

# Producer information disclosure decision of remanufactured product under licensing

## Online Supplement

Y. Jiang, W. Li, G. Zaccour

G-2025-04

January 2025

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**Citation suggérée :** Y. Jiang, W. Li, G. Zaccour (Janvier 2025). Producer information disclosure decision of remanufactured product under licensing, Online Supplement. Rapport technique, Les Cahiers du GERAD G- 2025-04, GERAD, HEC Montréal, Canada.

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**Suggested citation:** Y. Jiang, W. Li, G. Zaccour (January 2025). Producer information disclosure decision of remanufactured product under licensing, Online Supplement. Technical report, Les Cahiers du GERAD G-2025-04, GERAD, HEC Montréal, Canada.

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# Producer information disclosure decision of remanufactured product under licensing

## Online Supplement

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January 2025  
Les Cahiers du GERAD  
G-2025-04

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**Abstract :** We investigate the impact of information asymmetry regarding the producer of remanufactured products on the decisions of a manufacturer and an authorized remanufacturer (AR) in a competitive closed-loop supply chain. Information asymmetry affects consumers' perceived value of new and remanufactured products, thereby influencing market dynamics. In our model, the manufacturer licenses the AR to remanufacture products, with both parties independently setting prices and deciding on the policy of information disclosure about the remanufactured products' producer. The analysis shows that information symmetry generally results in higher prices for new products and lower prices for remanufactured products. However, when the AR's remanufacturing capacity is bound, there is a threshold in consumer recognition of remanufactured products (net gain value of product perceived value under information symmetry) which alters the price relationship. When the AR employs a partial remanufacturing strategy, information symmetry leads to a reduction in the manufacturer's licensing fee. Additionally, we find that information symmetry can cause the AR to adjust its remanufacturing strategies under certain conditions. Due to the perceived value effects, sometimes there is no threshold for product cost that can cause changes in the information disclosure policies. Furthermore, the independent decision-making of supply chain members sometimes can allow the manufacturer to benefit as a free rider. These findings highlight the complex interplay between information asymmetry, remanufacturing strategies, and remanufacturing licensing.

**Keywords :** Supply chain management, remanufacturing licensing, information disclosure, free rider, perceived value

In this Appendix, we provide the proofs of propositions and lemmas in the paper.

## Proof of Propositions 1 and 2

The Hessian matrix of the AR's decision problem is:  $\begin{pmatrix} \frac{-2}{\theta-\delta\rho} & \frac{-2}{\theta-\delta\rho} \\ \frac{2}{\theta-\delta\rho} & \frac{-2\theta}{\delta\rho(\theta-\delta\rho)} \end{pmatrix}$ , and the first and second order leading principal minors are:  $H_1 = \frac{-2}{\theta-\delta\rho} < 0$  &  $H_2 = \frac{4}{\delta\rho(\theta-\delta\rho)} > 0$ , so  $\pi_R^B$  is jointly concave in  $p_n$  and  $p_r$ . For the convenience of analysis and calculation, we transform the quantity constraints into price constraints. Therefore, by combining Equations (2) and  $0 \leq q_r \leq q_n$ , we can obtain  $\frac{2\delta\rho p_n + \delta\rho(\delta\rho - \theta)}{\theta + \delta\rho} \leq p_r \leq \frac{\delta\rho p_n}{\theta}$ .

The Lagrangian and Karush-Kuhn-Tucker optimality conditions of the AR's objective functions are as follows:

$$L = (p_n - w_n)q_n + (p_r - c_r - r)q_r + \lambda_1 \left( p_r - \frac{2\delta\rho p_n + \delta\rho(\delta\rho - \theta)}{\theta + \delta\rho} \right) + \lambda_2 \left( \frac{\delta\rho p_n}{\theta} - p_r \right),$$

$$s.t. \begin{cases} \frac{\partial L}{\partial p_n} = \frac{(\theta - \delta\rho) - 2(p_n - p_r) + w_n - c_r - r}{\theta - \delta\rho} + \lambda_1 \frac{2\delta\rho}{\theta + \delta\rho} + \lambda_2 \frac{\delta\rho}{\theta} = 0, \\ \frac{\partial L}{\partial p_r} = \frac{2(\delta\rho p_n - \theta p_r) - \delta\rho w_n + \theta(c_r + r)}{\delta\rho(\theta - \delta\rho)} + \lambda_1 - \lambda_2 = 0, \\ (p_r - \frac{2\delta\rho p_n + \delta\rho(\delta\rho - \theta)}{\theta + \delta\rho}) = \lambda_2 \left( \frac{\delta\rho p_n}{\theta} - p_r \right) = 0, \\ p_r \geq \frac{2\delta\rho p_n + \delta\rho(\delta\rho - \theta)}{\theta + \delta\rho}, \\ p_r \leq \frac{\delta\rho p_n}{\theta}. \end{cases}$$

**Scenario 1:**  $\lambda_1 = 0, \lambda_2 > 0, p_n^+ = \frac{\theta + w_n}{2}$ , and  $p_r^+ = \frac{\delta\rho(\theta + w_n)}{2\theta}$ . To ensure  $\lambda_2 > 0$ , we can infer that  $c_r > \frac{\delta\rho w_n - \theta r}{\theta}$ .

**Scenario 2:**  $\lambda_1 = \lambda_2 = 0, p_n^+ = \frac{\theta + w_n}{2}$ , and  $p_r^+ = \frac{\delta\rho + c_r + r}{2}$ . To ensure  $\frac{2\delta\rho p_n + \delta\rho(\delta\rho - \theta)}{\theta + \delta\rho} \leq p_r \leq \frac{\delta\rho p_n}{\theta}$ , we can infer that  $\frac{-\delta\rho(\theta - \delta\rho) - (\theta + \delta\rho)r - 2\delta\rho w_n}{\theta + \delta\rho} \leq c_r \leq \frac{\delta\rho w_n - \theta r}{\theta}$ .

**Scenario 3:**  $\lambda_1 > 0, \lambda_2 = 0, p_n^+ = \frac{(\theta + \delta\rho)(w_n + c_r + r) + \theta^2 - \delta^2\rho^2 + 4\delta\rho}{2(\theta + 3\delta\rho)}$ , and  $p_r^+ = \frac{\delta\rho(w_n + c_r + r + 2\delta\rho)}{\theta + 3\delta\rho}$ . To ensure  $\lambda_1 > 0$ , we can infer that  $c_r < \frac{-\delta\rho(\theta - \delta\rho) - (\theta + \delta\rho)r - 2\delta\rho w_n}{\theta + \delta\rho}$ .

**Scenario 4:** When  $\lambda_1 > 0$  and  $\lambda_2 > 0$ , it is impossible to have  $0 = q_r = q_n$  at the same time.

In response to the above three cases, the AR's response function is substituted into the manufacturer's profit to continue to solve the optimal decision of manufacturer.

Scenario 1.  $w_n^{B*} = \frac{c_n + \theta}{2}$ . Scenario 2.  $w_n^{B*} = \frac{c_n + c_r + \theta - \delta\rho}{2} + r$ . Scenario 3.  $w_n^{B*} = \frac{c_n - c_r + \theta + \delta\rho}{2} - r$ .

By substituting  $w_n^{B*}$  of the three scenarios into  $p_n^+$  and  $p_r^+$ , we can obtain the optimal retail price of the new product and the remanufactured product, i.e.,  $p_n^{B*}$  and  $p_r^{B*}$ . Furthermore, substituting  $p_n^{B*}$  and  $p_r^{B*}$  into  $\pi_M^B$  and  $\pi_R^B$ , we can obtain the optimal payoff of the manufacturer and AR, i.e.,  $\pi_M^{B*}$  and  $\pi_R^{B*}$ .

For Scenario 1,  $\pi_M^{B*}$  is not related to  $r$ . For Scenario 2,  $\frac{\partial^2 \pi_{M-P}^{B*}}{\partial r^2} = \frac{-1}{\delta\rho} < 0$  holds, so  $\pi_{M-P}^{B*}$  is concave in  $r$ , so the optimal royalty fee is  $r_{PR}^{B*} = \frac{\delta\rho - c_r}{2}$ . For Scenario 3,  $\pi_M^{B*}$  is not related to  $r$ .

Substituting  $r^{B*}$  of three scenarios (If it exists) into  $w_n^{B*}, p_n^{B*}, p_r^{B*}, \pi_M^{B*}$  and  $\pi_R^{B*}$ , we can acquire the equilibrium decisions of the game between the manufacturer and the AR in Proposition 1.

By substituting the equilibrium decisions into the demand function (Equations (2)), we can easily obtain three sets of optimal quantities. For Scenario 1,  $q_r^{B*} = 0, q_n^{B*} = \frac{\theta - c_n}{4\theta}$ . For Scenario 2,  $q_r^{B*} = \frac{\delta\rho c_n - \theta c_r}{4\delta\rho(\theta - \delta\rho)}, q_n^{B*} = \frac{\theta - \delta\rho - c_n + c_r}{4(\theta - \delta\rho)}$ . For Scenario 3,  $q_r^{B*} = q_n^{B*} = \frac{\theta + \delta\rho - c_n - c_r}{4(\theta + 3\delta\rho)}$ .

This completes the proof of [Propositions 1](#) and [2](#). The proof of [Propositions 3](#) to [5](#) is similar to that of the proof of [Propositions 1](#) and [2](#), so we omit the proof process.

## Proof of [Lemma 1](#)

In remanufacturing strategy NR,  $w_n^{B*} - w_n^{A*} = \frac{\theta-1}{4} > 0$ ,  $p_n^{B*} - p_n^{A*} = \frac{3(\theta-1)}{8} > 0$ ; In remanufacturing strategy PR,  $w_n^{B*} - w_n^{A*} = \frac{\theta-1+(1-\delta)\rho}{4} > 0$ ,  $p_n^{B*} - p_n^{A*} = \frac{3(\theta-1)+(1-\delta)\rho}{8} > 0$ ,  $p_r^{B*} - p_r^{A*} = \frac{(\delta-1)\rho}{4} < 0$ ; In remanufacturing strategy FR,  $w_n^{B*} - w_n^{A*} = \frac{\theta-1+(\delta-1)\rho}{4}$ ,  $w_n^{B*} > w_n^{A*}$  if  $\rho < \frac{\theta-1}{1-\delta}$ ; otherwise,  $w_n^{B*} < w_n^{A*}$ . Define  $\rho_1 = \frac{\theta-1}{1-\delta}$ . Considering that  $\rho \in (0, 1)$  and  $\rho_1 > 0$  always hold, we have: when  $0 < \rho_1 < 1$ ,  $w_n^{B*} > w_n^{A*}$  if  $\rho < \frac{\theta-1}{1-\delta}$ , otherwise,  $w_n^{B*} < w_n^{A*}$ ; when  $\rho_1 > 1$ ,  $w_n^{B*} > w_n^{A*}$  for any  $\rho \in (0, 1)$ . Define  $p_n^{BA} = p_n^{B*} - p_n^{A*}$ ,  $p_r^{BA} = p_r^{B*} - p_r^{A*}$ , and  $\frac{\partial p_n^{BA}}{\partial c_r} = \frac{\theta+\delta\rho}{4(\theta+3\delta\rho)} - \frac{(\theta+1)+(\delta+1)\rho}{4((\theta+1)+3(\delta+1)\rho)}$ , we have  $\frac{\partial p_n^{BA}}{\partial c_r} > 0$ , so  $p_n^{BA}$  increases with  $c_r$ .  $c_{r1}^{BA}$  is given by solving  $p_n^{BA} = 0$ .  $p_n^{B*} < p_n^{A*}$  if  $c_r < c_{r1}^{BA}$ ; otherwise,  $p_n^{B*} > p_n^{A*}$ .  $\frac{\partial p_r^{BA}}{\partial c_r} = \frac{\delta\rho}{2(\theta+3\delta\rho)} - \frac{(\delta+1)\rho}{2((\theta+1)+3(\delta+1)\rho)}$ , we have  $\frac{\partial p_r^{BA}}{\partial c_r} < 0$ , so  $p_r^{BA}$  decreases with  $c_r$ .  $c_{r2}^{BA}$  is given by solving  $p_r^{BA} = 0$ .  $p_r^{B*} > p_r^{A*}$  if  $c_r < c_{r2}^{BA}$ , otherwise,  $p_r^{B*} < p_r^{A*}$ .

This completes the proof of [Lemma 1](#). The proof of [Lemma 2](#) is similar to that of the proof of [Lemma 1](#), so we omit the proof process.

## Proof of [Corollary 1](#)

$r_{PR}^{B*} - r_{PR}^{A*} = \frac{\delta\rho - c_r}{2} - \frac{(\delta+1)\rho - 2c_r}{4} = \frac{\rho(\delta-1)}{4} < 0$ . This completes the proof of [Corollary 1](#).

## Proof of [Lemma 3](#) and [Corollary 2](#)

Define  $\pi_M^{MA} = \pi_M^{MD*} - \pi_M^{A*}$ ,  $\pi_R^{MA} = \pi_R^{MD*} - \pi_R^{A*}$  in different remanufacturing strategies. In remanufacturing strategy NR,  $\frac{\partial^2 \pi_M^{MA}}{\partial c_n^2} = \frac{1}{4\theta} - \frac{1}{2(\theta+1)} < 0$ . Two solutions are obtained by solving  $\pi_M^{MA} = 0$ , but one of them is less than 0, and  $c_{n1}^{MA}$  is the other one greater than 0, where  $c_{n1-NR}^{MA} = \frac{\sqrt{2\theta(\theta+1)}\sqrt{\frac{(\theta-1)(\theta-1-16C)}{\theta(\theta+1)}}}{2(\theta-1)}$ . Therefore,  $\pi_M^{MD*} < \pi_M^{A*}$  if  $c_n > c_{n1-NR}^{MA}$ , otherwise,  $\pi_M^{MD*} > \pi_M^{A*}$ .  $\frac{\partial^2 \pi_R^{MA}}{\partial c_n^2} = \frac{1-\theta}{8\theta(\theta+1)} < 0$ . Two solutions are obtained by solving  $\pi_R^{MA} = 0$ , but one of them is less than 0, and  $c_{n2-NR}^{MA}$  is the other one greater than 0, where  $c_{n2-NR}^{MA} = \frac{\sqrt{2\theta(\theta+1)}}{2}$ . Therefore,  $\pi_R^{MD*} < \pi_R^{A*}$  if  $c_n > c_{n2-NR}^{MA}$ , otherwise,  $\pi_R^{MD*} > \pi_R^{A*}$ . This completes the proof of [Lemma 3](#) and [Corollary 2](#).

## Proof of [Lemma 5](#) and [Corollary 3](#)

In remanufacturing strategy PR,  $\frac{\partial^2 \pi_M^{MA}}{\partial c_n^2} = \frac{1}{4(\theta-\delta\rho)} - \frac{1}{2((\theta+1)-(\delta+1)\rho)} < 0$ . Two solutions are obtained by solving  $\pi_M^{MA} = 0$ , but one of them is less than 0,  $c_{n1-PR}^{MA}$  is greater than 0. Therefore,  $\pi_M^{MD*} < \pi_M^{A*}$  if  $c_n > c_{n1-PR}^{MA}$ ; otherwise,  $\pi_M^{MD*} > \pi_M^{A*}$ .

$\frac{\partial^2 \pi_R^{MA}}{\partial c_n^2} = \frac{1}{8(\theta-\delta\rho)} - \frac{1}{4((\theta+1)-(\delta+1)\rho)} < 0$ . Two solutions are obtained by solving  $\pi_R^{MA} = 0$ , but one of them is less than 0,  $c_{n2-PR}^{MA}$  is greater than 0. Therefore,  $\pi_R^{MD*} < \pi_R^{A*}$  if  $c_n > c_{n2-PR}^{MA}$ ; otherwise,  $\pi_R^{MD*} > \pi_R^{A*}$ .

$\frac{\partial^2 \pi_M^{MA}}{\partial c_r^2} = \frac{\theta}{4\delta\rho(\theta-\delta\rho)} - \frac{(\theta+1)}{2(\delta+1)\rho((\theta+1)-(\delta+1)\rho)}$ . To compare the difference between  $\frac{\partial^2 \pi_M^{MA}}{\partial c_r^2}$  and 0, we can obtain  $\rho = \frac{(1-\delta)(\theta+1)\theta}{\theta(1-\delta)(1+\delta)+2\delta(\theta-\delta)} = \rho_2$  by solving  $\frac{\partial^2 \pi_M^{MA}}{\partial c_r^2} = 0$ . Due to the constraints of  $\delta \in (0, 1)$  and  $\theta \in (1, +\infty)$ ,  $\rho_2 > 0$  always holds. Then, considering  $\rho \in (0, 1)$ , we obtain two solutions  $\theta'_1 = \delta$  and  $\theta_1 = \frac{2\delta}{1-\delta}$  by solving  $\rho_2 = 1$ , which is a univariate quadratic equation of  $\theta$ . It is easy to obtain:

$0 < \theta'_1 < 1$  and  $\theta'_1 < \theta_1$  always hold;  $\theta_1 < 1$  if  $\delta < \frac{1}{3}$ , otherwise,  $\theta_1 > 1$ . According to the properties of quadratic function, we have: if  $0 < \delta < \frac{1}{3}$ , for any  $\theta > 1$ ,  $\rho_2 \in (1, +\infty)$ ; if  $\frac{1}{3} < \delta < 1$ , for  $1 < \theta < \theta_1$ ,  $\rho_2 \in (0, 1)$ , for  $\theta_1 < \theta < +\infty$ ,  $\rho_2 \in (1, +\infty)$ . Back to the relationship between  $\frac{\partial^2 \pi_M^{MA}}{\partial c_r^2}$  and 0, we have: when  $\rho_2 \in (0, 1)$ ,  $\frac{\partial^2 \pi_M^{MA}}{\partial c_r^2} > 0$  if  $0 < \rho < \rho_2$ ,  $\frac{\partial^2 \pi_M^{MA}}{\partial c_r^2} < 0$  if  $\rho_2 < \rho < 1$ ; when  $\rho_2 \in (1, +\infty)$ , for any  $\rho \in (0, 1)$ ,  $\frac{\partial^2 \pi_M^{MA}}{\partial c_r^2} > 0$ .  $\bar{c}_{r1-PR}^{MA}$  and  $c_{r1-PR}^{MA}$  are given by solving  $\pi_M^{MA} = 0$ . Thus, when  $\frac{\partial^2 \pi_M^{MA}}{\partial c_r^2} < 0$ ,  $\pi_M^{MD*} < \pi_M^{A*}$  if  $c_r < \min\{\bar{c}_{r1-PR}^{MA}, c_{r1-PR}^{MA}\}$  or  $c_r > \max\{\bar{c}_{r1-PR}^{MA}, c_{r1-PR}^{MA}\}$ , otherwise,  $\pi_M^{MD*} > \pi_M^{A*}$ . When  $\frac{\partial^2 \pi_M^{MA}}{\partial c_r^2} > 0$ ,  $\pi_M^{MD*} > \pi_M^{A*}$  if  $c_r < \min\{\bar{c}_{r1-PR}^{MA}, c_{r1-PR}^{MA}\}$  or  $c_r > \max\{\bar{c}_{r1-PR}^{MA}, c_{r1-PR}^{MA}\}$ , otherwise,  $\pi_M^{MD*} < \pi_M^{A*}$ . Then, we obtain: (a) When  $0 < \delta < \frac{1}{3}$ , for any  $\theta \in (1, +\infty)$ ,  $\rho \in (0, 1)$ ,  $\bar{c}_{r1-PR}^{MA} < 0$ ,  $0 < c_{r1-PR}^{MA} < c_n$ , so we eliminate  $\bar{c}_{r1-PR}^{MA}$ ; (b) when  $\frac{1}{3} < \delta < 1$ , if  $1 < \theta < \theta_1$ , for  $0 < \rho < \rho_2$ ,  $\bar{c}_{r1-PR}^{MA} < 0$ ,  $0 < c_{r1-PR}^{MA} < c_n$ , so we eliminate  $\bar{c}_{r1-PR}^{MA}$ ; (c) when  $\frac{1}{3} < \delta < 1$ , if  $1 < \theta < \theta_1$ , for  $\rho_2 < \rho < 1$ ,  $\bar{c}_{r1-PR}^{MA} > c_n$ ,  $0 < c_{r1-PR}^{MA} < c_n$ , so we eliminate  $\bar{c}_{r1-PR}^{MA}$ ; (d) when  $\frac{1}{3} < \delta < 1$ , if  $\theta > \theta_1$ , for any  $\rho \in (0, 1)$ ,  $\bar{c}_{r1-PR}^{MA} < 0$ ,  $c_{r1-PR}^{MA} > c_n$ , so we eliminate both of them.

$\frac{\partial^2 \pi_R^{MA}}{\partial c_r^2} = \frac{\theta}{8\delta\rho(\theta-\delta\rho)} - \frac{(\theta+1)}{4(\delta+1)\rho((\theta+1)-(\delta+1)\rho)}$ ,  $\frac{\partial^2 \pi_R^{MA}}{\partial c_r^2} = \frac{1}{2} \times \frac{\partial^2 \pi_M^{MA}}{\partial c_r^2}$ . Therefore, the relationship between  $\frac{\partial^2 \pi_R^{MA}}{\partial c_r^2}$  and 0 is exactly the same as that between  $\frac{\partial^2 \pi_M^{MA}}{\partial c_r^2}$  and 0.  $\bar{c}_{r2-PR}^{MA}$  and  $c_{r2-PR}^{MA}$  are given by solving  $\pi_R^{MA} = 0$ . The following proof is similar to the previous one, so we omit the process.

The specific expressions for all thresholds of product costs are very complex, so we omit them here.

On the basis of these, it is easy to prove [Lemma 4](#) and [Corollary 3](#).

## Proof of [Lemma 5](#) and [Corollary 4](#)

In remanufacturing strategy FR,  $\frac{\partial^2 \pi_M^{MA}}{\partial c_n^2} = \frac{1}{4(\theta+3\delta\rho)} - \frac{1}{2((\theta+1)+3(\delta+1)\rho)}$ . To compare the difference between  $\frac{\partial^2 \pi_M^{MA}}{\partial c_n^2}$  and 0, we can obtain  $\rho = \frac{(\theta-1)}{3(1-\delta)} = \frac{1}{3}\rho_1$  by solving  $\frac{\partial^2 \pi_M^{MA}}{\partial c_n^2} = 0$ . We have: when  $0 < \rho_1 < 3$ ,  $\frac{\partial^2 \pi_M^{MA}}{\partial c_n^2} < 0$  if  $0 < \rho < \frac{1}{3}\rho_1$ ,  $\frac{\partial^2 \pi_M^{MA}}{\partial c_n^2} > 0$  if  $\frac{1}{3}\rho_1 < \rho < 1$ ; when  $\rho_1 > 3$ ,  $\frac{\partial^2 \pi_M^{MA}}{\partial c_n^2} < 0$  for any  $\rho \in (0, 1)$ .  $\bar{c}_{n1-FR}^{MA}$  and  $c_{n1-FR}^{MA}$  are given by solving  $\pi_M^{MA} = 0$ . Thus, when  $\frac{\partial^2 \pi_M^{MA}}{\partial c_n^2} > 0$ ,  $\pi_M^{MD*} > \pi_M^{A*}$  if  $c_n < \min\{\bar{c}_{n1-FR}^{MA}, c_{n1-FR}^{MA}\}$  or  $c_n > \max\{\bar{c}_{n1-FR}^{MA}, c_{n1-FR}^{MA}\}$ , otherwise,  $\pi_M^{MD*} < \pi_M^{A*}$ . When  $\frac{\partial^2 \pi_M^{MA}}{\partial c_n^2} < 0$ ,  $\pi_M^{MD*} < \pi_M^{A*}$  if  $c_n < \min\{\bar{c}_{n1-FR}^{MA}, c_{n1-FR}^{MA}\}$  or  $c_n > \max\{\bar{c}_{n1-FR}^{MA}, c_{n1-FR}^{MA}\}$ , otherwise,  $\pi_M^{MD*} > \pi_M^{A*}$ . Then, we can obtain:

- (a) When  $0 < \frac{\theta-1}{1-\delta} < 3$ , for  $0 < \rho < \frac{1}{3}\rho_1$ , both  $c_{n1-FR}^{MA}$  and  $\bar{c}_{n1-FR}^{MA} < c_r$ , so we eliminate both of them.
- (b) when  $0 < \frac{\theta-1}{1-\delta} < 3$ , for  $\frac{1}{3}\rho_1 < \rho < 1$ ,  $\bar{c}_{n1-FR}^{MA} < c_r$ ,  $c_{n1-FR}^{MA}$  is too large, so we eliminate both of them.
- (c) when  $\frac{\theta-1}{1-\delta} > 3$ , for any  $\rho \in (0, 1)$ , both  $c_{n1-FR}^{MA}$  and  $\bar{c}_{n1-FR}^{MA} < c_r$ , so we eliminate both of them.

$\frac{\partial^2 \pi_R^{MA}}{\partial c_n^2} = \frac{1}{8(\theta+3\delta\rho)} - \frac{1}{4((\theta+1)+3(\delta+1)\rho)}$ . To compare the difference between  $\frac{\partial^2 \pi_R^{MA}}{\partial c_n^2}$  and 0, we can obtain  $\rho = \frac{(\theta-1)}{3(1-\delta)} = \frac{1}{3}\rho_1$  by solving  $\frac{\partial^2 \pi_R^{MA}}{\partial c_n^2} = 0$ . We have: when  $0 < \rho_1 < 3$ ,  $\frac{\partial^2 \pi_R^{MA}}{\partial c_n^2} < 0$  if  $0 < \rho < \frac{1}{3}\rho_1$ ,  $\frac{\partial^2 \pi_R^{MA}}{\partial c_n^2} > 0$  if  $\frac{1}{3}\rho_1 < \rho < 1$ ; when  $\rho_1 > 3$ ,  $\frac{\partial^2 \pi_R^{MA}}{\partial c_n^2} < 0$  for any  $\rho \in (0, 1)$ .  $\bar{c}_{n2-FR}^{MA}$  and  $c_{n2-FR}^{MA}$  are given by solving  $\pi_R^{MA} = 0$ . Thus, when  $\frac{\partial^2 \pi_R^{MA}}{\partial c_n^2} > 0$ ,  $\pi_R^{MD*} > \pi_R^{A*}$  if  $c_n < \min\{\bar{c}_{n2-FR}^{MA}, c_{n2-FR}^{MA}\}$  or  $c_n > \max\{\bar{c}_{n2-FR}^{MA}, c_{n2-FR}^{MA}\}$ , otherwise,  $\pi_R^{MD*} < \pi_R^{A*}$ . When  $\frac{\partial^2 \pi_R^{MA}}{\partial c_n^2} < 0$ ,  $\pi_R^{MD*} < \pi_R^{A*}$  if  $c_n < \min\{\bar{c}_{n2-FR}^{MA}, c_{n2-FR}^{MA}\}$  or  $c_n > \max\{\bar{c}_{n2-FR}^{MA}, c_{n2-FR}^{MA}\}$ , otherwise,  $\pi_R^{MD*} > \pi_R^{A*}$ . Then, we can obtain: (a) When  $0 < \frac{\theta-1}{1-\delta} < 3$ , for  $0 < \rho < \frac{1}{3}\rho_1$ ,  $\bar{c}_{n2-FR}^{MA} < c_r$ , so we eliminate it; (b) when  $0 < \frac{\theta-1}{1-\delta} < 3$ , for  $\frac{1}{3}\rho_1 < \rho < 1$ ,  $\bar{c}_{n2-FR}^{MA}$  is too large, so we eliminate it; (c) when  $\frac{\theta-1}{1-\delta} > 3$ , for any  $\rho \in (0, 1)$ ,  $\bar{c}_{n2-FR}^{MA} < c_r$ , so we eliminate it.

$\frac{\partial^2 \pi_M^{MA}}{\partial c_r^2} = \frac{1}{4(\theta+3\delta\rho)} - \frac{1}{2((\theta+1)+3(\delta+1)\rho)}$ ,  $\frac{\partial^2 \pi_R^{MA}}{\partial c_r^2} = \frac{1}{8(\theta+3\delta\rho)} - \frac{1}{4((\theta+1)+3(\delta+1)\rho)}$ . The following proof is similar to the previous one, so we omit the process.

The specific expressions for all thresholds of product costs are very complex, so we omit them here.

On this basis, it is easy to prove [Lemma 5](#) and [Corollary 4](#).

## Proof of [Lemma 6](#)

Since  $\pi_M^*$  and  $\pi_R^*$  always have multiple relationships in model RD and model A, i.e.  $\pi_M^{RD*} = 2\pi_R^{RD*}$  and  $\pi_M^{A*} = 2\pi_R^{A*}$ , the comparative relationship between  $\pi_R^{RD*}$  and  $\pi_R^{A*}$  is completely consistent with  $\pi_M^{RD*}$  and  $\pi_M^{A*}$ . Define  $\pi_M^{RA} = \pi_M^{RD*} - \pi_M^{A*}$  in different remanufacturing strategies, The following proof is similar to the previous one, so we omit the process. This completes the proof of [Lemma 7](#).

The proof of [Lemmas 7](#) to [9](#) and [Corollaries 5](#) to [6](#) is similar to the above Lemma and Corollary, so we omit it.