\left(Rs^o_{t-1} - m^o_t\right) - b\left[(1-\delta)P_{t-1} + \phi\psi_y\left(m^o_t\right) - \rho m^o_t\right].
$$

The optimality condition is

$$
\begin{aligned}
\frac{\partial W^o}{\partial m^o_t} &= \frac{-1}{Rs^o_{t-1} - m^o_t} - b\phi\psi'_y\left(m^o_t\right) + b\rho = 0, \\
&\Leftrightarrow \psi'_y\left(m^o_t\right) = \frac{-1}{b\phi\left(Rs^o_{t-1} - m^o_t\right)} + \frac{b\rho}{b\phi}.
\end{aligned}
$$

Imposing

$$
m^o_t = \hat{m}^o_t = Rs^o_{t-1} - \frac{1}{b\rho + \rho\beta b(1-\delta)},
$$

we get

$$
\psi'_y\left(\hat{m}^o_t\right) = \frac{-\rho\beta(1-\delta)}{\phi}.
$$

Substituting $\psi_o(l^y_t)$ for m^0_t, the young-generation problem becomes

$$
W^y = u(w_t l^y_t - s^y_t) - v(l^y_t) + \beta\left\{u(Rs^y_t - m^y_{t+1})\right.
$$

$$-b\left[(1-\delta)\left((1-\delta)P_{t-1}+\phi l_t^y-\rho\psi_o(l_t^y)\right)+\phi l_{t+1}-\rho m_{t+1}^y\right]\right\}.$$

The optimality condition with respect to l_t^y is given by

$$\frac{\partial W^y}{\partial l_t^y}\quad=\quad\frac{w_t}{w_t l_t^y-s_t^y}-l_t^y-\beta b(1-\delta)\left(\phi-\rho\psi_o'(l_t^y)\right)=0,$$

$$\Longleftrightarrow\quad\psi_o'(l_t^y)=\frac{\phi}{\rho}-\frac{w_t}{\rho\beta b(1-\delta)\left(w_t l_t^y-s_t^y\right)}+\frac{l_t^y}{\rho\beta b(1-\delta)}.$$

Imposing

$$l_t^y=\hat{l}_t^y=w_t\beta b\rho R-b\phi-\beta b\phi(1-\delta),$$

leads to

$$\psi_o'(\hat{l}_t^y)=-\frac{\phi}{\rho\beta(1-\delta)}.$$

<div align="right">□</div>

What is worth noting here is that, whatever the functional form of the incentive strategies, they have to satisfy the requirement of having the product of their derivatives equal to one, i.e., $\psi_y'(\hat{m}_t^o)\,\psi_o'(\hat{l}_t^y)=1$.

To illustrate, let us consider the simplest possible case, where the players adopt linear incentive strategies. In such a context, they would read as follows:

$$\psi_y(m_t^o)\quad=\quad\hat{l}_t^y+v^o(\hat{m}_t^o-m_t^o),\tag{1}$$

$$\psi_o(l_t^y)\quad=\quad\hat{m}_t^o+v^y\left(l_t^y-\hat{l}_t^y\right),\tag{2}$$

where the coefficients v^o and v^y are constants satisfying $v^o v^y=1$. They can be interpreted as penalties applied by the two generations to observed deviations from the cooperative or desired solution. Note, since we have shown that

$$\tilde{m}_t^o-\hat{m}_t^o<0\text{ and }\tilde{l}_t^y-\hat{l}_t^y>0,$$

we would expect that, if the old generation deviates from the cooperation, then it will invest less in the maintenance of the environment. Therefore, it is intuitive to consider only the case where $\hat{m}_t^o-m_t^o\geq 0$. This deviation is of course equal to zero when the old generation implements the cooperative solution. On the other hand, the young generation works less under cooperation; hence, the difference term in (2) is expected to be nonnegative.

As mentioned in the introduction, these incentive strategies will be effective only if they are credible, i.e., if it is more advantageous for a generation to apply its incentive strategy than to adopt the cooperative one, when the other generation deviates from the cooperative solution. A formal definition of credibility and a proposition providing conditions to guarantee it follow.

Definition 8 *The incentive equilibrium strategy pair* $(\psi_y \in \Psi_y, \psi_o \in \Psi_o)$ *at* $\left(\hat{l}^y, \hat{m}^o\right)$ *is credible in* R^+xR^+ *if the following conditions are satisfied:*

$$W^o\left(l^y, \psi_o(l^y)\right) \geqslant W^o\left(l^y, \hat{m}^o\right), \qquad \forall l^y \in R^+,$$
$$W^y\left(\psi_y(m^o), m^o\right) \geqslant W^y\left(\hat{l}^y, m^o\right), \qquad \forall m^o \in R^+.$$

Proposition 9 *For* $m_t^o \leq \hat{m}_t^o$, *and* $l_t^o \geq \hat{l}_t^o$, *the incentive equilibrium strategy pair* $(\psi_y \in \Psi_y, \psi_o \in \Psi_o)$ *at* $\left(\hat{l}^y, \hat{m}^o\right)$ *is credible if*

$$\psi_o'(l_t^y)\left[b\rho\left(Rs_{t-1}^o - \psi_o(l_t^y)\right) - 1\right] \geqslant 0, \tag{3}$$

and

$$\psi_y'(m_o)\left[-\psi_y(m_o) + \beta\phi\right] \leq 0. \tag{4}$$

Proof. Define by

$$\begin{aligned}
h(l_t^y) &= W^o\left(l^y, \psi_o(l^y)\right) - W^o\left(l^y, \hat{m}^o\right) \\
&= \log\left[Rs_{t-1}^o - \psi_o(l_t^y)\right] + b\rho\psi_o(l_t^y) + \log\left(b\rho\left(1 + \beta(1-\delta)\right)\right) \\
&\quad - b\rho\left(Rs_{t-1}^o - \frac{1}{b\rho\left(1 + \beta(1-\delta)\right)}\right).
\end{aligned}$$

The first condition for credibility becomes thus:

$$h(l_t^y) \geq 0, \quad \forall l^y \in R^+.$$

Clearly, $h(\hat{l}_t^y) = 0$. Compute the derivative of $h(l_t^y)$:

$$h'(l_t^y) = \frac{-\psi_o'(l_t^y)}{Rs_{t-1}^o - \psi_o(l_t^y)} + b\rho\psi_o'(l_t^y).$$

The first condition is thus:

$$h'(l_t^y) \geqslant 0 \Longleftrightarrow \psi_o'(l_t^y)\left[b\rho\left(Rs_{t-1}^o - \psi_o(l_t^y)\right) - 1\right] \geqslant 0.$$

Define by

$$\begin{aligned}
g(m^o) &= W^y\left(\psi_y(m^o), m^o\right) - W^y\left(\hat{l}^y, m^o\right) \\
&= -\frac{(\psi_y(m_o))^2}{2} + \beta\left(\phi\psi_y(m_o) - \rho m_o - \rho m_{t+1}^y\right) - \frac{Z^2}{2} \\
&\quad - \beta\left\{\phi\left[w_t\beta b\rho R - b\phi - \beta b\phi(1-\delta)\right] - \rho m_o - \rho\hat{m}_{t+1}^y\right\}.
\end{aligned}$$

Clearly $g(\hat{m}^o) = 0$. Compute the derivative of $g(m^o)$:

$$
\begin{aligned}
g'(m_o) &= -\psi'_y(m_o)\psi_y(m_o) + \beta\phi\psi'_y(m_o). \\
g'(m_o) &\le 0 \Longleftrightarrow \psi'_y(m_o)\left[-\psi_y(m_o) + \beta\phi\right] \le 0.
\end{aligned}
$$

$\square$

To remain in line with the example of linear incentive strategies, the proposition is stated under the conditions $m_t^o \le \hat{m}_t^o$ and $l_t^y \ge \hat{l}_t^y$. Again, for the sole purpose of illustration, let us assume that the players adopt the linear incentive strategies in (1)-(2). After some straightforward calculations, the credibility conditions become

$$
\begin{aligned}
0 &\le l_t^y - \hat{l}_t^y \le \frac{\beta^2(1-\delta)^2}{(1+\beta(1-\delta))\,b\phi}, \\
0 &\le \hat{m}_t^o - m_t^o \le \left((\beta+b)\,\phi - \beta b\,(w_t\rho R - \phi(1-\delta))\right)\frac{\phi}{\rho\beta(1-\delta)}.
\end{aligned}
$$

These conditions involve all of the model's parameters. Clearly, the ranges of the above two intervals depend on the actual values of these parameters. Looking more closely at the environmental parameters, we can make the following observations. The higher the labor externality ϕ (i.e., emissions due to working activities), the narrower is the first interval and the wider is the second. Whereas the efficiency parameter of the technology used for environmental maintenance ρ does not affect the range of the first interval, it negatively affects the range of the second.

To conclude, as has been shown in Martín-Herrán and Zaccour (2005), one cannot expect to have credibility of linear incentive strategies for any possible deviation. Following the guidelines provided by these authors, one can construct non-linear strategies which are credible for any deviation from the cooperative solution.

5   Conclusion

In this article, we showed how one can implement a desired coordinated strategy in the area of overlapping-generations models, using incentive strategies. We also established conditions for the credibility of such strategies. The main purpose of the paper being to illustrate such methodology, our setting was deliberately simple. The following extensions are of interest:

1. Consider a non-linear damage cost; this seems to be more realistic with regards to the literature in environmental economics and would lead to richer strategies, i.e., strategies that depend on the stock of pollution;

2. Drop the assumption of a representative agent for each generation and let there be a game within each generation on the top of the intergenerational one. This, however,

would require the tackling of the complex (and not yet considered) issue of how to extend the theory of incentive strategies to more than two players.

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