

# Unboundedness in bilevel optimization

## Supplementary materials

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G-2025-22

January 2025

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**Citation suggérée :** B. Rodrigues, M. Carvalho, M. F. Anjos, N. Sugishita (Janvier 2025). Unboundedness in bilevel optimization, Supplementary materials. Rapport technique, Les Cahiers du GERAD G- 2025-22, GERAD, HEC Montréal, Canada.

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**Suggested citation:** B. Rodrigues, M. Carvalho, M. F. Anjos, N. Sugishita (January 2025). Unboundedness in bilevel optimization, Supplementary materials. Technical report, Les Cahiers du GERAD G-2025-22, GERAD, HEC Montréal, Canada.

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Dépôt légal – Bibliothèque et Archives nationales du Québec, 2025  
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# Unboundedness in bilevel optimization

## Supplementary materials

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January 2025  
Les Cahiers du GERAD  
G–2025–22

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**Abstract :** Bilevel optimization has garnered growing interest over the past decade. However, little attention has been paid to detecting and dealing with unboundedness in these problems, with most research assuming a bounded high-point relaxation. In this paper, we address unboundedness in bilevel and multilevel optimization by studying its computational complexity. We show that deciding whether an optimistic linear bilevel problem is unbounded is strongly NP-complete, even without linking constraints. Furthermore, we extend the hardness result to the linear multilevel case, by showing that for each extra level added, the decision problem of checking unboundedness moves up a level in the polynomial hierarchy. Deciding unboundedness of a mixed-integer multilevel problem is shown to be one level higher in the polynomial complexity hierarchy than the decision problem for linear multilevel problem with the same number of levels. Finally, we introduce two algorithmic approaches to determine whether a linear bilevel problem is unbounded and, if so, return a certificate of unboundedness. This certificate consists of a direction of unboundedness and corresponding bilevel feasible point. We present a proof of concept of these algorithmic approaches on some relevant examples, and provide a brief computational comparison.

**Keywords :** Computational complexity, unbounded, bilevel optimization, multilevel optimization

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**Acknowledgements:** Our manuscript has an accompanying supplementary file containing the names of the BOBILib instances used in our computational experiments as detailed in Section ?? . This work was partially funded by the FRQ-IVADO Research Chair in Data Science for Combinatorial Game Theory.

This document contains the names of the BOBILib instances used in our computational experiments.

**Table 1: Instance Names on BOBILib Library.**

| Instance Names           |                          |
|--------------------------|--------------------------|
| general20-20-10-20-20-10 | general30-20-10-20-20-6  |
| general20-20-10-20-20-3  | general30-20-10-20-20-7  |
| general20-20-10-20-20-4  | general30-20-10-20-20-8  |
| general20-20-10-20-20-5  | general30-20-10-20-20-9  |
| general20-20-10-20-20-7  | general30-20-20-20-20-1  |
| general20-20-10-20-20-8  | general30-20-20-20-20-4  |
| general20-20-10-20-20-9  | general30-30-10-20-20-1  |
| general20-30-10-20-20-1  | general30-30-10-20-20-10 |
| general20-30-10-20-20-10 | general30-30-10-20-20-2  |
| general20-30-10-20-20-2  | general30-30-10-20-20-3  |
| general20-30-10-20-20-3  | general30-30-10-20-20-4  |
| general20-30-10-20-20-4  | general30-30-10-20-20-5  |
| general20-30-10-20-20-5  | general30-30-10-20-20-6  |
| general20-30-10-20-20-6  | general30-30-10-20-20-7  |
| general20-30-10-20-20-7  | general30-30-10-20-20-8  |
| general20-30-10-20-20-8  | general30-30-10-20-20-9  |
| general20-30-10-20-20-9  | general30-30-20-20-20-10 |
| general20-30-20-20-20-3  | general30-30-20-20-20-2  |
| general30-20-10-20-20-1  | general30-30-20-20-20-3  |
| general30-20-10-20-20-10 | general30-30-20-20-20-4  |
| general30-20-10-20-20-2  | general30-30-20-20-20-5  |
| general30-20-10-20-20-3  | general30-30-20-20-20-6  |
| general30-20-10-20-20-4  | general30-30-20-20-20-9  |
| general30-20-10-20-20-5  | milp_10_20_50_2310       |