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Abstract : We investigate the impact of information asymmetry regarding the producer of remanufactured products on the decisions of a manufacturer and an authorized remanufacturer (AR) in a competitive closed-loop supply chain. Information asymmetry affects consumers' perceived value of new and remanufactured products, thereby influencing market dynamics. In our model, the manufacturer licenses the AR to remanufacture products, with both parties independently setting prices and deciding on the policy of information disclosure about the remanufactured products' producer. The analysis shows that information symmetry generally results in higher prices for new products and lower prices for remanufactured products. However, when the AR's remanufacturing capacity is bound, there is a threshold in consumer recognition of remanufactured products (net gain value of product perceived value under information symmetry) which alters the price relationship. When the AR employs a partial remanufacturing strategy, information symmetry leads to a reduction in the manufacturer's licensing fee. Additionally, we find that information symmetry can cause the AR to adjust its remanufacturing strategies under certain conditions. Due to the perceived value effects, sometimes there is no threshold for product cost that can cause changes in the information disclosure policies. Furthermore, the independent decision-making of supply chain members sometimes can allow the manufacturer to benefit as a free rider. These findings highlight the complex interplay between information asymmetry, remanufacturing strategies, and remanufacturing licensing.

Keywords : Supply chain management, remanufacturing licensing, information disclosure, free rider, perceived value

In this Appendix, we provide the proofs of propositions and lemmas in the paper.

Proof of Propositions 1 and 2

The Hessian matrix of the AR's decision problem is: $\begin{pmatrix} \frac{-2}{\theta-\delta\rho} & \frac{-2}{\theta-\delta\rho} \\ \frac{2}{\theta-\delta\rho} & \frac{-2\theta}{\delta\rho(\theta-\delta\rho)} \end{pmatrix}$, and the first and second order leading principal minors are:$H_1 = \frac{-2}{\theta-\delta\rho} < 0$ & $H_2 = \frac{4}{\delta\rho(\theta-\delta\rho)} > 0$, so π_R^B is jointly concave in p_n and p_r. For the convenience of analysis and calculation, we transform the quantity constraints into price constraints. Therefore, by combining Equations (2) and $0 \leq q_r \leq q_n$, we can obtain $\frac{2\delta\rho p_n+\delta\rho(\delta\rho-\theta)}{\theta+\delta\rho} \leq p_r \leq \frac{\delta\rho p_n}{\theta}$.

The Lagrangian and Karush-Kuhn-Tucker optimality conditions of the AR's objective functions are as follows:

$$L = (p_n - w_n)\, q_n + (p_r - c_r - r)\, q_r + \lambda_1 \left(p_r - \frac{2\delta\rho p_n + \delta\rho(\delta\rho-\theta)}{\theta+\delta\rho}\right) + \lambda_2 \left(\frac{\delta\rho p_n}{\theta} - p_r\right),$$

$$s.t. \begin{cases} \frac{\partial L}{\partial p_n} = \frac{(\theta-\delta\rho)-2(p_n-p_r)+w_n-c_r-r}{\theta-\delta\rho} + \lambda_1 \frac{2\delta\rho}{\theta+\delta\rho} + \lambda_2 \frac{\delta\rho}{\theta} = 0, \\ \frac{\partial L}{\partial p_r} = \frac{2(\delta\rho p_n-\theta p_r)-\delta\rho w_n+\theta(c_r+r)}{\delta\rho(\theta-\delta\rho)} + \lambda_1 - \lambda_2 = 0, \\ (p_r - \frac{2\delta\rho p_n+\delta\rho(\delta\rho-\theta)}{\theta+\delta\rho}) = \lambda_2 \left(\frac{\delta\rho p_n}{\theta} - p_r\right) = 0, \\ p_r \geq \frac{2\delta\rho p_n+\delta\rho(\delta\rho-\theta)}{\theta+\delta\rho}, \\ p_r \leq \frac{\delta\rho p_n}{\theta}. \end{cases}$$

Scenario 1: $\lambda_1 = 0, \lambda_2 > 0$, $p_n^+ = \frac{\theta+w_n}{2}$, and $p_r^+ = \frac{\delta\rho(\theta+w_n)}{2\theta}$. To ensure $\lambda_2 > 0$, we can infer that $c_r > \frac{\delta\rho w_n - \theta r}{\theta}$.

Scenario 2: $\lambda_1 = \lambda_2 = 0$, $p_n^+ = \frac{\theta+w_n}{2}$, and $p_r^+ = \frac{\delta\rho+c_r+r}{2}$. To ensure $\frac{2\delta\rho p_n+\delta\rho(\delta\rho-\theta)}{\theta+\delta\rho} \leq p_r \leq \frac{\delta\rho p_n}{\theta}$, we can infer that $\frac{-\delta\rho(\theta-\delta\rho)-(\theta+\delta\rho)r-2\delta\rho w_n}{\theta+\delta\rho} \leq c_r \leq \frac{\delta\rho w_n-\theta r}{\theta}$.

Scenario 3: $\lambda_1 > 0, \lambda_2 = 0$, $p_n^+ = \frac{(\theta+\delta\rho)(w_n+c_r+r)+\theta^2-\delta^2\rho^2+4\delta\rho}{2(\theta+3\delta\rho)}$, and $p_r^+ = \frac{\delta\rho(w_n+c_r+r+2\delta\rho)}{\theta+3\delta\rho}$. To ensure $\lambda_1 > 0$, we can infer that $c_r < \frac{-\delta\rho(\theta-\delta\rho)-(\theta+\delta\rho)r-2\delta\rho w_n}{\theta+\delta\rho}$.

Scenario 4: When $\lambda_1 > 0$ and $\lambda_2 > 0$, it is impossible to have $0 = q_r = q_n$ at the same time.

In response to the above three cases, the AR's response function is substituted into the manufacturer's profit to continue to solve the optimal decision of manufacturer.

Scenario 1. $w_n^{B*} = \dfrac{c_n+\theta}{2}$. Scenario 2. $w_n^{B*} = \dfrac{c_n+c_r+\theta-\delta\rho}{2}+r$. Scenario 3. $w_n^{B*} = \dfrac{c_n-c_r+\theta+\delta\rho}{2}-r$.

By substituting $w_n^{B*}$of the three scenarios into p_n^+ and p_r^+, we can obtain the optimal retail price of the new product and the remanufactured product, i.e., p_n^{B*} and p_r^{B*}. Furthermore, substituting p_n^{B*} and p_r^{B*} into π_M^B and π_R^B, we can obtain the optimal payoff of the manufacturer and AR, i.e., π_M^{B*} and π_R^{B*}.

For Scenario 1, π_M^{B*} is not related to r. For Scenario 2, $\frac{\partial^2 \pi_{M-PR}^{B*}}{\partial r^2} = \frac{-1}{\delta\rho} < 0$ holds, so π_{M-PR}^{B*} is concave in r, so the optimal royalty fee is $r_{PR}^{B*} = \frac{\delta\rho-c_r}{2}$. For Scenario 3, π_M^{B*} is not related to r.

Substituting r^{B*} of three scenarios (If it exists) into w_n^{B*}, p_n^{B*}, p_r^{B*}, π_M^{B*} and π_R^{B*}, we can acquire the equilibrium decisions of the game between the manufacturer and the AR in Proposition 1.

By substituting the equilibrium decisions into the demand function (Equations (2)), we can easily obtain three sets of optimal quantities. For Scenario 1, $q_r^{B*} = 0$, $q_n^{B*} = \frac{\theta-c_n}{4\theta}$. For Scenario 2, $q_r^{B*} = \frac{\delta\rho c_n-\theta c_r}{4\delta\rho(\theta-\delta\rho)}$, $q_n^{B*} = \frac{\theta-\delta\rho-c_n+c_r}{4(\theta-\delta\rho)}$. For Scenario 3, $q_r^{B*} = q_n^{B*} = \frac{\theta+\delta\rho-c_n-c_r}{4(\theta+3\delta\rho)}$.

This completes the proof of Propositions 1 and 2. The proof of Propositions 3 to 5 is similar to that of the proof of Propositions 1 and 2, so we omit the proof process.

Proof of Lemma 1

In remanufacturing strategy NR, $w_n^{B*} - w_n^{A*} = \frac{\theta-1}{4} > 0$, $p_n^{B*} - p_n^{A*} = \frac{3(\theta-1)}{8} > 0$; In remanufacturing strategy PR, $w_n^{B*} - w_n^{A*} = \frac{\theta-1+(1-\delta)\rho}{4} > 0$, $p_n^{B*} - p_n^{A*} = \frac{3(\theta-1)+(1-\delta)\rho}{8} > 0$, $p_r^{B*} - p_r^{A*} = \frac{(\delta-1)\rho}{4} < 0$; In remanufacturing strategy FR, $w_n^{B*} - w_n^{A*} = \frac{\theta-1+(\delta-1)\rho}{4}$, $w_n^{B*} > w_n^{A*}$ if $\rho < \frac{\theta-1}{1-\delta}$; otherwise, $w_n^{B*} < w_n^{A*}$. Define $\rho_1 = \frac{\theta-1}{1-\delta}$. Considering that $\rho \in (0,1)$ and $\rho_1 > 0$ always hold, we have: when $0 < \rho_1 < 1$, $w_n^{B*} > w_n^{A*}$ if $\rho < \frac{\theta-1}{1-\delta}$, otherwise, $w_n^{B*} < w_n^{A*}$; when $\rho_1 > 1$, $w_n^{B*} > w_n^{A*}$ for any $\rho \in (0,1)$. Define $p_n^{BA} = p_n^{B*} - p_n^{A*}$, $p_r^{BA} = p_r^{B*} - p_r^{A*}$, and $\frac{\partial p_n^{BA}}{\partial c_r} = \frac{\theta+\delta\rho}{4(\theta+3\delta\rho)} - \frac{(\theta+1)+(\delta+1)\rho}{4((\theta+1)+3(\delta+1)\rho)}$, we have $\frac{\partial p_{n-FR}^{BA}}{\partial c_r} > 0$, so p_n^{BA} increases with c_r. c_{r1}^{BA} is given by solving $p_n^{BA} = 0$. $p_n^{B*} < p_n^{A*}$ if $c_r < c_{r1}^{BA}$; otherwise, $p_n^{B*} > p_n^{A*}$. $\frac{\partial p_{r-FR}^{BA}}{\partial c_r} = \frac{\delta\rho}{2(\theta+3\delta\rho)} - \frac{(\delta+1)\rho}{2((\theta+1)+3(\delta+1)\rho)}$, we have $\frac{\partial p_{r-FR}^{BA}}{\partial c_r} < 0$, so p_r^{BA} decreases with c_r. c_{r2}^{BA} is given by solving $p_r^{BA} = 0$. $p_r^{B*} > p_r^{A*}$ if $c_r < c_{r2}^{BA}$, otherwise, $p_r^{B*} < p_r^{A*}$.

This completes the proof of Lemma 1. The proof of Lemma 2 is similar to that of the proof of Lemma 1, so we omit the proof process.

Proof of Corollary 1

$r_{PR}^{B*} - r_{PR}^{A*} = \frac{\delta\rho-c_r}{2} - \frac{(\delta+1)\rho-2c_r}{4} = \frac{\rho(\delta-1)}{4} < 0$. This completes the proof of Corollary 1.

Proof of Lemma 3 and Corollary 2

Define $\pi_M^{MA} = \pi_M^{MD*} - \pi_M^{A*}$, $\pi_R^{MA} = \pi_R^{MD*} - \pi_R^{A*}$ in different remanufacturing strategies. In remanufacturing strategy NR, $\frac{\partial^2 \pi_M^{MA}}{\partial {c_n}^2} = \frac{1}{4\theta} - \frac{1}{2(\theta+1)} < 0$. Two solutions are obtained by solving $\pi_M^{MA} = 0$, but one of them is less than 0, and c_{n1}^{MA} is the other one greater than 0, where $c_{n1-NR}^{MA} = \frac{\sqrt{2}\theta(\theta+1)\sqrt{\frac{(\theta-1)(\theta-1-16C)}{\theta(\theta+1)}}}{2(\theta-1)}$. Therefore, $\pi_M^{MD*} < \pi_M^{A*}$ if $c_n > c_{n1-NR}^{MA}$, otherwise, $\pi_M^{MD*} > \pi_M^{A*}$. $\frac{\partial^2 \pi_R^{MA}}{\partial {c_n}^2} = \frac{1-\theta}{8\theta(\theta+1)} < 0$. Two solutions are obtained by solving $\pi_R^{MA} = 0$, but one of them is less than 0, and c_{n2-NR}^{MA} is the other one greater than 0, where $c_{n2-NR}^{MA} = \frac{\sqrt{2\theta(\theta+1)}}{2}$. Therefore, $\pi_R^{MD*} < \pi_R^{A*}$ if $c_n > c_{n2-NR}^{MA}$, otherwise, $\pi_R^{MD*} > \pi_R^{A*}$. This completes the proof of Lemma 3 and Corollary 2.

Proof of Lemma 5 and Corollary 3

In remanufacturing strategy PR, $\frac{\partial^2 \pi_M^{MA}}{\partial {c_n}^2} = \frac{1}{4(\theta-\delta\rho)} - \frac{1}{2((\theta+1)-(\delta+1)\rho)} < 0$. Two solutions are obtained by solving $\pi_M^{MA} = 0$, but one of them is less than 0, $c_{n1-PR}^{MA}$is greater than 0. Therefore, $\pi_M^{MD*} < \pi_M^{A*}$ if $c_n > c_{n1-PR}^{MA}$; otherwise, $\pi_M^{MD*} > \pi_M^{A*}$.

$\frac{\partial^2 \pi_R^{MA}}{\partial {c_n}^2} = \frac{1}{8(\theta-\delta\rho)} - \frac{1}{4((\theta+1)-(\delta+1)\rho)} < 0$. Two solutions are obtained by solving $\pi_R^{MA} = 0$, but one of them is less than 0, $c_{n2-PR}^{MA}$is greater than 0. Therefore, $\pi_R^{MD*} < \pi_R^{A*}$ if $c_n > c_{n2-PR}^{MA}$; otherwise, $\pi_R^{MD*} > \pi_R^{A*}$.

$\frac{\partial^2 \pi_M^{MA}}{\partial {c_r}^2} = \frac{\theta}{4\delta\rho(\theta-\delta\rho)} - \frac{(\theta+1)}{2(\delta+1)\rho((\theta+1)-(\delta+1)\rho)}$. To compare the difference between $\frac{\partial^2 \pi_M^{MA}}{\partial {c_r}^2}$ and 0, we can obtain $\rho = \frac{(1-\delta)(\theta+1)\theta}{\theta(1-\delta)(1+\delta)+2\delta(\theta-\delta)} = \rho_2$ by solving $\frac{\partial^2 \pi_M^{MA}}{\partial {c_r}^2} = 0$. Due to the constraints of $\delta \in (0,1)$ and $\theta \in (1,+\infty)$, $\rho_2 > 0$ always holds. Then, considering $\rho \in (0,1)$, we obtain two solutions $\theta_1' = \delta$ and $\theta_1 = \frac{2\delta}{1-\delta}$ by solving $\rho_2 = 1$, which is a univariate quadratic equation of θ. It is easy to obtain:

$0<\theta_1^{'}<1$ and $\theta_1^{'}<\theta_1$ always hold; $\theta_1<1$ if $\delta<\frac{1}{3}$, otherwise, $\theta_1>1$. According to the properties of quadratic function, we have: if $0<\delta<\frac{1}{3}$, for any $\theta>1$, $\rho_2\in(1,+\infty)$; if $\frac{1}{3}<\delta<1$, for $1<\theta<\theta_1$, $\rho_2\in(0,1)$, for $\theta_1<\theta<+\infty$, $\rho_2\in(1,+\infty)$. Back to the relationship between $\frac{\partial^2\pi_M^{MA}}{\partial {c_r}^2}$ and 0, we have: when $\rho_2\in(0,1)$, $\frac{\partial^2\pi_M^{MA}}{\partial {c_r}^2}>0$ if $0<\rho<\rho_2$, $\frac{\partial^2\pi_M^{MA}}{\partial {c_r}^2}<0$ if $\rho_2<\rho<1$; when $\rho_2\in(1,+\infty)$, for any $\rho\in(0,1)$, $\frac{\partial^2\pi_M^{MA}}{\partial {c_r}^2}>0$. $\bar{c}_{r1-PR}^{MA}$ and c_{r1-PR}^{MA} are given by solving $\pi_M^{MA}=0$. Thus, when $\frac{\partial^2\pi_M^{MA}}{\partial {c_r}^2}<0$, $\pi_M^{MD*}<\pi_M^{A*}$if $c_r<\min\left\{\bar{c}_{r1-PR}^{MA},c_{r1-PR}^{MA}\right\}$ or $c_r>\max\left\{\bar{c}_{r1-PR}^{MA},c_{r1-PR}^{MA}\right\}$, otherwise, $\pi_M^{MD*}>\pi_M^{A*}$. When $\frac{\partial^2\pi_M^{MA}}{\partial {c_r}^2}>0$, $\pi_M^{MD*}>\pi_M^{A*}$if $c_r<\min\left\{\bar{c}_{r1-PR}^{MA},c_{r1-PR}^{MA}\right\}$ or $c_r>\max\left\{\bar{c}_{r1-PR}^{MA},c_{r1-PR}^{MA}\right\}$, otherwise, $\pi_{MR}^{MD*}<\pi_M^{A*}$. Then, we obtain: (a) When $0<\delta<\frac{1}{3}$, for any $\theta\in(1,+\infty)$, $\rho\in(0,1)$, $\bar{c}_{r1-PR}^{MA}<0$, $0<c_{r1-PR}^{MA}<c_n$, so we eliminate $\bar{c}_{r1-PR}^{MA}$; (b) when $\frac{1}{3}<\delta<1$, if $1<\theta<\theta_1$, for $0<\rho<\rho_2$, $\bar{c}_{r1-PR}^{MA}<0$, $0<c_{r1-PR}^{MA}<c_n$, so we eliminate $\bar{c}_{r1-PR}^{MA}$; (c) when $\frac{1}{3}<\delta<1$, if $1<\theta<\theta_1$, for $\rho_2<\rho<1$, $\bar{c}_{r1-PR}^{MA}>c_n$, $0<c_{r1-PR}^{MA}<c_n$, so we eliminate $\bar{c}_{r1-PR}^{MA}$; (d) when $\frac{1}{3}<\delta<1$, if $\theta>\theta_1$, for any $\rho\in(0,1)$, $\bar{c}_{r1-PR}^{MA}<0$, $c_{r1-PR}^{MA}>c_n$, so we eliminate both of them.

$\frac{\partial^2\pi_R^{MA}}{\partial {c_r}^2}=\frac{\theta}{8\delta\rho(\theta-\delta\rho)}-\frac{(\theta+1)}{4(\delta+1)\rho((\theta+1)-(\delta+1)\rho)}$, $\frac{\partial^2\pi_R^{MA}}{\partial {c_r}^2}=\frac{1}{2}\times\frac{\partial^2\pi_M^{MA}}{\partial {c_r}^2}$. Therefore, the relationship between $\frac{\partial^2\pi_R^{MA}}{\partial {c_r}^2}$ and 0 is exactly the same as that between $\frac{\partial^2\pi_M^{MA}}{\partial {c_r}^2}$ and 0. $\bar{c}_{r2-PR}^{MA}$ and c_{r2-PR}^{MA} are given by solving $\pi_R^{MA}=0$. The following proof is similar to the previous one, so we omit the process.

The specific expressions for all thresholds of product costs are very complex, so we omit them here.

On the basis of these, it is easy to prove Lemma 4 and Corollary 3.

Proof of Lemma 5 and Corollary 4

In remanufacturing strategy FR, $\frac{\partial^2\pi_M^{MA}}{\partial {c_n}^2}=\frac{1}{4(\theta+3\delta\rho)}-\frac{1}{2((\theta+1)+3(\delta+1)\rho)}$. To compare the difference between $\frac{\partial^2\pi_M^{MA}}{\partial {c_n}^2}$and 0, we can obtain $\rho=\frac{(\theta-1)}{3(1-\delta)}=\frac{1}{3}\rho_1$by solving $\frac{\partial^2\pi_M^{MA}}{\partial {c_n}^2}=0$. We have: when $0<\rho_1<3$, $\frac{\partial^2\pi_M^{MA}}{\partial {c_n}^2}<0$ if $0<\rho<\frac{1}{3}\rho_1$, $\frac{\partial^2\pi_M^{MA}}{\partial {c_n}^2}>0$ if $\frac{1}{3}\rho_1<\rho<1$; when $\rho_1>3$, $\frac{\partial^2\pi_M^{MA}}{\partial {c_n}^2}<0$ for any $\rho\in(0,1)$. $\bar{c}_{n1-FR}^{MA}$ and $c_{n1-FR}^{MA}$are given by solving $\pi_M^{MA}=0$. Thus, when $\frac{\partial^2\pi_M^{MA}}{\partial {c_n}^2}>0$, $\pi_M^{MD*}>\pi_M^{A*}$ if $c_n<\min\left\{\bar{c}_{n1-FR}^{MA},c_{n1-FR}^{MA}\right\}$ or $c_n>\max\left\{\bar{c}_{n1-FR}^{MA},c_{n1-FR}^{MA}\right\}$, otherwise, $\pi_M^{MD*}<\pi_M^{A*}$. When $\frac{\partial^2\pi_M^{MA}}{\partial {c_n}^2}<0$, $\pi_M^{MD*}<\pi_M^{A*}$ if $c_n<\min\left\{\bar{c}_{n1-FR}^{MA},c_{n1-FR}^{MA}\right\}$ or $c_n>\max\left\{\bar{c}_{n1-FR}^{MA},c_{n1-FR}^{MA}\right\}$, otherwise, $\pi_M^{MD*}>\pi_M^{A*}$. Then, we can obtain:

(a) When $0<\frac{\theta-1}{1-\delta}<3$, for $0<\rho<\frac{1}{3}\rho_1$, both c_{n1-FR}^{MA} and $\bar{c}_{n1-FR}^{MA}<c_r$, so we eliminate both of them.

(b) when $0<\frac{\theta-1}{1-\delta}<3$, for $\frac{1}{3}\rho_1<\rho<1$, $\bar{c}_{n1-FR}^{MA}<c_r$, c_{n1-FR}^{MA} is too large, so we eliminate both of them.

(c) when $\frac{\theta-1}{1-\delta}>3$, for any $\rho\in(0,1)$, both c_{n1-FR}^{MA} and $\bar{c}_{n1-FR}^{MA}<c_r$, so we eliminate both of them.

$\frac{\partial^2\pi_R^{MA}}{\partial {c_n}^2}=\frac{1}{8(\theta+3\delta\rho)}-\frac{1}{4((\theta+1)+3(\delta+1)\rho)}$. To compare the difference between $\frac{\partial^2\pi_R^{MA}}{\partial {c_n}^2}$ and 0, we can obtain $\rho=\frac{(\theta-1)}{3(1-\delta)}=\frac{1}{3}\rho_1$ by solving $\frac{\partial^2\pi_R^{MA}}{\partial {c_n}^2}=0$. We have: when$0<\rho_1<3$, $\frac{\partial^2\pi_R^{MA}}{\partial {c_n}^2}<0$ if $0<\rho<\frac{1}{3}\rho_1$, $\frac{\partial^2\pi_R^{MA}}{\partial {c_n}^2}>0$ if $\frac{1}{3}\rho_1<\rho<1$; when $\rho_1>3$, $\frac{\partial^2\pi_R^{MA}}{\partial {c_n}^2}<0$ for any $\rho\in(0,1)$. $\bar{c}_{n2-FR}^{MA}$ and $c_{n2-FR}^{MA}$are given by solving $\pi_R^{MA}=0$. Thus, when $\frac{\partial^2\pi_R^{MA}}{\partial {c_n}^2}>0$, $\pi_R^{MD*}>\pi_R^{A*}$ if $c_n<\min\left\{\bar{c}_{n2-FR}^{MA},c_{n2-FR}^{MA}\right\}$ or $c_n>\max\left\{\bar{c}_{n2-FR}^{MA},c_{n2-FR}^{MA}\right\}$, otherwise, $\pi_R^{MD*}<\pi_R^{A*}$. When $\frac{\partial^2\pi_R^{MA}}{\partial {c_n}^2}<0$, $\pi_R^{MD*}<\pi_R^{A*}$ if $c_n<\min\left\{\bar{c}_{n2-FR}^{MA},c_{n2-FR}^{MA}\right\}$ or $c_n>\max\left\{\bar{c}_{n2-FR}^{MA},c_{n2-FR}^{MA}\right\}$, otherwise, $\pi_R^{MD*}>\pi_R^{A*}$. Then, we can obtain: (a) When $0<\frac{\theta-1}{1-\delta}<3$, for $0<\rho<\frac{1}{3}\rho_1$, $\bar{c}_{n2-FR}^{MA}<c_r$, so we eliminate it; (b) when $0<\frac{\theta-1}{1-\delta}<3$, for $\frac{1}{3}\rho_1<\rho<1$, $\bar{c}_{n1-FR}^{MA}$ is too large, so we eliminate it; (c) when $\frac{\theta-1}{1-\delta}>3$, for any $\rho\in(0,1)$, $\bar{c}_{n1-FR}^{MA}<c_r$, so we eliminate it.

$\frac{\partial^2 \pi_M^{MA}}{\partial {c_r}^2} = \frac{1}{4(\theta+3\delta\rho)} - \frac{1}{2((\theta+1)+3(\delta+1)\rho)}$, $\frac{\partial^2 \pi_R^{MA}}{\partial {c_r}^2} = \frac{1}{8(\theta+3\delta\rho)} - \frac{1}{4((\theta+1)+3(\delta+1)\rho)}$. The following proof is similar to the previous one, so we omit the process.

The specific expressions for all thresholds of product costs are very complex, so we omit them here.

On this basis, it is easy to prove Lemma 5 and Corollary 4.

Proof of Lemma 6

Since π_M^* and π_R^* always have multiple relationships in model RD and model A, i.e. $\pi_M^{RD*} = 2\pi_R^{RD*}and\pi_M^{A*} = 2\pi_R^{A*}$, the comparative relationship between $\pi_R^{RD*}$and$\pi_R^{A*}$is completely consistent with π_M^{RD*} and π_M^{A*}. Define $\pi_M^{RA} = \pi_M^{RD*} - \pi_M^{A*}$in different remanufacturing strategies, The following proof is similar to the previous one, so we omit the process. This completes the proof of Lemma 7.

The proof of Lemmas 7 to 9 and Corollaries 5 to 6 is similar to the above Lemma and Corollary, so we omit it.