Les Cahiers du GERAD ISSN: 0711–2440

An Economic and Mathematical Analysis of Gasoline Station Location: A Case Study

R. Nguimbus
A. Bass Bagayogo

G–2004–67

August 2004

An Economic and Mathematical Analysis of Gasoline Station Location: A Case Study

Raphaël Nguimbus
Collège Universitaire de Saint-Boniface
Administration des Affaires
200 Avenue de la Cathédrale
Saint-Boniface (Manitoba) Canada, R2H 0H7
RNguimbus@ustboniface.mb.ca

Aboubacar Bass Bagayogo
Faculté des arts, des sciences et d'administration des affaires
Administration des Affaires
200 Avenue de la Cathédrale
Saint-Boniface (Manitoba) Canada, R2H 0H7
Abagayogo@ustbboniface.mb.ca

August 2004

Les Cahiers du GERAD

G–2004–67

Abstract

In this paper we use Noboru (2000) convenience-store location model, to describe a case study for locating gasoline stations (i.e. the distance between two neighboring stations) and its size, which are operated by one or more petroleum companies in a different urban market structures in Montreal (Canada). In order to maximize its profit, a petroleum company selects as its decision variables the distance between the two neighboring stations (or n, the number of its stations located within a given linear market area of fixed length N) and the size of an individual station. The outcomes of the monopolistic, duopolistic and oligopolistic equilibra are compared with the "socially optimal" outcome. Such solutions are dependent on the reservation price of the gasoline held by automobilists, the marginal supply cost of gasoline and the maintenance cost of an individual station. After the analytical presentation, numerical simulations are given, including a sensitivity analysis with regard to the time service rate. A large discrepancy between the equilibrium and the optimum is shown to exist as in Noboru (2000).

Key Words: Location Competition, Consumer surplus, Social optimum, Reservation Price, Profit Maximization.

Résumé

Dans ce papier, nous utilisons le modèle de Nobru (2000) de la localisation des magasins de détail de convenance, pour décrire à partir dune étude de cas, la localisation des stations services (i.e. la distance entre deux stations service voisines) et sa taille, qui sont exploitées par une ou plusieurs compagnies pétrolières dans différentes structures de marché urbain de carburant automobile à Montréal (Canada). Dans lobjectif de maximiser son profit, une compagnie pétrolière sélectionne comme variables de décisions : la distance entre deux stations service voisines (ou n, le nombre de stations localisées dans un segment de route de longueur fixe N) et la taille de la station service. Les résultats des équilibres monopolistique, duopolistique, et oligopolistique sont comparés avec ceux de "loptimum social". De telles solutions sont dépendantes du prix de réservation du carburant affiché dans la station, du coût marginal doffre de carburant et du coût de maintenance dune station service. Après la présentation analytique du modèle, des simulations numériques sont faites, incluant une analyse de sensibilité par rapport au taux du temps de service. Une grande différence persiste entre les différents équilibres et loptimum social comme la montré Noburu (2000) dans le cas des magasins de détail de convenance.

Acknowledgments: A grant of the Collège Universitaire de Saint-Boniface supported this research.

This research started during a visit of Raphaël Nguimbus to GERAD.

1 Introduction

This paper describes a case study for locating and setting the size concomitantly of gasoline stations in different retailing market structures (monopoly, duopoly and oligopoly) and provides some economic and mathematic insights for dealing with this type of problem. While mathematical programming approaches for facility location for manufacturing firms have been extensively used, there has been very little treatment for the specific, often simpler case, of locating convenience stores as gasoline stations for a petroleum company. Through the vehicle of this case study, this paper attempts to:

- Indicate how a gasoline station location problem can be formulated mathematically and economically and solved in different market structures (monopoly, duopoly and oligopoly).
- Indicate how to handle the complex problems of equation resolution. These lessons are applied to any network distribution problem. These are exemplified by the amount of automobiles traffic in a linear market, gasoline size, quantities of gasoline sell, the constant marginal supply costs, mark-up ratio, the maintenance cost of a station, markets equilibriums for multiple candidate station locations or closing.
- Provide insights on decision-making process and its relationship to the model development task.

The focus and contributions of this paper is to model economically and mathematically, the location problem of gasoline station, when gasoline retailers choose concomitantly stations locations and size along the line market, and don't compete on price (Noboru 2000) in different markets structures. Can Gasoline retailers choose location and station size concomitantly in the market place to bridge the gap between net profit of the station, consumer surplus and social surplus? Consumers have a finite gasoline reservation price, knowing how much they are willing to pay for gasoline offered in the site. This idea of introducing a finite reservation price goes back to Lerner and Singer (1937) and Smithies (1941). If a customer buys gasoline, he purchases it from the site that offers the highest indirect utility. The indirect utility function of a customer incorporates both fixed and variable costs of station and reservation price. Gasoline retailers are aware of the reservation price, margins, marginal costs and take into account the impact of their location and size decisions on their profits. In gasoline retailing we assume that the reservation price is relatively low, and the degree of differentiation between stations is lower. We adapt Noboru's theoretical model and equations in convenience gasoline stores industry where station location and size are strategic endogenous variables.

The rest of the paper is organizing as follows: Montreal gasoline market is presented in Section 2, the model in Section 3. In Section 4, outcomes of market equilibrium (which forms the basis of comparative analysis) are examined. The model is extended to a duopolistic situation Section 5 and to an oligopolistic situation in Section 6, and a contrasting social optimum is discussed in Section 7. Section 8 is devoted to numerical simulation of the models discussed in the previous sections, in order to gain some intuitive insight into the different systems, to understand how gasoline marketers minimize gas station size and distance and maximize each station sales and profits. We conclude in Section 9.

2 The Market for Gasoline Stations Retailing in Montreal

The Market for Gasoline Stations Retailing in Montreal is mature, highly competitive with too many outlets and relatively low average pump turnovers at 2.3 MM litres/year during in 1993-1997. Comparatively in the Toronto market the average is 4 MM litres/year. During that period (1993-1997), Montreal (metropolitan area) gasoline consumption represented on average a bit more than 41% of total demand in Quebec. The annual rate of growth of demand was around 1%.

The Market for Gasoline Stations Retailing in Montreal is segmented, for basically managerial reasons, by companies into three sub-markets: Montreal Centre (60% of sales), Montreal West (18%) and Montreal East (22%). Outlets repartition is very much in line with sales: Montreal Centre (59% of total number of outlets), Montreal West (17.5%) and Montreal East (23.5%). In 1997, averages of debits by gas station were similar in the three markets: 2.3 MM litres/year in Montreal Center and respectively 2.4 and 2.2 in Montreal West and Montreal East. The declared aim of retailers is to increase significantly the average debit by site.

Spatial competition between gasoline stations in the Market for Gasoline Stations Retailing in Montreal is structured in monopoly, duopoly and oligopoly light differently in Montreal Centre, Montreal West and Montreal East (see Table 1).

Gasoline station posted prices in Montreal Market (1993-1997) fluctuated less during this period in the three market regions between 48.90 to 67.90 cents per litre. Consumers reservation prices varies with the highest gasoline price posted in the market place.

Gasoline station fixed and variables costs in East Canada (Montreal) represent in average, according professionals 93% of gasoline sales volumes (1993-1997) or 7% detail margins per station.

According to Kent Marketing, a consultancy, in 1997 more than 50 different banners which sold gasoline and gas oil were listed. But this apparent market atomization should not hide the real level of concentration of this market. Indeed, Esso (Exxon), Petro-Canada, Shell and Ultramar are responsible of 85% of gasoline sales and control 64% of outlets. Since the beginning of the eighties, the gasoline market in Quebec has been subject to a deep restructuring. The number of outlets decreased year after year, and outlets with self-service increased. From 1981 to 1997, the number of gasoline outlets was cut 31% from 7,334 to 5,059. The rationalization of gasoline outlets had a positive direct effect on site gasoline sales volume. The average annual gasoline sales volume by outlet increased from 0.6 MM litres in 1984 to 1.4 MM litres in 1996. In spite of this rationalization, the average gasoline throughput in Quebec is still below of Ontario's, where the average was above 3.5 MM litres per station in 1996.

Table 1: Gasoline Market Station Location Configuration in Montreal (1993–1997)

	Monopoly	Duopoly	Oligopoly
Montreal Center	5%	80%	15%
Montreal West	8%	75%	17%
Montreal East	35%	58%	7%

3 The Model

Gas stations are convenience stores and spatial profit-maximizing institutions. When gas retailer locate a new station in the market place, he or she determines concomitantly its location and size in function of local market structure (monopoly, duopoly and oligopoly), and tries to minimizing station costs and maximizing consumer surplus, social surplus and net profit of the station. Gas stations are horizontally differentiated by size (service capacity) and location. Neighboring competing gas stations sell an homogeneous product (gasoline) at the same mill price. Each gas station charges a single mill price to all its customers. All costs in the station are passed on to the customers. Clearly, mill pricing is the first-best optimal pricing policy with the mill price equal to marginal cost. Petroleum firms are rational agents, facing high fixed costs in an homogeneous spatial market where they must sell more fuel to maximize profits, consumer surplus and social surplus. The following assumptions are adapted from the basic model developed by Noboru (2000) for convenience-store location[1].

1. The potential market of a gasoline station is a straight line segment or a trading area. The amount of automobile traffic$N$ crossing a given segment of route during a period of time is the potential market of the station. This assumption is realistic because gasoline stations sales depend first on location, vehicular traffic volume crossing the station during a period of time. It is why gasoline stations are usually located along major urban routes with high amount of traffic volumes (Eiselt and Laporte, 1988)[2].
2. The density of automobilists crossing a station along the linear market is unity. Anderson et al. (1992) used the same hypothesis in their framework of analysis. We

[1] We use the same notations as in Noboru's basic model.

[2] Analytic models of spatial competition, largely in the Losch-Hotelling tradition, have concentrated on competition along linear streets.

assume that there is a uniform distribution of consumers (with unit density) over a linear market normalized (without loss of generality) to [0, 1].

3. Each gasoline station has a fixed size or service capacity, the number of gasoline bays. We assume that this service capacity depends on site potential market (cars traffic volume). The gasoline station is located in the center of the linear market.
4. Each automobilist purchases a quantity of gasoline q when he or she stops at gasoline station and when its c.i.f.[3] price is below a common reservation price[4] r. This assumption of a "reservation-price demand function" is indispensable in order to render the analysis tractable. If following Eaton (1976), a linear demand function was introduced; the analysis would become excessively complicated. Such assumption is suitable to gasoline products.
5. The constant marginal supply cost of gasoline q is expressed as $\lambda - \mu x$ where x is the size of a single gasoline station. This assumption is based on the assertion that the larger a gasoline station, the lower the price of gasoline which is supplied by that gasoline station[5]. Gasoline stations must sell large quantities of fuel to cover fixed costs. It is why gasoline retailing is quantity-oriented.
6. A monopolistic petroleum retailer which; operates many gas stations in an isolated region, never adopts a strategic pricing policy. That is, the retailer accepts a predetermined mark-up ratio m which; is publicly decided. Therefore, the decision variables for the retailer are limited to n, the number of stations in the market, and x, the size of each station. This assumption of a "normal rate of profit" represents an important part of the public regulation which; accompanies the granting of permission to operate monopolistic convenience stores. In particular, the assumption implies the absence of "cut-throat" competition among different petroleum retailers.
7. The maintenance cost of a station is expressed as $\gamma + \delta x + \varepsilon x^2$, where γ is the set-up fixed cost and $\delta x + \varepsilon x^2$ is the additional operating cost that is dependent on the size of gasoline station. This assumption implies that the marginal size related cost of a station is an increasing function of x that is, of $\delta + 2\varepsilon x$.

[3]Optimal spatial price patterns derived on the basis of welfare maximization (in terms of consumer surplus plus profit are investigated in economics under alternative spatial pricing policies: 1. discriminatory, 2. f.o.b. (free on board), and 3. c.i.f. (cost, insurance, and freight). The economic analysis focuses on a linear and bounded market in which a single firm sells a delivered product to buyers who are dispersed over space. Under all forms of spatial pricing policies, optimal spatial prices should equate marginal costs of production with the mill price. Transportation costs incurred are always charged to customers, even in the case of c.i.f. pricing. Society never prefers c.i.f. to any other pricing policies and has no preference between discriminatory and f.o.b. pricing. For more details see Hsu, Song-Ken (1983) "Social Optimal Pricing in a Spatial Market", *Regional Science and Urban Economics*, Vol. 13 (2), pp. 401-411.

[4]Reservation price is the highest price one is willing to pay for a product.

[5]This assumption is realistic because, when hypermarkets enter in this industry to sell gas, they tend to lower gas price because they are too bigger to support low price cost, the constant marginal supply cost is close to zero.

8. By assumption (4), the c.i.f. price of gasoline q for customers is expressed as $(1 + m)(\lambda - \mu x) + \beta\nu$; where β is the unit of time a customer spends at a station during gasoline purchasing[6]; ν is the distance between that customer's home or office and the nearest gasoline station.

Taking the above hypotheses together, the basic model is defined by the following nine parameters: N, r, λ, μ, m, γ, δ, ε, and β. As previously mentioned, the decision variables for the monopolistic retailer are n, the number of stations (or the distance s between two neighboring stations, where $s = N/n$); and x, the size of a station.

4 First case: monopolistic market equilibrium[7]

In Montreal market place, few isolated gas stations often control alone a linear market street without no visible competitors. When retailers locate gas stations in monopolistic spaces, the following condition regarding ν (the distance between customer's home or office) must be satisfied for any realized demand, according to the definition of reservation price r :

$$(1 + m)(\lambda - \mu x) + \beta\nu \quad \leq r \tag{1}$$

The value $\bar{s}$, the length of the effective market area for a gasoline station, is given by

$$\bar{s} = \min\left\{\frac{N}{n}, \frac{2\left[r - (1 + m)(\lambda - \mu x\right]}{\beta}\right\} \tag{2}$$

When n is small, $\bar{s} = 2[r - (1 + m)(\lambda - \mu x)]/\beta$. And when n is large $\bar{s} < 2[r - (1 + m)(\lambda - \mu x)]/\beta$.

When n is small, the net profit per station is expressed as:

$$\rho(x) = \frac{2m[r - (1 + m)(\lambda - \mu x)](\lambda - \mu x)}{\beta} - (\gamma + \delta x + \varepsilon x^2) \tag{3}$$

We assume that $\rho(0) > 0$ and that $d\rho/dx(0) > 0$ with the conditions $2m[r-(1+m)\lambda]\lambda > \beta\gamma$ and $2m\mu[2(1+m)\lambda - r] > \beta\delta$. Now, maximizing $\rho(x)$ with respect to x under a fixed small n, the optimizing value of x is calculated from equation (3) as:

$$x^* = \frac{m\mu[2(1 + m)\lambda - r] - \beta\delta}{2m(1 + m)\mu^2 + \beta\varepsilon} \tag{4}$$

[6]In many retail businesses, sellers serve randomly arriving customers from fixed capacities. From time to time queues form, and as emphasized by Becker (1965), customers pay two prices, an (explicit) price to the seller, and in addition, an (implicit) price in the time spent waiting (For more details see Png and Reitman, 1994).

[7]We conserve the same notations as Noboru (2000).

From the two assumptions given above, it is obvious that $\rho(x^*) > 0$. In this situation of an effective market area, the petroleum firm total profit π is expressed as $\pi = n\rho(x^*)$, so that

$$\frac{\partial \pi(x^*, n)}{\partial n} = \frac{\partial [n\rho(x^*)]}{\partial n} > 0 \tag{5}$$

There exists a strong incentive, therefore, for the gasoline retailer in a monopolistic market, to increase the number of gasoline stations until N/n becomes the size of the effective market area. Because of this, there is no possibility for the situation to become the final equilibrium.

Next, if $\bar{s} = N/n$ in equation (2), total profit for the gasoline retailer is:

$$\pi(n, x) = n\left[\frac{N}{n}m(\lambda - \mu x) - (\gamma + \delta x + \varepsilon x^2)\right] = Nm(\lambda - \mu x) - n(\gamma + \delta x + \varepsilon x^2) \tag{6}$$

So that, apparently,

$$\frac{\partial \pi}{\partial n} < 0 \text{ and } \frac{\partial \pi}{\partial x} < 0 \tag{7}$$

If n is treating as a continuous variable , the total net profit π of the gasoline retailer becomes only a function of n.

Thus:

$$\pi(x) = Nm(\lambda - \mu x) - \frac{\beta N}{2\left[r - (1+m)(\lambda - \mu x)\right]}(\gamma + \delta x + \varepsilon x^2) \tag{8}$$

We can determine the equilibrium by maximizing $\pi(x)$ with respect to x, and then obtaining n from expression:

$$n = \frac{\beta N}{2\left[r - (1+m)(\lambda - \mu x)\right]} \tag{9}$$

This case is rarely observed generally in large metropolis, like Montreal or Toronto where gasoline retailers compete on duopolistic or oligopolistic market structures. But in Montreal North with few populations and low incomes, gasoline retailers deal with monopolistic market structure. They locate more isolated stations to cover the whole market.

5 Second case: duopolistic equilibrium

In the same Montreal market area, the monopolistic equilibrium results in a market area of length s for a single station. This case is paramount in this market between Major's gasoline stations which compete side by side. The first station is located in the middle of the market. Then a second gasoline retailer attempts to enter the market. For this second gasoline retailer, there is no alternative but to locate its stations in the middle of the market area of length s[8]. In this duopolistic case, a symmetric solution is inevitable as the final equilibrium. Net profit for individual station of the second retailer, as well as that of the threatened station of the first retailer, is:

$$\rho(y) = \frac{m(\lambda - \mu y)s}{2} - (\gamma + \delta y + \varepsilon y^2) \tag{10}$$

where, y is the size of a station of the first or the second gasoline retailer.

Since $\rho(y)$ is a decreasing function of y, the optimal size of a station in this case is $y = 0$ (the minimum size)[9]. This is a natural result for a fixed market area. It becomes clear that if the two retailers compete in a sequential manner, the size of a station for each retailer becomes minimal (Noboru, 2000).

In the case of duopoly, for small y:

$$y* = \frac{-mr\mu + 2m\lambda\mu - 8\sigma\beta + 2m^2\lambda\mu}{2(m\mu^2 + m^2\mu^2 + 8\varepsilon\beta} \tag{11}$$

$$n* = \frac{N\beta}{(r - \lambda + \mu y - m\lambda + m\mu y)} \tag{12}$$

$$\varphi(y) = m\left[\frac{r - (1+m)(\lambda - \mu y)}{4\beta}\right](\lambda - \mu y) - (\gamma + \delta y + \varepsilon y^2) \tag{13}$$

In this case, the net profit by station decreases with the new comer in the market. The duopolistic case (two stations visible back to back) is paramount in Montreal Gasoline Market.

6 Third case: oligopoly equilibrium

In Montreal gasoline market, we observe the third case competition at three. Three Gasoline stations are located side by side. The duopolistic equilibrium results in a market area of length s for two gasoline stations. Then the third retailer attempts to enter the market. The first station is located at the middle of the market; the second station is located in the middle of the market. For the third retailer, there is no alternative but to locate its gasoline station in the middle of the market area. In this oligopolistic case, a symmetric solution is inevitable as the final equilibrium. Net profit of the third retailer, as well as that of the threatened stations of the two first retailers, is

$$\rho(z) = \frac{m(\lambda - \mu z)s}{3} - (\gamma + \delta z + \varepsilon z^2) \tag{14}$$

[8]This is the basic assumption in Hotelling (1929) theory to stabilize competition between competitive outlets in a segment route of the market area. When two competitive outlets locate in the middle of the segment of market, they maximize their gains and each one takes the half of the market.

[9]If y is too small, $\rho(y) = \frac{m\lambda s}{2} - \gamma$, it is better for the second retailer to minimize the size of his outlet. But gasoline station profit will depend definitively on the market size.

where, z is the size of a station of the first, second or third retailer. Since $\rho(z)$ is a decreasing function of z, the optimal size of a station in this case is $z = 0$ (the minimum size). It becomes clear that if the three retailers compete in a sequential manner, the size of a station for each retailer becomes minimal.

In the case of oligopoly for small z:

$$z* = \frac{2m\lambda\mu - mr\mu + 2m^2\lambda\mu - 18\sigma\beta}{2(m\mu^2 + m^2\mu^2 + 18\varepsilon\beta)} \tag{15}$$

$$n* = \frac{3N\beta}{2(r - \lambda + \mu z - m\lambda + m\mu z)} \tag{16}$$

$$\varphi(z) = m\left[\frac{r - (1+m)(\lambda - \mu z)}{9\beta}\right](\lambda - \mu z) - (\gamma + \delta z + \varepsilon z^2) \tag{17}$$

The net profit of each station decreases, more stations in a same linear market kill and lower gasoline sales. This case is exceptional in Gasoline Montreal Market.

7 Social Optimum

When $m = 0$ (so that marginal cost pricing exists), other conditions being equal, the gross social surplus is maximized. The effective market area then becomes:

$$\hat{s} = \min\left\{\frac{N}{n}, \frac{2\left[r - (\lambda - \mu x\right]}{\beta}\right\} \tag{18}$$

and the corresponding net social surplus, S, is

$$S = \frac{n}{2}\left\{\hat{s}\left[r - (\lambda - \mu x)\right] - 2(\gamma + \delta x + \varepsilon x^2)\right\} \tag{19}$$

When n is small, in this case, the net social surplus per station and its first and second derivatives with respect to x are as follows:

$$\theta(x) = \frac{\left[r - (\lambda - \mu x)\right]^2}{\beta} - (\gamma + \delta x + \varepsilon x^2) \tag{20}$$

$$\frac{d\theta}{dx} = \frac{2\mu\left[r - (\lambda - \mu x)\right]}{\beta} - (\delta x + 2\varepsilon x) \tag{21}$$

$$\frac{d^2\theta}{dx^2} = 2(\frac{\mu^2 - \beta\varepsilon}{\beta}) \tag{22}$$

Making realistic assumptions concerning parameters values, we may suppose that:

$$\theta(0) = \frac{(r - \lambda)^2}{\beta} - \gamma > 0 \tag{23}$$

$$\frac{d\theta}{dx}(0) = \frac{2\mu(r-\lambda)}{\beta} - \delta > 0 \quad (24)$$

$$\frac{d^2\theta}{dx^2} < 0 \quad (25)$$

Under these conditions, the optimizing value of x for small n is deduced as

$$x' = \frac{2\mu(r-\lambda) - \beta\delta}{2(\beta\varepsilon - \mu^2)} \quad (26)$$

and the corresponding value of θ_{max} is shown to be positive:

$$\theta_{max} = \theta(x') > 0 \quad (27)$$

In this case, the total net social surplus S (where $S = n \quad \theta_{\max}$) is apparently an increasing function of n, so that it is desirable for company to increase the number of stations until N/n becomes the effective market area. When this occurs, the total net social surplus S as a function of n and x will be:

$$S(n,x) = \frac{n}{2}\left[\frac{N}{n}\{[r-(\lambda-\mu x)] + [r-(\lambda-\mu x)-(\beta/2)(N/n)]\}\right] - n(\gamma+\delta x+\varepsilon x^2) \quad (28)$$

Rearranging equation (22), $S(n,x)$ becomes

$$S(n,x) = \frac{N}{2}[2(r-\lambda) + 2\mu x - (\beta/2)(N/n)] - n(\gamma+\delta x+\varepsilon x^2) \quad (29)$$

and the maximizing conditions for $(S(n,x))$ are given by

$$\frac{\partial S}{\partial n} = \frac{\beta}{4}\left(\frac{N}{n}\right)^2 - (\gamma+\delta x+\varepsilon x^2) = 0 \quad (30)$$

and

$$\frac{\partial S}{\partial x} = N\mu - n(\delta + 2\varepsilon x) = 0 \quad (31)$$

Eliminating n from equations (30) and (25), an equation in single variable x is obtained:

$$4\varepsilon(\mu^2-\beta\varepsilon)x^2 + 4\delta(\mu^2-\beta\varepsilon)x + (4\mu^2\gamma - \beta\delta^2) = 0 \quad (32)$$

We can calculate the optimal $x : (x'')$ from equation (32), and the optimal $n : (n'')$ is then obtained by

$$n'' = \frac{N\mu}{\delta + 2\varepsilon x''} \quad (33)$$

$$x'' = -\frac{\delta}{\varepsilon} + \left[\frac{\delta(\mu^2 - \beta\varepsilon - 16\varepsilon\mu^2 + 4\beta\varepsilon\delta^2)}{2\varepsilon(\mu^2-\beta\varepsilon)}\right]^{\frac{1}{2}} \quad (34)$$

An interesting feature of x'' and n'' is their independence of r and λ. These are determined in order to minimize the negative welfare effect of cost of time and station size on the price of the gasoline and on the set-up cost of a station.

Next, as the size-related part of the station maintenance cost approaches zero (as δ and ε approach zero), n approaches $(\beta/4\gamma)^{1/2}N$. In addition, if the set-up cost approaches zero, then n becomes infinite, implying the emergence of 'backyard stations', corresponding to customers on a one-to-one basis. Finally, since x'' is independent of N, as shown by equation (34), n'' becomes strictly proportional to N (Noboru Sakashita, 2000). Table 2 resumes all equations.

8 Numerical Simulations

In this section, we use data from Gasoline Market in Montreal City, Canada (1993–1997). The following parameter values are given:

$$N = 60; r = 9; \lambda = 5; \mu = 0.1; m = 0.3; \gamma = 10; \delta = 0.01; \varepsilon = 1; \beta = 0.2$$

Theses parameters were discussed and validated by five managers in Montreal Gasoline Market (ESSO, PETRO-CANDA, and SHELL). Each professional gave us parameters configuration from our model on Montreal Gasoline Retailing Market of: margins, reservation price, time spent waiting, constant supply cost, set-up fixed cost, operating costs. We calculate parameters average and deviations. In good locations, 40 to 80 automobiles in average crossing a station by minute during peak demand periods. Gasoline station size is determined by peak demand quantities, the market structure and the nature of local competitors in the linear market. In Montreal gasoline market a reservation price was in average 9 cents over the posted price of a litre of unleaded gasoline (1993-1997), and the average gasoline margins in the site was around 0.3 during the same period. Tables 3 & 4 give the results of our simulations.

9 Conclusion

In this case study, first, we have considered the case at equilibrium for a service station chain operated by a monopolistic firm which selected as decision variables the number of stations and the size of a single station. Secondly, a duopolistic market solution was examined, and a social optimum under the same service quality conditions was then analyzed. Here, the social objective was to maximize the social surplus, allowing deficits in gasoline retailing Montreal Market. Thirdly, an oligopolistic market solution was examined, and a social optimum under the same service quality conditions was then analyzed. Here, the social objective was to maximize the social surplus, allowing deficits in gasoline retailing Montreal Market. A distinctive feature of this analysis, which confirms Noboru (2000) analysis on convenience-store location, is the remarkable asymmetry between the monopolistic market

Table 2: Equations synthesis

Variables	Monopolistic equilibrium	Duopolistic equilibrium
Station size, x	$\frac{4m\lambda\mu-2mr\mu+4m^2\lambda\mu-\sigma\beta}{2(2m\mu^2+2m^2\mu^2+\varepsilon\beta)}$	$\frac{-mr\mu+2m\lambda\mu-8\sigma\beta+2m^2\lambda\mu}{2(m\mu^2+m^2\mu^2+8\varepsilon\beta)}$
Number of stations, n	$\frac{N\beta}{2(r-\lambda+\mu x-m\lambda+m\mu x)}$	$\frac{N\beta}{(r-\lambda+\mu y-m\lambda+m\mu y)}$
Station maintenance cost	$\gamma \quad + \quad \delta\mathrm{x} + \varepsilon\mathrm{x}^2$	$\gamma \quad + \quad \delta\mathrm{y} + \varepsilon\mathrm{y}^2$
Net profit of the firm per station	$\varphi(x) = 2m\left[\frac{r-(1+m)(\lambda-\mu x)}{\beta}\right](\lambda-\mu x) - (\gamma+\delta x+\varepsilon x^2)$	$\varphi(y) = m\left[\frac{(\lambda-\mu y)S}{2}\right] - (\gamma+\delta y+\varepsilon y^2)$
Consumer surplus	$[r - -(1+m)(\lambda-\mu x) \times S$	$\frac{3}{8}[r-(1+m)(\lambda-\mu y)] \times S$
Social surplus	$\varphi(x) + [r - -(1+m)(\lambda-\mu x) \times S$	$\varphi(x) + \frac{3}{8}[r-(1+m)(\lambda-\mu y)] \times S$

Variables	Oligopolistic equilibrium	Social optimum
Station size, x	$\frac{2m\lambda\mu-mr\mu+2m^2\lambda\mu-18\sigma\beta}{2(m\mu^2+m^2\mu^2+18\varepsilon\beta)}$	$-\frac{\delta}{\varepsilon} + \left[\frac{\delta(\mu^2-\beta\varepsilon-16\varepsilon\mu^2+4\beta\varepsilon\delta^2)}{2\varepsilon(\mu^2-\beta\varepsilon)}\right]^{\frac{1}{2}}$
Number of stations, n	$\frac{3N\beta}{2(r-\lambda+\mu z-m\lambda+m\mu z)}$	$\frac{N\mu}{\delta+2\varepsilon x}$
Station maintenance cost	$\gamma \quad + \quad \delta z + \varepsilon z^2$	$\gamma \quad + \quad \delta x + \varepsilon x^2$
Net profit of the firm per station	$\varphi(z) = m\left[\frac{(\lambda-\mu z)S}{3}\right] - (\gamma+\delta z+\varepsilon z^2)$	$S(n,x) = -n(\gamma+\delta x+\varepsilon x^2)$
Consumer surplus	$\frac{2}{9}[r-(1+m)(\lambda-\mu z)] \times S$	$N(r-\lambda+\mu x-\frac{\beta N}{4n})$
Social surplus	$\varphi(z) + \frac{2}{9}[r-(1+m)(\lambda-\mu z)] \times S$	$S(n,x) + N(r-\lambda+\mu x-\frac{\beta N}{4n})$

Table 3: Results of the numerical simulations

Variables	Monopolistic equilibrium	Duopolistic equilibrium	Oligopolistic equilibrium	Social optimum
Station size, x	0.57	0.03	0.01	0.64
Number of stations, n	2	5	7	5
Station maintenance cost	20,7	50	70	52
Net profit of the firm	55,7	40	20	-52
Consumer surplus	77,2	112,7	100	208
Social surplus	132,9	152,7	120	156

Table 4: Results of the sensitivity analysis

	Value of β									
	0.05		0.1		0.2		0.3		0.4	
	ME	SO	ME	SO	ME	SO	ME	SO	ME	SO
Station size, x	2	2	1,1	1	0.57	0.64	0,38	0,5	0.28	0,44
Number of stations, n	1	1	1	3	2	5	4	6	5	7
Station maintenance cost	14	14	11,2	33	20,7	52	40,6	61,5	50,4	71,4
Net profit of the firm	145	-14	66,3	-33	55,7	-52	60,6	-61,5	44,2	-71,4
Consumer surplus	83	207	79,3	216	77,2	208	76,5	198	76,1	191,2
Social surplus	228	193	145,6	183	132,9	156	137,1	136,5	120,3	119,8

Notes: ME = monopolistic equilibrium; SO = social optimum.

equilibrium and the social optimum. Because of this asymmetry, it is virtually impossible to find an economic policy in gasoline retailing which gasoline low price, which bridges the gap between equilibrium and optimum. As we seen with the numerical examples involving the sequential entry of duopolistic and oligopolistic firms, the introduction of competition does not seem to solve Noboru's dilemma. Gasoline retailers cannot choose location and station size concomitantly in the market place to bridge the gap between net profit of the station, consumer surplus and social surplus. Gasoline price at the pump cannot be set highly to increase gasoline margins. More stations in the market place kill profits by station. It is why, since a couple of years, majors gasoline retailers in United States and Canada, are restructuring their gasoline stations networks to increase sales volume site by site. Since 1980, according professionals, Montreal Gasoline Retailing Market outlets decrease in average 5% each year.

References

ANDERSON, S.P., DE PALMA, A., THISSE, J.-F. (1992) "Social Surplus and Profitability under Different Spatial Pricing Policies", *Southern Economic Journal*, 58(4), 934–950.

CRAIG, C.S., GHOSH, A., MCLAFFERTY, S. (1984) "Models of the Retail Location Process: a Review", *Journal of Retailing*, 60(1), 5–36.

EATON, B.C. (1976) "Free entry in one-dimensional models: pure profits and multiple equilibra", *Journal of Regional Science*, 16, 21–33.

EATON, B.C., LIPSEY, R.G. (1979) "The theory of market pre-emption: the persistence of excess capacity and monopoly growing spatial markets", *Economica*, 49, 149–158.

GABSZEWICZ, J.J., THISSE, J.-F. (1986) "Spatial competition and the location of firms", in: J.J. GABSZEWICZ, J.-F. THISSE, M. FUJITA and U. SCHWEIZER (Eds.), *Location Theory*, pp. 1–71. Chur: Harwood Academic Publishers.

HOTELLING, H. (1929) "Stability in Competition", *Economic Journal*, 39, 41–57.

JURION, B.J. (1983) "A theory of public services with distance-sensitive utility", in: J.F. THISSE and H.G. ZOLLER (Eds.), *Locational analysis of Public facilities.* North Holland.

NOBORU, S. (2000) "An economic analysis of convenience-store location", *Urban Studies*, 37(3), 471–479.

PETERS, J., FREI, B., ANDRE, B. (2000) "New Millennium Gasoline Retailing: Challenge to the incumbents", *National Petroleum News*, 92(5), 54–58.

PNG, I.P.L., REITMAN, D. (1994) "Service Time Competition", *Rand Journal of Economics*, 25(4), 619–634.

SHEPPARD, A. (1990) "Price Behavior and Vertical Contracts in Retail Markets", *The American Economic Review*, 80(2), 427–431.

SHEPPARD, A. (1991) "Price Discrimination and Retail Configuration", *Journal of Political Economy*, 99(1), 30–53.

STAHL, K. (1987) "Theories of urban business location", in: E.S. MILLS (Ed.), *Handbook of Regional and Urban Economics*, Vol. II, pp. 759–820. Oxford: Elsevier Science Publishers.

VARIAN, H.R. (1990) *Intermediate Microeconomics*, New York: W.W. Norton & Co.

WINGO, G.L. (1997) "The Store Size Dilemma: Is Bigger Better?", *National Petroleum News*, July, 40–44.